Complementarity in language : toward a general understanding

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Complementarity in Language
Toward a General Understanding

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A latest proof of the paper is what follows.

Abstract

Ever since its first conception in 1907 by Bergson, "complementarity" has come to represent extremely wholistic situations, for which fragmentability into parts turn out unsuccessful. In 1927, Bohr used the term complementarity within quantum mechanics, with profound consequences, for a principally unsuccessful fragmentability into independent observability and definability concepts.

The paper objectifies language, in a very general understanding, as a complementaristic phenomenon. Language is thereby conceived as a whole of description and interpretation processes, such that fragmentation in these parts is in principle impossible within the language itself, but possible in a metalanguage if one such exists. The linguistic complementarity is an ultimate form to which particular complementarity conceptions can be reduced.

In a basic understanding, the linguistic complementarity refers to the impossibility of describing the constituents of a language, its description and interpretation processes, in the language itself. As such, the complementarity obtains for every language, from genetic language over programming and formal languages, to external communication languages. The argument is based on a factual function of every language, namely to admit communication or control, whereby descriptions are bound to be finitely representable and static, whereas the corresponding interpretations may be infinite of any order as well as dynamic.

Further understandings of the linguistic complementarity are developed by utilizing specific knowledge of languages. With reference to languages for formal set theories, we develop the complementarity as a tension between describability and interpretability. With reference to a processual function concept, with origins in recursive function theory and lambda calculus, we develop complementarity in terms of the unavoidable partiality of the self-references that a language may permit.

The reducibility, to the linguistic complementarity, of the specific complementarity conceptions by Bergson and Bohr is investigated with positive results. For the reducibility of Bohr complementarity, as a tension between definability and observability, to the linguistic complementarity, as a tension between describability and interpretability within a language, we develop observability as interpretability in an observation language. Furthermore, we suggest that the self–reference problem for quantum mechanical measurement be naturally resolved in terms of the linguistic complementarity, thereby pointing at a possibility of developing linguistic models for quantum mechanics, for which mechanistic models do not suffice.

It is suggested that the real value in finding entities complementary may not be fully revealed until a reduction is carried out to the complementarity for an embracing language. Not until then, we may know if the complementarity is transcendable or not, and understand the possibilities which correspond to a weighing of describability against interpretability.

Contents

1 *Introduction*

It is my impression that complementarity, in its early conceptions as well as in current parlance, refers to situations where fragmentation does *not* succeed. Such a — *wholistic* — situation may be thought of in different ways, and apparently distinct views of complementarity do result. However, with reference to an ultimately wholistic conception, one of language, a general concept of complementarity is reached.

Fragmentation is what we use in description as well as in perception and conceptualization. Every description, even a whole descriptive theory, is a description of something, not of everything (a description of everything must describe also how it is to be interpreted, or understood, and there can be no such complete description according to the linguistic complementarity to be developed in the paper). Every perception is a local affair taking place in the mind of a body in individual interaction. Every concept, however general, comprehends the attributes of some class. Were it not for the remarkable property of nature that it allows fragmentation, as in the isolation of a particular physical phenomenon in the experimental set up for measuring an observable, or when we become conscious of a particular phenomenon as target for description, every attempt at describing nature would fail. Chew (1968; page 763) explains as follows:

> "A key discovery of Western culture has been the discovery that different aspects of nature can be individually 'understood' in an approximate sense without everything's being understood at once. All phenomena ultimately are interconnected, so an attempt to understand only a part necessarily leads to some error, but the error is sufficiently small for the partial approach to be meaningful. Save for this remarkable and far from obvious property of nature, scientific progress would be impossible."

The question whether it is the fragmentability of nature that causes its describability, or conversely, if it is our ways of describing nature that make us see it as fragmentable, is to be answered in terms of a mutual interdependence.

Notice, that the perspective does not rule out wholistic situations which are not fragmentable, not fully describable, yet — *sic* — conceivable. This is where the concept of complementarity has its place.

Depending upon whether an attempted — but failing — fragmentation is thought of primarily in ontological and semantical terms, or in epistemological and descriptional, apparently different complementarity views result. Yet, the fragmentation types are not independent, and an autological closure onto language, in its ultimate wholistic conception, will yield a general type of complementarity, to which others can be reduced.

