Rate-Compatible Spatially-Coupled LDPC Code Ensembles With Nearly-Regular Degree Distributions

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Rate-Compatible Spatially-Coupled LDPC Code Ensembles With Nearly-Regular Degree Distributions

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***Abstract*—Spatially-coupled regular LDPC code ensembles have outstanding performance with belief propagation decoding and can perform arbitrarily close to the Shannon limit without requiring irregular graph structures. In this paper, we are concerned with the performance and complexity of spatially-coupled ensembles with a rate-compatibility constraint. Spatially-coupled regular ensembles that support rate-compatibility through extension have been proposed before and show very good performance if the node degrees and the coupling width are chosen appropriately. But due to the strict constraint of maintaining a regular degree, there exist certain unfavorable rates that exhibit bad performance and high decoding complexity. We introduce an altered LDPC ensemble construction that changes the evolution of degrees over subsequent incremental redundancy steps in such a way, that the degrees can be kept low to achieve outstanding performance close to Shannon limit for all rates. These ensembles always outperform their regular counterparts at small coupling width.**

I. Introduction

Low-density parity-check (LDPC) codes are well-known for their good performance with the use of a belief-propagation (BP) decoder. Although the BP decoder is sub-optimal, its advantage over the optimal MAP decoder lies in the great reduction of decoding complexity. Asymptotically, in terms of iterative decoding thresholds, the sub-optimaliy of the BP decoder was overcome by LDPC convolutional codes [1] and the remarkable phenomenon of threshold saturation. Due to the introduced coupling of regular LDPC ensembles, the BP threshold of the coupled ensemble converges to the MAP threshold of the underlying uncoupled ensemble. This threshold improvement was observed first in [2], [3]. Proofs for this behavior on the binary erasure channel (BEC) [4] and the additive white Gaussian noise (AWGN) channel [5] emphasize the universality of this underlying performance improvement through coupling.

Rate-compatible spatially-coupled regular ensembles were first introduced in [6]. The proposed ensemble construction can in principle achieve every rational rate. Because of threshold saturation, the iterative decoding performance can be pushed arbitrarily close to the Shannon limit when the node degrees and the coupling width are chosen sufficiently large.

On the other hand, due to the regularity constraints on the overall node degrees, certain rates are only achievable with high degrees. While this is unproblematic in the infinite coupling width regime, at finite and small coupling width the performance is significantly reduced. Additionally, the complexity of ensembles with high variable node degree is higher. The performance loss can only be overcome by increasing the coupling width. This circumstance was already pointed out in [7] for regular spatially-coupled LDPC codes and an ensemble with slight irregularity was proposed that yielded performance closer to the Shannon limit together with low complexity. Although not proven for irregular spatially-coupled LDPC codes, the threshold saturation phenomenon is also observed in this case. It is worth to point out that the irregularity in this construction was not introduced for improving the iterative decoding thresholds but for increasing the flexibility in achieving different rates while maintaining low variable node degrees.

In this paper, we relax the degree regularity constraints of a rate-compatible spatially-coupled LDPC code ensemble as introduced in [6] to counteract the severe variable node degree increase of the regular construction. We use multi-edge type (MET) LDPC code ensembles to model regular rate-compatible LDPC code ensembles as well as a new proposed rate-compatible LDPC code ensemble with an altered degree evolution. In our ensemble construction the check node degree is not kept constant but decreased with every incremental redundancy (IR) step to allow for a sub-linear increase of variable node degrees. With this change, the decoding complexity can be reduced and a smaller coupling width for similar performance can also be achieved.

While the regular construction in [6] is based on graph extension, some rate-compatible regular spatially-coupled LDPC codes by means of puncturing were investigated in [8]. An asymptotic analysis of the effect of random puncturing is given in [9]. Rateless spatially-coupled ensembles, on the other hand, were introduced and analyzed in [10].

The paper is structured as follows. Section II introduces the MET model for rate-compatible spatially-coupled LDPC code ensembles. Section III focuses on the problem of increasing variable node degrees for regular rate-compatible code ensembles and discusses the complexity. The focus of Section IV is on the construction of ensembles with relaxed regularity constraints. The complexity of these ensembles is analyzed. In Section V, the iterative decoding thresholds for the newly constructed ensembles on the BEC as well as the AWGN channel are discussed and compared against their regular counterparts. Finally, Section VI concludes the paper.

