A conceptual study of OFDM-based multiple access schemes: Part 2 - Channel estimation in the uplink

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A Conceptual Study of OFDM-based Multiple Access Schemes

Part 2: Channel Estimation in the Uplink

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Abstract: **This paper deals with the design of channel estimators in the uplink of an OFDM-based multi-user system. First, we propose a user allocation scheme that is based on the aim to use the correlation of the radio channel for channel estimation purposes. Further, we present the linear MMSE channel estimator based on a few pilot tones. We present an error analysis that proves valuable in the evaluation of linear channel estimators under mismatch. Finally, we present some results on the design of linear channel estimators and pilot patterns, showing by means of examples the importance of careful pilot positioning.**

I. INTRODUCTION

The OFDM technique [1][2] has proved its potential in wired systems and in broadcast low-rate wireless systems employing differential modulation schemes [3][4]. Coherent OFDM transmission, using multi-amplitude modulation schemes [5], requires estimation and tracking of the fading channel.

The aim of this paper is to evaluate some channel estimation issues in multi-user wireless communication systems based on OFDM. In the down-link of such a system all users can use pilot symbols transmitted by the base station on all sub-carriers and the channel estimation resembles the broadcast case. In the up-link, however, the base station needs to track one channel for each user and can only exploit the pilot symbols transmitted by this particular user. The design of such a system involves two important steps. First, the allocation of sub-carriers to the users is crucial with regard to the channel estimation. Second, both the pattern and amount of pilots is important. This poses some extra challenges, which we address in this paper.

The OFDM modulation technique allows a channel estimator to use both the time and the frequency correlation between the channel attenuations. In [6] the time and frequency correlations have been used separately – a combined scheme using two separate FIR-Wiener-filters, one in the frequency direction and the other in the time direction.

In this paper, we present and analyse a class of block-oriented channel estimators in the up-link of an OFDM system. This class of linear estimators uses pilot symbols transmitted by the user to smooth the channel estimate.

After presenting our system model in Section II, we outline the access scheme and its relation to channel estimation in Section III. In Section IV we derive the channel estimator structure and its corresponding mean-squared error (MSE). These MSE expressions are then used in the BER evaluation of Section V, where we show the importance of careful pilot positioning. In Section VI we briefly discuss diversity and other aspects on system design. Suggestions for further work and concluding remarks appear in Section VII.

II. SYSTEM MODEL

Figure 1 illustrates the baseband OFDM system model we will use in the sequel. We consider the transmission of complex numbers x_k, taken from some signal constellation. Specifically, we will concentrate on 4-PSK.

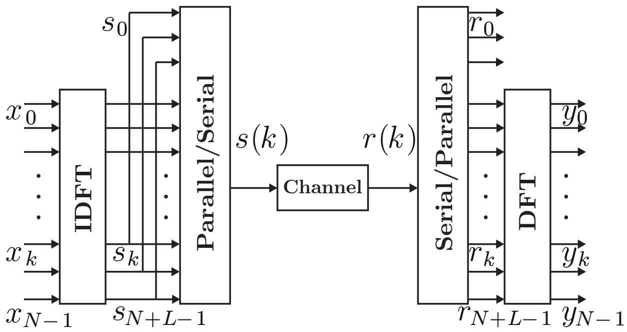

Figure 1: Baseband model of an OFDM system.

The data x_k are modulated on N sub-carriers by an inverse discrete Fourier transform (IDFT) and the last L samples are copied and put as a preamble (cyclic prefix) [7] to form the OFDM symbol s_k. This data vector is serially transmitted over the channel, whose impulse response is shorter than L samples. The cyclic prefix is removed at the receiver and the signal r_k is demodulated with a discrete Fourier transform (DFT). The insertion of a cyclic prefix avoids ISI and preserves the orthogonality between the tones, resulting in the simple input-output relation

$$y_k = h_k x_k + n_k, \qquad k = 0, \ldots, N-1, \tag{1}$$

where h_k is the channel attenuation at the kth sub-carrier and n_k is additive white Gaussian noise. In spite of the loss of transmission power and bandwidth associated with the prefix, the preservation of the orthogonality in the system and the simple channel equalization generally motivate the use of the cyclic prefix.