This is the *linguistic complementarity* (see also Löfgren 1984, 1988, 1989), which refers to language as an ultimate whole, and its nonfragmentability within itself into parts, descriptions and interpretations — which may yet be fully described if a metalanguage is available.

One way of looking at this complementarity is to say that the descriptions and interpretations, which constitute a language, are "complementary parts" of the language, i.e., parts that are not visible in the language itself in the sense that they cannot be fully described as such parts there — only in a metalanguage provided one such exists.

Another way of looking at the linguistic complementarity is to conceive of it as a tension between describability and interpretability within a language.

A third, with reference to the necessary partiality of self–reference in any language, conceives of complementarity in terms of a processual function concept. The essential reference function of a language (reference by description, reference by interpretation) is too complex to be modelled in the classical way mathematical functions are perceived, namely in terms of a mapping between a predefined domain and range. However, with a processual conception of the reference function, whereby processes (like computation processes) can be referred to in the interpretation of function descriptions, a nondestructive view of the linguistic complementarity is maintained.

That various such aspects of the linguistic complementarity may be developed is natural. The complementarity itself refers to a unique phenomenon, the autological nonfragmentability of language. Yet, since any description, and in particular one of complementarity (or of language in its complementaristic conception) requires fragmentation, we understand that a complete description of complementarity is not possible (in the same sense that no language can be completely described in itself). Still, with reference to some agreed–upon conceptualization, like the processual conception of functions, or other metamathematical concepts like describability and interpretability, we may provide quite satisfactory aspects of the phenomenon of complementarity.

Even if some such metamathematical concepts may not be interdisciplinary well known due to the fragmentation of knowledge, there is one property of language that may be argued within any discipline, a property stemming from the role of any language as a means of communication or of control. Accordingly, descriptions (theories, programs, DNA–strings) are always finitely representable and locally independent of time, whereas what the descriptions describe, the interpretations (models, computer behaviours, phenotypes), may be infinite of any order as well as dynamic. This is a decisive point, which is at the root of our knowledge of lack of knowledge, as well as of our conception of language. *This property of any language, is alone responsible for its complementarity* — in a crudest conception like the impossibility of giving a complete description of the interpretation processes of a language in the language itself. The further knowledge we have of languages, however, the more can that knowledge be used for a further study of complementarity, like for example in terms of the partiality of self–reference. But for no language can a complete description of its complementarity be formulated in the language itself, and the self–reference which is possible in a language is always partial.

There are contexts of an apparently nonlinguistic nature, within which the noun complementarity has been suggested. As we are about to argue in a following section, such uses of complementarity do, after all, presuppose an underlying linguistic context and are in fact reducible to the linguistic complementarity. Or else, they are reducible to the simple set theoretic notion of complementary sets — for which there is no need for a complementarity concept. Indeed, in set theory, the noun complementarity is not even defined; cf Section 3.

According to Grand Larousse (1972), the first occurrence of the noun complémentarité (complementarity) is in Bergson (1907), translated into English in Bergson (1911).

Bergson here uses the term complementarity in various contexts of attempted fragmentation. By way of example, in his account of Leibniz' metaphysics for monads (Bergson 1911; page 351):

> "the real Whole has no parts, but is repeated to infinity, each time integrally (though diversely) within itself, and that all these repetitions are complementary to each other."

Indeed, a metaphysical way of suggesting a whole which both has, and does not have, parts. In a later section, we will explain in terms of the linguistic complementarity.

Later on, in 1927, Bohr uses the term complementarity, in his "Como–lecture", in explaining a characteristic incompleteness situation in quantum mechanics, thus a context not as wide as, and more well defined than, those of Bergson — although not without ontological innuendo. Furthermore, he quests for a "theory of complementarity"; cf Bohr's written account of the Como–lecture, Bohr (1928; page 580):

> "Indeed, in the description of atomic phenomena, the quantum postulate presents us with the task of developing a 'complementarity' theory the consistency of which can be judged only by weighing the possibilities of definition and observation."

At this time, however, the epoch making discoveries of the thirties concerning definability and formalizability within mathematical logics and metamathematics had yet to come, and no further understanding beyond Bohr's primary view on complementarity developed. That is, with respect to epistemological insights.

