Precise Orbit Determination for Low Earth Orbit Satellites

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Precise Orbit Determination for Low Earth Orbit Satellites

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Abstract

The precise orbit determination problem is to accurately determine the position and velocity vectors of an orbiting satellite. In this paper we review the general contents of the orbit determination process, with particular emphasis on Low Earth Orbit (LEO) satellites. The different orbit determination approaches and the different force models contained within are briefly discussed. The two main estimation techniques, the batch least squares approach and the Kalman filter, are reviewed and their relative merits discussed. Numerical results are used to consider the impact of different modeling errors on the final orbit accuracy, in addition to errors due to the nature of near real-time processing conditions.

1 INTRODUCTION

With the ever increasing sophistication of satellites in low Earth orbit the ability to precisely predict the position and velocity of the satellite is extremely important. This need has been amplified in recent years by the development of Earth observation and meteorological satellites, which need to make measurements of the Earth's atmosphere, gravity field, sea surface height, etc. with unprecedented precision. This has led to the development of advanced numerical methods, which allow a precise orbit to be calculated on the basis of observation data from GPS signals as measured by the LEO satellite. Additionally, the observation data is often required by the user in "near real-time" (for use in, for example, numerical weather prediction simulations). This means that the data must be downloaded from the satellite every orbital revolution, and must then be processed to an accuracy of less than one meter in position and distributed to users within approximately one hour of downloading. This task presents a formidable challenge, but, with the use of numerical estimation methods, accuracies of 2-3 cm in position can be achieved, well within requirements.

The precise orbit determination problem is to accurately determine the ephemeris of an orbiting satellite. To achieve this, estimates of the position and velocity of the orbiting vehicle are made based on a sequence of observations. This is usually accomplished by integrating the equations-of-motion, starting from a reference epoch to produce predicted observations. This initial orbit is generally not very accurate but is a good starting point from which to construct a state vector which can then be used in a least squares algorithm to obtain a better estimate of the observations. The state vector is composed of the position vector, the velocity vector, empirical forces and measurement model parameters. Components of the satellites state vector at a reference epoch are then adjusted to minimize the observation residuals in a least squares sense. Thus, to solve the orbit determination problem one needs:

- Equations-of-motion describing the forces acting on the satellite and possibly the covariance of the process noise if a Kalman filter is employed.
- The relationship between the observed parameters and the satellite's state vector.
- A least squares estimation algorithm.

Finally, we illustrate how orbit determination methods can be verified by means of independent measurements. An analysis of worst-case scenarios, in which the usual error sources are maximized, is presented. Additionally, the impact of processing the data under near real time conditions is illustrated. Despite such adverse conditions the orbital accuracy is still shown to be sufficient, for example, for radio occultation experiments.

2 ORBIT DETERMINATION

There are three main approaches to this problem: the kinematic or geometric approach, the dynamical approach and the reduced dynamic approach. The kinematic approach does not include a dynamical description of the spacecraft's motion except for interpolation between solution points, but instead relies purely upon observation data. This approach generally assumes the availability of highly accurate measurements, a high data sampling rate and for GPS techniques at least four simultaneous measurements per epoch depending on the requirements of the mission. The dynamical approach uses dynamical models of the forces acting on a satellite. This approach is commonly used because the accuracy is mostly limited by the modeling errors in, for example, the atmospheric drag model (which decreases with altitude) and the geopotential model (whose errors have been reduced in recent years by the incorporation of satellite data). Finally, the reduced dynamical orbit determination uses a process noise model to absorb dynamic model errors by optimally weighting observational and dynamical errors.

