The German 3G Licence Auction: Did the Government's Stake in Deutsche Telekom Influence the Outcome?

Rander, Robin

2004

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

The German 3G Licence Auction: Did the Government's Stake in Deutsche Telekom Influence the Outcome?

Robin Rander*

14th April 2004

Abstract

The spectrum for third generation (3G) mobile communications for the German market was alloted to operators by means of an auction. This resulted in a highly competitive outcome: six operators were given rights to provide 3G services. Government revenues from this auction were a staggering €50.8 Bn. As the German government stands as majority shareholder in one of the strongest participants, Deutsche Telekom (DT), it was argued that DT had an incentive to push prices to higher levels than had otherwise been motivated, thereby servicing the interest of the majority shareholder. This paper provides a theoretical model which shows that the German auction rules were indeed vulnerable to such a conflict of interests. However, the only equilibrium of the model consistent with observed behaviour corresponds to a conflict of interest (in the context of the model) too small to have any impact on the behaviour of DT.

Keywords: Auction theory, Telecommunications, Game theory, Toehold
JEL Codes: D43, D44, D45, L13, H82

1 Introduction

The advent of the much ballyhooed third generation (3G) mobile telecommunications systems was, naturally, preceded by the allotting of government licenses for operating such networks. Since the electro-magnetic spectrum available for such operations is a scarce resource, it has to be allocated among potential operators by some mechanism. This mechanism would ideally yield an allocation which is, in some sense, efficient. Many would also argue that it should raise significant government revenues as well.

How licenses are allocated is crucial in forming the structure of the telecommunications services industry: once licences have been sold or handed out, entry into the market is hard or impossible, at least by other means than as a customer of or a partner to one of the license holders. It is thus obvious that the

*Correspondence to: Robin Rander, Department of Economics, Lund University, P.O. Box 7082, S-220 07 Lund, Sweden. Phone +46-46-222 79 09, e-mail: robin.rander@nek.lu.se.

mechanism used for allocation of licenses must be carefully designed in order to yield a viable industry structure.

A number of different methods for allocating licenses of this nature have been employed, the three main categories being lotteries, auctions and "beauty contests"[1]. Beauty contests and auctions were the prevalent mechanisms used when allocating 3G-licenses in Europe. Countries such as Sweden, Finland, Norway, France and Spain opted for beauty contests. Others, such as the UK, the Netherlands, Germany, Switzerland, Italy, Denmark and Austria chose to use auctions of various designs to allocate the licenses.

The question of whom to award a license is related to the question of how many licenses to award. There is only so much electro-magnetic spectrum suitable for telecommunication and a service provider needs a certain minimum amount of spectrum to be able to provide sensible services. Hence there is an upper limit on the number of licenses which can be awarded. In most countries this issue was resolved by bureaucratic procedure: the number of licenses was determined before the actual allocation procedure started. Germany and Austria, however, settled for a solution where the number of licenses was determined endogenously by participants.

A problem facing any allocation mechanism, be it a lottery, a beauty contest or an auction, is the absence of direct consumer participation. Thus consumer interest cannot be captured in the allocation process in a direct manner. Many oligopoly models, such as the canonical Cournot model, predict that consumer surplus as well as efficiency increase with increased competition. The goal of achieving a competitive industry structure can thus be considered a proxy for consumer interest.

The various auction designs used across Europe in the late 1990s and early 2000s to allocate licenses for 3G services have received a fair amount of attention from auction theorists. E.g. Klemperer (2002) and Jehiel and Moldovanu (2001) provide overviews of the various designs used and their outcomes. Both papers also describe some of the problems which are specific to the allocation of telecommunications licenses through auctions. General discussions on efficient outcomes in auctions can be found in e.g. Klemperer (2001) and Ausubel and Cramton (2002). For a general introduction to the theory of auctions, see Klemperer (1999).

When analysing the consequences a license auction, no matter what its rules are, will have on the structure of the industry it must be recognised that entrants and incumbents are, potentially, in very different positions. For the case of wireless telecommunication services it can reasonably be assumed that 3G services are a substitute for 2G services. Hence 3G service providers will compete for (at least some of) the same customers as the 2G service provider. Moreover, a license for 3G operations generally gives the holder the right to provide 2G services as well.

This implies that any holder of a 3G-license which is not already present in the market as a holder of a 2G-license is a new competitor to holders of 2G-licenses. Industry profits which decrease with the number of competitors may create an incentive for incumbents to block entry if possible. The classical case of a monopolist threatened by entry is analysed in Gilbert and Newbery (1982).

[1] A beauty contest is a bureaucratic procedure in which the regulating body allocates licenses to interested parties based on its evaluation of a pre-determined set of criteria, e.g. network coverage and financial viability.

This result is generalised to an oligopolistic industry threatened by entry in Jehiel and Moldovanu (2000).

It seems reasonable that the cost of setting up a 3G network would be greater for a new provider than for an established provider of 2G services. The established company has an existing physical infrastructure as well as a recognised brand and an administrative apparatus adapted for providing telecommunications services to the market in question.

The above considerations imply that given the same estimate of how profitable the market for 3G communications is, an incumbent will value a 3G license higher than a company which is not presently a provider of 2G services (an entrant, in this context). Provided licenses are allocated through an auction mechanism assigning licenses to participants with the highest valuations, as most forms of auctions in common use do, a likely outcome would be an industry structure much like the existing 2G structure if the number of 3G licenses sold are equal to or less than the number of 2G licenses. Since incumbents *cetris paribus* will have higher valuations than entrants, the incumbents would obtain the licenses effectively blocking any new competition.

The most obvious and probably most effective means of ensuring a more competitive 3G market is selling more 3G licenses than there are 2G licenses. For example, the UK had four incumbents and opted to auction out five 3G licenses so as to ensure at least one entrant to obtain a license.

Proponents of auctions are often seen to criticise beauty contests on the grounds of them being less transparent than auctions, their argument being that in an auction with sensible rules participants will have no or little incentive to make bolder promises than they are capable of delivering on. It is also easy to see why a particlular participant was awarded a license, since the determining ciriterion for being awarded a license is the willingness to pay. In this sense, a beauty competition is a less transparent allocation mechanism than an auction.

It is not entirely obvious that an auction actually *is* more transparent, since a bidder's motive for bidding in a certain way is often more or less opaque: Börger and Dustmann (2002) analyse the bidding behaviour of some of the participants in the UK auction and conclude that the rationale behind the behaviour is not easily seen. It appears that bidders do not have fixed valuations of licences, but rather that these change during the course of the auction. It is unclear what causes these changes in the valuation. A number of potential explanations have been proposed:

1. Bidders may learn from each other: i.e. if A has a large valuation of a license, the licenses is probably worth a lot to B as well. Hence, when B observes A placing a high bid, B will increase its valuation. (This is essentially a common-value assumption. More on this subject in section 4.)

2. Bidders may attempt to bid in a manner which does *not* reveal their true valuations, at least not until the end of the auction.

3. There may be *allocative externalities* present, e.g. A values a license higher when B also holds a license but C does not than when B does not hold a license but C does.

4. There may be financial constraints involved. A certain bidder may at the beginning of the auction have secured a certain amount of money

for buying a license. As prices increase when the auction proceeds, this constraint may rule certain bids out. If and when the bidder manages to raise more money bidding behaviour may change abruptly as more higher bids are suddenly made possible.

The presence of budget constraints also opens the possibility of bidders attempting to weaken their rivals' financial strength through driving prices to high levels.

5. Shareholder pressure. If the price of a bidder's shares change during the course of the auction and the bidder attributes this change to its bidding behaviour, the bidder may well find it prudent to revise its bidding behaviour in a manner more pleasing to shareholders.

6. Conflicting opinions among the management. It seems plausible that not all members of a bidder's management share the same opinion on how which bidding strategy should be pursued. If the "balance of power" in the management shifts during the course of the auction, it is not unlikely that bidding behaviour changes accordingly.

Börger and Dustmann consider all of the above. None of them fully rationalises the bidding behaviour of the bidders studied. The interesting issue of allocative externalities are investigated in e.g. Jehiel and Moldovanu (1996).

To varying degrees these potential bidding rationales determine a participants behaviour in any given round and for how long a certain bidder will be prepared to stay in the auction.

Since the number of licenses to be awarded in the German auction was not determined in advance it can plausible be assumed that some bidders had an incentive to minimise the number of licenses awarded in order to reap the benefits of a less competetive industry, i.e. there was an incentive for predation. As a consequence, some bidders will under reasonable circumstances be prepared to pay higher prices than they had if the number of licenses had been *a priori* determined and greater than the number of incumbents. This avenue is explored in Ewerhart and Moldovanu (2001) and Grimm, Riedel, and Wolfstetter (2002).

It is easy to think of scenaria where this purported transparency would be further muddled. One such example is the case of a partially government-owned participant in an auction where the government acts as seller. If the partially government-owned company wins the auction, its payment to the seller will to some extent be the government shifting money from one pocket to the other. Under these circumstances, the participant under partial government ownership can be considered as essentially holding a stake in the object for sale. It is obvious that partial ownership in the item for sale, often called a *toehold*, makes the partial owner less averse to paying a high price for the item. It is less obvious that the presence of a toehold will make bidders with no toehold more averse to paying a high price for the item.[2] The subject will be elaborated on in section 4. It appears that an otherwise sensibly designed auction will fail to generate efficient outcomes in the presence of a toehold.