It must be mentioned, however, that von Neumann already in 1925 had a clear view of the shortcomings of a purely descriptional and definitional attempt at set–theory (cf the end of section 3.4). Furthermore, both his axiomatic formulation of quantum mechanics in 1932, and his later automata studies, reflect a fundamental understanding of the self–referential possibilities, and limitations, of formal describability. All these contributions of von Neumann (cf also sections 4.3 and 6.3) are essential for the understanding of complementarity itself, let be that he did not use this very term. Neither did Gödel use the term, when he in 1931 provided his fundamental insight into the incompleteness of formal systems. Yet, his very conception of formal system, on which their incompleteness is based, shows a complementaristic feature as will be explained in section 4.2.

Within quantum mechanics, Pauli (1928) suggested a use of the term complementarity, with acknowledgement to Bohr, however in a sense that abstracted away essentials from Bohr's epistemological insights (section 5.4). Namely, for the noncompatibility of observables corresponding to the noncommutability of operators in Hilbert space. This is indeed a central idea of quantum mechanics which, however, today is more frequently referred to by terms like noncompatibility and noncommutability, than by complementarity.

In his primary view, Bohr maintains a genuine aspect of complementarity. Yet he is able to narrow the concept down, from ontological contemplations, to a tension or weighing between definability and observability, where observability is developed in terms of physical measurability. Even so, in arguing his position, Bohr ultimately refers to the problem of subject and object (Bohr 1928; page 590):

> “Indeed, we find ourselves here on the very path taken by Einstein of adapting our modes of perception borrowed from the sensations to the gradually deepening knowledge of the laws of Nature. The hindrances met with on this path originate above all in the fact that, so to say, every word in the language refers to our ordinary perception. In the quantum theory we meet this difficulty at once in the question of the inevitability of the feature of irrationality characterising the quantum postulate. I hope, however, that the idea of complementarity is suited to characterise the situation, which bears a deep–going analogy to the general difficulty in the formation of human ideas, inherent in the distinction between subject and object.”

In a later section, where we relate definability to describability, and observability to interpretability, we will look upon Bohr’s primary view on complementarity as a special case of the linguistic complementarity.

At this point, the reader may have noticed that we, when talking of language, are not referring to its everyday meaning as a set of sentences or descriptions endowed with grammar or syntax. Rather, we have its *wholistic* conception in mind, namely as a phenomenon of sentences *with* their meanings. After all, a sentence is not a sentence if not associated with a meaning. Likewise, a description is not a description if not associated with an interpretation and, again, a theory without model is like an artificial game devoid of epistemological significance.

Within this general conception of language, which includes formal languages, programming languages, genetic language, as well as natural communication languages, the context which naturally embeds a particular language may be known (in a metalanguage) to such an extent to be influential for what further understanding of its complementarity that may appear natural.

By way of example, formal languages are naturally studied in terms of their reflexive powers. Accordingly, their complementarity, as a tension between describability and interpretability, may here be further understood with reference to the metamathematics of the partiality of self–reference.

For languages like genetic language, where genotype and phenotype is the biological way of speaking of descriptions and interpretations, or programming languages with programs and computer behaviour the respective counterparts, it may be natural to try to understand the complementarity in terms of available metalanguages, i.e., in terms of hierarchies of language. Compare the hierarchical structure of the whole epigenesis complex, or how a computer language connects from below to the metamathematical knowledge of hierarchies of formal languages — and how these in turn connect to the hierarchy of inner cerebral languages leading up to phenomena of consciousness.

Quantum mechanics is, in virtue of its characteristic self–referential measurement problem, a phenomenon closing in on the phenomena of language. Of particular interest is to see how some physicists, in trying to resolve the measurement problem, are considering extensions all the way up to phenomena of consciousness of an ultimate human observer. As we will argue in a later section, extension into language, in its wholistic conception of course, is what suffices. Whether such complex languages are considered, that phenomena of consciousness are developed, or only simpler forms of languages, is of no vital importance. What is important, however, is the *complementaristic* nature of any language.

