Trilateral Trade and Asset Allocation - extending the Grossman-Hart-Moore model

Rehn, Eric

2007

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

Trilateral Trade and Asset Allocation
- *extending the Grossman-Hart-Moore model*

Eric Rehn*

December 20, 2007

Abstract

This paper extends the Grossman-Hart-Moore model to suite a specific trilateral trade transaction. In this transaction a downstream producer produces the final good using inputs from two different upstream suppliers. Moreover one of the upstream supplier needs an input from the other upstream supplier for its production. The optimal way to organize this transaction depend on the characteristics of assets, human capital and investments. The general finding is that it is more demanding to find a unique Pareto optimal organization in the trilateral model than in the bilateral Grossman-Hart-Moore model. This paper also produces a number of other potentially useful results.

Keywords: Trilateral Trade, Property Rights, Partial Integration
JEL Classifications: D23, L23

1 Introduction

The property rights approach to organizations was developed in Grossman & Hart (1986), Hart & Moore (1990) and Hart (1995). This approach focuses on the importance of asset ownership for the investments made in bilateral trade relationships (herein called the *bilateral model*). This is done in a world of incomplete contracting - all relevant outcomes are not verifiable for a third party. Generally, the aim is to find the optimal organizational structure for the analyzed transaction. Each party in the transaction is assumed to own a physical asset and have some human capital. In this setup integration is essentially the acquisition of a trading party's assets, and with this acquisition follows residual control rights over contingencies, concerning the assets, not specified in the contract. Greater control makes the investing party less vulnerable to hold-ups and thus provide incentives for greater investments in the trade relationship. These investments are beneficial for the trading parties and may either be investments in human capital or in physical assets. The investments may be thought of as

*eric.rehn@nek.lu.se, Department of Economics, Lund University, Box 7082, 220 07 Lund, Sweden

modifications that ensure a smoother (more efficient) trade between the trading parties. Hence, the organizational structure that best supports relationship-specific investments is optimal.

This paper extends the bilateral model to the case of trilateral trade. The analysis is concerned with a specific trilateral trade case, described in detail in section 2, where one downstream party produces the final good and two upstream parties supply inputs to this production. One of the upstream suppliers also supply the other upstream supplier with an input. It might simplify intuition to think about the following situation: a downstream producer of cars (M) needs both circuit boards and products containing circuit boards e.g. a travel computer for the production of cars. Let S be the producer of circuit boards and A be the producer of travel computers. Now trilateral trade, as depicted here, is the situation where S supplies circuit boards, significantly different types of circuit boards, to M and A respectively, while A supplies travel computers to M. Herein the optimal organization for this kind of transaction is analyzed by extending the bilateral model, as presented by Hart (1995), to incorporate this situation.

The main conclusion is that finding an unique Pareto optimal organization, for a given set of assumptions, is more demanding in the trilateral model than in the bilateral model. Moreover the relative productivity of investments, when applicable, may be of greater importance in the trilateral model - not the least as a tie-breaker between two or more Pareto optimal organizational forms. Moreover the tendency is that some form of partial integration, if any integration, is optimal in most cases. Full integration is more of an exception. This paper also addresses a couple of modelling issues: it analyzes two different negotiation scenarios i.e. sequential and simultaneous negations, and discusses the starting point for the analysis. It is found that the sequence of negotiations does not matter for the investments incentives and that the starting point for the analysis matters. Finally, it is shown that the incentives for investments in the simultaneous negotiations variant of the model are created off the equilibrium path i.e. are entirely given by the (best) outside options.

The paper is organized as follows: section 2 provides the basics and section 3 deals with sequential negotiations. Section 4 deals with the simultaneous negotiations case and it is in this setting most of the discussion and analysis are carried out. Section 5 concludes the paper.

2 Basics

A firm M produces a final good which is sold on the market for final goods. For this production M needs input from two other firms A and S. Moreover, for A to be able to produce its input A also needs input from S. Thus S produces input for both M and A. Now assume that the production of these two inputs make use of different parts of S's human capital, such that the cost of producing one is independent of producing the other (separability assumption). Note also that the input produced for A cannot be used by M and *vice versa*.[1] Following the basic story provided by Hart (1995) each party initially owns only one asset p_M, p_S and p_A, respectively. Figure 1 gives a schematic presentation of the trade relationships, and also introduces the realtionship-specific investments (returned to below).

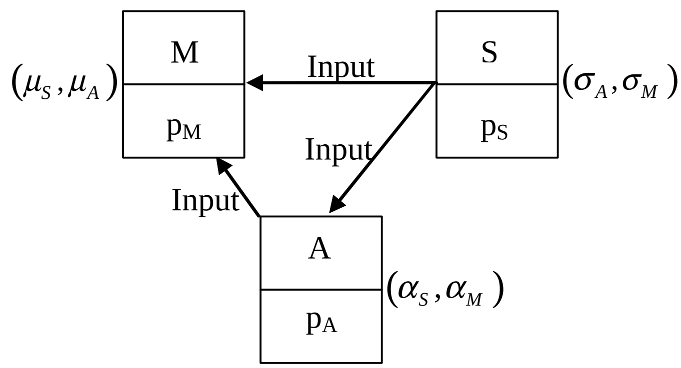

Figure 1: The trilateral trade

The model presented here follows the assumption made by Hart (1995) unless differently stated.[2] However, some notions are worth repeating. The timing of the model is the following: in period 0 each party make investments in their human capital that are relationship-specific to each relation M make the investments μ_A and μ_S, A α_M and α_S and S σ_A and σ_M.[3] The investment cost per unit of investment is assumed to be one, thus μ_A represents both the level and the cost of this investment.

In period 1 the trade is realized and after negotiations S and A is reimbursed for their inputs. One may ask why there are negotiations over the reimbursement, why is the price of the input contracted on in advance? The simple reason is that there is *ex ante* uncertainty about relevant characteristics of a suitable input, it cannot be described in a contract and thus not priced in a relevant manner. This uncertainty makes effective long-term contracts infeasible. Nevertheless, the uncertainty is resolved in period 1 and the parties then negotiate

[1] One may think of S as e.g. a computer support company that provides hardware programming to A's machines and IT-applications to M's sales division. The separability assumption creates a situation that is similar to M and A having one supplier each, with the important difference that the same asset p_S is used in both production processes which would be unnatural if the model was dealing with two suppliers.

[2] E.g. there is no uncertainty about costs and benefits, and no asymmetric information in this model. Moreover, the parties can make correct calculations about expected return of any action (c.f. Hart 1995).

[3] The investments are observable to all parties, but not verfiable to outsiders (not enforceable) (c.f. Hart 1995).

over price of the input - this negotiation is returned to below. The reimbursement from M to S is called v, the reimbursement from M to A is called m and the reimbursement from A to S is called y.

In this trilateral trade model eight possible patterns of (no-) trade may be realized in period 1, here the two extreme cases (full trade and no-trade) and one intermediate case (partial trade) is presented, all the intermediate cases follow the same basic outline. Now the benefit from full trade for M is denoted $T(\mu_A, \mu_S)$ while the production costs under full trade for A is denoted $K(\alpha_M, \alpha_S)$, both M's benefit and A's cost depend on the their relationship-specific investments. S's production cost is separable in the two inputs S's produces, the cost for the input to M's production is denoted $C(\sigma_M)$ and the cost of producing A's input is $G(\sigma_A)$. The no-trade counterparts to these benefits and costs is denoted by $t(\mu_A, \mu_S; P_M)$, $k(\alpha_M, \alpha_S; P_A)$, $c(\sigma_M; P_S)$ and $g(\sigma_A; P_S)$, respectively. Under full trade all assets are available to all parties (but not necessarily jointly owned), in the no-trade case the allocation of assets will matter for the benefit/cost that each party receives - P_i, $i = M, A, S$, denotes the assets owned by each party. To limit the number of feasible ownership structures are cases where a party does not own its own asset but owns other assets abstracted from, this will be discussed further below. [4] In the intermediate cases trade realized between some but not all parties, e.g. it could be that A trades with both S and M, but M does not trade with S - M's benefit from trade is then called $T_A(\mu_A, \mu_S; P_M)$ and depend not only the relationship-specific investment but also on the assets owned by M. It is important to note that the investments are beneficial in all trading patterns but to different degrees, as will be seen below, the positive effect of investments depends on assets owned and number of parties involved in trade. The payoffs from the different trade patterns are exemplified by:

i. Full trade (Trilateral trade): all parties trade with each other, and all parties assets and human capital are available to the other parties:

$$\begin{aligned} U_M &= T(\mu_A, \mu_S) - v - m \\ U_S &= v - C(\sigma_M) + y - G(\sigma_A) \\ U_A &= m - K(\alpha_M, \alpha_S) - y \end{aligned}$$

ii. No-trade: none of the parties trade with each other:

$$\begin{aligned} u_M &= t(\mu_A, \mu_S; P_M) - \bar{v} - \bar{m} \\ u_S &= \bar{v} - c(\sigma_M; P_S) + \bar{y} - g(\sigma_A; P_S) \\ u_A &= \bar{m} - k(\alpha_M, \alpha_S; P_A) - \bar{y} \end{aligned}$$

[4] The possible ownership configurations dealt with are the following:
$P_M = \begin{matrix} \{p_M, p_S, p_A\}, \{p_M, p_S\} \\ \{p_M, p_A\}, \{p_M\}, \{\varnothing\} \end{matrix}$, $P_S = \begin{matrix} \{p_S, p_M, p_A\}, \{p_S, p_M\} \\ \{p_S, p_A\}, \{p_S\}, \{\varnothing\} \end{matrix}$,
$P_A = \begin{matrix} \{p_A, p_S, p_M\}, \{p_A, p_S\} \\ \{p_A, p_M\}, \{p_A\}, \{\varnothing\} \end{matrix}$

Here the benefit function, $t(\mu_A, \mu_S; P_M)$, reflect that neither S or A's human capital is available to M. $\bar{v}$ is the market price for a generic input (of the type that S could have provided M with), likewise $\bar{m}$ is the market price for a generic "A-type" input. The cost functions $c(\sigma_M; P_S)$ and $g(\sigma_A; P_S)$ are S's cost for producing generic inputs and reflects that the other parties human capital is not available to S. Similarly, $k(\alpha_M, \alpha_S; P_A)$ is A's cost for producing a generic "A-type" input and $\bar{y}$ is the market price for the input needed for this production.

iii. Example of partial trade: M & A, A & S trades, but not M & S :

$$\begin{aligned} \dot{U}_M &= T_A(\mu_A, \mu_S; P_M) - \bar{v} - m \\ \ddot{U}_S &= \bar{v} - c(\sigma_M; P_S) + y - G(\sigma_A) \\ U_A &= m - K(\alpha_M, \alpha_S) - y \end{aligned}$$

Here T_A indicates that A's human capital is available to M but not S's human capital. As mentioned above the payoffs from all partial trades are similarly configured, e.g. $K_S(\alpha_M, \alpha_S; P_S)$ is A's production cost when only trading with S (all the patterns of trade and the resulting payoff structures can be found in the appendix). Concerning the functional form of the benefit functions and cost functions, it is assumed that $T(\cdot)$ is strictly concave in both its arguments and that $K(\cdot)$,$C(\cdot)$ and $G(\cdot)$ are strictly convex in all their arguments. Furthermore, $T_i(\cdot)$, $i = A, S$ and $t(\cdot)$ are concave in both their arguments, while $K_i(\cdot)$, $i = M, S$, $k(\cdot)$, $c(\cdot)$ and $g(\cdot)$ are convex in all their arguments. For formal statements of these assumptions see appendix ((A1)-(A10)).

Assume, as in the bilateral model, that trade is beneficial for all parties reflecting that the investments are relationship-specific. Full trade is the most beneficial form of trade i.e. produces the greatest total surplus. That is, partial trade is also beneficial but to a lesser extent and assume that the total surplus, under partial trade, is growing in the number of trading parties. These assumptions give a partial ranking of the total surplus from trade (see appendix (A11)), partial since the relation between two surpluses with the same number of trading parties cannot be determined.

The marginal conditions, i.e. how the relationship-specific investments affect the marginal benefit, reflect that investments are relationship-specific and also show that these investments are more valuable in trade than in no-trade. Moreover they show that the relationship-specific investments are at least partly specific to the other parties' assets. The assumptions about the marginal conditions is presented in the appendix ((A12)-(A17))and their interpretation is here exemplified by M's investment in the relationship with S.

The marginal benefit of the investment, μ_S, is at least as high or higher (under full trade) in all types of trade than in no-trade irrespective of ownership. In all forms of trade the marginal benefit of the investment is growing in the number of asset that M owns. Furthermore the marginal benefit of an investment in the relationship with S when trading with S is at least as high as the marginal benefit of this investment when trading with A and this holds for all ownership structures - here called the *trade effect*. The investment is at least partly specific to S's asset. For an equal number of assets owned the marginal benefit of μ_S, when trading with S and owning p_S, is greater or equal to the marginal benefit when not owning p_S. This is named the *asset effect*. The asset effect is weaker than the trade effect i.e. the marginal benefit when trading with S is greater or equal to the marginal benefit when trading with A even if M owns p_S in the latter case. The marginal conditions for the other investments can be interpreted in a similar way.[5]

If the parties decide to trade with each other, which they do in equilibrium, there is, by assumption, a gain from trade to be divided between the parties. The division of this surplus is decided by bargaining, since the parties have symmetric information (see Hart, 1995). The next two sections present two different bargaining regimes: sequential and simultaneous negotiation.