In Germany, one of the participants in the auction for 3G licences was T-Mobil, a subsidiary of the partially state-owned telecommunications giant

[2]More generally, if there are many bidders with toeholds, then all but the bidder with the greatest toehold will be more averse to paying higher prices than if there had been no toe holds.

Deutsche Telekom. This paper extends a previously existing model of the German UMTS auction in an attempt to analyse the potential impact of this state of affairs. This is accomplished by introducing a toehold, the magnitude of which is unknown to all participants except T-Mobil. Hence we add to the plethora of potential rationales behind bidding behaviour the possibility of a bidder trying to increase government revenues from selling the licenses.

A brief description of the rules and peculiarities, as well as some of their consequences, is provided in section 2. The actual events of the auction are briefly described in section 3. Section 4 outlines a previously existing model of the auction. This model is modified to account for the aforementioned ownership issues in section 4.2, which also contains the main result of this paper. The main result consists of a complete characterisation of subgame perfect equilibria of the game which is used to model the auction. It will be found that the equilibria depend on the magnitude of the toehold, but not on players' beliefs about its magnitude. Whether the assumptions of the model are reasonable or not is discussed in 4.2.5. The results from the model are discussed and put into context in section 5, where it is argued that the only equilibrium of the model consistent with observed behaviour corresponds to a toehold which is too small to have any impact on the behaviour of the bidders. Section 6 concludes.

2 Rules of the German Auction

The German auction for 3G-licenses had two stages, each in the form of an ascending simultaneous auction with a number of added features. What follows is an outline of the rules of the German auction as described in e.g. Grimm, Riedel, and Wolfstetter (2002) and Ewerhart and Moldovanu (2001). The full set of rules is available from the Regulierungsbehörde für Telekommunikation und Post[3].

When compared to other auctions used to allocate spectrum rights the German auction had a remarkable feature: the number of licenses to be sold was not *a priori* given but rather endogenously determined by the bidders. This was accomplished by dividing the total spectrum available for 3G services into twelve identical blocks of *paired spectrum*[4]. Each such block was given a number between one and twelve. The first stage of the auction gave each participant the opportunity to buy two or three such blocks.

In each round participants placed bids on two or three blocks. Note that bids were placed on individual blocks.[5] If a participant in any round chose to bid on only two blocks the participant in question was not allowed to bid on three blocks in future rounds. In what follows, the choice of moving from bidding on three blocks to bidding on only two blocks will be called *demand reduction*.

[3] The German Regulatory Authority for Telecommunications and Posts. Auction rules are available in both German and English on their web site, www.regtp.de.

[4] "Paired spectrum" is telecommunication jargon for a slice of the electromagnetic spectrum which has been subdivided into two parts in order to make two-way (i.e. full duplex) communication possible. The paired-spectrum blocks in the German auctions were 2x5 MHz. A two-block licence thus corresponds to having 2x10 MHz of paired spectrum, while a three-block licence corresponds to having 2x15 MHz of paired spectrum. Having only 2x5 MHz of paired spectrum was deemed as insufficient for providing useful services

[5] This differs from a common model of multi-unit auctions as described in e.g. Ausubel and Cramton (2002), where each bidder submits her demand function in each round. In such an auction, bids are not placed on specific blocks, but rather on quantities of blocks.

Minimum increments on the bid for a certain block between rounds were ten per cent of the participants current bid on the block in question. The auctioneer had the option to reduce the minimum increment at any round of the auction. The reserve price for each block was set to DM 100 million.

Participants with winning bids on two or three blocks at the end of the first stage were given licenses to provide 3G telecommunications services. A participant holding a winning bid on one or no blocks were not given a license but was on the other hand not obliged, or even allowed, to buy the block. Potentially unsold blocks of paired spectrum plus a number of blocks of *unpaired* spectrum[6] were sold in the second stage of the auction. Only participants who obtained a license in the first stage were allowed to participate in the second stage. The reserve price for blocks of paired spectrum sold in the second auction was reset to DM 100 million. The outcome of the first auction did thus not directly impact on the prices in second auction.

It is clear that this design allows anywhere between none and six licensees. In practice four, five or six licensees seemed the most reasonable outcomes.

2.1 Some Consequences of the German Rules

Possibly the most important consequence of the rules is the following: there exist complementarities among the items for sale. The value of a bidder's first block is essentially nil, since at least two blocks are required for a license and bidders are not even allowed to buy single blocks. The second block is worth a lot, since it guarantees entry to the industry. Two blocks of paired spectrum provide enough capacity to provide 3G telecommunications services. The third block is certainly worth something as well: the owner of three blocks will have access to more spectrum, which means the third block has an intrinsic value. But the third block also has an indirect value: if at least one bidder obtains three blocks a maximum of five licences can be sold, resulting in a less competitive industry and thus higher profits for licensees. Ironically, this implies that a bidder obtaining a third block will be of value to all bidders except the one that gets pushed out of the industry by the bidder obtaining three blocks. Less competition is a common good.

The act of buying a third block in order to prevent another firm from entering the industry is a form of predation. Note that the possibility of predation may create an incentive for a firm to acquire more spectrum than had otherwise been optimal for its planned operations. It can thus be argued that the division of licences into blocks in conjunction with the manner in which these could be aggregated into licenses under the German rules inflated the demand for blocks. At the same time, the fact that a single firm crowding out a weaker firm benefits all remaining bidders opens the field for "free riding", i.e. reducing demand in the hope that another firm will bear the cost of obtaining a third block in order to reduce competition in the industry.

Under certain circumstances it may also be beneficial for a bidder to reduce demand in the first auction and attempt to obtain additional capacity in the second auction. One may expect the price of a block of paired spectrum to be lower in the second stage than it is at the end of the first stage on the

[6] As the allocation of unpaired spectrum seems to be of lesser importance than the allocation of paired spectrum, the potential impact of unpaired spectrum will not be mentioned further in this text.

grounds that in the first round, bidders are forced to pay a, potentially high, per-block price to enter the industry. If only one block of paired spectrum remains unsold after the first auction the second auction will essentially be a conventional ascending auction for a single item. It will be argued in section 4.1.1 that it is reasonable to expect that the per-block price will be lower in the second auction than in the first auction.

2.2 Some Comments Regarding the Rules of the German Auction

The rules of the German auction have been harshly criticised as well as praised.

Praise of the German auction, as found in e.g. Grimm, Riedel, and Wolfstetter (2002), generally focus on the flexibility of the number of licences: since bidders are more well informed as to the expected revenues and costs of setting up operations, it seems reasonable to let the participants choose the future structure of the industry. It is also noted out that the auction actually resulted in a highly competitive industry as well as high government revenues (see section 3.1 for details of the outcome). The authors argue that an auction for a fixed number of licenses will never attract more participants than the number of licenses for sale if potential bidders are well enough informed about each others' valuations. Many companies are, or at least had ambitions to be, present as providers of telecommunications services in many countries in Europe. The wave of license auctios that swept across Europe between 1999 and 2001 would have provided ample opportunities for these companies to infer each others' valuations. Hence, after sufficiently many auctions have been settled an auction for a fixed number of licenses will not acctract more participants than the number of licenses. An auction for a flexible number of license, on the other hand, can even in the long run attract as many participants as the maximum number of licenses which can be sold in the auction.

Klemperer (2002), among others, argue that the arguments as to why the industry should choose its structure by itself is erroneous: since consumer interest is not represented in the auction, the outcome will maximise industry revenue rather than economic efficiency. Moreover, it is argued that rules encourage collusion since the actual goods the participants bid for (i.e. licenses for 3G operations) are divided into smaller units.

Ewerhart and Moldovanu (2002) argue that the division of licenses into smaller parts also cause inefficient allocations. Ewerhart and Moldovanu (2001) also show that auctions under the German rules can result in regretful outcomes, i.e. outcomess where a bidder *ex post* will find itself having paid too much for a license. This uncertainty can have a deterring effect, resulting in an auction with fewer participants than under a different set of rules. Of course, this possibility is inherent in all almost-common-value auctions when bidders are uncertain about their own valuations of the items for sale. In this case, however, the possibility of a regretful outcome for the strongest bidder is a consequence of its uncertainty regarding the weakest bidder's valuation rather than its own valuation.

3 A Summary of Events

There are four operators of GSM networks in Germany, listed in table 1. Three potential entrants chose to enter the auction. These are listed in table 2.

Incumbent	Market share	Backer
T-Mobil	40%	Deutsche Telekom
Mannesmann-Vodafone	40%	
e-plus	15%	
Viag Interkom	5%	BT

Table 1: Incumbents on the German market. Data on market shares taken from Grimm, Riedel, and Wolfstetter (2002). Mobilcom has changed its name to "0_2" since the auction.