2 *Terminology relating to Describability, Interpretability, and Language*

An abstract noun like describability (interpretability) is formed from the adjective describable (interpretable), essentially by addition of the suffix "-ity". In general, this suffix is used to form abstract nouns from adjectives, the nouns denoting state, condition, quality, or degree, as in, by way of further examples, computabil*ity*, complementar*ity*. Knowledge of such an abstract noun, as knowledge of computability, involves knowledge of computable objects as well as of noncomputable and the suffixed noun is often more complex than its forming adjective.

Unlike computability, which is invariant over programming languages, describability and interpretability are frequently used with reference to some particular language. For example, describability (interpretability) in a language L, refers to the possibility of being describable (interpretable) in L.

The negated states, nondescribability and noninterpretability, may likewise refer to some particular language. However, they are also used in a sense that is independent of the choice of some particular language. For example, nondescribability (noninterpretability) may refer to the impossibility of being describable (interpretable) in *any* language.

To clarify the issues, we will use a terminology which distinguishes between processes for the generation of descriptions and interpretations as input–directed processes, as enumerative processes, and as autonomous linguistic processes.

A *description process in* L refers to a process, in the language L, which produces, or fails to produce, descriptions to exposed objects (with which it interacts). If the description process converges into a description, i.e., a description is generated, the exposed object is describable in L.

An *interpretation process in* L refers to a process, in the language L, which produces, or fails to produce, interpretations to exposed objects, namely sentences (with which it interacts). If the interpretation process converges into an interpretation, the exposed sentence is a description and thus interpretable in L.

A *sentence–enumeration process for* L refers to a generation, in an enumerative sense, of sentences as well formed objects, suitable for exposure to an interpretation process in L . If, for an exposed sentence, the interpretation process converges into an interpretation, the sentence is a description, and thus interpretable in L. In some languages, like the natural language, there are sentences for which no interpretation process converges, i.e., sentences which are not descriptions.

An *object–enumeration process for* L refers to the production, in an enumerative sense, of objects, suitable for exposure to a description process in L. If, for an exposed object, the description process converges into a description, the object is describable in L. Even though the domain of interpretation may be nondenumerable, like the set of real numbers, only an enumerable subset of the real numbers, like the computable reals, can be exposed as inputs to a description process.

Although both description processes and sentence–enumeration processes may produce descriptions, the essential difference is that a description process works on an object, exposed to it, attempting to describe it, whereas a sentence–enumeration process for L generates — in an enumerative sense — sentences irrespective of what they may describe. A corresponding difference obtains between interpretation processes and object–enumeration processes.

A description–interpretation process in a language system LS refers to a systemic process, which is *autonomous* in the sense that there are no particular items, like objects or sentences, predetermined on which to work. Rather, the system fragmentizes, out of the environment with which it is in interaction, *relevant* items on which it focuses the description and interpretation processes in a language L in LS (the environment may include parts of the linguistic process itself). This is a highly complex inductive activity, where objects that are relevant for description on an L–level, may be generated as existential perceptions on a previous level in LS (see Löfgren 1977).

At this point, we want to stress a difference between the *classical view of language* as a reference relation between preconceived sets of descriptions and interpretations, and *language in its processual conception*, namely as a description–interpretation process (a complementaristic conception of section 4).

According to the classical view, knowledge of language is knowledge of the reference relation with its domain and range. It is compatible with a complete picture of the reference relation as a set of ordered pairs of descriptions and interpretations, representable at will as an enumeration of the set of all descriptions in the language together with the related interpretations. In this complete picture, all pairs of the reference relation are comparable, and a notion like relevant pairs is irrelevant.

By contrast, in the processual conception, knowledge of language is primarily knowledge of the description and interpretation processes — through their generative structures. The linguistic behaviour is the behaviour of these structures and, as such, of larger complexity than that of the structures (see Löfgren 1987b). As we are about to see, knowledge of language in terms of knowledge of its generative structure is akin to a complementaristic conception. For example, to know if an entity is describable requires an implementation of the description structure *and using* it to see if it converges for the exposed entity and, if so, to what. The domain and range of the reference relation are thus here determined by the structure, whereas they are primary in the classical conception of language.

In a natural sense, the processual view of a language L determines complexity impressions such that, for example, an entity is deemed complex if the description process, resulting by exposing the entity to the description structure, has a long duration (cf Löfgren 1987b). If the process does not converge at all, which may be known on a higher level in LS, the process better not be tried on the entity which, on the higher level may be deemed irrelevant for investigation in the object language L, and thus be fragmentized away.