II. Rate-Compatible Multi-Edge Type LDPC Code Ensembles

In this paper we consider MET ensembles as introduced in [11] which consist of m_e different edge types. A *degree type* of a check node is a vector of integers of length m_e. The i-th entry of this vector represents the number of sockets that are connected to edge type i. The degree type of a variable node consists of two parts. A length m_e vector fulfills the same purpose as on the check degree side. Additionally, variable nodes are related to the respective channel on which the codeword is transmitted. As a slight simplification we assume the transmission of all variable nodes over the same channel. Therefore, we define only one *received edge-type* contrary to [11]. The representation of the graph structure is done via a multinomial representation. We assume $\underline{d} = (d_1, \ldots, d_{m_e})$ to be a MET degree and let $\underline{x} = (x_1, \ldots, x_{m_e})$. We use $\underline{x}^{\underline{d}}$ to denote $\prod_{i=1}^{m_e} x_i^{d_i}$. The received edge-type is represented by r. With these definitions a MET ensemble is defined by the multinomial $\nu(r, \underline{x})$ for the variable node side and $\mu(\underline{x})$ for the check node side as

$$\nu(r, \underline{x}) = \sum \nu_{\underline{d}} r \underline{x}^{\underline{d}} \tag{1}$$

$$\mu(\underline{x}) = \sum \mu_{\underline{d}} \underline{x}^{\underline{d}} \tag{2}$$

with $\nu_{\underline{d}}$ and $\mu_{\underline{d}}$ being nonnegative reals. Assuming a block length n, the quantity $n\nu_{\underline{d}}$ represents the number of variable nodes of degree type $\underline{d}$. Similarly, $n\mu_{\underline{d}}$ is the number of check nodes of degree type $\underline{d}$. Every variable (check) node is therefore member of a specific variable (check) node degree type that we with slight abuse of notation also refer to as $\underline{d}_V$ ($\underline{d}_C$). We additionally define $\nu_{x_i}(r, \underline{x}) = \frac{d\nu(r,\underline{x})}{dx_i}$ and $\mu_{x_i}(\underline{x}) = \frac{d\mu(\underline{x})}{dx_i}$ and with this the multinomial representation from an edge perspective as

$$\underline{\lambda}(r, \underline{x}) = \left(\frac{\nu_{x_1}(r, \underline{x})}{\nu_{x_1}(1, \underline{1})}, \frac{\nu_{x_2}(r, \underline{x})}{\nu_{x_2}(1, \underline{1})}, \ldots, \frac{\nu_{x_{m_e}}(r, \underline{x})}{\nu_{x_{m_e}}(1, \underline{1})} \right) \tag{3}$$

$$\underline{\rho}(\underline{x}) = \left(\frac{\mu_{x_1}(\underline{x})}{\mu_{x_1}(\underline{1})}, \frac{\mu_{x_2}(\underline{x})}{\mu_{x_2}(\underline{1})}, \ldots, \frac{\mu_{x_{m_e}}(\underline{x})}{\mu_{x_{m_e}}(\underline{1})} \right). \tag{4}$$

The design rate of a MET ensemble is defined as

$$R = \nu(1, \underline{1}) - \mu(\underline{1}). \tag{5}$$

In our considerations of rate-compatible LDPC code ensembles we focus on rate-compatibility produced by extension as it was considered in [6]. A precode is subsequently extended by adding variable and check nodes to produce new IR bits and therefore lower the rate. The MET LDPC ensemble that models this structure is constructed as follows. The IR chain is started by a base code of given degree profile that is the code with the highest rate in the IR family. We therefore set a variable node and a check node type that are connected by an edge type $x_{1,\mathrm{p}}$ which can be referred to as the parity edge-type. At each IR step a, we now add an additional variable and check node type that will be connected with an additional edge type $x_{a,\mathrm{p}}$. To add the connectivity between different IR steps, at every IR step $a = 2, \ldots, \alpha$ we add an additional edge type $x_{a,\mathrm{b}}$ that connects the check node type at IR step a with all variable node types at previous IR steps and can be referred to as the back-connection edge type. Important to note is that to ensure rate-compatibility, back-connections from variable node