Entrant	Backer
Mobilcom	France Telecom
Debitel	Swisscom
3G	Telefonica and Sonera

Table 2: Potential entrants to the German market. Mobilcom has left the maket for 3G telecommunication services in Germany since the auction.

Two of the entrants, Debitel and Mobilcom, were already present on the German market as service providers, although they did not operate their own networks. It was generally perceived that T-mobil and Mannesmann-Vodafone were the strongest bidders, i.e. the bidders with the highest valuations of licenses.

3.1 The Auction

The auction begun on July 31, 2000 and lasted for 173 rounds. Since only the winning bids of each round have been made public it is not possible to pinpoint with certainty the round at which a given bidder reduces its demand from three to two blocks. Conditions under which it can with certainty be concluded that a bidder has reduced its demand are presented in Grimm, Riedel, and Wolfstetter (2002). The summary of events presented here is based on the same article.

Bidding started at or slightly above the reserve price of DM 100 million with the exception of Mobilcom whose initial bid was DM 501 million each on two blocks (nos. 4 and 5).[7] At the beginning, all bidders submitted bids for three blocks each. Total demand was thus for 21 blocks. The first bidder to reduce demand was Debitel, who did so no later than in round 115. Debitel dropped out of the auction at round 127, when the price per block was about DM 5 billion.

[7] This may have been an attempt to signal to other bidders that Mobilcom intended to bid for these blocks for the rest of the auction, thereby urging remaining bidders to focus on other blocks in order to avoid unnecessary competition. This kind of collusive coordination among participants is one of the drawbacks of an ascending auction.

None of the remaining bidders, with the possible exception of Viag Interkom, appeared to have reduced its demand at this point. By round 146 all participants except T-Mobil and Mannesmann-Vodafone had reduced their demands. Given this situation only outcomes with five or six licensees were possible.

At round 147 and forward, the decision whether to continue the auction or not was left to T-Mobil and Mannesmann-Vodafone. These continued to demand three blocks for quite a while: T-Mobil chose to reduce its demand in round 167 and Mannesmann-Vodafone did the same in round 172. The price per block was at this point roughly DM 8.2 billion.

3.2 The Outcome

The industry structure resulting from the outcome of the auction is the same as the structure that would have resulted if the auction had terminated in round 127. The prices paid, however, were approximately DM 3.2 billion higher per block at the end of the auction than in round 127. This means that the winners were in total DM 38.4 billion (€19.5 bn) poorer than they had they been if the auction had ended in round 127.

Most theories presented explain the phenomenon in terms of bidders' uncertainty regarding the weakest bidders valuation of a license and that this turned out to be greater than expected. Jehiel and Moldovanu (2001) mention the hypothesis of Deutsche Telekom having acted to raise per-block prices on account of it being majority owned by the German government, although the authors do not promote this theory themselves. Apparently this explanation of the high per-block prices was put forward by some of the smaller participants. It is also noted that a lawsuit was initiated by one of the small incumbents but was not further pursued.

4 Theoretical Analysis of the German Auction

Which types of theoretical models could plausibly be argued to provide a good description of an auction for the rights to provide telecommunications services?

It is clear that the value of holding the rights to provide such services cannot differ between potential holders in an entirely arbitrary way. Rather, it seems plausible to assume that two hypothetical and in all respects identical (in the sense that they both have access to the same technology and face the same costs) potential holders would place the same value on the license, provided they both have the same estimates of the various parameters of the market for telecommunications services which affect the holder's profit. The assumption of bidders having *cetris paribus* identical valuations of the items for sale is commonly known as a *common-value* assumption.

Let $V_i(t_i)$ be bidder i's valuation of the items for sale as a function if the bidder's *signal* t_i. A bidder's signal can be considered to represent her estimates of the parameters which affect the value of the items. Assume also that all t_i:s are drawn from the same distribution and that $V_i(t_i)$ is increasing in t_i. In this notation the common-value assumption in a situation with n bidders amounts to $V_1(t) = \ldots = V_i(t) = \ldots = V_n(t)$ for any given signal t.

The common-value assumption is of course unrealistically strong in the case at hand. For example, it seems reasonable that incumbents value licenses higher than entrants as argued in section 1.

The potential holders of telecommunications licenses in the present case can reasonably be believed to have access to similar technology and face roughly the same prices. There is no reason to believe that the estimates of the market parameters a particular bidder makes is greatly biased away from the estimates of other bidders. It follows that the valuation functions, given identical estimates of relevant parameters, cannot differ much.

It thus appears reasonable to require that $V_i(t) \approx V_j(t)$ for any signal t and any bidders i and j. The difference in valuation between any two bidders with identical information must, in some suitable sense, be small. This amounts to what is usually called an *almost-common-value* assumption. It appears that an almost-common-value model would be appropriate for the situation at hand.

The presence of a bidder with only a slightly higher valuation can greatly impact on the outcome of an auction. Assume, for example, that $V_2(t) = \ldots = V_i(t) = \ldots = V_n(t)$ but $V_1(t) > V_i(t)$ for all bidders $i = 2, \ldots, n$ and any signal t. Under such circumstances winning a single unit-auction is essentially bad news for any participant but participant no. 1, which is assumed to have an advantage. Bidder $i \neq 1$ will win the auction only if t_i is such that $V_i(t_i) > V_1(t_1)$. Since $V_i(t_t)$ is assumed to be increasing in t_i for all $i = 1, \ldots, n$, it must follow that $t_i > t_1$. As signals are drawn from the same distribution the bidder i is thus likely to have over-estimated the value of the item, which is a bad thing. This phenomenon is known as the *winner's curse*.

In the presence of an advantage bidder in a single unit-auction the non-advantaged bidders' bidding should thus be cautious since they will be wary of falling victim to this curse. It is widely conceded that the advantaged bidder will almost always win the auction. Moreover, the price paid for the item by the winner will be low because of the cautions bidding. This is an intricate phenomenon which is lucidly explained in Bulow and Klemperer (2002).

4.1 Previous Models of the German Auction

Because of the peculiar design of the German auction standard models from the theory of auctions are not directly applicable.

In section 2.1, it is pointed out that the items for sale in the auction have complementarities between them. Theoretical results, as presented in e.g. Milgrom (2000), indicate that auctions in general do not yield efficient allocations[8] in the presence of complementarities between the objects for sale. As Ewerhart and Moldovanu (2002) point out, there exist very few results regarding the efficiency of the allocations created by multi-unit auctions.

Two models of the German auction have been proposed so far: one by Ewerhart and Moldovanu (2001), which does not take the consequences of the second stage of the auction into account, and one by Grimm, Riedel, and Wolfstetter (2002) which does. The model of Grimm, Riedel, and Wolfstetter is described briefly in section 4.1.1 and then extended in section 4.2.

Why extend the model of Grimm, Riedel, and Wolfstetter rather than that of Ewerhart and Moldovanu? The rationale behind this choice is that while the latter model describes the interplay between the weakest bidder and the

[8] In this context an efficient allocation is an allocation which maximises seller revenue. This should not be confused with the economic efficiency spoken of in section 1.

strongest bidder, the former describes the interplay between the weakest bidder and the two strongest bidders (Mannesmann-Vodafone and T-Mobil). We wish to introduce a parameter, the value of which is not directly observable, which potentially makes one of the two strongest bidders even stronger. It is thus reasonable to choose to extend the model which models the two strongest bidders.

4.1.1 The Model of Grimm, Riedel and Wolfstetter

In Grimm, Riedel, and Wolfstetter (2002) the auction from round 147 and forward is modelled as a game of incomplete information with two participants: Mannesmann-Vodafone (M) and T-Mobil (T). At this point there are six bidders and only M and T, regarded as equally strong[9], demand three blocks each. The remaining participants, denoted 3 through 6, demand only two blocks each and are considered weaker than M and T with bidder no. 6 being the weakest. Only the interaction between M, T and the weakest bidder is explicitly modelled.

At the stage of the auction just described an outcome with four licensees is no longer possible, since this would require at least three operators to hold at least three blocks each, and bidders are not allowed to bid for three blocks once they have placed bids on only two blocks.

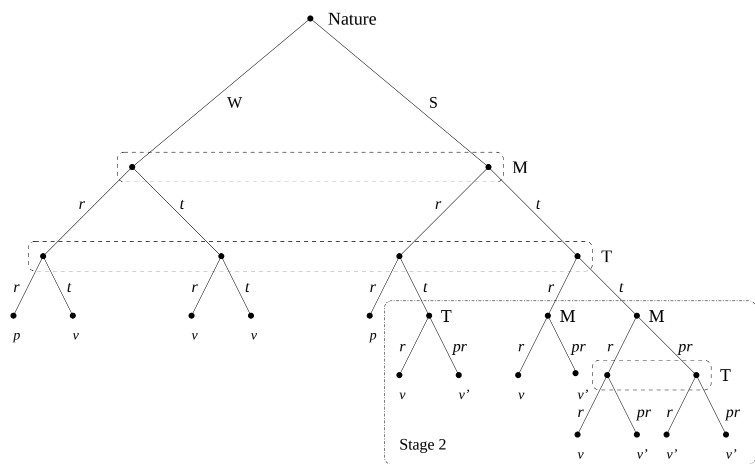

Figure 1: The event-tree of the game in the Grimm-Riedel-Wolfstetter model. The symbols r, t and pr denote reducing demand, trying predation and actually predating. The values p, v and v' at the terminal nodes denote the per-unit prices that result from the corresponding outcome. Subgames found in the dash-dotted box at the lower right are stage-two subgames.