2.1 Force Models

The equations-of-motion of a LEO satellite are usually described in an inertial reference frame as being composed of a sum of gravitational, non-gravitational and empirical or un-modeled forces [1]. The gravitational forces are primarily composed of a series of perturbations from the following sources:

$$F_G = P_{Geo} + P_{SET} + P_{OT} + P_{RD} + P_{SMP} + P_{Rel}$$

P_{Geo} is the geopotential force due to the gravitational attraction of the Earth and can be expressed as a spherical harmonic expansion of the gradient of the Earth's solid body mass distribution. P_{SET} is the Solid Earth Tide term as the Earth is not a rigid body its mass distribution changes due to gravitational perturbations, e.g., the Sun and the Moon. P_{OT} is the Ocean Tide term and is due to tidal perturbations, which are dominated by the Sun and the Moon. P_{RD} is the term due to Rotational Deformation caused by a variable centrifugal force, which has it origins in the elastic nature, and fluid components, of the Earth. The temporal variation of the geopotentials induced from solid Earth tides, ocean tides and the rotational deformation are also derived from spherical harmonic expansions. P_{SMP} is the gravitational perturbation of the Sun, Moon and planets, which are usually modeled as point masses within the Newtonian framework. The planetary ephemerides are readily obtained from the Jet Propulsion Laboratory. P_{Rel} is the general relativity correction to the above Newtonian approximation and includes corrections for Schwarzschild motion, effects due to precession of perigee, geodesic (or de Sitter) precession and Lense-Thirring precession, due to the angular momentum of the rotating Earth.

Non-gravitational forces acting on the LEO satellite are:

$$F_{NG} = P_{Drag} + P_{Solar} + P_{Earth} + P_{Thermal}$$

P_{Drag} is the atmospheric drag force acting on a satellite. This force depends on the ratio of the satellite's mass and surface area in the direction of motion (the ballistic coefficient), and on the density of the atmosphere. This is an important quantity to model correctly, particularly for satellites in lower orbits. The density of the atmosphere is usually obtained from models, which have increased in sophistication over the years. Nevertheless, this is a field where new and better models are still being developed. The combined effect of atmospheric density and ballistic coefficient is usually estimated for low Earth orbiters as part of the state vector as this is a major error source. P_{Solar} is the force exerted by solar radiation pressure and a simple model can be constructed [1]. Again, the area to mass ratio is a key quantity for solar radiation pressure. P_{Earth} is the radiation pressure imparted by the energy flux of the Earth and this clearly must take into account the shadowing effects of the Earth. $P_{Thermal}$ is the force due to a net thermal radiation imbalance caused by non-uniform temperatures on different satellite surfaces.

Finally, empirical forces can be introduced to account for any un-modeled or incorrectly modeled forces, which may still affect the solution. Two sets of empirical parameters are customarily introduced. Firstly, empirical tangential perturbations cope with un-modeled forces in the tangential direction for example by estimating a set of piecewise constants, either along the inertial velocity or the spacecraft-centered velocity. Secondly, once-per-revolution perturbations and constant empirical accelerations in the radial, transverse and normal directions can be computed in the spacecraft frame and transformed into geocentric inertial components when necessary. The use of such empirical parameters has been shown to significantly reduce orbit errors [1].

2.2 Observation Models

While, in principle, any observations of a satellite may be suitable for orbit determination (with differing degrees of accuracy) the global availability and high accuracy of GPS signals to satellites in low Earth orbit makes them ideal for the solution of the precise orbit determination problem. There are two types of GPS observations: Pseudoranges and carrier phases. Either can be used independently in the orbit determination process, but carrier phases must be used for high precision applications. Combinations of the two observables are often employed for orbit determination problems.

The pseudorange is a measure of the distance between the GPS satellite and the LEO's receiving antenna. The pseudorange measurement model may be expressed as:

$$\rho_{PR} = \rho - c \cdot \delta t_{GPS} + c \cdot \delta t_{LEO} + \delta\rho_{Trop} + \delta\rho_{Iono} + \delta\rho_{Rel} + \varepsilon$$

The first term ρ is the true geometric range between the two satellites. The second term represents the clock error on the transmitting GPS satellite, and the third term is the clock error on the receiving LEO satellite. Next is the troposphere term due to the Earth's atmosphere and is only relevant if a ground station is used. The fifth term is the ionosphere path delay and is relevant for satellites below about 1000 km, unless an ionosphere-free linear combination of data is used, or if a ground station is used instead of a LEO satellite. Clearly the troposphere and the ionosphere terms are zero for satellites in higher orbits. The penultimate term is a correction for relativistic effects, while the final term includes all other effects such as electronic hardware delays, multipath effects and random measurement noise.