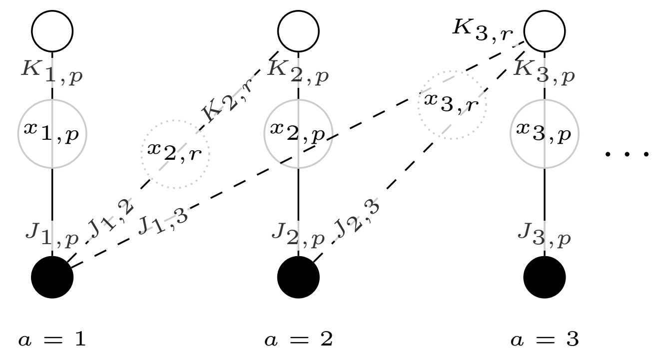


Fig. 1: Graphical representation of a rate-compatible MET LDPC code ensemble with edge type variables (denoted by circles) and check and variable node type degrees attached to the edge types.

types at IR step a to previous check node types are not allowed. The graphical representation of this subsequently constructed IR chain is shown in Fig. 1. The corresponding multinomials for a rate-compatible MET LDPC code ensemble are given in the following definition.

Definition II.1 (($\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x})$) Rate-compatible MET ensemble). A rate-compatible MET ensemble with rate R_a at IR step a is defined by the variable node degree multinomial

$$\begin{aligned} \nu^{(a)}(r, \underline{x}) =& \nu_1 r x_{1,\mathrm{p}}^{J_{1,\mathrm{p}}} \prod_{i=2}^{a} x_{i,\mathrm{b}}^{J_{1,i}} + \\ &+ r \sum_{j=2}^{a-1} \nu_j x_{j,\mathrm{p}}^{J_{j,\mathrm{p}}} \prod_{k=j+1}^{a} x_{k,\mathrm{b}}^{J_{j,\mathrm{b}}} + \nu_a r x_{a,\mathrm{p}}^{J_{a,\mathrm{p}}} \end{aligned} \tag{6}$$

and the check degree multinomial

$$\mu^{(a)}(\underline{x}) = \mu_1 x_{1,\mathrm{p}}^{K_{1,\mathrm{p}}} + \sum_{i=2}^{a} \mu_i x_{i,\mathrm{b}}^{K_{i,\mathrm{b}}} x_{i,\mathrm{p}}^{K_{i,\mathrm{p}}} \tag{7}$$

with $\underline{x} = \{x_{1,\mathrm{p}}, \ldots, x_{a,\mathrm{p}}, x_{1,\mathrm{b}}, \ldots, x_{a,\mathrm{b}}\}$ and rate $R_a = \nu^{(a)}(1, \underline{1}) - \mu^{(a)}(\underline{1})$.

The average variable and check node degrees are denoted by $\bar{J}$ and $\bar{K}$.

So far, we considered an uncoupled rate-compatible MET LDPC code ensemble. A spatially-coupled $(\nu(\underline{x}), \mu(\underline{x}), L, w)$ rate-compatible MET LDPC code ensemble can be described as follows. We assume a sequence of L time instants indexed by a time index $t \in [0, L-1]$ with L as the *coupling length*. At each time instant t, a $(\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x}))$ LDPC code ensemble with n_t variable nodes (code bits) and $(1 - R_a)n_t = m_t$ check nodes is located. For the remainder of the paper, we assume n_t to be constant for $t \in [0, L-1]$ as well as m_t being constant for the complete sequence. Currently, the sequence of codes (or codewords) is a sequence of $(\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x}))$ block codes that do not interact with each other. As the idea of spatial coupling is to interconnect block codes over different time instants, it remains to define how interconnections between code ensembles at different time instants are chosen, i.e., how edges are distributed over time instants. Edges that connect variable node type $\underline{d}_V$ with check node type $\underline{d}_C$ in the uncoupled setting will independently and uniformly distribute the connections of variable node type $\underline{d}_V$ at time instant t to the check node types $\underline{d}_C$ in the range $\{t, \dots, t+w-1\}$, were w denotes the *coupling width* and is a measure of the strength of coupling.