Bidder 6 can be of two types: weak or strong. The difference between these two types is that the strong type has a higher marginal valuation v'_6 of the blocks than the weak type, whose valuation is $v_6 < v'_6$. A weakest bidder of the strong type is thus costlier to force out of the industry, since it will be willing to pay more for the rights to provide 3G telecommunications services.

[9]In this context, *strong* should be taken to mean a preparedness to pay a high price for the items for sale, while *weak* is the lack of preparedness to do so.

The game, depicted in figure 1, begins with nature choosing the type of bidder 6. Both M and T then simultaneously choose whether to *try predation* (t) or *resign* (r). Chosing to resign means resigning from trying predation, not resigning from the auction all together. If M and T both choose r the auction ends, with six licensees holding two blocks each. Should at least one of M and T choose to try predation the per-block price goes up from p, the price in round 147, to v_6. The type of bidder 6 is now revealed through its response to this increase: the weak-type bidder, whose per-block valuation is v_6, quits the auction and the strong-type bidder, whose per-block valuation is v_6', remains.

The action (t), trying predation, amounts to *not* reducing demand at an early stage in the game. This leaves the door open for predation (playing pr), i.e. forcing the weakest bidder out of the market through buying three blocks of paired spectrum.

If either M, T or both chose to play t and the weakest bidder is weak the game ends. If at least one of M and T play t and the weakest bidder is strong play moves on to one of three possible subgames, which will hence forth be referred to as stage-two subgames. Any bidder still having a high demand (i.e. a bidder that has not previously chosen to resign) may now choose to *predate* (pr) or resign (r). In the case of neither M nor T having reduced their demands the choice will be a simultaneous one. If a bidder chooses to predate, the per-block price rises to v_6' and bidder 6 is forced out. The predating bidder obtains three blocks at the per-block price v_6'. *Every* bidder (except bidder 6, who has been forced out) also receives a bonus utility $b > 0$ as a consequence of the resulting less competitive industry. A bidder, either M or T, that chooses not to predate faces the possibility of obtaining a third block of paired spectrum at a potentially lower price p_2 (if the weakest bidder is weak) or p_2' (if the weakest bidder is strong) in the auction for unpaired and left-over paired spectrum. There is thus a very real possibility for free riding: assume that e.g. T chooses to predate and M chooses to reduce its demand. M can then obtain an additional block of paired spectrum in the second auction at a potentially lower cost while reaping the benefits of a less competitive industry.

The sequence of events described in section 3.1 corresponds to both M and T trying to predate (choosing t) on the weakest bidder, which turned out to be of the strong type. Facing this, both M and T chose to resign from predation.

Since the model of Grimm, Riedel, and Wolfstetter will be extended in section 4.2 some attention will be paid to its technical details. In order to do so the following notation is needed: w_n^q denotes player q's ($q \in \{M, T\}$) valuation of an n-block license ($n \in \{2, 3\}$) and ρ is the probability M and T attribute to the weakest bidder being of the weak type. Both M and T have intrinsic marginal valuation v_{13} of a third block, i.e. $w_3^q - w_2^q = v_{13}$ for $q \in \{M, T\}$.

It can be shown under quite reasonable assumptions that demand reduction for both players constitutes a unique equilibrium of all stage-two subgames. The assumptions are the following:

1. Unilateral predation is strictly unprofitable for T-Mobil.

   Formally, this condition is the following:

$$w_T^3 - 3v_6' + b \leq w_T^2 - 2v_6 \qquad \text{(GRW-1)}$$

   In short, T-Mobil finds it more profitable to pay a lower price for a small license in a competitive market than paying a higher price for larger license in a less competitive market.

2. Free riding is strictly profitable. Formally:

$$v_6' > p_2' \tag{GRW-2}$$

where p_2' is the expected price (conditional on all available information) of a remaining block of paired spectrum sold in the second auction in the case where the weakest bidder is strong. This condition simply states that obtaining a third block in the second auction is expected to be less costly than obtaining it in the first at price v_6'.

If the inequality (GRW-1) holds then strict demand reduction is the unique equilibrium strategy for both Mannesmann-Vodafone and T-Mobil.

On a first glance, (GRW-2) may seem an *ad hoc* assumption. There are, however, a strong case for making this assumption: if all but one block of paired spectrum are sold in the first auction, then the second auction is essentially an almost-common-value, single-item auction. The reserve price is reset to DM 100 million. It follows from the winner's-curse argument presented at the beginning of section 4 that the strongest bidder will win the auction and pay a low price for the remaining block.

Once this has been concluded, it can be shown that both M and T playing the strategy *(t, r, r)* is a unique subgame perfect equilibrium of the game iff, in addition to (GRW-1) (strictly) and (GRW-2), the following conditions are fulfilled:

1. Free riding does not pay in stage one. Formally

$$\rho(v_6 - p_2) \leq 0 \tag{GRW-3}$$

2. Unilaterally trying predation pays. Formally:

$$\rho(v_{13} + b - v_6) - 2(v_6 - p) > 0 \tag{GRW-4}$$

Under assumptions (GRW-1)-(GRW-4) it thus holds that *(t, r, r)* is a unique equilibrium strategy for both players and the outcome of both players following this strategy yields the sequence of events described in section 3.1. This observation is, in short, the rationale behind the above assumptions.

4.2 A Toe-hold Model of the German Auction

In the model described above the per-block price is driven by the possibility of predation. As described in section 3.2 some of the participants accused T-Mobil of deliberately raising prices on account of it being partially owned by the German government, which was also the seller of the 3G licenses.

If Deutsche Telekom indeed had such vested interests, the situation has similarities to the case of *toeholds* in company takeovers. A toehold is said to be present when e.g. two agents A and B bid for a company and both potential buyers hold stakes θ_A and θ_B, with at least one being strictly positive, in the company. Bulow, Huang, and Klemperer (1999) model such a situation as an ascending auction and find that the case of common values with toe holds is strongly reminiscent of the case of almost-common values. The bidder with the larger toehold will face the same advantage as a bidder with a slightly higher valuation of an almost-common-value asset, i.e. he or she will be able to bid more aggressively thereby increasing the winners curse of the other bidders, making them less aggressive.

The intuition behind this result is that since a bidder with a toe hold will receive a fraction (e.g. θ_A) of the final price the bidders payment will in practice be smaller than it had been in the absence of a toehold. In the case of not winning the auction, the payoff from letting the other agent buy the company will be greater the higher the price at which the auction ends. These two effects make the bidder with a toehold less averse to high prices.

Can such a toehold be a partial explanation to why the prices in the German auction rose the way they did? A toehold can be incorporated into the model of Grimm, Riedel and Wolfstetter in order to analyse this possibility.

Some general assumptions are made in section 4.2.1. Section 4.2.2 introduces some notation and specific assumptions about certain relations between the parameters of the model. The lemmata on which the main result rests are found in sections 4.2.3 and 4.2.4. A longish proof has been relegated to appendix A and a number of helpful, but rather large, tables have been relegated to appendix B. The findings are summarised in theorem 1. Some aspects and potential problems of the modelling approach taken are discussed in section 4.2.5.

4.2.1 Assumptions of the Toe-Hold Model

In the model of Bulow, Huang, and Klemperer (1999) all bidders are perfectly informed about the magnitudes of each other's toeholds, i.e. θ_A and θ_B are publicly known. The case of toeholds in the German auction is less obvious: was there a toehold at all, and if so, how large a toehold?

The toehold is incorporated through assuming that Deutsche Telekom receives a fraction Θ of the total revenues from the auction. This Θ is unknown to Mannesmann-Vodafone, which has a belief $F_\Theta(\theta) = P(\Theta \leq \theta)$ about the magnitude of Deutsche Telekom's toehold.

Note that the toehold in this model is, conceptually, slightly different from that in the model of Bulow, Huang, and Klemperer, which has a very clear cut interpretation. In the case presented here, there are conflicting interests between different share holders and it is not obvious that the company will be interested in running errands for its majority shareholder. The parameter θ should, in the present context, be interpreted as *a measure* of the extent to which the company is willing to please its majority shareholder.

The assumption of Mannesmann-Vodafone and T-Mobil having identical valuations will be maintained. Most notably, this means they have the same marginal valuation v_{13} of a third block of capacity and have the same valuation b of a less competitive industry[10].

[10] The two entities v_{13} and b can be given a more precise meaning: assume a company with $k \in \{2, 3\}$ blocks of paired spectrum has fixed costs C (for building masts for antennae, setting up a billing system, advertising etc.), variable cost $c(k)$ (for purchasing, installing and maintaining antennae etc.) assumed to be increasing in k and revenue $r(k)$ (from selling services utilising the company's bandwidth) assumed to be increasing in k (since having more bandwidth means the company can put through more traffic or sell services requiring more bandwidth at higher prices). Further more, assume that in a less competitive industry, the revenues are $\hat{r}(k)$ and that $\hat{r}(k) > r(k)$ for $k \in \{2, 3\}$.