III. USER ALLOCATION AND CHANNEL CORRELATION

As a frame-work for the oncoming analysis, we adopt the following scheme: Each user is assigned K simultaneous rectangular transmission symbol blocks, (size n_t symbols and n_f sub-carriers in each block). After the transmission of n_t OFDM symbols, the K transmission blocks are re-located in frequency, similar to a frequency hopping system. Figure 2 shows the relevant parameters in the context of the user allocation structure.

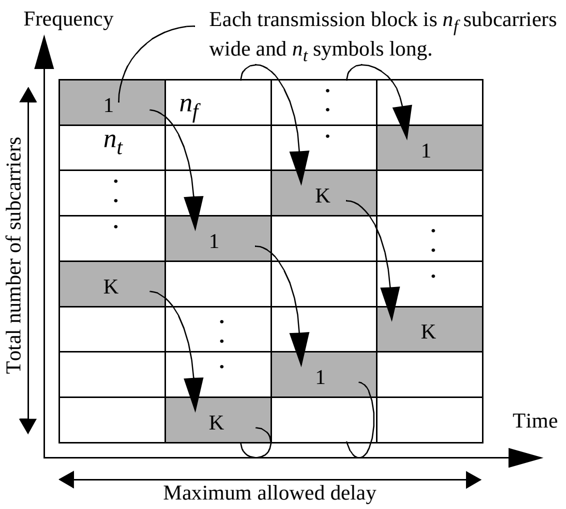

Figure 2: Characteristics of the allocation design in a multi-user OFDM system.

The problem is now to choose the block size, n_f and n_t, so that efficient channel estimation can be performed at the same time as the diversity of the channel is exploited in an efficient manner. A full analysis of the two contradicting aims is beyond the scope of this paper. We will mainly focus on the channel estimation and briefly discuss the diversity problem.

Before we start the channel estimation analysis we introduce two channel types and their corresponding statistical properties. Consider the following two examples, elaborating on two different types of channel correlations.

Example 1: Channel with uniform power-delay profile
Assume that the channel consists of independent impulses, each fading according to Jakes' model [8] with a maximal relative Doppler frequency f_d and that the channel has a uniform power-delay profile of the same length as the cyclic prefix. Further, assume that the time-delay of each impulse is uniformly distributed over the cyclic prefix and independent of its fading amplitude. Then the correlation between channel attenuations separated by Δ_t symbols and Δ_f sub-carriers is

$$r(\Delta_t, \Delta_f) = r_t(\Delta_t) r_f(\Delta_f), \tag{2}$$

where the correlation depending on the time separation is [8]

$$r_t(\Delta_t) = J_0\left(2\pi f_d \Delta_t \frac{N+L}{N}\right) \tag{3}$$

and the correlation depending on the frequency separation is

$$r_f(\Delta_f) = \begin{cases} 1 & \text{if } \Delta_f = 0 \\ \dfrac{1 - e^{-j2\pi L \frac{\Delta_f}{N}}}{j2\pi L \dfrac{\Delta_f}{N}} & \text{if } \Delta_f \neq 0 \end{cases}. \tag{4}$$

Figure 3 displays this time-frequency correlation for N=1024, L=128, and f_d=2%.

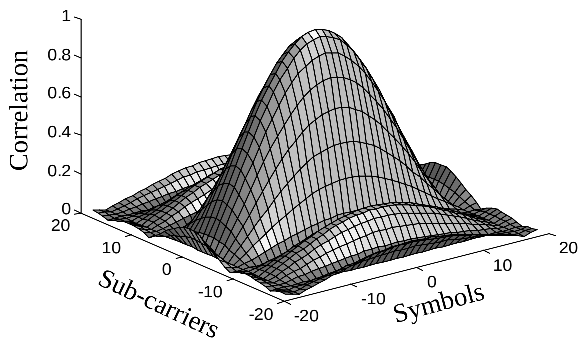

Figure 3: Time-frequency correlation (magnitude) for a uniform channel. N=1024, L=128, f_d=2%.