The iterative decoding thresholds of $(\nu^{(a)}(r,\underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensembles can be numerically calculated using density evolution (DE) as given in the following definition.

Definition II.2 (DE of a $(\nu(r,\underline{x}), \mu(\underline{x}), L, w)$ spatially-coupled LDPC code ensemble). Given a $(\nu(r,\underline{x}), \mu(\underline{x}), L, w)$ spatially-coupled LDPC code ensemble with coupling width w, coupling length L and associated multinomials $\underline{\lambda}(r,\underline{x})$ and $\underline{\rho}(\underline{x})$ from an edge perspective, the vector $\underline{\mathsf{x}}_t^{(l)} = (\mathsf{x}_{t,1}^{(l)}, \mathsf{x}_{t,2}^{(l)}, \dots, \mathsf{x}_{t,m_e}^{(l)})$ represents the densities emitted from variable nodes at time instant t where $\mathsf{x}_{t,i}^{(l)}$ denotes the density emitted from variable nodes of edge type i at time instant t in iteration l. The recursion for $\underline{\mathsf{x}}_{t,i}^{(l)}$ is then given as

$$\underline{\mathsf{x}}_{t,i}^{(l+1)} = \underline{\lambda}\left(\mathsf{c}, \frac{1}{w}\sum_{j=0}^{w-1} \underline{\rho}\left(\frac{1}{w}\sum_{k=0}^{w-1} \underline{\mathsf{x}}_{t-k+j,i}^{(l)}\right)\right). \quad (8)$$

where c is the density emitted by the respective channel and computations with densities on variable and check node side are carried out according to [11].

III. Regular Spatially-Coupled Rate-Compatible LDPC Code Ensembles

To construct a rate-compatible $(\nu^{(a)}(r,\underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensemble with overall regular degree distribution, the variable and check node degrees have to be chosen appropriately. Due to the rate-compatible structure a single check node type of the precode exists that does not change its degree throughout subsequent IR steps as no back-connections are allowed to previous check node types. The degree of this check node type dictates the check degree of the complete regular $(\nu^{(a)}(r,\underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensemble to be K. To lower the rate for subsequent IR steps, the variable node degree J has to be increased linearly with every IR step. The choice of K is crucial for the granularity of the rates that can be achieved with the ensemble as well as the variable node degree, needed to produce these rates. We exemplarily construct two regular rate-compatible LDPC code ensemble families with different check degree to emphasize this behavior. We choose $K = 10$ and $K = 25$ as the constant check node degree for the ensembles. The former one with $K = 10$ resembles the ensemble from [6]. The according degrees for the ensemble with $K = 10$ are shown in Table I. Note that in the regular case, J (K) is equal to the average variable (check) degree $\bar{J}$ $(\bar{K})$. As the decoding of LDPC codes is predominantly done with BP decoders, we seek for a simple decoding complexity metric. The decoding complexity is determined by the number of edges in the graph which stems from the fact that BP update operations have to be done per edge. The complexity in terms of average operations per bit is then given as follows.

Definition III.1 (Decoding complexity of an LDPC code ensemble). The complexity $\mathfrak{C}$ of an LDPC code ensemble with average variable node degree $\bar{J}$ and rate R is defined as

$$\mathfrak{C} = \frac{\bar{J}}{R} \quad (9)$$

a	$J_{a,\mathrm{p}}$	$J_{k,a}$	J	$K_{a,\mathrm{b}}$	$K_{a,\mathrm{p}}$	K	R_a
1	3	-	3	-	10	10	0.7
2	4	1	4	6	4	10	0.6
3	5	1,1	5	5	5	10	0.5
4	6	1,1,1	6	4	6	10	0.4
5	7	1,1,1,1	7	3	7	10	0.3
6	8	1,1,1,1,1	8	2	8	10	0.2
7	9	1,1,1,1,1,1	9	1	9	10	0.1

TABLE I: Degree evolution of a regular $(\nu^{(a)}(r,\underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code with $K = 10$ and $L \to \infty$. Note that for $J_{k,a}$ always all degrees for the back-connections $k = 1, \dots, a$ are shown. The corresponding graph is shown in Fig. 1.