The fragmentation occurring in LS is a reflexion of an evolutionary process whereby the language system and its natural environment become adapted. Features of the environment, which are vital for the species of planning organisms that interact with themselves and the environment, become predictable in their language system to the same extent that the system — in its actual processual form — is adapted to the environment (which includes the organisms).

On the lower levels of the language system LS, we have *observation languages*, aiming at finding relations between data generated by sensors, or by measurement. In each case, the observation language faces self–reference problems when directed at the goal of a complete description of the sensing, or the measuring, process. This is due to the complementaristic nature of language, and will be further elaborated in section 6.3 on the quantum mechanical measurement problem.

In fact, also the above use of both language and language system, in dealing with fragmentation and relevance, is due to the linguistic complementarity — which, basically, refers to the impossibility of fragmentizing a language within itself, but where the description and interpretation parts may be fully seen in a metalanguage.

3 *From Set Complement to Complementarity*

As suggested in Section 2, the noun "complementarity" is more complex than its forming adjective, complementary (complemental). Let us illustrate the point in terms of a complementarity formation out from the set–theoretical adjective complementary.

In elementary set theory, the complement of a set is relativized to a universal set which is part of the theory. Two disjoint sets, the union of which is the universal set, are said to be complementary, or complemental, and there is no need to set this part–whole relation apart as a complementarity, simply because both parts and whole are on a par within the theory as well conceived sets. In set theory, complementarity is not even defined — in spite of its verbal affinity to the adjectives complementary and complemental.

A real need for introducing complementarity within the set conceptions does not occur until the conceptions are allowed to become wholes of such a magnitude that the above universal sets no longer can be retained as sets within the theory.

By way of intermediary example, consider a whole that comprizes all sets. As we are about to see, that collection cannot itself be a set. It is a proper class, the conception of which requires a more powerful language than that of a pure set theory. We have here a whole, the class of all sets, which is beyond the nature of its parts, the sets, and the formation of complements is no longer a set–theoretic operation. Yet, with a class theory at hand, we are able to handle the complement of a set (of sets), namely as a class.

In progressing from set to class there is, however, a natural further step, namely to language. This step is final in the sense that we have to conceive of language in — language. This is the so called autological predicament, which is unavoidable as soon as we start reflecting upon what we are doing as the reflecting creatures we are. True that there are lower forms of language that can be described in higher languages. But at every moment of development we are confined to some language as a furthest tool of conception and description. That language plays the role of an ultimate universal, which is unattainable for description — namely within itself — and a genuine concept of complementarity suggests itself.

The linguistic complementarity refers to a whole, the language itself, the constituents of which, descriptions and interpretations, cannot be fully seen (described) in the language. In this section on sets, we want to explain this subtle point by recalling the basic set–symbolism

$$S = \{x\colon\ Px\}$$

with the observation that we have here — in one and the same formula — both description and interpretation represented. The predicate Px is a description, and its interpretation is the set S. Or, Px is interpreted (extensionally) as the set S.

This is an exceptional state of affairs within mathematics, which otherwise tends to use formalisms where interpretations are abstracted away (together with complementarity). Theoretical physics, in particular quantum mechanics, is another (exceptional) area which is in need of representations, within a single theory, of both descriptions (like the state–function ψ and its determining differential equations) and interpretations (of ψ in terms of measurement equations). As expected, quantum mechanics is a domain which is germane for complementarity conceptions.

The above set–symbolism may be looked at as a symbolization of *comprehension*, namely of how to comprehend, or interpret, the description Px as a set S. In early set theory it was thought that every predicate Px which could be formulated in a language could also be interpreted in it as a set. But, with a beginning with Russell's predicate, $x \notin x$ (the set x is not an element of itself), which cannot be comprehended as a set, it was gradually understood that (set–)comprehension itself ought to be the object of set–theoretical analysis. In other words, a goal was to reach a set theory, in a formal language for sets, which describes also its comprehensions (set–interpretations).

The goal may be compared with a situation in quantum mechanics where one tries to describe physical phenomena which include their measurement processes (their interpretation processes; cf section 6.2). In view of the linguistic complementarity difficulties are likely to occur in these situations, because in no language can its interpretation process be completely described in the language itself.