3 Sequential negotiation

In this section it is assumed that the reimbursements in the bilateral relationships are decided by negotiations in a predetermined, exogenously given, order. Six different negotiation sequences are possible, these six sequences yields four different scenarios i.e. four different reimbursement patterns. Table 1 contains the six sequences, the four different scenarios and shows the outside options available in each bilateral negotiation under the heading "other trades". Nash bargaining is assumed for the negotiations. Thus the reimbursements m, y and v are decided using the Nash Bargaining Product for the different negotiations.

Using the reimbursements the individual payoffs from trilateral trade may be calculated. Let R_i^J, $i = 1, 2, 3, 4$ and $J = M, A, S$ be the individual payoffs in the different scenarios of sequential negotiations. Table 2 presents the individual payoffs. Now the three parties will choose their relationship-specific investments to maximize the *ex post* benefit from trade. The *ex post* benefit from trade is given by[6]:

$$R_i^M - \mu_A - \mu_S \tag{1}$$

where R_i^M is the individual payoff in scenario $i = 1, 2, 3, 4$ for M.

$$R_i^S - \sigma_M - \sigma_A \tag{2}$$

where R_i^S is the individual payoff in scenario $i = 1, 2, 3, 4$ for S.

$$R_i^A - \alpha_M - \alpha_S \tag{3}$$

[5] Notably the trade effect expresses itself differently in S's marginal conditions than in M's and A's since S's payoff function is separable in the arguments connected to M and A respectively. It becomes the basic effect that an relationship-specific investment is, on the marginal, more beneficial under bilateral trade than under no-trade. Moreover the trade effect does not vary with asset allocation and does, therefore, not affect the optimal organization decision.

[6] Here the analysis differs from the basic model in Hart (1995) in an non-substantial way. Hart calculates the *ex post* benefit by first subtracting the payoff from no-trade from the payoff from trade to create the surplus from trade and then adds this to payoff from no-trade minus the investments costs - getting the same expression for the *ex post* benefit.

where R_i^A is the individual payoff in scenario $i = 1, 2, 3, 4$ for A.

Neg. parties	Other trades	Reimbursement m, y and v	Scenario
M, A	none	$m = \bar{m} + \frac{T_A(\cdot) - t(\cdot) + K_M(\cdot) - k(\cdot)}{2}$	1
M, S	M, A	$v = \bar{v} + \frac{T(\cdot) - T_A(\cdot) + C(\cdot) - c(\cdot)}{2}$	
A, S	M, A & M, S	$y = \bar{y} + \frac{K_M(\cdot) - K(\cdot) + G(\cdot) - g(\cdot)}{2}$	
M, S	none	$v = \bar{v} + \frac{T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot)}{2}$	2
A, S	M, S	$y = \bar{y} + \frac{k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot)}{2}$	
M, A	M, S & A, S	$m = \bar{m} + \frac{T(\cdot) - T_S(\cdot) + K(\cdot) - K_S(\cdot)}{2}$	
A, S	none	$y = \bar{y} + \frac{k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot)}{2}$	3
M, A	A, S	$m = \bar{m} + \frac{T_A(\cdot) - t(\cdot) + K(\cdot) - K_S(\cdot)}{2}$	
M, S	A, S & M, A	$v = \bar{v} + \frac{T(\cdot) - T_A(\cdot) + C(\cdot) - c(\cdot)}{2}$	
M, A	none	$m = \bar{m} + \frac{T_A(\cdot) - t(\cdot) + K_M(\cdot) - k(\cdot)}{2}$	1
A, S	M, A	$y = \bar{y} + \frac{K_M(\cdot) - K(\cdot) + G(\cdot) - g(\cdot)}{2}$	
M, S	M, A & A, S	$v = \bar{v} + \frac{T(\cdot) - T_A(\cdot) + C(\cdot) - c(\cdot)}{2}$	
M, S	none	$v = \bar{v} + \frac{T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot)}{2}$	4
M, A	M, S	$m = \bar{m} + \frac{T(\cdot) - T_S(\cdot) + K_M(\cdot) - k(\cdot)}{2}$	
A, S	M, S & M, A	$y = \bar{y} + \frac{K_M(\cdot) - K(\cdot) + G(\cdot) - g(\cdot)}{2}$	
A, S	none	$y = \bar{y} + \frac{k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot)}{2}$	2
M, S	A, S	$v = \bar{v} + \frac{T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot)}{2}$	
M, A	M, S & A, S	$m = \bar{m} + \frac{T(\cdot) - T_S(\cdot) + K(\cdot) - K_S(\cdot)}{2}$	

Table 1: Reimbursements under different negotiation sequences

Solving the maximization problem for the three parties yields the same first order conditions for the respective parties in all four scenarios. This since the investments enter in the parties gain from trade in the same way in all scenarios. The first order conditions become:

for M:

$$\frac{1}{2}\frac{\partial T(\cdot)}{\partial \mu_A} + \frac{1}{2}\frac{\partial t(\cdot)}{\partial \mu_A} - 1 = 0 \quad (4)$$

$$\frac{1}{2}\frac{\partial T(\cdot)}{\partial \mu_S} + \frac{1}{2}\frac{\partial t(\cdot)}{\partial \mu_S} - 1 = 0 \quad (5)$$

for S:

$$-\frac{1}{2}\frac{\partial C(\cdot)}{\partial \sigma_M} - \frac{1}{2}\frac{\partial c(\cdot)}{\partial \sigma_M} - 1 = 0 \quad (6)$$

$$-\frac{1}{2}\frac{\partial G(\cdot)}{\partial \sigma_A} - \frac{1}{2}\frac{\partial g(\cdot)}{\partial \sigma_A} - 1 = 0 \quad (7)$$

for A:

$$-\frac{1}{2}\frac{\partial K(\cdot)}{\partial \alpha_M} - \frac{1}{2}\frac{\partial k(\cdot)}{\partial \alpha_M} - 1 = 0 \quad (8)$$

$$-\frac{1}{2}\frac{\partial K(\cdot)}{\partial \alpha_S} - \frac{1}{2}\frac{\partial k(\cdot)}{\partial \alpha_S} - 1 = 0 \quad (9)$$

Party	Scenario	Individual payoffs
M	1	$\frac{T(\cdot)+t(\cdot)-C(\cdot)+c(\cdot)-K_M(\cdot)+k(\cdot)}{2}-\bar{m}-\bar{v}$
S	1	$\frac{-C(\cdot)-c(\cdot)-G(\cdot)-g(\cdot)+T(\cdot)-T_A(\cdot)+K_M(\cdot)-K(\cdot)}{2}+\bar{v}+\bar{y}$
A	1	$\frac{-K(\cdot)-k(\cdot)+T_A(\cdot)-t(\cdot)-G(\cdot)+g(\cdot)}{2}+\bar{m}-\bar{y}$
M	2	$\frac{T(\cdot)+t(\cdot)-C(\cdot)+c(\cdot)-K(\cdot)+K_S(\cdot)}{2}-\bar{m}-\bar{v}$
S	2	$\frac{-C(\cdot)-c(\cdot)-G(\cdot)-g(\cdot)+T_S(\cdot)-t(\cdot)-K_S(\cdot)+k(\cdot)}{2}+\bar{v}+\bar{y}$
A	2	$\frac{-K(\cdot)-k(\cdot)+T(\cdot)-T_S(\cdot)-G(\cdot)+g(\cdot)}{2}+\bar{m}-\bar{y}$
M	3	$\frac{T(\cdot)+t(\cdot)-C(\cdot)+c(\cdot)-K(\cdot)+K_S(\cdot)}{2}-\bar{m}-\bar{v}$
S	3	$\frac{-C(\cdot)-c(\cdot)-G(\cdot)-g(\cdot)+T(\cdot)-T_A(\cdot)-K_S(\cdot)+k(\cdot)}{2}+\bar{v}+\bar{y}$
A	3	$\frac{-K(\cdot)-k(\cdot)+T_A(\cdot)-t(\cdot)-G(\cdot)+g(\cdot)}{2}+\bar{m}-\bar{y}$
M	4	$\frac{T(\cdot)+t(\cdot)-C(\cdot)+c(\cdot)-K_M(\cdot)+k(\cdot)}{2}-\bar{m}-\bar{v}$
S	4	$\frac{-C(\cdot)-c(\cdot)-G(\cdot)-g(\cdot)+T_S(\cdot)-t(\cdot)+K_M(\cdot)-K(\cdot)}{2}+\bar{v}+\bar{y}$
A	4	$\frac{-K(\cdot)-k(\cdot)+T(\cdot)-T_S(\cdot)-G(\cdot)+g(\cdot)}{2}+\bar{m}-\bar{y}$

Table 2: Individual payoff from trade in the different scenarios

Obviously the sequence of negotiation does not affect the incentives for investments for a given ownership structure. Hence the asset allocation determines the investment incentives, in together with characteristics of assets, investments and human capital, just as in the bilateral model and, as will be seen below, in the simultaneous negotiations model. An effect of the sequence on the investment incentive would have provided rationale for further investigation of the sequential model, e.g. an analysis of the determinants of the sequence and how different sequences affect the organizational choice. Since no "sequencing effect" is found the sequential model is left at this stage, and should be considered a reference case for the coming analysis. The next section deals with simultaneous negotiations which is the focus of the analysis in the paper.

4 Simultaneous negotiations

All parties negotiate bilaterally at the same time, with rational expectations about the outcome in the other negotiations. The negotiations are bilateral since it is assumed that the agreement between two parties cannot be conditioned on the participation of the third party, i.e. M negotiate with S and A at the same time but not at the same "table". Notably this negotiation differs from a setup where all three parties are involved in a "common" negotiation, all three parties sit at the same table at the same time, with one third of the surplus each as the solution.[7]

In these negotiations the threat points/ outside options for each party are *no trade* and *third party trade* i.e. only trade with the party not involved in the particular bilateral negotiation. The question that arise is: which outside option is most credible i.e. rational to expect? It turns out that third party trade is the best outside option for all parties. To see this, start by looking at the bilateral negotiation between M and A (from this general conclusions about the other negotiations may be drawn). In the negotiation with A, M has two possible outside options:

1. trade with S only - third party trade
2. not trade at all - no trade

In the negotiation M would like to have the best possible outside option, possible in the sense that it is credible - i.e. that it is expected to be realized in the case negotiations break down. Which of the two outside options is best for M and is this outside option credible? To only trade with S gives M the following benefit:

$$T_S(\cdot) - \bar{m} - v$$

where $v = \bar{v} + (T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot))/2$ see scenario 2 in table 1 above. Furthermore the benefit from no trade is :

$$t(\cdot) - \bar{m} - \bar{v}$$

Thus the gain from trading with S compared to no trade is $T_S(\cdot) - \bar{m} - v - (t(\cdot) - \bar{m} - \bar{v}) = (T_S(\cdot) - t(\cdot) - C(\cdot) + c(\cdot))/2$ and this is greater than zero since $T_S(\cdot) - C(\cdot) - g(\cdot) - k(\cdot) > t(\cdot) - c(\cdot) - g(\cdot) - k(\cdot)$ according to the assumptions in (A11). Thus it is better for M to trade with S than not trade at all if the negotiations with A breaks down. Now, will S prefer to trade with M to not trade with M in all settings - i.e. is the outside option credible? It is obvious that it is better for S to trade with M than not at all both when S negotiates with A and when this negotiation has broken down (see appendix). This implies that trade with S is a credible outside option for M in the negotiation with A. The next step is to investigate whether trade with S is a credible outside option for A in the negotiation with M. Using the same kind of reasoning as for M it is found that this is the case. Thus both M and A has trade with S as

[7] Applying the Nash bargaining solution to the common negotiation gives:
$\frac{1}{3}\begin{bmatrix} (T(\cdot) - t(\cdot)) - (G(\cdot) - g(\cdot)) + \\ -(K(\cdot) - k(\cdot)) - (C(\cdot) - c(\cdot)) \end{bmatrix}$
to each party.

outside option in their bilateral negotiation. The Nash Bargaining product for this negotiation is:

$$[(T(\cdot) - v - m) - (T_S(\cdot) - v - \bar{m})] \times [(m - y - K(\cdot)) - (\bar{m} - y - K_S(\cdot))] \tag{10}$$

The Nash bargaining product is used to determine the optimal reimbursement between M and A i.e. the optimal m, in this case m is found to be:

$$m = \bar{m} + (T(\cdot) - T_S(\cdot) + K(\cdot) - K_S(\cdot))/2 \tag{11}$$

Notably this is the same m as in scenario 2 with sequential negotiation. In the same manner v and y for the simultaneous negotiation can be found, it is straightforward to see that:

$$v = \bar{v} + (T(\cdot) - T_A(\cdot) + C(\cdot) - c(\cdot))/2 \tag{12}$$

is the outcome from the bilateral negotiation between M and S when both parties has trade with A as outside option (scenario 3), and that:

$$y = \bar{y} + (K_M(\cdot) - K(\cdot) + G(\cdot) - g(\cdot))/2 \tag{13}$$

is the outcome from the bilateral negotiation between A and S when both parties has trade with M as outside option (scenario 1).