Since the type of T-mobil (i.e. the value of Θ) is not revealed to Mannesmann-Vodafone at any stage of the game, the extended game does not have subgames in the technical sense. What was referred to as stage-two subgames in the original Grimm-Riedel-Wolfstetter model will be denoted by a sequence of three letters, the first denoting whether nature played strong (S) or weak (W), the second denoting whether M chose to play demand reduction (r) or trying predation (t) and the third indicating the same for T. For example, the Stt-game refers to the case of the weakest bidder being strong and both M and T trying predation. All possible combinations of such choices will be referred to as stage-two games.

For details on notation in the following analysis, the reader is referred to section 4.1.1.

4.2.2 Preliminaries

In what follows, the follwing notation will be used extensively:

Definition 1 (Some useful notation).

1. *Let θ denote a realisation of the random variable Θ, the event space of which is $[0, 1]$.*
2. *Let $U_2^T(\theta) = w_2^T - 2v_6 + 12\theta v_6$ be T-Mobils pay-off from the outcome (r, r), i.e. T obtaining two blocks, at the price v_6, in a market with six licensees.*
3. *Let $U_3^T(\theta) = w_3^T - 3v_6' + b + \theta(11v_6' + p_2')$ be T-Mobils pay-off from the outcome (pr, r), i.e. T obtaining three blocks at the price v_6' in a market with five licensees.*[11]
4. *Let $\Delta = U_2^T(0) - U_3^T(0)$.*
5. *Let $\delta = 12v_6 - 11v_6' - p_2'$.*
6. *Let $U_2^M = w_2^M - 2v_6$ be Mannesmanns pay-off from the outcome (r, r), i.e. M buying a two-block license at per-block price v_6.*

Under these assumptions the company's valuations will be $W_2 = r(2) - c(2) - C$ and $W_3 = \hat{r}(3) - c(3) - C$. It follows that $W_3 - W_2 = \hat{r}(3) - r(2) - (c(3) - c(2))$. This can be rewritten in the following manner:

$$\hat{r}(3) - r(2) - (c(3) - c(2)) = \hat{r}(3) - (r(2) - r(3) + r(3)) - (c(3) - c(2)) =$$
$$= \underbrace{\hat{r}(3) - r(3)}_{b} + \underbrace{(r(3) - r(2)) - (c(3) - c(2))}_{v_{13}}$$

The difference associated with b in this equation clearly relates to increased revenues from a less competitive industry, whilst the terms associated with v_{13} relate to the valuation of an extra block of bandwidth *had the competition in the industry remained unchanged.* Note that v_{13} by this reasoning need not even be positive in order for a company to prefer a three-block license as long as b is large enough.

This has the implication that the valuations w_k^p of the analysis in section 4.2 must be taken to be valuations *excluding* the gains from a less competitive industry.

[11] The term within parentheses derives from all blocks but one being sold in the first stage of the auction and the remaining block being sold as "left over" paired spectrum at price p_2' in the second stage of the auction.

7. *Let* $U_3^M = w_3^M - 2v_6' - p_2' + b$ *be Mannesmanns pay-off from the outcome* (pr, r)*, i.e. M buying two blocks at the price* $2v_6'$ *and a third block at price* p_2' *in the second stage of the auction.*

It follows from the above definitions that

$$U_2^T(\theta) - U_3^T(\theta) = \Delta + \theta\delta \tag{1}$$

and that

$$U_2^M - U_3^M = -v_{13} - b + p_2' - 2v_6 + 2v_6' = \Delta + p_2' - v_6' \tag{2}$$

.

In a manner much like that of Grimm, Riedel, and Wolfstetter, a number of assumptions about the parameters of the model will be made. Unless otherwise mentioned, the following will be assumed throughout the analysis.

Assumption 1 (Maintained Assumptions).

1. *The strong-type weakest bidder has a higher per-block valuation than the weak-type weakest bidder. Formally:*

$$v_6' > v_6 \tag{MA-1}$$

2. *Free-riding is strictly profitable. Formally:*

$$v_6' > p_2' \tag{MA-2}$$

Assumption (MA-2) corresponds exactly to (GRW-2) and assumption (MA-1) is an obvious requirement for the model to be sensible.

4.2.3 Analysis of the Stage-Two Games Under the Toe-Hold Assumption.

A cautionary note on nomenclature: the stage-two games must not be confused with the second stage of the auction, i.e. the auction for leftover paired spectrum and unpaired spectrum.

Payoffs of the stage-two games under this assumption are given in tables 3 and 4. The toe-hold assumption breaks the symmetry between the two strongest bidders, even though the assumption of their valuations of various licenses being the same is maintained.

It is fairly easy to derive at the payoffs in table 3: $U_2^T(\theta)$, $U_3^T(\theta)$, U_2^M and U_3^M have been defined previously as T's and M's payoffs from various outcomes. The payoffs from the outcome *(pr, pr)* are much like the payoffs from the outcome *(pr, r)* with the exception that M gets to pay v_6' rather than p_2' for its third block. T gets a corresponding increase in its payoff through its toehold. The payoffs from the outcome *(r, pr)* are essentially anti-symmetric to those of the outcome *(pr, r)*: T gets to pay the lower price p_2' rather than v_6' for its third block and M gets to pay the higher rather than the lower price for its third block.

We now proceed to show that essentially the same strategic equilibrium exists in the extended model as in the model of Grimm, Riedel, and Wolfstetter.

Lemma 1. *Demand reduction* r *is a dominant strategy for T in the Stt-game if*

$$\Delta + \theta\delta > 0 \tag{3}$$

Proof. Since $\Delta + \theta\delta = U_2^T(\theta) - U_3^T(\theta)$ it follows from (3) that $U_2^T(\theta) > U_3^T(\theta)$, i.e. given that M plays r, T strictly prefers playing r to playing pr.

Since it holds by definition that $\theta \in [0, 1]$, it follows from (MA-2) that $U_3^T(\theta) + v_6' - p_2' \geq U_3^T(\theta) + \theta(v_6' - p_2')$, i.e. T prefers playing r to pr given that M plays pr. □

Corollary 1.1. *The outcome* (r, r) *is a unique equilibrium of the game if in addition to inequalities (3) it holds that*

$$U_2^M > U_3^M - v_6' + p_2' \tag{4}$$

Proof. Since r is a dominant strategy for T by lemma 1 it follows that M:s payoff from playing r is U_2^M which by the inequality (4) is strictly greater than $U_3^M - v_6' + p_2'$ which is M:s payoff from playing pr. M will thus strictly prefer playing r to playing pr. □

The inequalities (3) and (4) can be interpreted as *unilateral predation being unprofitable for both M and T.* These assumptions essentially correspond to the inequality (GRW-1) assumed in the original Grimm-Riedel-Wolfstetter model.

Corollary 1.2. *The inequality (4) implies that* $\Delta > 0$*.*

Proof. It is apparent from (2) that $\Delta > 0$ iff inequality (4) holds. □

Corollary 1.3. *Under assumptions (3)-(4) demand reduction,* r*, is a dominant strategy for both M and T in all stage-two games.*

Proof. Consider the *Str*-game. In this part of the game, M has the choice between playing demand reduction r, yielding the payoff U_3^M, and predation pr, yielding the payoff $U_3^M - v_6' + p_2'$. It follows from (MA-2) that M strictly prefers playing r.

Now, consider the *Srt*-game. Here, T has the choice between playing r, yielding the payoff $U_2^T(\theta)$, and pr, yielding the payoff $U_3^T(\theta)$. From (3) it follows that T strictly prefers playing r.

That r is a dominant strategy for both M and T has been shown in corollary 1.1. Thus r is the dominant strategy for both M and T in all stage-two games. □

What if condition (3) doesn't hold? Let

$$\theta^* = -\Delta/\delta$$

and

$$Q_{(\Delta,\delta)} = \{\theta \in [0, 1] | \Delta + \theta\delta > 0\}$$

As seen in table 5, nine distinct cases emerge depending on the signs of Δ and δ.

The value of θ^* must of course be appropriately restricted to the interval $[0, 1]$, e.g. if $\Delta < 0$ and $\delta > 0$ then θ^* must be taken to be zero if $-\Delta/\delta < 0$.

For the rest of this text, it will be assumed that θ^* denotes an appropriately restricted value.

Note that assumption (GRW-1) in the model of Grimm, Riedel, and Wolfstetter is essentially a special case of the case presented here, since for $\Delta > 0$ it holds that $0 \in Q_{(\Delta,\delta)}$ for all $\delta \in \mathbf{R}$.

When (3) does not hold, i.e. when $\theta \notin Q_{(\Delta,\delta)}$, it is apparent (consult tables 3 and 4) that demand reduction is still a dominant strategy for M in all stage-two games. Once this has been concluded, it follows that it is optimal for T to predate in all stage-two games, i.e. *(pr, r)* is a strategic equilibrium of the stage-two games, since condition (3) not being true is equivalent to $U_3^T(\theta) \geq U_2^T(\theta)$.