The view of the linguistic complementarity as a tension between describability and interpretability within a language, is well apt for understanding set theoretic developments in formal language. In particular, concerning description of set–comprehension, as in axioms of comprehension, and the necessary incompleteness of decriptions of infinite sets, as in the axiom of infinity.

3.1 *The Tension Aspect of the Linguistic Complementarity*

Naturally, describability as well as interpretability are language dependent, and may be increased by passing from one language to a higher language. But also *within* a language, describability and interpretability may be varied — but here in *opposite* directions.

Let us use the following terminology, with reference to the one already introduced in Section 2. *Within* a language, describability and interpretability may be varied with varying fragmentation, obtained by variation of the sentence and object processes (short for sentence–enumeration, and object–enumeration, processes). If the syntactical criteria of well–formedness are widened, such that further sentences (theories) are allowed as inputs to the interpretation process, and this process converges for some of these new inputs, then the *interpretability is increased*. If the object process is widened such that further inputs to the description process are generated, and this process converges for some of these new inputs (not necessarily as complete descriptions but at least by producing some of the properties of the new object), then the *describability is increased*.

In the processual view of language any finite string of letters may be tried as input to the interpretation process. Most such strings will be rejected on the ground of syntax error. That is, they will not produce interpretations because of violation of syntax rules which are built into the interpretation process. Strings which pass the syntax rules may, or may not, lead to convergent interpretation processes, i.e., to interpretations.

By way of example, if $x \notin x$ is allowed as a syntactically well formed string and fed (as part of a set theory) to the interpretation process in a set–language (a language where the intended interpretations are sets), the process will not converge. For if it did converge in a set, say R, a contradiction arizes: $R \in R$ would imply $R \notin R$, and $R \notin R$ would imply $R \in R$.

In general, there is no way to exclude, only by further syntactical restrictions, just those strings (theories) which make the interpretation process diverge. In the above example, if $x \notin x$ were excluded from interpretation, the effect would be that we also excluded some consistent theories which do describe sets which are self–belonging ($S \in S$) as well as sets which are not.

We may view this situation as a tension, or mutual opposition, between describability and interpretability within a language. In a language, describability and interpretability may be varied with varying fragmentation (obtained by variation of the sentence and object processes). The larger the describability, i.e., the larger the domain of objects for exposure to the description process, the smaller the interpretability (understandability) of the descriptions produced (which become more partial).

> *The Tension Aspect of Complementarity.* There is a tension, or mutual opposition, between describability and interpretability within a language. An increasing describability, like covering more of the interpretation process as object for description, implies a decreasing interpretability (understandability) of the descriptions. Conversely, a decreasing describability allows increasing interpretability. In particular, no language can describe its own interpretation process. That is, if the object process for a language L is made so wide as to include also the interpretation process in L as an object relevant for description, then the description process in L will not converge for that object (but may generate partial descriptions of the interpretation process).

Let us see how the developments in set theory support the tension aspect of complementarity for formal set languages.

To argue this point for general languages, we refer to Löfgren (1988). In this general case we have to look into the nature of the object–enumeration process, and distinguish between two cases. One, for a mathematical or set–theoretical context like the one considered here. The other, for a context of natural science, where the objects are exposed to the description process through observation or measurement processes involving a lower level observation language.

3.2 *Comprehension Versus Describability — an Example of the Tension Aspect*

In the set theories we meet different ways of dealing with the problem of set interpretability of predicates. Attempts at describing this interpretability have resulted in various axioms of comprehension.

Axiom of Typed Comprehension (used in Russell's theory of types; cf Mendelson 1987). All variables are here typed, such that, if $x \in y$, and x is of type n (an integer), then y is of type $n+1$. A well formed predicate must here respect this type condition. Thus, none of the predicates $x \in x$, its negation $x \notin x$, or $x \in y \;\&\; y \in x$, is well formed. Any well formed predicate Px is comprehensible as a set $S = \{x\colon\ Px\}$; if x is of type n, then S is of type $n+1$.