It might be helpful, for the intuition in the following sections, to consider some interpretations of the three reimbursements above. Besides the market price, m consists of half of $M:s$ benefit from trilateral trade minus half of $M:s$ benefit from only trading with S. Moreover m also covers half of $A:s$ trilateral production cost minus half the production cost when A only trades with S. In a similar manner v consists of the market price and half of $M:s$ benefit from trilateral trade minus half of $M:s$ benefit from only trading with A. Through v S is also reimbursed for half its production cost when trading with M minus half the production cost when not trading with M. Thus in the negotiations over the benefit from trade M gives up its entire benefit from trilateral trade but for half of the benefit from each of the bilateral trades, moreover M also cover some its trading parties costs. Finally, y, the reimbursement from A to S, covers half of S production cost when trading with A minus half of the cost when not trading with A. Moreover, S also receive half of the cost difference, for A, between only trading with M and trilateral trade. The next subsection show the individual payoffs from trade and how the reimbursements affect these.

4.1 Individual Payoffs from Trade

Similarly to the sequential negotiations case the payoffs from trade can be calculated by inserting the reimbursements into the payoff functions. With some abuse of notation the resulting payoffs from trade are denoted by U_i $i = M, A, S$, and are given by:

For M

$$T(\cdot) - v - m \text{ where}$$
$$m = \bar{m} + \frac{T(\cdot) - T_S(\cdot) + K(\cdot) - K_S(\cdot)}{2} \text{and } v = \bar{v} + \frac{T(\cdot) - T_A(\cdot) + C(\cdot) - c(\cdot)}{2}$$
$$\Rightarrow U_M = \frac{T_S(\cdot) + T_A(\cdot) - K(\cdot) + K_S(\cdot) - C(\cdot) + c(\cdot)}{2} - \bar{m} - \bar{v} \tag{14}$$

For S

$$v - C(\cdot) + y - G(\cdot) \text{ where}$$
$$v = \bar{v} + \frac{T(\cdot) - T_A(\cdot) + C(\cdot) - c(\cdot)}{2} \text{ and } y = \bar{y} + \frac{K_M(\cdot) - K(\cdot) + G(\cdot) - g(\cdot)}{2}$$
$$\Rightarrow U_S = \frac{-C(\cdot) - c(\cdot) + T(\cdot) - T_A(\cdot)}{2} + \frac{-G(\cdot) - g(\cdot) + K_M(\cdot) - K(\cdot)}{2} + \bar{v} + \bar{y} \tag{15}$$

For A

$$m - y - K(\cdot) \text{ where}$$
$$m = \bar{m} + \frac{T(\cdot) - T_S(\cdot) + K(\cdot) - K_S(\cdot)}{2} \text{and } y = \bar{y} + \frac{K_M(\cdot) - K(\cdot) + G(\cdot) - g(\cdot)}{2}$$
$$\Rightarrow U_A = \frac{-K_S(\cdot) - K_M(\cdot) + T(\cdot) - T_S(\cdot) - G(\cdot) + g(\cdot)}{2} + \bar{m} - \bar{y} \tag{16}$$

The individual *ex post* benefits from trade thus become:

$$U_M - \mu_A - \mu_S \text{ for } M \tag{17}$$
$$U_S - \sigma_M - \sigma_A \text{ for } S \tag{18}$$
$$U_A - \alpha_M - \alpha_S \text{ for } A \tag{19}$$

4.2 Optimal Investment Decision

4.2.1 First-Best Choice of Investments

The *ex post* negotiations are always efficient under any organizational structure, however the investments in date 0 might not be efficient (c.f. Hart, 1995). In a first-best situation the parties can coordinate their investments to maximize the net present value of their trading relationship at date 0. Consequently they choose their investments to maximize:

$$T(\mu_A, \mu_S) - \mu_A - \mu_S - K(\alpha_M, \alpha_S) - \alpha_M - \alpha_S - C(\sigma_M) - \sigma_M - G(\sigma_A) - \sigma_A \tag{20}$$

Coordination of investments increases the benefit that is divided between the actors *ex post*, any choice of investments that does not maximize (20) can be improved on by choosing to maximize (20) (c.f. ibid). The first order conditions for this maximization are:

$$\frac{\partial T(\mu_A,\mu_S)}{\partial \mu_A}-1 = 0 \quad (21)$$
$$\frac{\partial T(\mu_A,\mu_S)}{\partial \mu_S}-1 = 0 \quad (22)$$
$$-\frac{\partial C(\sigma_M)}{\partial \sigma_M}-1 = 0 \quad (23)$$
$$-\frac{\partial G(\sigma_A)}{\partial \sigma_A}-1 = 0 \quad (24)$$
$$-\frac{\partial K(\alpha_M,\alpha_S)}{\partial \alpha_M}-1 = 0 \quad (25)$$
$$-\frac{\partial K(\alpha_M,\alpha_S)}{\partial \alpha_S}-1 = 0 \quad (26)$$

For M, S and A respectively. For future reference let $\mu_A^*, \mu_S^*, \sigma_M^*, \sigma_A^*, \alpha_M^*, \alpha_S^*$ denote the first-best investments.

4.2.2 Second-Best Choice of Investments

The incomplete contracting in the model renders the first-best impossible, thus the model depicts a second-best world. In the second-best each of the trading parties will chose period 0 investments to maximize their *ex post* benefit, this produces the following first order conditions:

For M

$$\frac{1}{2}\frac{\partial T_S(\mu_A,\mu_S;P_M)}{\partial \mu_A}+\frac{1}{2}\frac{\partial T_A(\mu_A,\mu_S;P_M)}{\partial \mu_A}-1 = 0 \quad (27)$$
$$\frac{1}{2}\frac{\partial T_S(\mu_A,\mu_S;P_M)}{\partial \mu_S}+\frac{1}{2}\frac{\partial T_A(\mu_A,\mu_S;P_M)}{\partial \mu_S}-1 = 0 \quad (28)$$

For S[8]

$$-\frac{1}{2}\frac{\partial C(\sigma_M)}{\partial \sigma_M}-\frac{1}{2}\frac{\partial c(\sigma_M;P_S)}{\partial \sigma_M}-1 = 0 \quad (29)$$
$$-\frac{1}{2}\frac{\partial G(\sigma_A)}{\partial \sigma_A}-\frac{1}{2}\frac{\partial g(\sigma_A;P_S)}{\partial \sigma_A}-1 = 0 \quad (30)$$

For A

$$-\frac{1}{2}\frac{\partial K_S(\alpha_M,\alpha_S;P_A)}{\partial \alpha_M}-\frac{1}{2}\frac{\partial K_M(\alpha_M,\alpha_S;P_A)}{\partial \alpha_M}-1 = 0 \quad (31)$$
$$-\frac{1}{2}\frac{\partial K_S(\alpha_M,\alpha_S;P_A)}{\partial \alpha_S}-\frac{1}{2}\frac{\partial K_M(\alpha_M,\alpha_S;P_A)}{\partial \alpha_S}-1 = 0 \quad (32)$$

[8] Notably if the production cost for S was not separable in the two inputs and instead given by a function $C(\sigma_M,\sigma_A)$ (just as for A) the first order conditions for S would have the same form as the first order conditions for A and M i.e. :
$-\frac{1}{2}\frac{\partial C_A(\sigma_M,\sigma_A;P_S)}{\sigma_j}-\frac{1}{2}\frac{\partial C_M(\sigma_M,\sigma_A;P_S)}{\sigma_j}-1=0$ for $j=A,M$.

The fact that the first-order conditions for M and A depend only on the outside options can be attributed to the Nash bargaining solution. Given the non-separable benefit/cost functions and the fact that there are three parties in the transaction - each party is involved in two simultaneous bilateral negotiations - Nash bargaining implies that each party gives up half their trilateral trade benefit/cost in each negotiation. In general it should be noted that bilateral and symmetric Nash bargaining always imply that the surplus from trade with the other party is split in half, this can be seen in the bilateral model, in the sequential negotiation case and here under simultaneous negotiations.[9] This pattern also holds for multilateral trade, with for example four parties in the transaction it implies that each party gives up three halves of the benefit from full trade (all four trades).

Here the division of surplus, described above, implies that the equilibrium outcome (trilateral trade) does not affect the incentives for investments (for M and A). This contrasts with the bilateral model and the sequential case, see above, where the incentives are affected by both the equilibrium outcome and the outside option (bilateral trade and no-trade in the bilateral model, trilateral trade and no-trade in the sequential case). Thus the strongest positive effect on investments (see the marginal conditions in appendix) is absent from the first order conditions, implying that the incentives would have been stronger if the bargaining had not entailed a transfer of the entire trilateral trade benefit. Notably, if the transaction had involved four parties the impact of the full trade benefit on the first order conditions would have been negative.[10] Thus Nash bargaining in this variant of multilateral transactions affect incentives by either muting the effect of the equilibrium outcome (three parties) or by making this effect negative (more than three parties).

4.2.3 Inherent Underinvestments in the Second-Best

As in the original model presented by Hart (1995) the trilateral variant of the model exhibits underinvestments in second-best. The proof follows Hart (see Hart 1995:41) and is here only presented for μ_A and σ_M since the reasoning is identical for the other investments.

First μ_A, it is assumed above that $\frac{\partial T(\mu_A,\mu_S)}{\partial \mu_A} > 0$, $\frac{\partial^2 T(\mu_A,\mu_S)}{\partial \mu_A \partial \mu_A} < 0$ and that $\frac{\partial T(\mu_A,\mu_S)}{\partial \mu_A} > \frac{\partial T_i(\mu_A,\mu_S;P_M)}{\partial \mu_A}$ $i = S, A$. Now suppose that $\hat{\mu}_A$ solves the second-best problem then:

$$\frac{\partial T(\mu_A,\mu_S)}{\partial \hat{\mu}_A} > \frac{1}{2}\frac{\partial T_S(\mu_A,\mu_S;P_M)}{\partial \hat{\mu}_A} + \frac{1}{2}\frac{\partial T_A(\mu_A,\mu_S;P_M)}{\partial \hat{\mu}_A} = 1$$

[9] Obviously, this holds also for S in this case, that is can be seen in S's payoffs which among other things consist of: $-\frac{1}{2}C(\sigma_M) - \frac{1}{2}c(\sigma_M;P_S)$ and $-\frac{1}{2}G(\sigma_A) - \frac{1}{2}g(\sigma_A;P_S)$ implying that half the payoff difference from a certain trade is given away.

[10] For example, if adding a party H to the transaction and making suitable adjustments of the model, then M's F.O.C w.r.t μ_A would be:
$-\frac{1}{2}\frac{\partial T(\cdot)}{\partial \mu_A} + \frac{1}{2}\frac{\partial T_{AH}(\cdot)}{\partial \mu_A} + \frac{1}{2}\frac{\partial T_{SH}(\cdot)}{\partial \mu_A} + \frac{1}{2}\frac{\partial T_{AS}(\cdot)}{\partial \mu_A} = 1$
Where the subscripts AH, SH and AS denotes the trades going on in the outside options.

and it follows that

$$\frac{\partial T(\mu_A,\mu_S)}{\partial \hat{\mu}_A} > \frac{\partial T(\mu_A,\mu_S)}{\partial \mu_A^*} = 1$$
$$\Rightarrow \mu_A^* > \hat{\mu}_A \text{ since } \frac{\partial^2 T(\mu_A,\mu_S)}{\partial \mu_A \partial \mu_A} < 0$$

Second σ_M, here it is assumed that $\frac{\partial C(\sigma_M)}{\partial \sigma_M} < 0$, $\frac{\partial^2 C(\sigma_M)}{\partial \sigma_M \partial \sigma_M} > 0$ and that $\frac{\partial C(\sigma_M)}{\partial \sigma_M} < \frac{\partial c(\sigma_M;P_S)}{\partial \sigma_M}$. Now suppose that $\hat{\sigma}_M$ solves the second-best problem then:

$$\frac{\partial C(\sigma_M)}{\partial \hat{\sigma}_M} < \frac{1}{2}\frac{\partial C(\sigma_M)}{\partial \hat{\sigma}_M} + \frac{1}{2}\frac{\partial c(\sigma_M;P_S)}{\partial \hat{\sigma}_M} = -1$$

and from this it follows that:

$$\frac{\partial C(\sigma_M)}{\partial \hat{\sigma}_M} < \frac{\partial C(\sigma_M)}{\partial \sigma_M^*} = -1$$
$$\Rightarrow \sigma_M^* > \hat{\sigma}_M \text{ since } \frac{\partial^2 C(\sigma_M)}{\partial \sigma_M \partial \sigma_M} > 0$$

showing that second-best leads to underinvestments. This holds for all investments by all parties.

4.3 Optimal Organization Decision

There are a number of ways to organize the transaction analyzed in this paper. Organization in this setting is the allocation of physical assets i.e. ownership structure. The optimal ownership structure is the ownership structure that supports the greatest relationship-specific investments. Any change in ownership structure, that entails higher investments from one or more of the parties and equal investments from the others, is an improvement, because it implies a move towards the first-best. This due to the inherent underinvestments in the model.