The carrier phase observable is the difference between the received GPS carrier phase and the phase of the internal LEO receiver oscillator. The carrier phase measurement is likewise modeled as follows:

$$\phi_l^g = \phi^g(t_{GPS}) - \phi_l(t_{LEO}) + N_l^g(t_0) + \varepsilon$$

The carrier phase contains an initial integer ambiguity N, which is an arbitrary counter of the number of whole carrier waves between the transmitter and receiver at the start of observations (phase lock). This is an important quantity because it is unknown. Processing of carrier phases makes use of the fact that the received phase was emitted at some point in the past, and that the temporal variation of the carrier phase relates to changes in the topocentric distance in a similar manner to pseudorange measurement. Different strategies are used to either eliminate the initial ambiguity - as in epoch-by-epoch phase differencing - or to estimate the ambiguities as part of the least squares solution.

3 LEAST SQUARES ESTIMATION

There are two commonly used classes of least squares algorithms: firstly, the batch least squares approach where all the data for a fixed period is collected and processed together; secondly, the Kalman filter which sequentially updates the state vector to produce a better estimate at each epoch using process noise information. The batch least squares approach is commonly employed for off-line processing of trajectories from LEO spacecraft as the tracking data is typically downloaded once per revolution. On the other hand, in applications involving on-board navigation of spacecraft in real time, the Kalman filter (in various guises) is typically used for the least squares estimation algorithm. In recent years the need has increased for precise orbit determination for LEO spacecraft in near-real time. In this case, either the Kalman filter or the batch approach can be used, with the majority now choosing to use the Kalman filter approach. Alternative methods may also be employed, a good example being the square root information filter, which is a special solution technique for the Kalman filter and the related smoothing problem. Although this is a less popular choice it can increase computational accuracy by guaranteeing positive definiteness of the covariance matrix and decreasing the condition number of the manipulated matrices.

3.1 Batch Least Squares

Least squares estimation involves finding the trajectory and model parameters for which the square of the difference between modeled observations and actual measurements, i.e. residuals, becomes as small as possible. As different measurements have different units and reliability, a weighting factor is applied to each residual, and it is the square of the weighted residuals, which is minimized. In this section we discuss the general least squares problem. Note that the formalism is a batch least squares when all the data is processed in a single step to obtain a better estimate, as opposed to sequential least squares which will be discussed under the heading of Kalman filters in the next section.

Following Montenbruck and Gill [2], let x denote an m-dimensional time-dependent vector comprising the satellite's position r and velocity v along with the free parameters p and q which affect the force and measurement models. Likewise, let z denote an n-dimensional vector of measurements taken at each epoch.

$$x(t) = \begin{pmatrix} r(t) \\ v(t) \\ p \\ q \end{pmatrix}; \qquad z = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix} = h(x_0) + \rho$$

where the h vector denotes the model values as a function of the state vector x_0 at the reference epoch and the vector ρ denotes the difference between actual and modeled observations due to measurement errors.

The least squares orbit determination problem is then defined as finding the state, x_0^{lsq}, that minimizes the loss function for the sum of the square residuals for given measurements z.

$$J(x_0) = \rho^T \rho = (z - h(x_0))^T (z - h(x_0))$$

As h is a non-linear function of the unknown vector x_0, locating the global minimum of the loss function is not straight-forward. We may reformulate the orbit determination problem as an iterative solution to a linear least squares problem. Utilizing approximate orbit information for x_0^{apr} at each epoch, the residual is approximately given by:

$$\rho = z - h(x_0) \approx z - h(x_0^{ref}) - \frac{\partial h}{\partial x_0}(x_0 - x_0^{ref})$$
$$= \Delta z - H\Delta x_0$$

where $\Delta x_0 = x_0 - x_0^{ref}$; $\Delta z = z - h(x_0^{ref})$ and the Jacobian $H = \left.\frac{\partial h(x_0)}{\partial x_0}\right|_{x_0 = x_0^{ref}}$ gives the partial derivatives of the observations with respect to the state vector at the reference epoch. The linearization error is highly dependent on the selection of the reference state vector, however during the iterative solution this will converge quickly to the best estimated state and thus the error is negligible. The loss function is then defined as:

$$J(\Delta x_0) = \rho^T \rho = (\Delta z - H\Delta x_0)^T (\Delta z - H\Delta x_0)$$