Example 2: Channel with exponentially decaying power-delay profile

We modify the channel from Example 1 so that it has an exponentially decaying power-delay profile. The correla-

tion between channel attenuations separated by Δ_t *symbols and* Δ_f *sub-carriers is given by (2) where* $r_t(\Delta_t)$ *is the same as in the previous example and*

$$r_f(\Delta_f) = \frac{1 - e^{-L\left(\frac{1}{\tau_{rms}} + j2\pi\frac{\Delta_f}{N}\right)}}{\left(\frac{1}{\tau_{rms}} + j2\pi\frac{\Delta_f}{N}\right)\left(1 - e^{-\frac{L}{\tau_{rms}}}\right)} . \quad (5)$$

The parameter τ_{rms} *in (5) determines the decay of the power-delay profile. Figure 4 displays this time-frequency correlation for* N=1024, L=128, f_d=1%, τ_{rms} =16. *Note that the Doppler frequency is lower than in Example 1.*

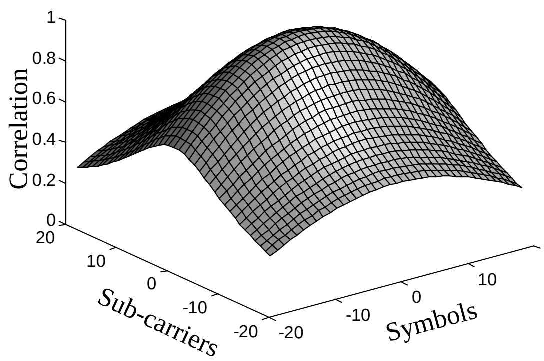

Figure 4: Time-frequency correlation (magnitude) for a channel with exponentially decaying power-delay profile. N=1024, L=128, f_d=1%, τ_{rms} =16.

The coherence time and coherence bandwidth of the channels are given by the -3dB-crossings of the two factors $r_t(\Delta_t)$ and $r_f(\Delta_f)$, respectively. For the channel of Example 1 these widths become

$$\Delta_{t0} \approx \frac{0.24N}{(N+L)f_d} = 10.7 \text{ and} \quad (6)$$

$$\Delta_{f0} \approx \frac{0.6N}{L} = 4.8 , \quad (7)$$

respectively. The -3dB contour is displayed in Figure 5.

For the sake of channel estimation, we wish to shape each block (size $n_t \times n_f$) such that the correlation between channel attenuations within each block is exploited in a good way. Intuitively, the transmission block should be shaped so that it resembles the shape of the peak of the correlation function as much as possible. Let us express the shape of the channel correlation function by the ratio

$$C \equiv \frac{\Delta_{t0}}{\Delta_{f0}} = \frac{0.4L}{(N+L)f_d} = 0.4\frac{\tau_c}{f_d} , \quad (8)$$

where $\tau_c = L/(N+L)$. Following the intuition, we require that the shape of the transmission block, n_t/n_f, reflects the correlation shape, *i.e*,

$$\frac{n_t}{n_f} \approx C . \quad (9)$$

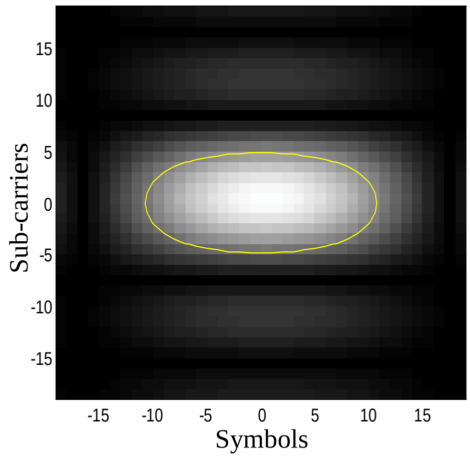

Figure 5: Pseudo colour image of channel time-frequency correlation function (magnitude). Curve shows the -3dB-crossing of the correlation. Uniform channel with N=1024, L=128, f_d=2% (c.f. Figure 3).

Notice that relation (9) only affects the shape of the block. The absolute size of each block and the number of blocks per user are not affected by the relation.

Example 3: Choosing block shape

Consider a system with N=1024 tones and L=128 samples in the cyclic prefix. The channel is assumed to have a uniform delay spread in the interval [0,L] and a maximal relative Doppler frequency f_d *=2%. Thus* $\tau_c = 1/9$ *and, from expression* (8),

$$C = 0.4\frac{1/9}{0.02} \approx 2.2 . \quad (10)$$

Based on this we choose transmission blocks that are about twice as long in the time direction as wide in the frequency direction (measured in the number of sub-carriers and symbols).