4.3.1 Ten Ways to Organize the Transaction

Here, as in the bilateral model , are the cases when party J, $J = M, A, S$, does not own its own asset but some, or all, of the others' assets abstracted from. Intuitively this is reasonable if one considers that all parties only have one physical asset and this asset is the used for each party's production, thus each party can be expected to be more productive owning its own asset than when only owning other assets (c.f. Hart, 1995). In spite of this limitation there are ten feasible ownership structures:

1 M-integration where M owns all assets, first order conditions in this are the following, let the superscript 1 denoted the investments under M-

integration.

$$\frac{1}{2}\frac{\partial T_S\left(\mu_A^1,\mu_S^1;p_M,p_S,p_A\right)}{\partial \mu_A}+\frac{1}{2}\frac{\partial T_A\left(\mu_A^1,\mu_S^1;p_M,p_S,p_A\right)}{\partial \mu_A} = 1 \quad (33)$$

$$\frac{1}{2}\frac{\partial T_S\left(\mu_A^1,\mu_S^1;p_M,p_S,p_A\right)}{\partial \mu_S}+\frac{1}{2}\frac{\partial T_A\left(\mu_A^1,\mu_S^1;p_M,p_S,p_A\right)}{\partial \mu_S} = 1 \quad (34)$$

$$-\frac{1}{2}\frac{\partial C\left(\sigma_M^1\right)}{\partial \sigma_M}-\frac{1}{2}\frac{\partial c\left(\sigma_M^1;\varnothing\right)}{\partial \sigma_M} = 1 \quad (35)$$

$$-\frac{1}{2}\frac{\partial G\left(\sigma_A^1\right)}{\partial \sigma_A}-\frac{1}{2}\frac{\partial g\left(\sigma_A^1;\varnothing\right)}{\partial \sigma_A} = 1 \quad (36)$$

$$-\frac{1}{2}\frac{\partial K_S\left(\alpha_M^1,\alpha_S^1;\varnothing\right)}{\partial \alpha_M}-\frac{1}{2}\frac{\partial K_M\left(\alpha_M^1,\alpha_S^1;\varnothing\right)}{\partial \alpha_M} = 1 \quad (37)$$

$$-\frac{1}{2}\frac{\partial K_S\left(\alpha_M^1,\alpha_S^1;\varnothing\right)}{\partial \alpha_S}-\frac{1}{2}\frac{\partial K_M\left(\alpha_M^1,\alpha_S^1;\varnothing\right)}{\partial \alpha_S} = 1 \quad (38)$$

2 S-integration where S owns all assets, the first order conditions for S-integration follow the same pattern but above with S owning all assets and the investments in this case are denoted by the superscript 2 i.e. $\mu_A^2, \mu_S^2, \sigma_M^2, \sigma_A^2, \alpha_M^2, \alpha_S^2$

3 A-integration where A owns all assets, once again the pattern from case 1 and 2 repeats itself, this time with A owning all the assets and investments denoted by: $\mu_A^3, \mu_S^3, \sigma_M^3, \sigma_A^3, \alpha_M^3, \alpha_S^3$

4 Non-integration where M, S and A own their respective assets p_M, p_S and p_A, in this case the first order conditions are somewhat different from the previous cases and investments in this case is denoted by the superscript 4.

$$\frac{1}{2}\frac{\partial T_S\left(\mu_A^4,\mu_S^4;p_M\right)}{\partial \mu_A}+\frac{1}{2}\frac{\partial T_A\left(\mu_A^4,\mu_S^4;p_M\right)}{\partial \mu_A} = 1 \quad (39)$$

$$\frac{1}{2}\frac{\partial T_S\left(\mu_A^4,\mu_S^4;p_M\right)}{\partial \mu_S}+\frac{1}{2}\frac{\partial T_A\left(\mu_A^4,\mu_S^4;p_M\right)}{\partial \mu_S} = 1 \quad (40)$$

$$-\frac{1}{2}\frac{\partial C\left(\sigma_M^4\right)}{\partial \sigma_M}-\frac{1}{2}\frac{\partial c\left(\sigma_M^4;p_S\right)}{\partial \sigma_M} = 1 \quad (41)$$

$$-\frac{1}{2}\frac{\partial G\left(\sigma_A^4\right)}{\partial \sigma_A}-\frac{1}{2}\frac{\partial g\left(\sigma_A^4;p_S\right)}{\partial \sigma_A} = 1 \quad (42)$$

$$-\frac{1}{2}\frac{\partial K_S\left(\alpha_M^4,\alpha_S^4;p_A\right)}{\partial\alpha_M}-\frac{1}{2}\frac{\partial K_M\left(\alpha_M^4,\alpha_S^4;p_A\right)}{\partial\alpha_M} = 1 \quad (43)$$

$$-\frac{1}{2}\frac{\partial K_S\left(\alpha_M^4,\alpha_S^4;p_A\right)}{\partial\alpha_S}-\frac{1}{2}\frac{\partial K_M\left(\alpha_M^4,\alpha_S^4;p_A\right)}{\partial\alpha_S} = 1 \quad (44)$$

There is, as can be seen, an obvious structure in the first order conditions for the different cases, therefore the first order conditions for partial integration cases will be represented by only one example even though there are six different cases. Notably the investments in each case are denoted by the case-number in the superscript, to make this clear the investments are presented in connection to each case.

5 Partial M-integration type one: M owns p_M and p_S, A owns p_A. Investments: $\mu_A^5,\mu_S^5,\sigma_M^5,\sigma_A^5,\alpha_M^5,\alpha_S^5$

6 Partial M-integration type two: M owns p_M and p_A, S owns p_S. Investments: $\mu_A^6,\mu_S^6,\sigma_M^6,\sigma_A^6,\alpha_M^6,\alpha_S^6$

7 Partial S-integration type one: S owns p_S and p_M, A owns p_A. Investments: $\mu_A^7,\mu_S^7,\sigma_M^7,\sigma_A^7,\alpha_M^7,\alpha_S^7$

8 Partial S-integration type two: S owns p_S and p_A, M owns p_M. Investments: $\mu_A^8,\mu_S^8,\sigma_M^8,\sigma_A^8,\alpha_M^8,\alpha_S^8$

9 Partial A-integration type one: A owns p_A and p_M, S owns p_S. First order conditions:

$$\frac{1}{2}\frac{\partial T_S\left(\mu_A^9,\mu_S^9;\varnothing\right)}{\partial\mu_A}+\frac{1}{2}\frac{\partial T_A\left(\mu_A^9,\mu_S^9;\varnothing\right)}{\partial\mu_A} = 1 \quad (45)$$

$$\frac{1}{2}\frac{\partial T_S\left(\mu_A^9,\mu_S^9;\varnothing\right)}{\partial\mu_S}+\frac{1}{2}\frac{\partial T_A\left(\mu_A^9,\mu_S^9;\varnothing\right)}{\partial\mu_S} = 1 \quad (46)$$

$$-\frac{1}{2}\frac{\partial C\left(\sigma_M^9\right)}{\partial\sigma_M}-\frac{1}{2}\frac{\partial c\left(\sigma_M^9;p_S\right)}{\partial\sigma_M} = 1 \quad (47)$$

$$-\frac{1}{2}\frac{\partial G\left(\sigma_A^9\right)}{\partial\sigma_A}-\frac{1}{2}\frac{\partial g\left(\sigma_A^9;p_S\right)}{\partial\sigma_A} = 1 \quad (48)$$

$$-\frac{1}{2}\frac{\partial K_S\left(\alpha_M^9,\alpha_S^9;p_A,p_M\right)}{\partial\alpha_M}-\frac{1}{2}\frac{\partial K_M\left(\alpha_M^9,\alpha_S^9;p_A,p_M\right)}{\partial\alpha_M} = 1 \quad (49)$$

$$-\frac{1}{2}\frac{\partial K_S\left(\alpha_M^9,\alpha_S^9;p_A,p_M\right)}{\partial\alpha_S}-\frac{1}{2}\frac{\partial K_M\left(\alpha_M^9,\alpha_S^9;p_A,p_M\right)}{\partial\alpha_S} = 1 \quad (50)$$

10 Partial A-integration type two: A owns p_A and p_S, M owns p_M. Investments: $\mu_A^{10},\mu_S^{10},\sigma_M^{10},\sigma_A^{10},\alpha_M^{10},\alpha_S^{10}$

4.3.2 The Basics of the Analysis

As already mentioned the optimal organization of the trilateral trade depends on how well the organization supports investments. Any of the ten structures above may be optimal, depending on the nature of the assets, human capital and investments involved in the transaction. Some possible characteristics of assets, human capital and investments are described in definition 1-5 (see appendix). In coming subsections the effects of these definitions on the optimal organizational structure is exemplified.

The analysis is divided into two analytical frameworks, a Pareto analysis and an unbounded analysis where the relative productivity of investments, for example M's investment being ten times more productive than S's investment in monetary terms i.e. in its contribution to total surplus, and related issues are included. The Pareto analysis produces results that hold no matter what the pattern of relative productivity looks like, but it only gives a partial ranking of organizational structures because it deals with Pareto improvements from a given starting point, within the limits imposed by assumptions. The unbounded analysis opens up for asset allocations in accordance with the relative productivity of investments, and this may settle the choice between organizational structures that are found optimal in the Pareto analysis i.e. complement the Pareto analysis. Moreover, in the unbounded analysis the starting point does not matter and generically a full ranking of organizational structures is obtained. The unbounded analysis may either be used as an extension to the Pareto analysis or as an analytical tool on its own. However, relative productivity of investments is an elusive concept in the model and therefore the results generally become less clear-cut. The demands on the parties' information and calculative abilities are also greater in the unbounded analysis than in the Pareto analysis, implying greater frictions in the allocation of assets.

This amounts to an important insight, namely that the characteristics of the asset market may affect the applicability of the unbounded analysis. While the Pareto analysis can be used on both rigid and more flexible asset markets, issues like the relative productivity of investments may only be taken into account on a frictionless asset market. If asset can be transferred without friction between the parties will be able, and willing, to do so to maximize the investments contribution to the trilateral trade surplus.

4.3.3 Examples of Pareto Analysis

Here the focus is on changes in organizational structure that are better, yielding a higher level of investments, than a given starting point irrespective of the relative productivity of assets, when assets and human capital have certain characteristics. To analyze these changes one has to find a natural starting point for the analysis. Here that starting point is non-integration (following Hart, 1995), the effects of changing the starting point are discussed below. The analysis asks the question asked whether some other organizational structure, given the assumptions imposed, would increase the level of investments in the trilateral trade relationship. The results are strong in the sense that the relative productivity of investments does not matter.

Full Strict Complementarity of Assets Assume that all three assets are strictly complementary and that this strict complementarity is characterized by A, in the partial and no-trade cases, being indifferent between all ownership structures that do not contain all three assets (see the second condition of definition 1 in the appendix). The formal statement of this assumption is:

$$\frac{\partial K_i\left(\alpha_M,\alpha_S;P_A\right)}{\partial\alpha_j}\equiv\frac{\partial K_i\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial\alpha_j}\text{ and }\frac{\partial k\left(\alpha_M,\alpha_S;P_A\right)}{\partial\alpha_j}\equiv\frac{\partial k\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial\alpha_j} \tag{51}$$

$$\text{where } i=M,S,\ j=M,S \text{ and } P_A=\{p_A\},\ \{p_A,p_S\}\ \text{ or }\ \{p_A,p_M\}$$

To investigate how this assumption affects the optimal organization of the trilateral trade scrutinize the first order conditions for the different ownership structures (1-10 above). First note that the assumption implies that all of A's first order conditions, except for case 3, are equal. This implies that $\alpha_M^i=\alpha_M^i$ and $\alpha_S^i=\alpha_S^i$ for $i=1,2,4,...,10$ and furthermore (A16) and (A17) give that $\alpha_M^3\geq\alpha_M^i$ and $\alpha_S^3\geq\alpha_S^i$ for $i=1,2,4,...,10$. The marginal conditions (A12)-(A15) provide the rationale for how M and S adapt their investment levels to changes in ownership structure. It is easily seen that M will invest more under M-integration (1) and both cases of partial M-integration (5 & 6) than under non-integration. M's investment level is the same for non-integration (4), partial S-integration of type two (8) and partial A-integration of type two (10). For the other cases (2,3,7, & 8) M will invest less than under non-integration. S, on the other hand, invests more than under non-integration in case 2, 7, and 8, less in case 1, 3, 5, and 10. Finally, S makes the same level of investments in case 6 and 9 as under non-integration.

This means that total level of investments is increased in two of the cases above, implying a move towards the first-best, namely partial M-integration of type two and partial S integration of type two. In the first case M will invest more than under non-integration while S and A's investments remain the same, in the second case S will invest more and A and M will keep their investments constant.

To determine whether case 6 or case 8 is optimal more information about the characteristics of the trilateral trade is needed. It could e.g. be the case that M's human capital is more valuable for the transaction than S's human capital and thus more beneficial with an increase of M's investment - the next example takes this to the extreme and assumes that M's human capital is *essential* for the production of the final good.