The derivative of the loss function is uniquely defined if the columns of H are linearly independent, i.e. H is full rank. The minimum of the loss function is uniquely defined when the partial derivatives of J with respect to Δx_0 vanish.

$$\frac{\partial J}{\partial \Delta x_0} = \left.\frac{\partial (\Delta z - H\Delta x_0)^T (\Delta z - H\Delta x_0)}{\partial \Delta x_0}\right|_{\Delta x_0 = \Delta x_0^{lsq}} = 0$$

Computing the partial derivatives and rearranging, we obtain the familiar least squares solution:

$$\Delta x_0^{lsq} = (H^T H)^{-1} (H^T \Delta z)$$

This correction may be adequate if the *a-priori* state vector is very close to the best estimate; if this is not the case several iterations may be required. Thus far we have treated all observations equally. However, in practice the observations will consist of different measurement types with different accuracies. This can easily be achieved by weighting all observations with the inverse of the mean measurement error σ_i (random and systematic errors) so the residuals become:

$$\hat{\rho}_i = \frac{1}{\sigma_i}\rho_i = \frac{1}{\sigma_i}(z_i - h_i(x_0))$$

where the subscript i denotes the ith data batch and the subscript zero indicates the initial state or epoch. The least squares expression remains unchanged except that we

replace H and Δz by $\hat{H} = SH$; $\Delta\hat{z} = S\Delta z$. Here S is a square diagonal matrix:

$$S = diag(\sigma_1^{-1},...,\sigma_n^{-1}) = \begin{pmatrix} \sigma_1^{-1} & & 0 \\ & \ddots & \\ 0 & & \sigma_n^{-1} \end{pmatrix}$$

which divides the ith row of a matrix or vector by σ_i upon multiplication. The solution of the weighted least squares may then be written as:

$$\Delta x_0^{lsq} = (H^T WH)^{-1}(H^T W\Delta z)$$

with the weighting matrix

$$W = S^2 = diag(\sigma_1^{-2},...,\sigma_n^{-2})$$

This is a widely used formula for least squares problems and can handle uncorrelated measurements and correlated measurements, in which case W becomes non-diagonal. A final improvement on the above formulation is to include information on the accuracy of the approximate state, x_0^{apr}, used to start the least squares orbit determination. The least squares expression can then be reformulated to include *a-priori* information in the form:

$$\Delta x_0^{lsq} = (\Lambda + H^T WH)^{-1}(\Lambda\Delta x_0^{apr} + H^T W\Delta z)$$

The information matrix Λ (or inverse covariance matrix)

$$\Lambda = (P_0^{apr})^{-1}$$

is used to penalize any deviations from $x_0^{apr}+\Delta x_0^{apr}$ by an appropriate contribution to the loss function. This feature is often used to avoid singularities in least squares problems by giving a small *a-priori* weight to each estimation parameter and adding the corresponding diagonal matrix Λ to the normal equation matrix. To conclude this section we should comment that the above represents a conceptual understanding of the least squares problem. The numerical solution of the least squares problem may be obtained by a number of standard mathematical approaches. Common approaches include QR factorization, which can be achieved using Householder transformations or Givens rotations. Alternatively, singular value decomposition may be used to analyze the least squares problem and to solve it in a numerically stable manner.

3.2 Kalman Filters

The Kalman filter is a set of mathematical equations that provide an efficient recursive solution to the least squares problem and is a common approach to sequential least squares. The filter is powerful in that it enables the use of dynamical (process) noise models in order to accommodate the use of models with known, and to some extent unknown, error characteristics. It is therefore of particular interest for orbit determination applications where the dynamical models are not completely known (for example the density of the atmosphere) and where a sequential update of the state vector is required.