4.2.4 Equilibria of the Stage-One Game

Given that $\theta \in Q_{(\Delta,\delta)}$ and the inequalities (MA-2) and (4) are fulfilled both M and T will play demand reduction in all stage-two games. In order to obtain a subgame perfect equilibrium the case $\theta \in Q_{(\Delta,\delta)}$, i.e. T playing *pr* in the stage-two games, must also be considered.

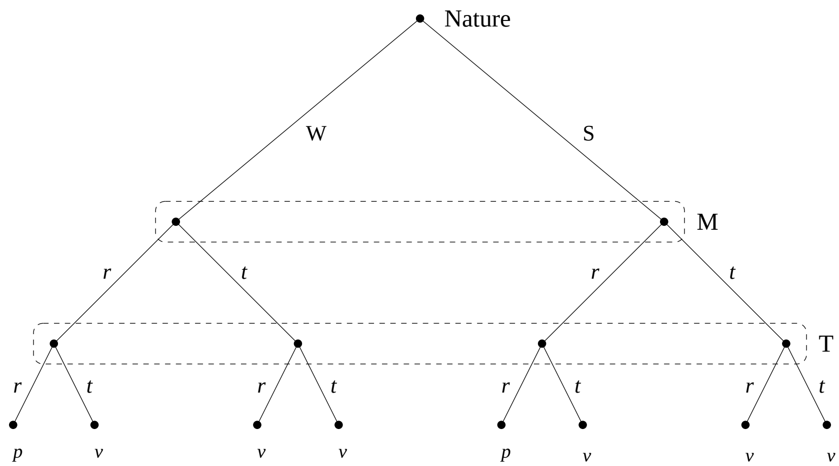

Figure 2: The event-tree of the reduced game in the modified Grimm-Riedel-Wolfstetter model under the assumptions leading to both M and T playing demand reduction in all stage-two games (i.e. $\theta \in Q_{(\Delta,\delta)}$). Nature's initial move, i.e. when nature chooses T:s type θ, has been omitted. The letters shown at the terminal nodes denote the per-block prices that arise as a result of the outcomes corresponding to the nodes.

Expected payoffs in the reduced-form game can, with some labour, be computed from the payoffs given in tables 6–8.[12] For the remainder of the analysis ρ will denote the probability that both M and T assign to the event of the weakest bidder being of the weak type.

Definition 2 (Critical probability). *The* critical probability ρ_c^i *of player* $i \in \{M, T\}$ *is the probability at which* i *is indifferent between playing* r *and playing* t *in the stage-one game.*

[12]The authour will upon request be happy to provide the reader with a complete description of the expected payoffs.

It is fairy obvious that for probabilities ρ below M:s critical probability, M will find it appropriate to reduce its demand comparatively early in the auction since the chance of actually succeeding in the predation are considered too small for it to be an attractive option. The same holds for T, but as will be shown in corollary 3.2, T:s critical probability depends on the value θ.

Lemma 2. *Trying predation, i.e. playing* t*, is a dominant strategy for M in the reduced form game regardless of the value of θ if it holds that*

$$p < v_6 < p_2 \tag{5}$$
$$v_{13} + b > 3v_6 - 2p \tag{6}$$
$$\rho > \frac{2(v_6 - p)}{v_{13} + b - v_6} \tag{7}$$

Proof. Let $U^i(s; s')$, where $i \in \{M, T\}$, denote i:s expected utility from playing the strategy $s \in \{r, t\}$ given that T plays the strategy $s' \in \{r, t\}$. From the expected payoffs in the reduced-form game it follows that

$$U^M(r; r) - U^M(t; r) = -\rho(v_{13} + b - v_6) + 2(v_6 - p) \tag{8}$$

Provided that $v_6 > p$, as in (5), and that (6) is true it follows that (8) is strictly negative whenever (7) holds, which is equivalent to M strictly preferring playing t over playing r given that T plays r.

Now, consider

$$U^M(r; t) - U^M(t; t) = \rho(v_6 - p_2) \tag{9}$$

This equality holds for both $\theta \in Q_{(\Delta,\delta)}$ and $\theta \notin Q_{(\Delta,\delta)}$. It follows that M strictly prefers playing t given that T plays t if $v_6 < p_2$ (as in (5)) and $\rho > 0$. Given that (7) holds it follows that $\rho > 0$ whenever $v_6 > p$ (as in (5)).

It is thus clear that conditions (5) and (7) are sufficient for t to be a dominant strategy for M regardless of the value of θ. □

Corollary 2.1. *If the inequalities of lemma 2 hold, then*

$$\rho_c^M = \frac{2(v_6 - p)}{v_{13} + b - v_6}$$

Proof. Follows from the proof of lemma 2. □

The leftmost inequality of condition (5) is a very natural requirement: it simply states that the weak type of the weakest (remaining) bidder has a per-block valuation which is greater than the per-block price at round 147 of the auction. Had this not been the case, the bidder would already have quit the auction *or* been revealed as being of the strong type. The rightmost inequality in (5) corresponds to condition (GRW-3) of the model of Grimm, Riedel, and Wolfstetter, i.e. it corresponds to free riding *not* paying in the first stage of the game. The condition (6) is clearly reminiscent of condition (GRW-4). It can reasonably be interpreted as unilateral predation being an attractive option for both M and T, *conditional on the weakest bidder being weak.*

Corollary 2.2. *Under the assumption that the rightmost part of (5) hold, it follows that $\delta < 0$ if*

$$p_2' \geq p_2 \tag{10}$$

Proof. By assumption it holds that $v_6' > p_2' \geq p_2 > v_6$. From the definition it follows that $\delta = 11(v_6 - v_6') + (v_6 - p_2') \leq 11(v_6 - v_6') + (v_6 - p_2)$ which is apparently strictly negative under the assumptions. ☐

It follows from corollary 2.2 that there may exist θ such that condition (3) does not hold. From table 5 it can be concluded that this will be the case when $\theta > \theta^*$. In this case, as previously argued, both M and T playing r is no longer a strategic equilibrium. Instead, T predating (playing pr) and M reducing demand is a strategic equilibrium. When $\theta > \theta^*$ unilaterally predating becomes profitable for T, which gives M the opportunity to free ride!

Lemma 3. *Assume that $\delta \in Q_{(\Delta,\delta)}$ and that (5) holds. If in addition it holds that*

$$\rho > 0 \tag{11}$$
$$\theta < 1 \tag{12}$$

then trying predation, i.e. playing t*, is the strategy preferred by T in the reduced form game given that M plays* t*.*

Proof. Consider the difference

$$U^T(r;t) - U^T(t;t) = \rho(1-\theta)(v_6 - p_2) \tag{13}$$

This is evidently strictly negative when inequalities (5), (11) and (12) hold. T thus strictly prefers playing t given that M plays t. ☐

Corollary 3.1. *If $\delta \in Q_{(\Delta,\delta)}$ and conditions (5) and (6) hold, and it also holds that*

$$\theta > \frac{1}{6} \tag{14}$$

then playing t *is a dominant strategy for T in the reduced form game regardless of the value of ρ.*

Proof. Consider the following difference:

$$\begin{aligned} U^T(r;r) - U^T(t;r) = &\rho\left(-v_{13} - 2p - b + 3v_6 - \theta\left(11v_6 + p_2 - 12p\right)\right) + \\ &+ (1-\rho)2(1-6\theta)(v_6 - p) \end{aligned} \tag{15}$$

It is clear that iff inequalities (5), (6) and (14) hold (15) is a convex combination of two negative quantities and can thus never be positive. T will thus strictly prefer playing t for all values of $\rho \in [0,1]$. ☐

Corollary 3.2. *If the inequalities (5) and (6) hold, the critical probability $\rho_c^T(\theta)$ of T is a decreasing function in θ s.t.*

$$\rho_c^T(0) = \rho_c^M \tag{16}$$

and

$$\rho_c^T(\theta) \leq \rho_c^M \tag{17}$$

with the inequality being strict for $\theta > 0$.

If $\theta \in Q_{(\Delta,\delta)}$ the critical probability is strictly *decreasing for $\theta < \frac{1}{6}$.*

Proof. See appendix A. □

Lemma 4. *If $\theta \notin Q_{(\Delta,\delta)}$ and (5) holds, then both M and T trying predation, i.e. playing* t*, is a strategic equilibrium of the corresponding games.*

Proof. It is known from lemma 2 that t is a dominant strategy for M. The expected payoffs in the reduced-form game yield

$$
\begin{aligned}
U^T(r;t) - U^T(t;t) = \rho\left(w_3^T - b - 2v_6 - p_2 + \theta(11v_6 + p_2)\right) + (1-\rho)U_2^T(\theta) - \\
- \rho\left(w_3^T + b - 3v_6 + 12\theta v_6\right) - (1-\rho)U_3^T(\theta) = \\
= \rho(1-\theta)(v_6 - p_2) + (1-\rho)\left(U_2^T(\theta) - U_3^T(\theta)\right) \quad (18)
\end{aligned}
$$

The quantity on the last line is clearly a convex combination of two negative quantities under the assumptions made and is thus negative. T will thus prefer playing t. □

There are clearly two qualitatively different possibilities as to the values of θ where the nature of the strategic equilibria changes:

1. The case of $\frac{1}{6} < \theta^*$. In this case, there are values of θ ($\theta \in (\frac{1}{6}, \theta^*)$) for which T regardless of the value of ρ finds trying predation profitable, but does not find predating profitable, should the weakest bidder be strong.
2. The case of $\theta^* < \frac{1}{6}$. In this case, there are no values of θ such that T, regardless of the value of ρ, finds it profitable to try predation but not actually predating should the weakest bidder turn out to be strong.