a	$J_{a,\mathrm{p}}$	$J_{k,a}$	J	$K_{a,\mathrm{b}}$	$K_{a,\mathrm{p}}$	K	R_a
1	3	-	3	-	25	25	0.88
2	3	1	3.955	21	3	24.719	0.84
3	3	1, 1	4.862	20	3	20.258	0.76
4	3	1, 1, 1	5.718	19	3	20.421	0.72
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
21	3	$1, \dots, 1$	7.382	2	3	8.024	0.08
22	3	$1, \dots, 1$	5.691	1	3	5.928	0.04

TABLE II: Degree evolution of a relaxed $(\nu^{(a)}(r,\underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code with $L \to \infty$. Note that for $J_{k,a}$ always all degrees for the back-connections $k = 1, \dots, a$ are shown.

The complexity of the regular $(\nu^{(a)}(r,\underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensembles with $K = 10$ and $K = 25$ is shown in Fig. 2. The complexity of the ensemble with $K = 10$ is smaller than for $K = 25$. This can be explained by the fact that, to achieve a similar rate, the variable node degree J has to be higher when K is higher. This produces higher complexity. For lower K, the complexity stays small but the rates that can be achieved are restricted. An increased check node degree increases the complexity but the granularity of rates is also increased. The iterative decoding thresholds for these ensembles are shown in Fig. 3.

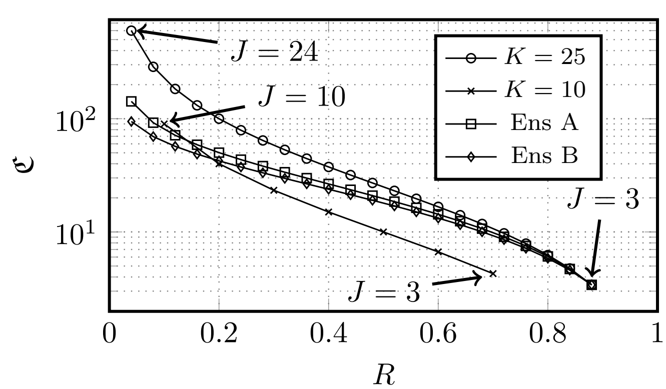


Fig. 2: Decoding complexity of regular $(\nu^{(a)}(r,\underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensembles with $K = 10$ and $K = 25$.

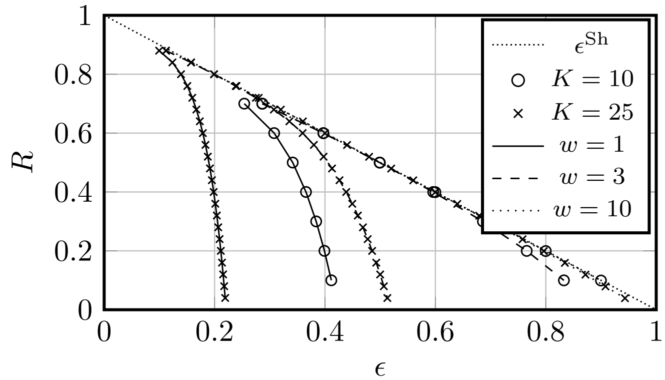


Fig. 3: Iterative decoding thresholds of regular $(\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensembles with $L \to \infty$ and $w = \{1, 3, 10\}$ on the BEC. Note that $w = 1$ refers to the uncoupled case.