Using the solution for the general least squares problem, and rewriting in terms of the covariance matrix [2]:

$$\Delta x_0^+ = P_0^+\left((P_0^-)^{-1}\Delta x_0^- + H^T W\Delta z\right) \qquad (1)$$

with *a-posteriori* (+ superscript) covariance matrix:

$$P_0^+ = \left((P_0^-)^{-1} + (H^T WH)\right)^{-1} \qquad (2)$$

which represents the increased knowledge of x_0 based on *a-priori* (- superscript) covariance information and the latest data batch. Rearranging (2) for the inverse covariance matrix and substituting into (1) one can relate the new estimate of x_0^+ to the previous estimate x_0^- yielding a recursive least squares formulation:

$$\Delta x_0^+ = \Delta x_0^- + K(\Delta z - H\Delta x_0^-)$$

The Kalman gain,

$$K = P_0^- H^T (W^{-1} + HP_0^- H^T)^{-1}$$

maps the residuals into an appropriate correction for the estimate x_0^-. The Kalman gain can also be used together with (2) to update the covariance matrix without inverting the normal equations, using:

$$P_0^+ = (1 - KH)P_0^-$$

In order to obtain estimates of the state vector at measurements times both the state vector and its covariance between observation times are required. The state transition matrix between epochs can be defined as:

$$\Phi_i = \Phi(t_i, t_{i-1}) = \frac{\partial x_i^{ref}}{\partial x_{i-1}^{ref}}$$

Using this, the state vector and the corresponding *a-priori* state covariance may be predicted at future epochs as:

$$x_i^- = x_i^{ref} + \Phi_i(x_{i-1}^+ - x_{i-1}^{ref})$$

$$P_i^- = \Phi_i P_{i-1}^+ \Phi_i^T + Q_{i-1}$$

Here Q has been introduced as the covariance matrix of the assumed white process noise model. To incorporate new observations z_i to update the *a-priori* information h_i is replaced by g_i to model the observations as a function of t_i instead of t_0. The observations in terms of the state vector at the measurement time and the Jacobian become:

$$g_i(t_i, x(t_i)) = h_i(t_i, x(t_0))$$

$$H_i = \frac{\partial z_i}{\partial x_0^{ref}} = \frac{\partial z_i}{\partial x_i^{ref}}\frac{\partial x_i^{ref}}{\partial x_0^{ref}} = G_i\Phi(t_i, t_0)$$

Deviations between the reference state and the estimated state must be small to avoid any non-linearities of the dynamical system and the measurement modeling. To avoid this the extended Kalman filter was developed by resetting the reference state, x_{i-1}^{ref}, to the estimate, x_{i-1}^+, at the start of each update step. The extended Kalman filter thus makes use of the latest estimate to propagate the state vector and the state transition matrix, which makes it less sensitive to non-linearities than the linearized version. The final expressions comprise a time update phase

$$x_i^- = x(t_i; x(t_{i-1}) = x_{i-1}^+)$$
$$P_i^- = \Phi_i P_{i-1}^+ \Phi_i^T + Q_{i-1}$$

followed by the measurement update phase:

$$K_i = P_i^- G_i^T (W_i^{-1} + G_i P_i^- G_i^T)^{-1}$$
$$x_i^+ = x_i^- + K_i \left(z_i - g_i(x_i^-)\right)$$
$$P_i^+ = (1 - K_i G_i) P_i^-$$

Both phases are then repeated cyclically. The addition of process noise gives the filter a fading memory behavior as past observations have a gradually decreasing effect on the state estimate. Another advantage of including process noise is that the covariance can be based on a physical model of the satellite and station clock errors. Incorporating a clock model with known statistics as process noise is expected to improve the orbit determination accuracy in future applications.