The larger the size of the blocks, the better the channel estimation will perform. However, there are other system requirements and components that will affect the block size. In the next two sections we assume that a block size is chosen and focus on the channel estimation. Primarily we will address the estimator structure and some design parameters, such as design correlation and pilot patterns.

IV. CHANNEL ESTIMATION

Consider one block consisting of n_f sub-carriers during n_t OFDM symbols. Assume that P of these $n_f n_t$ symbols are known pilot symbols with unit energy. Finally, assume that the channel covariance matrix R_{hh} is known. In the next section we will see that this last assumption is not critical.

The most straight-forward type of estimator is the least-squares (LS) estimator [9], which minimizes the criterion

$$J(h_P) = (y_P - X_P h_P)^H (y_P - X_P h_P)\,, \tag{11}$$

where subscript P denotes that we address pilot positions. The LS estimate of the channel attenuations in the pilot positions are

$$\hat{h}_{ls} = X_P^{-1} y_P\,, \tag{12}$$

where the matrix X_P consists of known pilot symbols. Since

$$\hat{h}_{ls} = X_P^{-1}(X_P h_P + n_P) = h_P + X_P^{-1} n_P = h_P + \tilde{n}_P\,, \tag{13}$$

where $\hat{n}_P$ is white Gaussian noise, the correlation properties of h_P are transferred to $\hat{h}_{ls}$ and its covariance matrix becomes

$$R_{\hat{h}_{ls}\hat{h}_{ls}} = R_{h_P h_P} + \sigma_n^2 I\,. \tag{14}$$

We now consider the class of linear estimators

$$\hat{h} = W\hat{h}_{ls}\,, \tag{15}$$

estimating all channel attenuations, h, within the block, where W is a weighting matrix. In particular we are interested in the linear estimator that minimizes the mean-squared error. The linear minimum mean-squared error (LMMSE) estimator is given by the weighting matrix [9]

$$W = R_{h\hat{h}_{ls}} R_{\hat{h}_{ls}\hat{h}_{ls}}^{-1}\,, \tag{16}$$

where $R_{h\hat{h}_{ls}}$ denotes the cross-correlation matrix between all attenuations h and the LS estimates of the attenuations at the pilot positions $\hat{h}_{ls}$.

To aid the evaluation of the channel estimators we calculate the performance of the estimator in terms of mean-squared error. The error associated with the linear estimator (15) has covariance matrix

$$K = R_{hh} - R_{h\hat{h}_{ls}} W^H - W R_{h\hat{h}_{ls}} + W R_{\hat{h}_{ls}\hat{h}_{ls}} W^H\,. \tag{17}$$

The mean-squared error of the estimate becomes

$$MSE = Trace(K)\,. \tag{18}$$

Notice that this expression is the mean-squared error for every linear estimator of the channel attenuations, *i.e.*, the form of (17) and (18) is independent of the design of the weighting matrix W. Since the true channel correlation properties usually are not known to the receiver, a mismatch between the design correlation and the true correlation is expected.

In the special case where we employ the true channel correlation (no mismatch) in the design of W expression (17) reduces to

$$K = R_{hh} - W R_{h\hat{h}_{ls}}\,. \tag{19}$$

Even though the mean-squared error is easily calculated by the above formulae, the bottom line evaluation of a communication system is its average bit-error rate (BER). Evaluating a fully coded system with interleaving is beyond the scope of this paper, but we will use the above formulae in conjunction with the analytical expressions from [10] to evaluate the performance in terms of uncoded BER.

V. PERFORMANCE EXAMPLES

In this section we show a few examples on linear channel estimators, based on the previously described concept, and their performance in terms of uncoded 4-PSK bit-error rate (BER). We have chosen to examplify by assuming that the exponentially decaying channel in Example 2 is the true channel of the system, starting with an ideal situation where there is no correlation mismatch and all transmitted symbols are pilots. Further, we have chosen to set the block size to 5 tones and 11 frames, which coincide with the coherence time and coherence bandwidth of Example 1. The SNR of the system is set to E_b/N_0=20 dB.

Example 4: All pilots and no correlation mismatch

In an ideal situation we know the true channel correlation and all data transmitted is known to the receiver (pilots). Figure 6 shows the 4-PSK BER, over one transmis-

sion block, under these ideal conditions.