Essential Human Capital In this example it is assumed that M's human capital is essential for the production of the final good. The formal implications of this assumption are found in the first part of definition 5 (see appendix):

$$\frac{\partial K_S\left(\alpha_M,\alpha_S;P_A\right)}{\partial\alpha_j}\equiv\frac{\partial K_S\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial\alpha_j}\text{ and }\frac{\partial k\left(\alpha_M,\alpha_S;P_A\right)}{\partial\alpha_j}\equiv\frac{\partial k\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial\alpha_j}$$
$$\text{where } j=M,S \text{ and } P_A=\{p_A,p_S,p_M\},\ \{p_A,p_S\},\ \{p_A,p_M\} \text{ or } \{p_A\} \quad (52)$$

and:

$$\frac{\partial c\left(\sigma_M;P_S\right)}{\partial\sigma_M}\equiv\frac{\partial c\left(\sigma_M;\varnothing\right)}{\partial\sigma_M}$$
$$\text{where } P_S=\{p_A,p_S,p_M\},\ \{p_S,p_M\},\{p_S,p_A\} \text{ or } \{p_S\} \quad (53)$$

As in the example above this assumption has implications for the parties' first order conditions. An obvious implication is that A will have weaker incentives for investments in all cases where A own assets. This stems from the basic assumption that $\frac{\partial K_S(\alpha_M,\alpha_S;\varnothing)}{\partial\alpha_j}\geq\frac{\partial K_S(\alpha_M,\alpha_S;P_A)}{\partial\alpha_j}$[11] for $j=M,S$ and P_A being a non-empty set, meaning that investments are at least as valuable under asset ownership as when not owning any assets, i.e. that incentives for investments are weakly higher when owning assets. Notably this changes when condition (52) is imposed on the problem, weakly lowering and equalizing the incentives for investments, that can be attributed to the trade with S, for all ownership structures. The incentives for investments that can be attributed to the trade with M are however unchanged and follow the general idea that more assets imply higher investments. Thus the incentives for investments are qualitatively the same, as if the assumption had not been imposed, but somewhat weaker. Compared to non-integration A will invest more in case 3, 9, and 10; less in case 1,2, 6, and 8; and make the same investments in case 5 and 7.

Condition (53) states that S will in this example make the same investment in the relation with M, σ_M, irrespective of ownership. The incentive for investments in the relation with A is however unaffected by the assumption M's human capital is essential. Hence definition 5 implicitly adds to the separability assumption about S's production cost, this by stating that if M's human capital is essential it is only so for the production of the input to M. S investment in the relationship with A follows ownership. S will invest more than under non-integration in case 2, 7 and 8; less in case 1, 3, 5, and 10; and at the same level in case 6 and 9. When it comes to M's investment it is obvious that M will invest more in case 1, 5, and 6; less in case 2, 3, 7, and 9; and equal amounts in case 8 and 10.

Given that the investments are made in human capital and that M's human capital is essential for the production of the final good, it seems reasonable to find an organizational structure that increases M's investment compared to non-integration. In the bilateral model with bilateral trade this follows immediately from the definition of essential human capital, that is if M's human capital is essential then M-integration is optimal (see Hart, 1995). In the trilateral trade

[11] Remember that K_S is convex i.e. $\frac{\partial K_S(\alpha_M,\alpha_S;P_A)}{\partial\alpha_j}$ $j=M,S$ is non-positive and the second derivative is non-negative.

case this is not possible without imposing further assumptions on the model. Specifically, all changes in ownership structure away from non-integration entail a lower level of investments by one or two of the parties (and a higher level for the third). Focusing on the cases that produces higher investments by M (case 1, 5, and 6) it is easily seen that: 1) M-integration gives the following relationships between investments: $\sigma_M^1 = \sigma_M^4$, $\sigma_A^1 \leq \sigma_A^4$, $\alpha_M^1 \leq \alpha_M^4$, $\alpha_S^1 \leq \alpha_S^4$ (and of course $\mu_A^1 \geq \mu_A^4$, $\mu_S^1 \geq \mu_S^4$), 2) partial M-integration of type 1 gives: $\sigma_M^5 = \sigma_M^4$, $\sigma_A^5 \leq \sigma_A^4$, $\alpha_M^5 = \alpha_M^4$, $\alpha_S^5 = \alpha_S^4$, and finally that partial M-integration of type 2 gives: $\sigma_M^6 = \sigma_M^4$, $\sigma_A^6 = \sigma_A^4$, $\alpha_M^6 \leq \alpha_M^4$, $\alpha_S^6 \leq \alpha_S^4$. The same story can be told for any organizational structure aiming at producing higher investments by the other two parties. It is apparent that no organizational structure entails an improvement over non-integration in this example. However if the assumption made here is coupled with an assumption that p_M and p_S strictly complementary such that S is indifferent between all ownership structures where S does not own p_M, as in the second condition of definition 2, the optimal organizational structure would be *partial M-integration of type one*.

Two Parties with Essential Human Capital Assume that both M and A's human capital is essential for the production of the final good, this leaves S indifferent over ownership i.e. S will make the same investments, σ_M and σ_A, irrespective of ownership structure. M and A will have somewhat weaker incentives for investments (see the discussion for A in the previous example), but they will invest more the more assets they own. Two ownership structures improve the level of investments compared to non-integration, namely partial A-integration of type two and partial M-integration of type one (case 10 and case 5). Once again it is difficult to get an clear-cut result by just imposing one definition on the problem. For example, if there is a complementarity between S's asset and A's asset implying that A becomes indifferent between all cases where A does not own p_S, then partial A-integration (case 10) would be optimal.

Independent assets, Independence of assets imply that two or all assets are strictly non-complementary in the transaction. Now, it is obvious that full independence (definition 3 in the appendix) implies that non-integration is optimal. This conclusion is straightforward since all parties are indifferent between owning one, two or all assets, while owning assets imply weakly greater incentives for investments than not owning any assets (see the marginal conditions). Partial independence is the situation where two of the assets are independent, e.g. p_A and p_S, and definition 4 gives the formal representation of this situation. Imagine that p_A and p_S are independent implying that S and A are indifferent between owning both assets and only owning their own asset, then non-integration is optimal. To see this consider the first-order conditions for the different organizational structures and note that:

- S will invest more in case 2, 7; less in case 1, 3, 5, 10; and make the same investments in case 6, 8, 9.

- A will invest more in case 3, 9; less in case 1, 2, 6, 8; and make the same investments in case 5, 7, 10
- M will invest more in case 1, 5, 6; less in case 2, 3, 7, 9; and make the same investments in case 8, 10

From this it is apparent that there is no change in ownership implying an obvious improvement over non-integration i.e. increasing the investments by one or more parties without lowering the investments by one or more of the other parties.[12]

Full Integration There is an apparent bias towards different types of partial integration in the examples above, begging the question what it takes to make "full" integration optimal. The answer is, while easy to find, not straight forward, it requires quite special combinations of partial complementarity. Take the case of M-integration it requires: 1) that assets p_M and p_S are complementary and that this complementarity implies that $\frac{\partial c(\sigma_M;P_S)}{\partial \sigma_M} \equiv \frac{\partial c(\sigma_M;\varnothing)}{\partial \sigma_M}$ and $\frac{\partial g(\sigma_A;P_A)}{\partial \sigma_A} \equiv \frac{\partial g(\sigma_A;\varnothing)}{\partial \sigma_A}$ where $P_A = p_S$ or $\{p_S, p_A\}$; 2) that assets p_M and p_A are complementary such that $\frac{\partial K_i(\alpha_M,\alpha_S;P_A)}{\partial \alpha_j} \equiv \frac{\partial K_i(\alpha_M,\alpha_S;\varnothing)}{\partial \alpha_j}$ where $i = M, S$, $j = M, S$ and $P_A = p_A$ or $\{p_A, p_S\}$. That is, both S and A are indifferent over ownership unless they own M's asset and a move from non-integration to M-integration, therefore, increases M's investment level without lowering A and S's investment levels. Examples of A- and S-integration can be constructed in a similar manner, i.e. by letting the other two parties be indifferent between all ownership structures where they do not own the acquiring party's asset. Compared to the bilateral case this naturally adds an extra requirement on the complementarity i.e. that both the other parties must be indifferent. A more interesting observation, see above examples, is that assumptions about essential human capital cannot ensure that full integration is optimal as it does in the more clear cut bilateral model, even in the presence of essential human capital the assumptions about partial complementarity of assets are needed to ensure that full integration is optimal.

4.3.4 Applying the Unbounded Analysis

As is seen above, e.g. in the example with full integration, it is somewhat cumbersome to reach clear cut conclusions about the optimal organizational structure in the Pareto analysis, however the conclusions are strong if reached i.e. the they are valid for the whole range of possible extensions. In this section two possible extensions are discussed, namely the asset effect and, most importantly, relative productivity of investments. In this section it is assumed that the asset market is sufficiently flexible for these kinds of extensions to be considered.

[12] If adding the assumption that M's asset and A's asset are strictly complementary to the problem the optimal organization would be partial M-integration of type two (case 6).

Relative Productivity of Investments Hart defines an investment as relative unproductive if its contribution to the net surplus from trade goes to zero (see Hart, 1995:44-46). This type of assumption can be made in the trilateral trade case as well, although making the formal statements of the kind made in Hart (1995) seems superfluous. Instead it is natural to assume that investments sometimes are misdirected and not as beneficial for the relationship as first believed and that this may be discerned by a low marginal contribution to the net benefit of trilateral trade. In the extreme this contribution goes to zero, implying that all marginal contributions, in partial and no-trade as well, of this investment goes to zero. This makes the other investments relatively more important for the transaction i.e. for the net benefit.

Now, return, for a moment, to the example where M's human capital is essential and assume that S investment in the relationship with A, σ_A, is relatively unproductive. This implies that the net benefit from trilateral trade is virtually unaffected by this investments. M's investments on the other hand contribute to this benefit and are thus relatively more important than σ_A. Taking this into consideration the fact that M's human capital is essential leads to M-integration of type 1, instead of non-integration, being optimal. Under M-integration of type 1 M invests more than under non-integration and A make the same investments, while S invests less in the relationship with A. The fact is S invests less is more than outweighed by M's increased investments since S investment is relatively unproductive.[13]

Relative productivity could also affect the conclusions in the other examples above. In the example with full strict complementarity the choice between case 6 and 8 could be made on basis of relative productivity. Moreover it could in fact, in this example, be optimal with A-integration if A's investments are relatively more productive than both S and M's investments, notably this conclusion can be reached irrespective of starting point and is totally dependent of the relative productivity and the mobility of assets. Here relative productivity alone produces the result. However, irrespective of this a move from non-integration to case 6 or 8 would still constitute a Pareto improvement since it would increase M or S's investments without decreasing A is investment. Thus if there is friction in the asset market that prohibits the transfer of M or S's asset (or both) to A then would 6 or 8 be the only feasible improvements.

For independent assets, above its assumed that p_A and p_S are independent, relative productivity of investments could foster other results than non-integration. For example, it could be the case that e.g. M's investment is relatively more productive than S's investment implying that case 5 could be an improvement over non-integration.

Clearly, the introduction of relative productivity opens up the analysis and

[13] Here is one case where the separability assumption, potentially, matters for the result. In most cases both specifications yields the same results when it comes to the optimal organizational structure, but this is an exception. With separability partial M-integration of type one is optimal if σ_A is relatively unproductive, if the production cost is non-separable then non-integration cannot be improved on (M-integration of type one is optimal if both of S's investments are unproductive).

provides new possibilities, but it also makes the analysis less straightforward and creates a need for more assumptions, e.g. that an investment is somewhat important, to narrow down the options. This might not always be intuitive and rewarding.

Asset Effect Finally, something needs to be said about the *asset effect* which appears in the marginal conditions. The asset effect reflects that an investment is partially specific an other party's asset e.g. that M's investment μ_A is partially specific to A's asset p_A. This is reflected by the assumption that $\frac{\partial T_j(\mu_A,\mu_S;P_M)}{\partial \mu_A} \geq \frac{\partial T_j(\mu_A,\mu_S;\bar{P}_M)}{\partial \mu_A}, |P_M| = |\bar{P}_M|, p_A \in P_M, p_A \notin \bar{P}_M$ irrespective of the asset replacing p_A, i.e. the marginal benefit of the investment, in the outside options, is weakly greater if M owns p_A than if M does not own this asset, given that the total number of assets owned is the same in both cases. With appropriate adoptions this assumption is imposed on all parties and their investments. This assumption becomes important for the optimal organizational structure if: 1) the choice is between one party's different forms of partial integration (e.g. partial M-integration of type 1 and type 2), 2) one of the party's investments is relatively unproductive or preferred over the other. In the discussion about a different starting point, than non-integration, below provides an example of this choice.

4.4 Changing the Starting Point

The examples in the Pareto analysis, above, use non-integration as the starting point for the analysis, following Hart (1995), in a rather unquestioning manner. In this section the choice of starting point will be discussed and some short examples of how this might affect the conclusions are provided.

Stigler (1951) suggests that non-integration is rarely manifested in infant industries, characterized by small scale production, because the value-added from specialization is small on such immature markets. Instead vertical integration of tasks is prevalent. However, when the market matures it becomes profitable for firms to specialize on certain tasks and sell their services to other firms, that is a disintegration of tasks is carried through. On the other hand a declining market, where the some tasks are not carried out at a sufficient rate to support an independent firm, is characterized by more vertical integration (c.f. Stigler, 1951).[14] Thus it is reasonable to suggest that the starting point of the analysis performed here should vary with type of market analyzed, if, as is done in many instances, one deals with a mature market then non-integration is the natural starting point.