4 APPLICATIONS

There is a wide range of orbit determination tools available, some used by space agencies and universities, others in commercial products. We shall briefly mention three of the software tools, but the reader should bear in mind that other high quality tools also exist. The European Space Agency (ESA) has traditionally used BAHN, which relies mainly on batch processing, although there is interest in incorporating a sequential filter. In the USA GIPSY/OASIS-II is widely used and employs the Kalman filter approach. Finally, we have relied on the Bernese tool [3] from the University of Bern in our own studies [4][5] for GPS based radio occultation missions. This tool uses the batch processing approach with optional sequential processing as well as stochastic force estimation. Additionally, custom developed software by AIUB [6] was used. All the packages are written in Fortran - the mainstay of scientific computing.

4.1 Verification Methods

To assess the accuracy of a GPS based POD software tool it is typical to choose an existing satellite, which has independent measurements of its orbit with an accuracy comparable to the orbit determination tool. Fortunately such data is available based on Satellite Laser Retro-reflector (SLR) measurements and Doppler based receivers (DORIS) for a number of satellites. The selected satellite must also have GPS receiving equipment onboard which facilitates the orbit determination process described in this paper. The TOPEX/Poseidon satellite has been chosen for this task as it carries both a GPS receiver and SLR/DORIS equipment, thus making it possible to have independent orbital comparisons. Since TOPEX/Poseidon is located at an altitude of 1335 km, additional analysis has to be made to demonstrate the feasibility of orbit determination for satellites in lower orbits. Three steps are required to accomplish this:

- The orbit determination tool is validated by comparing its results with those from an independent tool and based on independent measurements.
- The orbit for a satellite at lower altitude is simulated by use of an orbit integrator providing a reference orbit and realistic errors are injected both into the dynamical equations and the simulated observations.
- An orbit determination is performed using this set of simulated measurements and standard dynamical models. The result is compared to the reference orbit.

The SLR/DORIS equipment has been used to determine the TOPEX/Poseidon orbit to a high degree of accuracy. Using the Bernese tool, the comparison with SLR/DORIS data shows agreement at ~2-3 cm. Likewise, the GIPSY/OASIS tool, which uses the Kalman filtering approach, also shows agreement at this level.

4.2 Data Analysis

The design of a precise orbit determination tool is strongly motivated by accuracy and performance, allowing measurements to be processed for near-real time applications. Figure 1 gives an overview of a generic precise orbit determination process which includes the iterative process of integrating the orbit based on models and parameters, comparing with observations from the pre-processing module and finding better estimates for the model parameters using the least squares method. The process may be iterated until convergence is achieved.

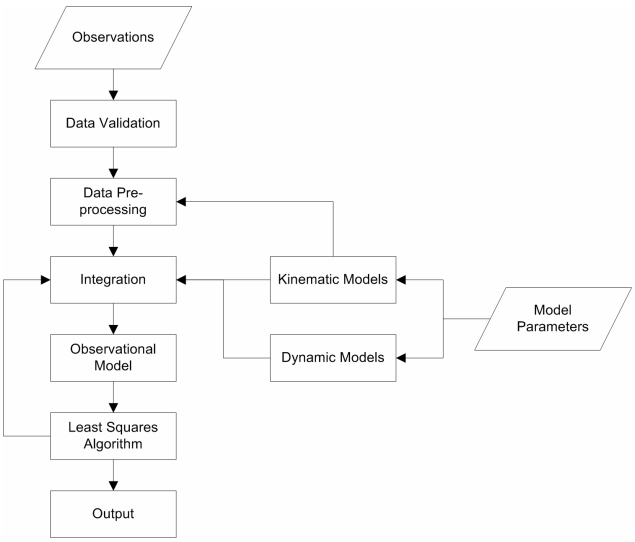

Figure 1 : POD overall functional flow

The main observation inputs for this process generally include: IGS (International GPS Service) Final or Ultra Rapid GPS orbits; IGS weekly ground station solutions; ground based 30-second GPS measurements and LEO satellite GPS measurements. The measurements generally consist of L1 and L2 pseudorange and carrier phase data; IGS configuration files containing ground station, receiver, antenna and antenna height information. The model parameters may include Earth Orientation parameters, luni-solar ephemerides, etc.