Note that since the value of θ^* depends only on quantities known to both M and T, M will be perfectly informed as to which of the two possibilities is the actual case.

The equilibrium strategy profile, then, does not only depend on ρ and the critical probabilities of M and T[13] but also explicitly on the value of θ.

Theorem 1. *Under the assumptions (4), (5) and (6), M's perfect Bayesian equilibrium strategy is playing* (r, r, r) *whenever $\rho < \rho_c^M$ and* (t, r, r) *when $\rho > \rho_c^M$* regardless *of the value of θ.*

The perfect Bayesian equilibrium strategy of T is the following:

1. *If $\theta < \frac{1}{6} < \theta^*$ or $\theta < \theta^* < \frac{1}{6}$ T plays* (r, r, r) *if $\rho < \rho_c^T(\theta)$ and* (t, r, r) *if $\rho > \rho_c^T(\theta)$.*
2. *If $\frac{1}{6} < \theta < \theta^*$ T plays* (t, r, r).
3. *If $\theta^* < \theta$ T plays* (t, pr, pr).

Proof. Follows from the definition of critical probability and previous lemmata and corollaries. □

Note that T's critical probability Note that since by corollary 3.2 T's critical probability ρ_c^T is bounded from above by ρ_c^M. Hence M finding it profitable to try predation is a sufficient condition for T finding the option of trying predation profitable.

[13] The equilibrium strategy depends implicitly on the value of θ, since ρ_c^T is a function of θ.

4.2.5 Comments Regarding the Model and its Assumptions

As the reader surely has noticed, the model above contains many assumptions in the form of inequalities. It should be pointed out that most of these are technical assumptions in order to assure that the players have strict preferences over their strategies. Those assumptions with an actual economic content are quite few, have very direct economic interpretations and are necessary to make the model interesting.

The convention of using strict inequalities is purely a technical detail: replacing the "strictly less than" in e.g. (12) of lemma 3 by a "less than or equal to" would result in T being indifferent between playing t and r in the game corresponding to $\theta = 0$. It would thus make the analysis bloated, apart from introducing ambiguities in the equilibrium of the game. Also, in the cases of inequalities regarding θ, it must be kept in mind that the random variable Θ has a continuous distribution, from which follows that the event of its value being in a singleton set is identically zero (in other words $P(\Theta \in \{\theta\}) = 0$ for all $\theta \in [0, 1]$). Although it is not stated explicitly in the model, one can also consider the other quantities upon which the model depends (such as ρ, b and v_{13}) as being random variables with continuous distributions. Replacing the "strictly less than" relations of the model by "less than or equal to" relations would then change the equilibrium by changing the assertion "$p \in \{\mathrm{T}, \mathrm{M}\}$ will play s" into "$p \in \{\mathrm{T}, \mathrm{M}\}$ will play s except in the case C where p will play either s or s'" where C is a zero-probability event and s and s' are the strategies between which p is indifferent.

Assumptions with more economic content appear less innocent. However, one must keep in mind that the models presented herein are attempts to explain observed events *after* they have occured. They do not aim to predict the behaviour of some hypothetical future situation, but rather to explain the rationale behind a the observed behaviour. In the light of this, the assumptions must be considered as a part of the explanation behind the behaviour. For example, one can not *a priori* know whether Mannesmann-Vodafone would find unilateral predation against a strong weakest bidder profitable or not (i.e. (4) being true or false). But no predation against the weakest bidder from Mannesmann-Vodafone was observed, hence it seems implausible that Mannesmann-Vodafone found the option of unilateral predation profitable. Assumption (4) is thus reasonable as a part of an explanation of the outcome. The same kind of argument can be applied to all non-technical assumptions.

The assumption of considering Mannesmann-Vodafone and T-Mobil as being equal, bar T-Mobil's toehold, in the sense that they share the same belief ρ and have the same valuations is arguably the most controversial assumption. This assumption is essential for theorem 1 to hold. Is this assumption reasonable?

Both companies have access to the same technologies and had about the same share of the market for 2G telecommunications services. It is less clear if they both faced the same costs. The fact that Mannesmann-Vodafone chose to reduce its demand so shortly after T-Mobil had done so points in the direction of their valuations being roughly the same. Had Mannesmann-Vodafone had significantly higher valuations (in the sense that either $v_{13}^M > v_{13}^T$ or $b^M > b^T$ or both, where the superscripted quantities denote the quantities previously denoted b and v_{13} in the case of them not being shared by M and T), there is *cetris paribus* no reason why Mannesmann-Vodafone should reduce its demand at approximately the same price as T-Mobil. The assumption of the companies being equal does thus not appear unreasonable.

Of course, share holder pressure at end the auction, exerted on all participants of the auction, makes this line of reasoning less credible. Also, one must keep in mind that as the prices in the auction rise, not only the price of the third block increases, but also the price of the two first blocks. It is not necessary to assume that Mannesmann-Vodafone's and T-Mobil's valuations of these blocks be the same in order to obtain the results of theorem 1. Rather, it would explain why Mannesmann-Vodafone chose to reduce its demand later than T-Mobil if $w_2^M > w_2^T$ but the marginal valuation of the third block is the same between the companies.

5 Discussion

What, then, can be concluded from the theoretical models of section 4 and the events described in section 3? Can the question posed in the title of this paper be answered?

Strictly speaking, one can't conclude if Deutsche Telekom had vested interests in the form of a non-zero toehold from the theoretical models and the cause of events. Nevertheless, the implications of theorem 1 are of some interest: the outcome corresponds, in the context of the models of section 4.2, to Mannesmann-Vodafone and T-Mobil both playing the strategies *(t, r, r)*. It follows from corollary 3.2 that T-Mobil's critical probability is bounded from above by Mannesmann-Vodafone's critical probability. If Mannesmann-Vodafone tries predation, then the probability that the weakest bidder is of the strong type, which by assumption is shared by both Mannesmann-Vodafone and T-Mobil, is greater than Mannesmann-Vodafones critical probability. Hence it must also be greater than T-Mobil's critical probability. It follows that is we observe Mannesmann-Vodafone trying predation then, to the extent we believe in the predictions of the model of section 4.2, we must also expect T-Mobil to try predation *regardless* of the value of θ.

In the context of the model, T-Mobil did try predation but once the weak bidder had revealed to be strong, demand was reduced. This rules the case $\theta > \theta^*$ out, since if this had been the case T-Mobil would by theorem 1 have chosen to predate.

It appears that what can be concluded regarding the magnitude of Deutsche Telekom's toehold is that it must have been smaller than θ^*. A lower bound can not be inferred from the model and the observed behaviour.

Thus one can, as far as the model is trusted, not accuse Deutsche Telekom of *unfairly* driving the prices to high levels, since it would have done so even if no toehold had been present (as would certainly be the case if the German government held no stake in the company).

That only small values of θ are consistent with observed behaviour is not a particularly surprising result, considering that the model of Grimm, Riedel, and Wolfstetter, which explains the outcome in terms of the weakest bidder being stronger than expected, arises as the special case $\theta = 0$ of the model presented herein. Adding a new parameter to a model which already explains a phenomenon is unlikely to add further explanatory power. What is more interesting is that the model shows that there are significant regions of the parameter space in which the toehold *does* affect the outcome.

In the light of the results developed in Bulow, Huang, and Klemperer (1999), one would expect a bidder with a toehold, i.e. T-Mobil, to be the strongest bidder. Following this line of reasoning, the mere fact that T-Mobil reduced its demand *before* Mannesmann-Vodafone, i.e. T-Mobil appearing to be the weaker bidder, would indicate that T-Mobil did *not* have a toe hold. However, the analysis developed in section 4.2 indicates that the results of Bulow, Huang, and Klemperer cannot be applied directly to a German-style auction. The reasoning is the following: the driving factor behind the outcome of stage-two subgames in the Grimm-Riedel-Wolfstetter model of section 4.1.1, namely the wish to "free ride" and acquire a third block of spectrum at a lower price in the second auction than in the first whilst still reaping the benefits of a less competitive market, remains unchanged under the assumption of a toehold for T-Mobil. In fact, for $\theta \in (0, 1]$ free riding is strictly more profitable than in the case $\theta = 0$ (i.e. no toehold), given that inequality GRW-2 holds.