There may also be other institutional reasons for choosing another starting point than non-integration. It could be that M is a public agency and that the analysis focuses on what parts of its operations that should/could be outsourced. Alternatively, there may be other reasons for a certain organizational structure e.g. inertia or tradition providing rationale for a different starting point than

[14] I'm grateful to prof. H J Holm for enlightening me about the ideas of Stiegler.

non-integration. Irrespective of the underlying rationale there is reason to consider other starting points than non-integration to investigate how this affects the analysis and conclusions.

Now, consider a situation where all assets are strictly complementary and that A as a consequence is indifferent between all ownership structures where A does not own all assets (as in the first example above). Furthermore, imagine that the starting point is M-integration i.e. that M owns all assets needed for the production of the final good.[15] In this situation there is no change in ownership structure that does not entail a lower investment from M, however, S will invest more or equal amounts if the ownership structure is changed. In fact, S will invest more in most cases, only in case 3, 5 and 10 will S make the same investment as under M-integration. Changing to a structure where M invests less, while A and S invests the same is clearly suboptimal - implying a move away from the first-best. Thus case 5 and 10 can be ruled out. Does any of remaining ownership structures imply an improvement over M-integration? This depends on the relative productivity of the investments. If all investments are equally productive, or if the asset market is rigid such that the possible gains of relative productivity cannot be reaped, then M-integration cannot be improved on. Infant industries may experience that kind of frictions in the asset markets, but as the industry evolves the asset markets become more flexible and eventual difference in relative productivity may be taken into consideration when choosing organizational structure.

For example, if M's investments is relatively unproductive compared to either S or A's investments then improvements can be made. If S's investments are relatively more productive than M's investments, but M's investments are still somewhat productive, then could either partial S-integration (of either type) or partial M-integration of type two be improvements, depending on how productive M's investments are.[16]

Finally one more example, again assets are strictly complementary in the manner described above, but now the starting point is partial M-integration of type one (case 5). In this setup S will invest more in a number of cases among them case 6. Case 6 is interesting since it entails that M owns the same number of assets as in case 5, implying that M's incentives for investments are basically the same in the both cases. Here basically means that the incentives in both cases are stronger than in the cases where M owns less assets and weaker than under M-integration, but there might be some difference in incentive strength for each of M's investments between the two cases. In fact the asset effect ensures that M invests weakly less in μ_S and weakly more in μ_A in case 6 compared to case 5. Thus, case 6 is an improvement compared to the starting

[15] It might be useful to think of M as a public agency controlling all assets needed for the production of a public service. But M does not control the human capital needed (besides its own human capital) i.e. A and S are free to use their human capital any way they want to. Allowing ownership by A or S (outsourcing) might be beneficial for the transaction since it increases the incentives for relationship-specific investments.

[16] Obviously may all structures where S's invests more, all structures where S owns one or more assets, constitute an improvement, but it seems realistic that M's investments are somewhat beneficial for the transaction since the whole transaction initially is integrated.

point, if the increase in S's investments outweigh the loss of investments in μ_S, i.e. if S's investments are relatively more productive than the investment μ_S.

It is apparent that the starting point for the analysis matters, an extensive list of examples may be constructed in a similar manner as above, and it is also obvious that relative productivity of investments become more important with other starting points than non-integration. Taking the starting point into account might enable a more thorough analysis of firm organization by heeding the institutional surroundings and how it affects the organization decision.

5 Concluding Discussion

A natural, but noteworthy, difference compared to the bilateral model is that it is more difficult to find a unique Pareto optimal organizational structure in the trilateral trade model. More difficult in the sense that more information about asset and investment characteristics is needed to pin down only one structure. In terms of the formal model this implies that, in most cases, at least two assumptions about e.g. assets need to be imposed for the analysis to produce a clear-cut result. In the bilateral model, on the other hand, one assumption, e.g. essential human capital, in most cases is sufficient. However, in the cases when it is possible to get clear-cut results these results are strong in the sense that relative productivity of investments does not matter. If it is reasonable to assume that asset markets work without friction then the relative productivity of investments may be used to choose between different organizational structures that each implies a Pareto improvement. Also in cases where there are no obvious Pareto improvements relative productivity could be used to find the optimal organizational structure, given low or no friction in the asset market.

By changing the starting point of the Pareto analysis the trilateral trade model may be able to mimic actual situations in different industries. Taking more of the institutional surroundings into account, when using the model, may potentially give new insights and produce better predictions about the optimal organizational structure for a certain transaction. Changing the starting point of the analysis is a small step in that direction, but still provide some useful results. Obviously more work needs to be done here.

There is also a tendency towards partial integration in the trilateral model, while full integration is more of an exception.[17] This result suggests to a more general finding i.e. that a downstream firm, in many instances, should not treat all of its suppliers the same way (integrate all or integrate none). Instead it is beneficial to integrate one party and let the other party remain an independent contractor, implying that the latter party's investment incentive generally is greater than it would be under full integration. Which party to integrate depends on the characteristics of assets, human capital and investments, generally the party that is least sensitive, indicated by unchanged incentives for

[17] Adding an extra assumption/characteristic to the examples where non-integration cannot be improved on in most cases leads to partial integration. Full integration requires as already mentioned two quite specific assumptions.

investments, to loosing control over its asset may be integrated.

The prevalence of partial integration is indicative of that the party's activities being important for the integration decision. Activities in terms of investments and human capital characteristics determine which party to integrate and which party not to integrate. Hence the trilateral model addresses the criticism by Holmström (1999), that the bilateral model does not say anything about the allocation of activities, by allowing for more than an analysis of the allocation of assets. The trilateral model provides some insight about the distribution of activities across firms, and the distribution of activities determines, according to Holmström (1999),[18] the boundaries of the firm.

In the bilateral model and the sequential variant of the trilateral model the incentives for investments are created partly by the marginal trade benefit and partly by the marginal no-trade benefit. Thus the marginal benefit in equilibrium and marginal benefit in the "worst " outside option create the incentives for investments. With simultaneous negotiations the incentives are created off the equilibrium path (if the benefit/cost functions are non-separable in investments) by the marginal benefits in the credible outside options i.e. third party trade. This is an effect of the Nash bargaining assumption (see section 4.2.2). Changing the assumptions bargaining may, of course, affect this result.[19]

When it comes to the difference between the sequential variant and simultaneous variant of the model it is reasonable ask: does sequential negotiation affect the organizational decision? Is there a difference between in the conclusions between the simultaneous and sequential case? Generally the answer is no. However, in one instance a difference is found and that is when M's human capital is essential - then sequential negotiation implies that case 6 or case 8 is optimal, while simultaneous negotiation implies that non-integration cannot be improved on. In all other cases studied in this paper there are no differences in the results. Thus the main contribution of the sequential negotiations case is that the sequence does not matter for the investment incentives.

The Grossman-Hart-Moore model is an attempt to formalize the arguments for and against integration in a bilateral trade relationship. Here an advance is made by allowing for trilateral trade. Many real world transactions involve multiple dependencies, e.g. the manufacturing industry and hospital care, the trilateral trade model presented here may shed some light on the complexities that arise in these situations. Obviously, it does not provide a complete picture an number of possible extensions could improve its intuitive appeal, among them it would be interesting to introduce the possibility for powerful parties in the transaction to change the their dependence on the other parties, i.e. change the technology. This would enable an analysis of corporate structures with one leading party that dominates over the others, i.e. the other parties are dependent on the leading party but not the other way around. Possibly answering the question whether it is beneficial to break the bilateral dependence

[18] see also Coase, 1988

[19] De Meza & Lockwood (1998) analyze the bilateral model under alternating-offers bargaining and find that the results of the bilateral model are fundamentally changed under certain circumstances.

or not? This, and many other things, are left for future research.

References

Coase, Ronald H, 1988, *The Firm, the Market and the Law*, Chicago: University of Chicago Press

De Meza, David & Lockwood, Ben, 1998, "Does Asset Ownership Always Motivate Managers? Outside Options and the Property Right Theory of the Firm, Quarterly Journal of Economics, 113:361-386

Grossman, Sanford J. & Hart, Oliver D., 1986, "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration", Journal of Political Economy, 94:691-719

Hart, Oliver & Moore, John, 1990, "Property Rights and the Nature of the Firm", Journal of Political Economy, 98:1119-1158

Hart, Oliver, 1995, *Firms, Contracts and Financial Structure*, Claredon Lectures in Economics, Claredon Press, Oxford

Holmström, Bengt, 1999, "The Firm as a Subeconomy", Journal of Law, Economics, & Organization, 15:74-102

Stigler, G., 1951, "The Division of Labor Is Limited by the Extent of the Market", Journal of Political Economy, 59:185-193

Appendix

I. Eight patterns of trade

i. All parties trade with each other (full trade or trade):

$$\begin{aligned} U_M &= T(\mu_A, \mu_S) - v - m \\ U_S &= v - C(\sigma_M) + y - G(\sigma_A) \\ U_A &= m - K(\alpha_M, \alpha_S) - y \end{aligned}$$

ii. None of the parties trade with each other (no-trade):

$$\begin{aligned} u_M &= t(\mu_A, \mu_S; P_M) - \bar{v} - \bar{m} \\ u_S &= \bar{v} - c(\sigma_M; P_S) + \bar{y} - g(\sigma_A; P_S) \\ u_A &= \bar{m} - k(\alpha_M, \alpha_S; P_A) - \bar{y} \end{aligned}$$

iii. M & A, A & S trades, but not M & S (no-trade M & S):

$$\begin{aligned} \dot{U}_M &= T_A(\mu_A, \mu_S; P_M) - \bar{v} - m \\ \ddot{U}_S &= \bar{v} - c(\sigma_M; P_S) + y - G(\sigma_A) \\ U_A &= m - K(\alpha_M, \alpha_S) - y \end{aligned}$$

where T_A indicates that A's human capital is available to M but not S's human capital.

iv. M & A, M & S trades, but not A & S (no-trade A & S):

$$\begin{aligned} U_M &= T(\mu_A, \mu_S) - v - m \\ \dot{U}_S &= v - C(\sigma_M) + \bar{y} - g(\sigma_A; P_S) \\ \ddot{U}_A &= m - K_M(\alpha_M, \alpha_S; P_A) - \bar{y} \end{aligned}$$

v. M & S, A & S trades, but not A & M (no-trade A & M):

$$\begin{aligned} \ddot{U}_M &= T_S(\mu_A, \mu_S; P_M) - v - \bar{m} \\ U_S &= v - C(\sigma_M) + y - G(\sigma_A) \\ \dot{U}_A &= \bar{m} - K_S(\alpha_M, \alpha_S; P_A) - y \end{aligned}$$

vi. A & S trades, but not A & M and not M & S (trade A & S):

$$\begin{aligned} u_M &= t(\mu_A, \mu_S; P_M) - \bar{v} - \bar{m} \\ \ddot{U}_S &= \bar{v} - c(\sigma_M; P_S) + y - G(\sigma_A) \\ \dot{U}_A &= \bar{m} - K_S(\alpha_M, \alpha_S; P_A) - y \end{aligned}$$

vii. M & S trades, but not A & M and not A & S (trade M & S):

$$\begin{aligned} \ddot{U}_M &= T_S(\mu_A, \mu_S; P_M) - v - \bar{m} \\ \dot{U}_S &= v - C(\sigma_M) + \bar{y} - g(\sigma_A; P_S) \\ u_A &= \bar{m} - k(\alpha_M, \alpha_S; P_A) - \bar{y} \end{aligned}$$

viii. M & A trades, but not M & S and not A & S (trade M & A):

$$\begin{aligned} \dot{U}_M &= T_A(\mu_A, \mu_S; P_M) - \bar{v} - m \\ u_S &= \bar{v} - c(\sigma_M; P_S) + \bar{y} - g(\sigma_A; P_S) \\ \ddot{U}_A &= m - K_M(\alpha_M, \alpha_S; P_A) - \bar{y} \end{aligned}$$

II. Assumptions about the Benefit Functions

$$T(\mu_A, \mu_S) \text{ is strictly concave in both } \mu_A \text{ and } \mu_S \text{ i.e.}$$
$$\frac{\partial T(\mu_A, \mu_S)}{\partial \mu_j} > 0 \text{ and } \frac{\partial^2 T(\mu_A, \mu_S)}{\partial \mu_j \partial \mu_j} < 0 \text{ for } j = A, S \tag{A1}$$

$$C(\sigma_M) \text{ is strictly convex in } \sigma_M \text{ i.e.}$$
$$\frac{\partial C(\sigma_M)}{\partial \sigma_M} < 0 \text{ and } \frac{\partial^2 C(\sigma_M)}{\partial \sigma_M \partial \sigma_M} > 0 \tag{A2}$$

$$G(\sigma_A) \;\; \textit{is strictly convex in } \sigma_A \;\; i.e$$

$$\frac{\partial G(\sigma_A)}{\partial \sigma_A} < 0 \;\; and \;\; \frac{\partial^2 G(\sigma_A)}{\partial \sigma_A \partial \sigma_A} > 0 \tag{A3}$$

$$K(\alpha_M, \alpha_S) \;\; \textit{is strictly convex in both } \alpha_M \textit{ and } \alpha_S \;\; i.e.$$

$$\frac{\partial K(\alpha_M, \alpha_S)}{\partial \alpha_j} < 0 \;\; and \;\; \frac{\partial^2 K(\alpha_M, \alpha_S)}{\partial \alpha_j \partial \alpha_j} > 0 \; for \; j = M, S \tag{A4}$$

$$T_i(\mu_A, \mu_S; P_M), \; i = A, S, \textit{ is concave in both } \mu_A \textit{ and } \mu_S \;\; i.e.$$

$$\frac{\partial T_i(\mu_A, \mu_S; P_M)}{\partial \mu_j} \geq 0 \;\; and \;\; \frac{\partial^2 T_i(\mu_A, \mu_S; P_M)}{\partial \mu_j \partial \mu_j} \leq 0 \; for \; j = A, S \tag{A5}$$

$$K_i(\alpha_M, \alpha_S; P_A) \;\; , \; i = M, S, \textit{ is convex in both } \alpha_M \textit{ and } \alpha_S \;\; i.e.$$