An important part of the process is data validation to ensure its quality and screening of poor data during a pre-processing phase. The screening of unusable observations often relies on an *a-priori* orbit, which may be derived from code observations or an earlier predicted orbit. The preprocessing algorithms provide a transparent layer to the observation data as seen from the least squares algorithm. They also estimate the initial parameters to be used in further processing and detect, remove or correct as many of the errors in the observation data as possible.

The observational model employed in a precise orbit determination tool describes the theoretical observables as a function of the state variables and time. The model depends on the types of data used, which in the current context are usually dual frequency pseudorange and carrier phase measurements. By appropriate differencing between phase measurements and separately pseudoranges at different receivers and/or for different satellites, it is possible to eliminate the first-order clock errors. This can be achieved if two or more receivers observe the same GPS satellite. A new observable can then be formed. This is known as single differencing but double differencing techniques, which form baselines from the LEO satellite to different ground stations, are also routinely employed [7]. This approach is often used to advantage but some effort must be put into selecting the appropriate observations for differencing. Another approach [6] employs zero-difference code observations and their epoch-by-epoch phase differences. Tracking data from ground stations is needed only for the independent generation of the GPS clock corrections. This results in simple and fast algorithms and is of particular interest in the context of numerical weather predictions where data processing is needed in near-real time. A review of precise orbit determination approaches may be found in reference [8].

Finally, the output data from the least squares algorithm depends on the user requirements. Examples include: Best orbit fits, using ephemerides to represent the satellite's position and velocity at equidistant time points; Estimated model parameters; Statistical analysis; Post-fit residuals, which are the difference between observables and the observation model using the estimated orbit; Precise orbit predictions for future epochs.

4.3 Worst Case Numerical Studies

Before building a new satellite for a specific application the space agencies concerned typically want to know how accurately they can predict the satellites orbit. This may have a huge impact on the scientific measurements of which the satellite is capable. Indeed, if the orbit accuracy is insufficient the satellite may have to be moved to a higher orbit or redesigned to reduce the perturbing forces acting on it. It is within this context that we have conducted a number of feasibility studies for potential new spacecraft [4][5]. The objective was not to determine the best orbit fit but to investigate the worst case error sources and then to combine the individual worst case errors in a number of simulations to see if the desired accuracy in position and velocity can still be achieved.

4.4 Error Sources

In terms of modeling errors, atmospheric drag is usually expected to give a large contribution for LEO satellites. Both the uncertainty in the way drag affects the satellite and the uncertainty in the density of the upper atmosphere contributes to the total error. Another large error source is solar radiation pressure, which is almost constant but its impact on the orbit is not well known due to inappropriate modeling of re-radiation from the spacecraft or temporally changing radiation parameters. Additionally, imprecisely known attitude information may also be a source of errors, depending on the shape of the satellite.

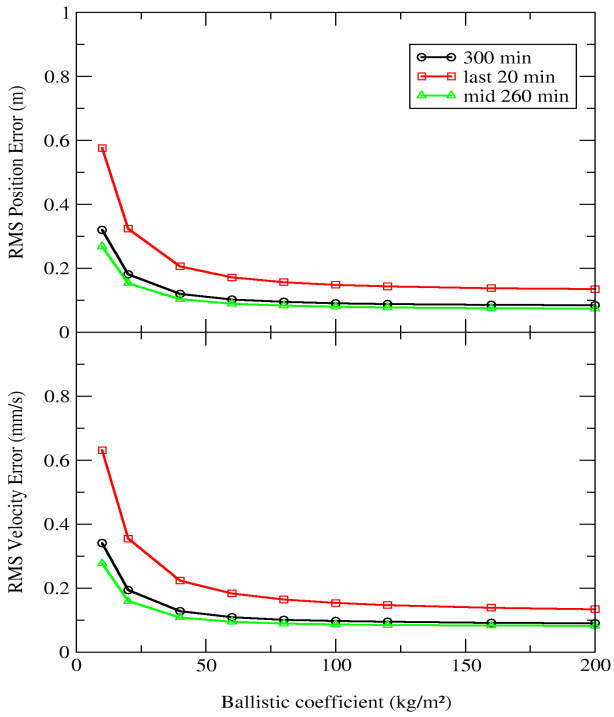

Figure 2 : Satellite position and velocity errors based on a tracking network of 57 ground stations with worst case errors.