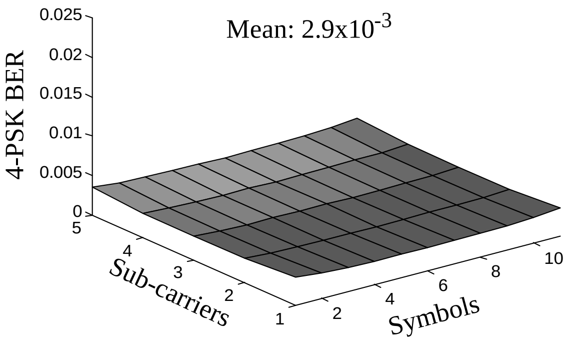

Figure 6: Uncoded 4-PSK BER when estimating the channel without correlation mismatch and using all symbols as pilots.

We can not expect to know the channel correlation beforehand and we therefore present an example where the estimator is designed for the uniform channel in Example 1, but applied to the exponentially decaying channel of Example 2. This implies a mismatch in both frequency correlation (different power-delay profiles) and time correlation (different Doppler frequencies).

Example 5: All pilots and correlation mismatch

Calculation of the BER under the same ideal condition on available pilots, but with design mismatch in both power-delay profile and Doppler frequency reveals what we can expect in terms of sensitivity to design errors. In Figure 7 we display the BER under these conditions.

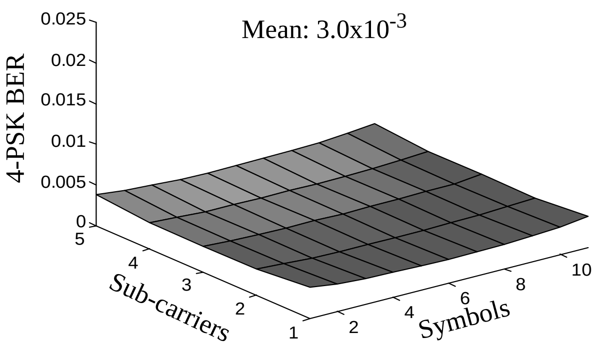

Figure 7: Uncoded 4-PSK BER when estimating the channel with correlation mismatch and using all symbols as pilots.

Comparing the BER of Example 5 with the BER of Example 4 we can see that a mismatched estimator design does not necessarily have any great effect on the performance of the system. It should be noted that the estimator is deliberately designed, as a rule of thumb, for the 'worst' correlation, and applied to a channel with stronger correlation. If the design correlation and the true correlation had been swapped in the example, the increase in BER due to mismatch would have been much higher.

Considering that, in a real system, we want a major part of the transmitted symbols to contain actual data, not pilots, we continue our series of examples by removing the ideal pilot condition. We evaluate the performance when only a small fraction of the symbols are pilots. We will use two different pilot patterns to illuminate the impact of pilot positioning. The general strategy we use when assigning the pilot patterns is based on the fact that no pilots in neighbouring blocks can be used in the estimation, since neighbouring blocks are transmitted from different terminals and received through different channels. Therefore, most of the pilots will reside on the edges of the blocks. The two chosen pilot patterns are displayed in Figure 8.

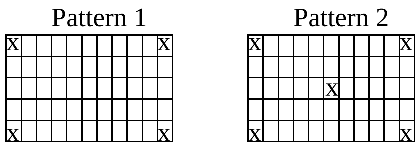

Figure 8: Pilot patterns used in Example 6 and Example 7. Pilot positions marked by 'x'.

The pilot densities of the two patterns are 7.3% and 9.1%, respectively. That is, about 93% and 91% of the positions are available for data transmission. Naturally we have to sacrifice in BER if we want a small number of pilots. In the following two examples we will show that the positioning of pilots is critical in the system design. We use the correlation mismatch from Example 5, and apply the two slightly different pilot patterns.

Example 6: Pilot pattern 1 and correlation mismatch

Using pilot pattern 1 in the channel estimation, the average BER is more than four times higher (Figure 9) com-

pared to the ideal situation with all pilots (Figure 7).