$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \leq 0 \;\; and \;\; \frac{\partial^2 K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j \partial \alpha_j} \geq 0 \; for \; j = M, S \tag{A6}$$

$$t(\mu_A, \mu_S; P_M) \;\; \textit{is concave in both } \mu_A \textit{ and } \mu_S \;\; i.e.$$

$$\frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_j} \geq 0 \;\; and \;\; \frac{\partial^2 t(\mu_A, \mu_S; P_M)}{\partial \mu_j \partial \mu_j} \leq 0 \; for \; j = A, S \tag{A7}$$

$$c(\sigma_M; P_S) \;\; \textit{is convex in } \sigma_M \;\; i.e.$$

$$\frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \leq 0 \;\; and \;\; \frac{\partial^2 c(\sigma_M; P_S)}{\partial \sigma_M \partial \sigma_M} \geq 0 \tag{A8}$$

$$g(\sigma_A; P_S) \;\; \textit{is convex in } \sigma_A \;\; i.e.$$

$$\frac{\partial g(\sigma_A; P_S)}{\partial \sigma_A} \leq 0 \;\; and \;\; \frac{\partial^2 g(\sigma_A; P_S)}{\partial \sigma_A \partial \sigma_A} \geq 0 \tag{A9}$$

$$k(\alpha_M, \alpha_S; P_A) \;\; \textit{is convex in both } \alpha_M \textit{ and } \alpha_S \;\; i.e.$$

$$\frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \leq 0 \;\; and \;\; \frac{\partial^2 k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j \partial \alpha_j} \geq 0 \; for \; j = M, S \tag{A10}$$

III. Ranking of Total Surplus from Trade

$$\begin{gathered}
T(\mu_A, \mu_S) - C(\sigma_M) - G(\sigma_A) - K(\alpha_M, \alpha_S) > \\
T_A(\mu_A, \mu_S; P_M) - c(\sigma_M; P_S) - G(\sigma_A) - K(\alpha_M, \alpha_S) \lesseqgtr \\
T(\mu_A, \mu_S) - C(\sigma_M) - g(\sigma_A; P_S) - K_M(\alpha_M, \alpha_S; P_A) \lesseqgtr \\
T_S(\mu_A, \mu_S; P_M) - C(\sigma_M) - G(\sigma_A) - K_S(\alpha_M, \alpha_S; P_A) > \\
t(\mu_A, \mu_S; P_M) - c(\sigma_M; P_S) - G(\sigma_A) - K_S(\alpha_M, \alpha_S; P_A) \lesseqgtr \\
T_S(\mu_A, \mu_S; P_M) - C(\sigma_M) - g(\sigma_A; P_S) - k(\alpha_M, \alpha_S; P_A) \lesseqgtr \\
T_A(\mu_A, \mu_S; P_M) - c(\sigma_M; P_S) - g(\sigma_A) - K_M(\alpha_M, \alpha_S; P_A) > \\
t(\mu_A, \mu_S; P_M) - c(\sigma_M; P_S) - g(\sigma_A; P_S) - k(\alpha_M, \alpha_S; P_A) \qquad \text{(A11)}
\end{gathered}$$

IV. Marginal Conditions

$$\begin{gathered}
\text{For } j = A, S \\
\frac{\partial T(\mu_A, \mu_S)}{\partial \mu_S} > \frac{\partial T_j(\mu_A, \mu_S; P_M)}{\partial \mu_S} \geq \frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_S}, \\
P_M = \begin{array}{c} \{p_M, p_S, p_A\}, \{p_M, p_S\} \\ \{p_M, p_A\}, \{p_M\}, \{\varnothing\} \end{array} \\
\frac{\partial T_j(\mu_A, \mu_S; P_M)}{\partial \mu_S} \geq \frac{\partial T_j\left(\mu_A, \mu_S; \bar{P}_M\right)}{\partial \mu_S}, \bar{P}_M \subset P_M \\
\frac{\partial T_j(\mu_A, \mu_S; P_M)}{\partial \mu_S} \geq \frac{\partial T_j\left(\mu_A, \mu_S; \bar{P}_M\right)}{\partial \mu_S}, |P_M| = \left|\bar{P}_M\right|, p_S \in P_M,\ p_S \notin \bar{P}_M \\
\text{and this holds irrespective of the asset replacing } p_S \qquad \text{(Asset effect)} \\
\frac{\partial T_S(\mu_A, \mu_S; P_M)}{\partial \mu_S} \geq \frac{\partial T_A(\mu_A, \mu_S; P_M)}{\partial \mu_S} \qquad \text{(Trade effect)} \\
\frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_S} \geq \frac{\partial t\left(\mu_A, \mu_S; \bar{P}_M\right)}{\partial \mu_S}, \bar{P}_M \subset P_M \qquad \text{(A12)}
\end{gathered}$$

$$
\begin{gathered}
\text{For } j = A, S \\
\frac{\partial T(\mu_A, \mu_S)}{\partial \mu_A} > \frac{\partial T_j(\mu_A, \mu_S; P_M)}{\partial \mu_A} \geq \frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_A}, \\
P_M = \begin{array}{c} \{p_M, p_S, p_A\}, \{p_M, p_S\} \\ \{p_M, p_A\}, \{p_M\}, \{\varnothing\} \end{array} \\
\frac{\partial T_j(\mu_A, \mu_S; P_M)}{\partial \mu_A} \geq \frac{\partial T_j\left(\mu_A, \mu_S; \bar{P}_M\right)}{\partial \mu_A}, \bar{P}_M \subset P_M \\
\frac{\partial T_j(\mu_A, \mu_S; P_M)}{\partial \mu_A} \geq \frac{\partial T_j\left(\mu_A, \mu_S; \bar{P}_M\right)}{\partial \mu_A}, |P_M| = \left|\bar{P}_M\right|, p_A \in P_M,\ p_A \notin \bar{P}_M \\
\text{and this holds irrespective of the asset replacing } p_A \qquad \text{(Asset effect)} \\
\frac{\partial T_A(\mu_A, \mu_S; P_M)}{\partial \mu_A} \geq \frac{\partial T_S(\mu_A, \mu_S; P_M)}{\partial \mu_A} \qquad \text{(Trade effect)} \\
\frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_S} \geq \frac{\partial t\left(\mu_A, \mu_S; \bar{P}_M\right)}{\partial \mu_S}, \bar{P}_M \subset P_M \qquad \text{(A13)}
\end{gathered}
$$

$$
\begin{gathered}
\frac{\partial C(\sigma_M)}{\partial \sigma_M} < \frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \leq \frac{\partial c\left(\sigma_M; \bar{P}_S\right)}{\partial \sigma_M}, \\
\bar{P}_S \subset P_S,\ P_S, \bar{P}_S = \begin{array}{c} \{p_S, p_M, p_A\}, \{p_S, p_M\} \\ \{p_S, p_A\}, \{p_S\}, \{\varnothing\} \end{array} \\
\frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \leq \frac{\partial c\left(\sigma_M; \bar{P}_S\right)}{\partial \sigma_M}, |P_S| = \left|\bar{P}_S\right|,\ p_M \in P_S,\ p_M \notin \bar{P}_S \\
\text{and this holds irrespective of the asset replacing } p_M \qquad \text{(A14)}
\end{gathered}
$$

$$
\begin{gathered}
\frac{\partial G(\sigma_A)}{\partial \sigma_A} < \frac{\partial g(\sigma_A; P_S)}{\partial \sigma_A} \leq \frac{\partial g\left(\sigma_A; \bar{P}_S\right)}{\partial \sigma_A}, \\
\bar{P}_S \subset P_S,\ P_S, \bar{P}_S = \begin{array}{c} \{p_S, p_M, p_A\}, \{p_S, p_M\} \\ \{p_S, p_A\}, \{p_S\}, \{\varnothing\} \end{array} \\
\frac{\partial g(\sigma_A; P_S)}{\partial \sigma_A} \leq \frac{\partial g\left(\sigma_A; \bar{P}_S\right)}{\partial \sigma_A}, |P_S| = \left|\bar{P}_S\right|,\ p_A \in P_S,\ p_A \notin \bar{P}_S \\
\text{and this holds irrespective of the asset replacing } p_A \qquad \text{(A15)}
\end{gathered}
$$

$$\text{For } i = M, S$$
$$\frac{\partial K(\alpha_M, \alpha_S)}{\partial \alpha_M} < \frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_M} \leq \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_M},$$
$$P_A = \begin{array}{c} \{p_A, p_M, p_S\}, \{p_A, p_S\} \\ \{p_A, p_M\}, \{p_A\}, \{\varnothing\} \end{array}$$
$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_M} \leq \frac{\partial K_i(\alpha_M, \alpha_S; \bar{P}_A)}{\partial \alpha_M}, \bar{P}_A \subset P_A$$
$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_M} \leq \frac{\partial K_i(\alpha_M, \alpha_S; \bar{P}_A)}{\partial \alpha_M}, |P_A| = |\bar{P}_A|,\ p_M \in P_A,\ p_M \notin \bar{P}_A$$
$$\text{and this holds irrespective of the asset replacing } p_M \qquad \text{(Asset effect)}$$
$$\frac{\partial K_M(\alpha_M, \alpha_S; P_A)}{\partial \alpha_M} \leq \frac{\partial K_S(\alpha_M, \alpha_S; P_A)}{\partial \alpha_M} \qquad \text{(Trade effect)}$$
$$\frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_M} \leq \frac{\partial k(\alpha_M, \alpha_S; \bar{P}_A)}{\partial \alpha_M}, \bar{P}_A \subset P_A \qquad \text{(A16)}$$

$$\text{For } i = M, S$$
$$\frac{\partial K(\alpha_M, \alpha_S)}{\partial \alpha_S} < \frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_S} \leq \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_S},$$
$$P_A = \begin{array}{c} \{p_A, p_M, p_S\}, \{p_A, p_S\} \\ \{p_A, p_M\}, \{p_A\}, \{\varnothing\} \end{array}$$
$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_S} \leq \frac{\partial K_i(\alpha_M, \alpha_S; \bar{P}_A)}{\partial \alpha_S}, \bar{P}_A \subset P_A$$
$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_S} \leq \frac{\partial K_i(\alpha_M, \alpha_S; \bar{P}_A)}{\partial \alpha_S}, |P_A| = |\bar{P}_A|,\ p_S \in P_A,\ p_S \notin \bar{P}_A$$
$$\text{and this holds irrespective of the asset replacing } p_S \qquad \text{(Asset effect)}$$
$$\frac{\partial K_S(\alpha_M, \alpha_S; P_A)}{\partial \alpha_S} \leq \frac{\partial K_M(\alpha_M, \alpha_S; P_A)}{\partial \alpha_S} \qquad \text{(Trade effect)}$$
$$\frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_S} \leq \frac{\partial k(\alpha_M, \alpha_S; \bar{P}_A)}{\partial \alpha_S}, \bar{P}_A \subset P_A \qquad \text{(A17)}$$

V. Credible Outside Option:

S prefers trade with M in all cases

i) It is better for S to trade with M than not trade at all (no negotiation/breakdown in the negotiation with A). This since trade with trade with M gives:

$$v - C(\cdot) + \bar{y} - g(\cdot)$$
$$\text{where } v = \bar{v} + (T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot))/2 \text{ see table 1}$$

and no trade gives:

$$\bar{v} - c - \bar{y} - g$$

thus the surplus from trade is:

$$\bar{v} + (T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot))/2 - C(\cdot) + \bar{y} - g(\cdot) - [\bar{v} - c(\cdot) - \bar{y} - g(\cdot)] =$$
$$(T_S(\cdot) - t(\cdot) - C(\cdot) + c(\cdot))/2 > 0$$

ii) It is better for S to trade with M than not trade at all (negotiation and trade with A). S's benefit from trading with A if trade with M is already is established is:

$$\begin{aligned} & v - C(\cdot) + y - G(\cdot) \\ \text{where } v &= \bar{v} + (T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot))/2 \\ \text{and } y &= \bar{y} + (k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot))/2 \text{ see table 1} \end{aligned}$$

and trade with A without trading with M gives:

$$\begin{aligned} & \bar{v} - c(\cdot) + y - G(\cdot) \\ \text{where } y &= \bar{y} + (k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot))/2 \text{ see table 1} \end{aligned}$$

thus the surplus from trade in this case is:

$$\bar{v} + (T_S(\cdot) - t(\cdot) + C(\cdot) - c(\cdot))/2 \;\; - C(\cdot) + \bar{y} + (k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot))/2 - G(\cdot)$$
$$- [\bar{v} - c(\cdot) + \bar{y} + (k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot))/2 - G(\cdot)] =$$
$$(T_S(\cdot) - t(\cdot) - C(\cdot) + c(\cdot))/2 > 0$$

Thus it is always better for S to trade with M.