How does the German-style auction compare to an auction of more conventional design in the presence of a potential toehold, e.g. a simultaneous ascending multiple-unit auction with unit-demands (such as the design chosen in the UK)? In an almost-common-value setting an auction of the conventional type, with N units for sale, the N bidders with highest valuations all pay the same price for one unit (as described in e.g. Ausubel and Cramton (2002)), namely the valuation of the bidder with the $N + 1$:th highest valuation contingent on the signals of the bidders who have revealed their signals by quitting and the inferred lowest possible signals of remaining bidders. Hence a bidder with a toehold has no opportunity to engage in price-driving behaviour. This reasoning, of course, rests on the commonly made assumption on bidders' valuations being symmetric in the signals of all participants.

The potential presence of a toehold may of course potentially deter potential bidders from entering a German-style auction. For example, in the light of the model in section 4.2, one can conclude that should the weakest bidder be of its weak type, it will not enter the auction if it places a high enough probability on the event $\theta > \frac{1}{6}$, since this leads to T-Mobil trying predation with certainty, thereby pushing the weakest bidder out of the auction.

6 Conclusions

It has been concluded that in the context of the model developed in this paper it cannot be ruled out that Deutsche Telekom had a toehold. On the other hand, it has also been concluded that if Deutsche Telekom had a toehold it was too small to significantly affect its behavious. One can thus not hold the company responsible for *unfairly* driving the prices to high levels. This result rests on the assumption of T-Mobil and Mannesmann-Vodafone being sufficiently symmetrically informed as to the strength of the weakest remaining bidder and having sufficiently identical marginal valuation of a third block of capacity.

It is further suggested that if a set of auction rules where trhe strong bidders have less influence over the price, such as rules where bidders bid for entire licenses, rather than parts of them, and the number of licenses are exogenously determined, the case for accusing Deutsche Telekom of driving prices to high levels is weakened further.

Finally, it is concluded that under certain circumstances, the possibility of a toehold will deter potential bidders from entering a German-style auction.

A Proofs

Proof of corollary 3.2. Set (15) equal to zero (i.e. consider the value of ρ which, given that Mannesmann-Vodafone plays r, makes T-Mobil indifferen between playing t and playing r) and take the total derivative with respect to θ to obtain the following equation:

$$\begin{aligned}\rho_c'^M(\theta)\left(-v_{13}-b+3v_6-2p-\theta\left(11v_6+p_2-12p\right)\right)+&\\ \rho_c^M(\theta)\left(12p-11v_6-p_2\right)-\rho_c'^M(\theta)\left(1(1-6\theta)(v_6-p)\right)&\\ -12(v_6-p)(1-\rho_c'^M(\theta))=0& \quad (19)\end{aligned}$$

This can be re-written as

$$\rho_c'^M(\theta)=-\frac{\rho_c^M(\theta)(v_6-p_2)-12(v_6-p)}{-v_{13}-b+3v_6-2p+\theta\left(12p-11v_6-p_2\right)-2(1-6\theta)(v_6-p)} \quad (20)$$

It follows from inequalities (5), (6) and the assumption $\theta<\frac{1}{6}$ that the denominator as well as the numerator of the fraction in the rhs of (20) are strictly negative (which in case of the numerator is shown by considering the upper bound given by the case $\rho_c^M=1$). It follows that the entire rhs is strictly negative. Hence the derivative of $\rho_c^M(\theta)$ is strictly negative and it follows by a well known result from calculus that the function is strictly decreasing.

By construction the game becomes entirely symmetric in the limit $\theta=0$. It follows that a reasonable requirement is that the boundary condition (16) be satisfied.

The bound (17) follows from the boundary condition (16) and the fact that $\rho_C^M(\theta)$ is strictly decreasing. □

B Some Tables

M \ T	r	pr
r	$(U_2^T(\theta),U_2^M)$	$(U_3^T(\theta),U_3^M)$
pr	$(U_3^T(\theta)+v_6'-p_2',U_3^M-v_6'+p_2)$	$(U_3^T+\theta(v_6'-p_2'),U_3^M-v_6'+p_2')$

Table 3: Payoffs in the *Stt*-game.

Strategy \ Game	Srt	Str
r	$(U_2^T(\theta), U_2^M)$	$(U_2^T(\theta), U_2^M)$
pr	$(U_3^T(\theta), U_3^M)$	$(U_3^T(\theta) + v_6' - p_2', U_3^M - v_6' + p_2')$

Table 4: Payoffs in the Str and Srt-games.

	$\Delta < 0$	$\Delta = 0$	$\Delta > 0$
$\delta > 0$	$Q_{(\Delta,\delta)} = (\theta^*, 1]$	$Q_{(\Delta,\delta)} = (0, 1]$	$Q_{(\Delta,\delta)} = [0, 1]$
$\delta = 0$	$Q_{(\Delta,\delta)} = \emptyset$	$Q_{(\Delta,\delta)} = \emptyset$	$Q_{(\Delta,\delta)} = [0, 1]$
$\delta < 0$	$Q_{(\Delta,\delta)} = \emptyset$	$Q_{(\Delta,\delta)} = \emptyset$	$Q_{(\Delta,\delta)} = [0, \theta^*)$

Table 5: The set $Q_{(\Delta,\delta)}$ in relation to the signs of Δ and δ.

M \ T	r	t
r	$(U_2^T(\theta) + 2(v_6 - p) - 12\theta(v_6 - p), U_2^M + 2(v_6 - p))$	$(U_2^T(\theta), U_2^M)$
t	$(U_2^T(\theta), U_2^M)$	$(U_2^T(\theta), U_2^M)$

Table 6: Payoffs in the reduced form game in the case of the weakest bidder being of the strong type and $\theta \in Q_{(\Delta,\delta)}$.

M \ T	r	t
r	$(U_2^T(\theta) + 2(v_6 - p) - 12\theta(v_6 - p), U_2^M + 2(v_6 - p))$	$(U_3^T(\theta), U_3^M)$
t	$(U_2^T(\theta), U_2^M)$	$(U_3^T(\theta), U_3^M)$

Table 7: Payoffs in the reduced form game in the case of the weakest bidder being of the strong type and $\theta \notin Q_{(\Delta,\delta)}$.

M \ T	r	t
r	$(w_2^T - 2p + 12\theta p,$ $w_2^M - 2p)$	$(w_3^T + b - 3v_6 + \theta(11v_6 + p_2),$ $w_3^M + b - 2v_6 - p_2)$
t	$(w_3^T + b - 2v_6 - p_2 + \theta(11v_6 + p_2),$ $w_3^M + b - 3v_6)$	$(w_3^T + b - 3v_6 + 12\theta v_6,$ $w_3^M + b - 3v_6)$

Table 8: Payoffs in the reduced form game in the case of the weakest bidder being of the weak type. These are the same regardless of whether $\theta \in Q_{(\Delta,\delta)}$ or not.

References

Ausubel, L. M., and P. Cramton (2002): "Demand Reduction and Inefficiency in Multi-Unit Auctions," Working Paper no. 96-07, Dept. of Economics, University of Maryland.

Börger, T., and C. Dustmann (2002): "Rationalizing the UMTS Spectrum Bids: The Case of the UK Auction," Working paper.

Bulow, J., M. Huang, and P. Klemperer (1999): "Toeholds and Takeovers," *Journal of Political Economy*, 107(3), 427–454.

Bulow, J., and P. Klemperer (2002): "Prices and the Winner's Curse," *RAND Journal of Economics*, 33(1), 1–21.

Ewerhart, C., and B. Moldovanu (2001): "A Stylized Model of the German UMTS Auction," Unpublished — paper provided by Sonderforschungsbereich 504, University of Mannheim in its series Sonderforschungsbereich 504 Publications with number 02-07.

——— (2002): "The German UMTS Design: Insight from Multi-Object Auction Theory," *IFO Studien*, (1), 157–174.

Gilbert, R. J., and D. M. Newbery (1982): "Preemptive Patenting and the Persistence of Monopoly," *American Economic Review*, 72(3), 514–526.

Grimm, V., F. Riedel, and E. Wolfstetter (2002): "The Third Generation (UMTS) Spectrum Auction in Germany," *IFO Studien*, (1), 124–144.

Jehiel, P., and B. Moldovanu (1996): "Strategic Nonparticipation," *RAND Journal of Economics*, 27(1), 84–98.

——— (2000): "License Auctions and Market Structure," Discussion paper.

——— (2001): "The European UMTS/IMT-2000 License Auctions," Unpublished — paper provided by Sonderforschungsbereich 504, University of Mannheim in its series Sonderforschungsbereich 504 Publications with number 01-20.

Klemperer, P. (1999): "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys*, 13(3), 227–286.

——— (2001): "What Really Matters in Auction Design," Unpublished — paper provided by Economics Working Paper Archive at WUSTL in its series Microeconomics with number 0004008.

——— (2002): "How (not) to run auctions: The European 3G telecom auctions," *European Economic Review*, 46(4–5), 829–845.

Milgrom, P. (2000): "Putting Auction Theory to Work: The Simultaneous Acending Auction," *Journal of Political Economy*, 108(2), 245–272.