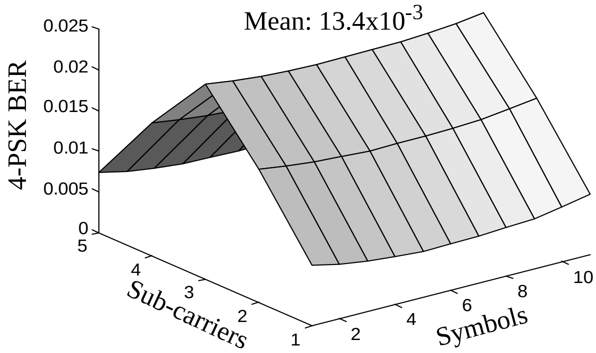

Figure 9: Uncoded 4-PSK BER when estimating the channel with correlation mismatch and pilot pattern 1.

Considering Example 6, where the largest BER is found as a ridge in the transmission block, we apply pilot pattern 2 in the next example. The difference between the two pilot patterns is that the second one contains one extra pilot symbol in the centre of the transmission block. This slightly increases the pilot density, but, as we will see, greatly reduces the BER of the system.

Example 7: Pilot pattern 2 and correlation mismatch

To lower the BER of Example 6 we add one pilot in the centre of the transmission block (pattern 2). The resulting BER is displayed in Figure 10.

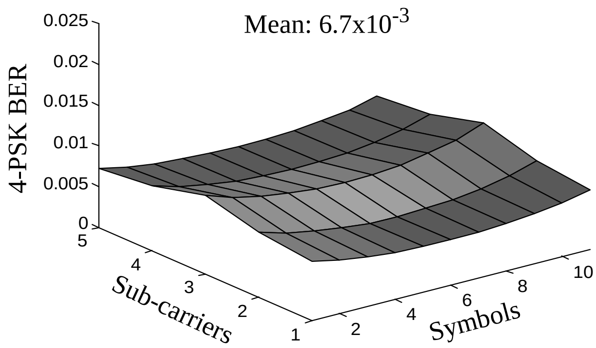

Figure 10: Uncoded 4-PSK BER when estimating the channel with correlation mismatch and using pilot pattern 2.

Comparing the last two examples, where only a single pilot symbol is the difference, we see that only small changes in the pilot pattern may result in drastic changes in the BER performance. Due to one extra pilot symbol in Example 7 the BER was reduced, compared to Example 6, by almost a factor two.

With the preceding four examples we have illustrated that choosing design correlation in the estimator is not as critical a task as positioning of the pilots.

VI. DIVERSITY AND OTHER ASPECTS

In the previous sections we have investigated the basic aspects of channel estimation in the up-link, such as block shape, correlation mismatch and pilot positioning. Apart from these aspects, the absolute size on the transmission blocks is of importance to the performance.

The larger the transmission blocks are, the more channel correlation can be used to smooth the channel estimate. However, in a real system, we want to exploit as much as possible of the available channel diversity. Therefore the optimal block size should be determined by evaluation of a system where interleaving and channel coding is included. Even if channel estimation performs best for large transmission blocks, the diversity gain obtained by choosing smaller blocks, thus allowing an increased hopping frequency, will improve the overall performance down to a certain block size.

Another aspect on the choice of block size is the granularity in the bit rate. Since one transmission block is the smallest unit allocated, the block size will usually determine the flexibility in the individual assignment of data rate to mobile terminals.

VII. CONCLUSIONS AND FURTHER WORK

This preliminary investigation of channel estimation in the up-link of an OFDM-based multi-access system has answered some questions. However, there are still questions to be answered and even more questions arose from the study. We believe that two of the most important questions for the future are:

- What transmission block size is optimal in terms of coded BER and bit-rate flexibility?
- What are the critical implementation issues, such as estimator complexity and quantization?

The conclusions we have drawn from this preliminary investigation of channel estimation in the up-link of an OFDM-based multi-access system are the following:

- Each user should be assigned continuous transmission blocks that are as large as possible, both in time and frequency, to allow the channel estimator to benefit from as much channel correlation as possible.Using a block shape proportional to the ratio between the coherence time and coherence bandwidth of the channel is a reasonable choice.

- Using an assumed uniform power-delay profile of the channel in the design of the channel estimator only has a minor affect on the performance when applied to a channel with an exponentially decaying power-delay profile.
- The positioning of pilot symbols within the transmission block is one of the crucial tasks in the system design.

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