S is a credible outside option for A

i) compare the case where A only trades with S with no trade at all i.e. $\bar{m} - y - K_S(\cdot)$ where $y = \bar{y} + (k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot))/2$ with $\bar{m} - \bar{y} - k(\cdot)$. The difference between the two benefits is:

$$\bar{m} - \bar{y} - (k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot))/2 - K_S(\cdot) - [\bar{m} - \bar{y} - k(\cdot)] =$$
$$(k(\cdot) - K_S(\cdot) + g(\cdot) - G(\cdot))/2 > 0$$

where the strict inequality stems from the assumption that $t(\cdot) - c(\cdot) - G(\cdot) - K_S(\cdot) > t(\cdot) - c(\cdot) - g(\cdot) - k(\cdot)$ see A11.

ii) compare the case where S only trades with A with no trade i.e. $\bar{v} - c + y - G(\cdot)$ where $y = \bar{y} + (k(\cdot) - K_S(\cdot) + G(\cdot) - g(\cdot))/2$ with $\bar{v} - c + \bar{y} - g(\cdot)$. It is now obvious from the preceding comparisons that the assumptions above ensures that trade is beneficial, in this case that difference between trade with A and no trade at all ($(k(\cdot) - K_S(\cdot) + g(\cdot) - G(\cdot))/2$) is strictly positive.

iii) compare the case where S trades with M and A to the case where S only trades with M. Following the same reasoning as above it is once again found that the gain from "adding" A to its trading parties is $(k(\cdot) - K_S(\cdot) + g(\cdot) - G(\cdot))/2$ and positive.

VI. Definitions - Optimal Ownership Structure

Definition 1 *Full Strict Complementarity (FSC) of assets: the assets p_M, p_S and p_A are strictly complementary if:*

$$\frac{\partial T_i\left(\mu_A,\mu_S;P_M\right)}{\partial \mu_j} \equiv \frac{\partial T_i\left(\mu_A,\mu_S;\varnothing\right)}{\partial \mu_j} \text{ and } \frac{\partial t\left(\mu_A,\mu_S;P_M\right)}{\partial \mu_j} \equiv \frac{\partial t\left(\mu_A,\mu_S;\varnothing\right)}{\partial \mu_j}$$

$$\text{where } i=S,A,\ j=S,A \text{ and } P_M=\{p_M\},\ \{p_M,p_S\} \text{ or } \{p_M,p_A\}$$

or if:

$$\frac{\partial K_i\left(\alpha_M,\alpha_S;P_A\right)}{\partial \alpha_j} \equiv \frac{\partial K_i\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial \alpha_j} \text{ and } \frac{\partial k\left(\alpha_M,\alpha_S;P_A\right)}{\partial \alpha_j} \equiv \frac{\partial k\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial \alpha_j}$$

$$\text{where } i=M,S,\ j=M,S \text{ and } P_A=\{p_A\},\ \{p_A,p_S\} \text{ or } \{p_A,p_M\}$$

or if:

$$\frac{\partial c\left(\sigma_M;P_S\right)}{\partial \sigma_M} \equiv \frac{\partial c\left(\sigma_M;\varnothing\right)}{\partial \sigma_M} \text{ and } \frac{\partial g\left(\sigma_A;P_A\right)}{\partial \sigma_A} \equiv \frac{\partial g\left(\sigma_A;\varnothing\right)}{\partial \sigma_A}$$

$$\text{where } P_A=\{p_S\},\ \{p_S,p_M\} \text{ or } \{p_S,p_A\}$$

Definition 2 *Partial Strict Complementarity (PSC) of assets: the assets p_M and p_S are strictly complementary if:*

$$\frac{\partial T_i\left(\mu_A,\mu_S;P_M\right)}{\partial \mu_j} \equiv \frac{\partial T_i\left(\mu_A,\mu_S;\varnothing\right)}{\partial \mu_j} \text{ and } \frac{\partial t\left(\mu_A,\mu_S;P_M\right)}{\partial \mu_j} \equiv \frac{\partial t\left(\mu_A,\mu_S;\varnothing\right)}{\partial \mu_j}$$

$$\text{where } i=S,A,\ j=S,A \text{ and } P_M=\{p_M\} \text{ or } \{p_M,p_A\}$$

or if:

$$\frac{\partial c\left(\sigma_M;P_S\right)}{\partial \sigma_M} \equiv \frac{\partial c\left(\sigma_M;\varnothing\right)}{\partial \sigma_M} \text{ and } \frac{\partial g\left(\sigma_A;P_A\right)}{\partial \sigma_A} \equiv \frac{\partial g\left(\sigma_A;\varnothing\right)}{\partial \sigma_A}$$

$$\text{where } P_A=\{p_S\} \text{ or } \{p_S,p_A\}$$

The assets p_M and p_A are strictly complementary if:

$$\frac{\partial T_i\left(\mu_A,\mu_S;P_M\right)}{\partial \mu_j} \equiv \frac{\partial T_i\left(\mu_A,\mu_S;\varnothing\right)}{\partial \mu_j} \text{ and } \frac{\partial t\left(\mu_A,\mu_S;P_M\right)}{\partial \mu_j} \equiv \frac{\partial t\left(\mu_A,\mu_S;\varnothing\right)}{\partial \mu_j}$$

$$\text{where } i=S,A,\ j=S,A \text{ and } P_M=\{p_M\} \text{ or } \{p_M,p_S\}$$

or if:

$$\frac{\partial K_i\left(\alpha_M,\alpha_S;P_A\right)}{\partial \alpha_j} \equiv \frac{\partial K_i\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial \alpha_j} \text{ and } \frac{\partial k\left(\alpha_M,\alpha_S;P_A\right)}{\partial \alpha_j} \equiv \frac{\partial k\left(\alpha_M,\alpha_S;\varnothing\right)}{\partial \alpha_j}$$

$$\text{where } i=M,S,\ j=M,S \text{ and } P_A=p_A \text{ or } \{p_A,p_S\}$$

The assets p_S *and* p_A *are strictly complementary if:*

$$\frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \equiv \frac{\partial c(\sigma_M; \varnothing)}{\partial \sigma_M} \text{ and } \frac{\partial g(\sigma_A; P_A)}{\partial \sigma_A} \equiv \frac{\partial g(\sigma_A; \varnothing)}{\partial \sigma_A}$$
$$\text{where } P_A = \{p_S\} \text{ or } \{p_S, p_M\}$$

or if:

$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial K_i(\alpha_M, \alpha_S; \varnothing)}{\partial \alpha_j} \text{ and } \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial k(\alpha_M, \alpha_S; \varnothing)}{\partial \alpha_j}$$
$$\text{where } i = M, S,\ j = M, S \text{ and } P_A = \{p_A\} \text{ or } \{p_A, p_M\}$$

Definition 3 *Full Independence (FI) of assets: the assets* p_M, p_S *and* p_A *are independent if:*

$$\frac{\partial T_i(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial T_i(\mu_A, \mu_S; p_M)}{\partial \mu_j} \text{ and } \frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial t(\mu_A, \mu_S; p_M)}{\partial \mu_j}$$
$$\text{where } i = S, A,\ j = S, A \text{ and } P_M = \{p_M, p_S, p_A\},\ \{p_M, p_S\} \text{ or } \{p_M, p_A\}$$

and

$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial K_i(\alpha_M, \alpha_S; p_A)}{\partial \alpha_j} \text{ and } \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial k(\alpha_M, \alpha_S; p_A)}{\partial \alpha_j}$$
$$\text{where } i = M, S,\ j = M, S \text{ and } P_A = \{p_M, p_S, p_A\},\ \{p_A, p_M\} \text{ or } \{p_A, p_S\}$$

and

$$\frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \equiv \frac{\partial c(\sigma_M; p_S)}{\partial \sigma_M} \text{ and } \frac{\partial g(\sigma_A; P_A)}{\partial \sigma_A} \equiv \frac{\partial g(\sigma_A; p_S)}{\partial \sigma_A}$$
$$\text{where } P_A = \{p_S, p_M, p_A\} \ \{p_S, p_M\} \text{ or } \{p_S, p_A\}$$

Definition 4 *Partial Independence (PI) of assets: the assets* p_M *and* p_S *are independent if:*

$$\frac{\partial T_i(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial T_i(\mu_A, \mu_S; p_M)}{\partial \mu_j} \text{ and } \frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial t(\mu_A, \mu_S; p_M)}{\partial \mu_j}$$
$$\text{where } i = S, A,\ j = S, A \text{ and } P_M = \{p_M, p_S\}$$

and

$$\frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \equiv \frac{\partial c(\sigma_M; p_S)}{\partial \sigma_M} \text{ and } \frac{\partial g(\sigma_A; P_A)}{\partial \sigma_A} \equiv \frac{\partial g(\sigma_A; p_S)}{\partial \sigma_A}$$
$$\text{where } P_A = \{p_M, p_S\}$$

The assets p_M *and* p_A *are independent if:*

$$\frac{\partial T_i(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial T_i(\mu_A, \mu_S; p_M)}{\partial \mu_j} \text{ and } \frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial t(\mu_A, \mu_S; p_M)}{\partial \mu_j}$$
$$\text{where } i = S, A,\ j = S, A \text{ and } P_M = \{p_M, p_A\}$$

and

$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial K_i(\alpha_M, \alpha_S; p_A)}{\partial \alpha_j} \text{ and } \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial k(\alpha_M, \alpha_S; p_A)}{\partial \alpha_j}$$

$$\text{where } i = M, S,\ j = M, S \text{ and } P_A = \{p_A, p_M\}$$

The assets p_S and p_A are independent if:

$$\frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \equiv \frac{\partial c(\sigma_M; p_S)}{\partial \sigma_M} \text{ and } \frac{\partial g(\sigma_A; P_A)}{\partial \sigma_A} \equiv \frac{\partial g(\sigma_A; p_S)}{\partial \sigma_A}$$

$$\text{where } P_A = \{p_S, p_A\}$$

and

$$\frac{\partial K_i(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial K_i(\alpha_M, \alpha_S; p_A)}{\partial \alpha_j} \text{ and } \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial k(\alpha_M, \alpha_S; p_A)}{\partial \alpha_j}$$

$$\text{where } i = M, S,\ j = M, S \text{ and } P_A = \{p_A, p_S\}$$

Definition 5 *Essential Human Capital: M's human capital is essential if:*

$$\frac{\partial K_S(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial K_S(\alpha_M, \alpha_S; \varnothing)}{\partial \alpha_j} \text{ and } \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial k(\alpha_M, \alpha_S; \varnothing)}{\partial \alpha_j}$$

$$\text{where } j = M, S \text{ and } P_A = \{p_A, p_S, p_M\},\ \{p_A, p_S\},\ \{p_A, p_M\} \text{ or } \{p_A\}$$

and:

$$\frac{\partial c(\sigma_M; P_S)}{\partial \sigma_M} \equiv \frac{\partial c(\sigma_M; \varnothing)}{\partial \sigma_M}$$

$$\text{where } P_S = \{p_A, p_S, p_M\},\ \{p_S, p_M\}, \{p_S, p_A\} \text{ or } \{p_S\}$$

S's human capital is essential if:

$$\frac{\partial T_A(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial T_A(\mu_A, \mu_S; \varnothing)}{\partial \mu_j} \text{ and } \frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial t(\mu_A, \mu_S; \varnothing)}{\partial \mu_j}$$

$$\text{where } j = S, A \text{ and } P_M = \{p_A, p_S, p_M\},\ \{p_M, p_S\},\ \{p_M, p_A\} \text{ or } \{p_M\}$$

and

$$\frac{\partial K_M(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial K_M(\alpha_M, \alpha_S; \varnothing)}{\partial \alpha_j} \text{ and } \frac{\partial k(\alpha_M, \alpha_S; P_A)}{\partial \alpha_j} \equiv \frac{\partial k(\alpha_M, \alpha_S; \varnothing)}{\partial \alpha_j}$$

$$\text{where } j = M, S \text{ and } P_A = \{p_A, p_S, p_M\},\ \{p_A, p_S\},\ \{p_A, p_M\} \text{ or } \{p_A\}$$

A's human capital is essential if:

$$\frac{\partial T_S(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial T_S(\mu_A, \mu_S; \varnothing)}{\partial \mu_j} \text{ and } \frac{\partial t(\mu_A, \mu_S; P_M)}{\partial \mu_j} \equiv \frac{\partial t(\mu_A, \mu_S; \varnothing)}{\partial \mu_j}$$

$$\text{where } j = S, A \text{ and } P_M = \{p_A, p_S, p_M\},\ \{p_M, p_S\},\ \{p_M, p_A\} \text{ or} \{p_M\}$$

and

$$\frac{\partial g(\sigma_A; P_S)}{\partial \sigma_A} \equiv \frac{\partial g(\sigma_A; \varnothing)}{\partial \sigma_A}$$

$$\text{where } P_A = \{p_A, p_S, p_M\},\ \{p_S, p_M\}, \{p_S, p_A\} \text{ or } \{p_S\}$$