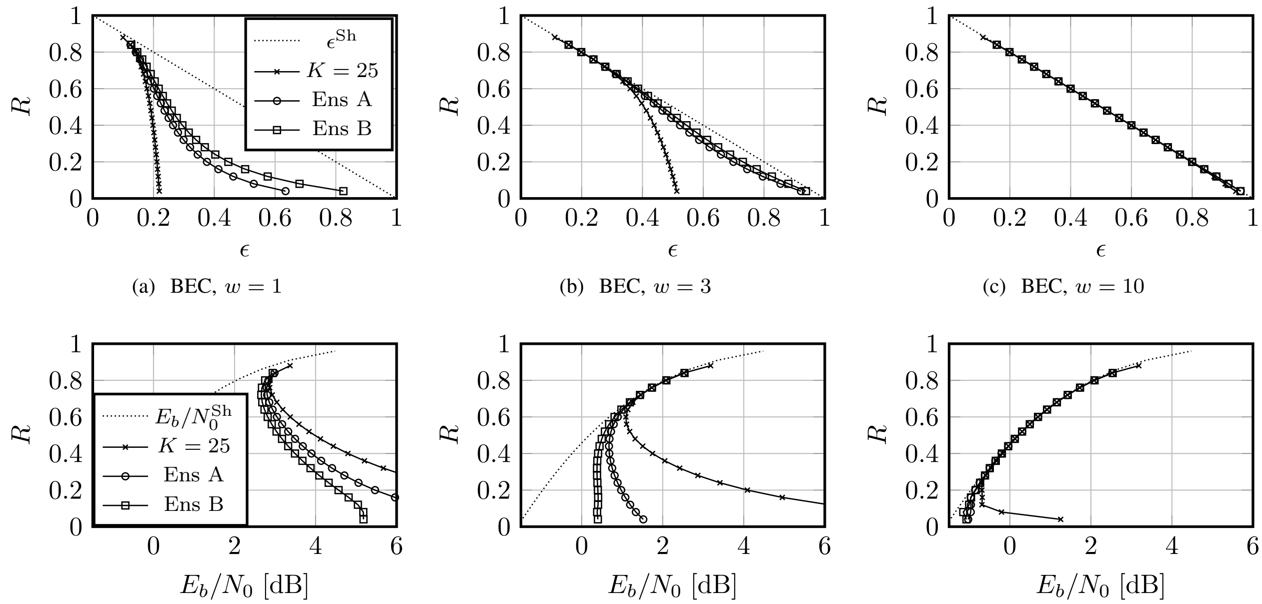
In the uncoupled case ($w = 1$), the performance is far away from the Shannon limit although the ensemble with $K = 10$ is closer to the limit. When coupling is introduced ($w = 3$), the performance gets pushed closer to the Shannon limit for both ensembles due to the threshold improvement of spatial coupling. The ensemble with $K = 10$ is already very close to the Shannon limit for all rates while the ensemble with $K = 25$ still shows a significant gap that can only be compensated when the coupling width is increased even more ($w = 10$). Ensembles with higher K generally perform worse because to achieve lower rates, the variable node degree has to be increased accordingly. The performance harm of high J can only be overcome by increased coupling width. We therefore seek for a construction that allows to keep the variable node degree small over all IR steps in the family of $(\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensembles. With the generic ensemble definition from Definition II.1 the remaining question is how to find suitable values for $J_{i,\text{p}}$, $J_{i,k}$, $K_{i,\text{p}}$ and $K_{i,\text{r}}$ to get the desired rates. Such a construction in form of altered degree evolutions will be introduced in the next section.

IV. Ensembles with Relaxed Degree Evolution

The performance gap of regular $(\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensembles stems from the fact that the variable node degree has to be increased to lower the rate with subsequent IR steps. In the following we relax the condition of overall regularity to adjust the degree evolutions on check and variable node side. To lower the rate of $(\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensembles, the ratio of average degrees $\bar{J}/\bar{K}$ has to be increased. In the regular setting this was accomplished by the increase of $J = \bar{J}$ while $K = \bar{K}$ remains constant. For our construction we change the degree evolution to lower the rate by decreasing $\bar{K}$ while trying to hold $\bar{J}$ small. In the regular setting, the check degrees of extension nodes had to be chosen to add up to the check degree of the precode ($K_{a,\text{p}} + K_{a,\text{b}} = K_{1,\text{p}}, \ \forall a$). This constraint formerly also dictated the choice of the newly appended variable node degrees $J_{a,\text{p}}$ to be $J_{a,\text{p}} = K_{a,\text{p}}$ for all $a > 1$. We change the assignment of degrees in the following way. The precode remains an arbitrarily chosen regular LDPC code of variable node degree $J_{1,\text{p}}$ and check node degree $K_{1,\text{p}}$. The variable node degree $J_{a,\text{p}}$ of extension nodes is set arbitrarily small to achieve a minimized growth of average variable node degree. To hold the condition to only produce IR, we restrict $J_{a,\text{p}} = K_{a,\text{p}}$ for all $a > 1$ similar to the regular case. Then we can choose $K_{1,\text{b}}$ in such a way to reduce the rate with every IR step. Practically, this means to subsequently lower $K_{1,\text{b}}$ at each IR step a. An exemplary assignment of degrees to form a relaxed $(\nu^{(a)}(r, \underline{x}), \mu^{(a)}(\underline{x}), L, w)$ LDPC code ensemble is shown in Table II and we refer to this ensemble as *Relaxed Ensemble A*. Additionally to this ensemble, we construct a second relaxed ensemble which inherits the same degree assignment except for an even further reduced variable node degree $J_{a,\text{p}} = 1$ at the extension variable nodes which we refer to as *Relaxed Ensemble B*. The complexities of the relaxed ensembles are additionally shown in Fig. 2. Clearly for the precode and high rates, the complexity is almost the same for regular as well as relaxed ensemble A and B but already at a rate around 0.7, the relaxed ensembles show lower complexity compared to the regular ones. For lower rates, the significant complexity drop is more severe. The regular ensemble with $\bar{K} = 10$ is also shown and exhibits the smallest complexity for rates down to $R = 0.2$. The relaxed ensemble B shows always slightly lower complexity compared to ensemble A as a smaller extension variable node degree of $J_{a,\text{p}} = 1$ keeps $\bar{J}$ even smaller. The reduction of average variable node degree overall IR steps showed lower complexity but how this change affects the iterative decoding performance will be discussed in the next section.