In these simulations we used the empirical MSIS90 atmosphere model, which describes factors such as temperature and density as a function of day of year, time of day, geodetic latitude and longitude, ultraviolet radiation linked to solar activity, and geomagnetic index. To obtain worst case results we choose large values for the geomagnetic index and the radiation pressure coefficient. Additionally, an extra error term, constructed from a high order polynomial, is added allowing up to 300% variation in the drag and radiation pressure models. The gravitational model employed is EGM96. As it is difficult to determine the true errors associated with using this model we have included as an extra source the difference between using this model and the older and less accurate JGM3 model. This is not a realistic error but

it does represent the model error to some extend. Realistic observation noise - based on residual statistics from the TOPEX/Poseidon orbit determination - is included for the pseudoranges and the carrier phases. Finally, near-real time (NRT) errors are also included. These errors arise as the orbit determination process is often required within a few hours of downloading the data. Final IGS precise orbits for the GPS network and estimated tropospheric zenith delays are not available in NRT. Thus, one has to rely on less accurate ultra-rapid GPS orbits and model troposphere parameters. The difference between these two data sets has been included as an extra error source.

Error Source	Position Error (m)	Velocity Error (mm/s)
Atmospheric Drag	0.023 (0.039)	0.020 (0.034)
Solar Radiation Pressure	0.018 (0.029)	0.020 (0.028)
Gravity Model (JGM3)	0.050 (0.064)	0.061 (0.067)
NRT Clock Errors	0.057 (0.107)	0.053 (0.089)
Observation Noise	0.022 (0.027)	0.013 (0.018)

Table 1 : Summary of impact of individual error sources on the satellite radial and velocity errors. A ballistic parameter of 80 kg/m^2 is assumed where applicable. Brackets imply the last 20 mins. of the orbit.

Figure 2 shows the worst case results, which include all the error sources mentioned above, while Table 1 shows the individual contributions. The results have been calculated for a relatively ideal tracking network consisting of 57 ground stations. Our studies [4][5], which use the epoch-by-epoch phase difference algorithms described in reference [6], have shown that the orbit determination process is fairly robust if smaller less ideal networks are chosen. The simulated orbit geometry is polar with an altitude of 833 km and no eccentricity. Results are calculated based on 30 s tracking data over a 300 min. data arc, which we have determined to be the optimal data arc length for the investigated errors and orbital geometry [4]. Additionally, as the processing error typically gets worse at the start and the end of the arc, the results from the last 20 minutes and the middle 260 minutes of the arc are shown. Clearly, the errors at the end of the arc are considerably worse. From the graph we see the errors increase dramatically for small ballistic parameters (mass/area). This is because a light satellite with a large surface area is strongly affected by atmospheric drag and solar radiation pressure. Such graphs are useful for satellite designers as they can place limits on the final mass and shape of the satellite for fixed altitudes. Accuracies of less than 1 m in position were desirable for the ACE study [4] and this is clearly achieved here under worst case conditions. The velocity error requirements are less well defined. In our studies [4][5], the velocity error drift during an instruments measurement interval was the driving requirement. For radio occultation measurements this interval is typically 2 minutes, and the corresponding worst case error was found to be ~0.035 mm/s compared to a requirement of 0.1 mm/s. As these results are worst case the prospect of designing such missions is quite feasible.

5 CONCLUSION

In this paper we have reviewed the two main methods of least squares parameter estimation and their application to the precise orbit determination problem. Several other methods exist but these two were chosen due to their widespread use and due to their different areas of applicability. The batch least squares solution is typically employed in post-processing while the Kalman filter finds its main use in real-time applications. The implementation of the Kalman filter is generally faster and more efficient in terms of storage capacity. Both the batch least squares and the extended Kalman filter approach can give equivalent results in terms of precision especially if the basic methods are augmented in order to add features such as smoothing to the Kalman filter and stochastic state parameters to the batch least squares method. Future research will thus be focused on developing better force models for the orbit integration and determining how they can best be incorporated into existing orbit determination packages.

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