V. Numerical Results

The iterative decoding thresholds for the BEC as well as the AWGN channel for relaxed Ensemble A and B are shown in Fig. 4. For the uncoupled case with $w = 1$, the performance is far away from the Shannon limit for all ensembles at lower rates, although the relaxed ensembles perform significantly better for these rates. This is due to the decreased average variable node degree for lower rates. Even Ensemble B is performing better as Ensemble A due to the further decreased variable node degrees of the parity bits at each IR step. Results for the AWGN channel reflect the same behavior as for the BEC. For increased coupling width $w = 3$ the performance generally gets closer to the Shannon limit for all ensembles but the gaps are significantly different. While Ensemble B is showing a gap of $\Delta E_b/N_0 \approx 2.2$dB at a rate $R = 0.2$, the regular case has a gap of $\Delta E_b/N_0 \approx 5.5$dB. For an increased coupling of $w = 10$ the performance starts to saturate very close to the Shannon limit for all considered ensembles. On the BEC, the performance is hardly distinguishable from the ultimate limit. On the AWGN channel only minor variations at very low rates can be determined. The relaxed ensembles do outperform the original regular rate-compatible ensembles for all considered coupling width at all rates. Additionally, the decoder complexity is significantly reduced especially for lower rates. A remaining gap to Shannon limit for the relaxed ensembles at $w = 3$ is still visible and it remains to push the performance closer to the limit at small coupling widths. Raptor-like code ensembles [12] are a promising approach in this direction and we consider trade-offs between these constructions as an interesting topic for further research.



(a) BEC, $w = 1$ (b) BEC, $w = 3$ (c) BEC, $w = 10$

(d) AWGN, $w = 1$ (e) AWGN, $w = 3$ (f) AWGN, $w = 10$

Fig. 4: BP thresholds for the relaxed rate-compatible LDPCC code ensembles for $L \to \infty$ on BEC and BIAWGN channel. Note that $w = 1$ refers to the uncoupled case.

VI. Conclusion

In this paper we investigate the performance and complexity of spatially-coupled rate-compatible MET LDPC code ensembles. Code ensembles with an overall regular degree distribution are considered as starting point. Due to threshold saturation, these code ensembles have been shown to achieve the capacity if coupling width and variable node degree are chosen sufficiently large. Our observation is that, if the coupling width is restricted to be finite and small, the situation changes such that smaller degrees are favorable for decoding thresholds close to the Shannon limit. In the construction of regular rate-compatible code ensembles the high variable node degrees inevitable occur at unfavorable rates due to the constraint of an overall degree regularity. In order to circumvent this problem, we introduce a new code ensemble with a relaxed degree evolution that is only slightly irregular. With this approach, variable node degrees can be kept lower to push performance closer to the Shannon limit at small coupling width. Additionally, due to minimized variable node degree the decoding complexity is decreased as well.

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