Axiom of Stratified Comprehension (used in Quine 1937). Any predicate Px, which is well formed in a stratified sense, is comprehensible as a set $S = \{x\colon\ Px\}$. Here the variables are not really typed, but the requirement of stratification on Px means, essentially, that in any subformula $x \in y$ of Px, it is possible to assign integers to the variables such that the integer for y is 1 greater than the integer for x. For an individual, however, and only for individuals, we have $x = \{x\}$ (which is impossible in the theory of types).

Axiom of Relative Comprehension. For any predicate Px which is well formed in a set–language without any type or stratification conditions, and with any already established *set* y, there exists a *set* S that contains just those elements x of y for which Px holds true, namely $S = \{x\colon\ Px \;\&\; x{\in}y\}$.

If y is not a set, neither is in general S.

The first two axioms try to secure set interpretability by restricting the predicates, as objects for interpretation, by syntactic criteria of well–formedness. By contrast, the axiom of relative comprehension may be looked at as a complementaristic resolution in that it refers not only to a pure descriptive part, Px (which is the only part in the first two axioms), but also to a semantic part, namely a *set* y.

With the goal to reach a *description* of *set interpretation* within a set theory, we would have to consider the axiom of relative comprehension circular in that it presupposes y as a set already in existence. The first two axioms, however, which only stipulate syntactical conditions, would be adequate as general descriptions of set interpretation provided that they were complete with respect to a natural conception of sets. They are not, however. Therefore, our interest is in relative comprehension, and *not* in trying to reach a set theory which allows a description of its interpretation — which is impossible. Rather, we want to understand this impossibility further in terms of the tension between describability and interpretability within a language. In this respect, the complementaristic view of the axiom of relative comprehension is enlightening.

The axiom of relative comprehension with a syntactic part, Px, and a semantic part stipulating reference to a set, y, shows that comprehension (set interpretation) is describable if and only if the semantic part is describable, i.e., if and only if the irreducible semantic part in the axiom can be diminished to zero. If this is the case, the axiom turns into a comprehension like that of the first two axioms with their comparatively strong syntactical constraints on well–formedness. These stronger constraints correspond to a diminished interpretability (relative to that of relative comprehension). For example, with comprehension according to either of the first two axioms, the interpretation process would not converge on inputs like $x \in x$, except possibly for the case where x is an individual — whereas many more such inputs would make the interpretation process converge in the case of relative comprehension (cf Löfgren 1979).

Thus, by a decreased interpretability we may reach increased describability like describing also comprehension (set interpretation). Or, in a perspective of fragmentation, if we diminish the interpretability, we may not have to fragment away comprehension as object of description.

Again, if we increase the interpretability too far within a language, we have to pay in terms of nondescribability. This we have already seen. If we allow $x \notin x$ unrestrictedly for set interpretation, we know that the resulting class cannot be described as a set — and this was the reason for introducing axioms of comprehension in the first place.

By way of a more subtle example, illustrating the tension view of complementarity, let us adhere to the axiom of relative comprehension. Let us try to increase the interpretability within a set language by widening well–

formedness in stretching both the syntactic part, Px, and the semantic part, $x \in y$, of the comprehension axiom. For Px we select $x \notin x$ (which is allowed in relative comprehension), and for the set y we select the set U of those sets onto which the interpretation process converges. But then, although U is a set (a denumerable set), this widened interpretability is beyond description in the language. Namely, because the relative comprehension, which is instrumental in the interpretation process, allows the production of a new set, z, which is beyond U, thus contradicting the stipulating property for U. In other words, we have widened interpretability to such an extent that the semantic part of relative comprehension, in its complementaristic conception, is beyond description in the language.

The more detailed argument is as follows. According to the premisses, $z = \{x\colon\ x \notin x \quad \& \quad x \in U\}$ is a set produced by the interpretation process. For this set we must have either $z \in z$ or $z \notin z$. If $z \in z$, it follows that $z \notin z$ and $z \in U$. Hence, it cannot be the case that $z \in z$. On the other hand, if $z \notin z$, we must have that either $z \in z$ or $z \notin U$. Hence, it must be the case that $z \notin z$ and $z \notin U$.

In other words, on the assumption that U is the set of all sets onto which the interpretation process converges, we have concluded that there is a new set z, which is not in U but still produced by the interpretation process. This contradiction shows that the set U is too complex for description in the language in question, and that the corresponding semantic part, $x \in U$, cannot be described (whereby relative comprehension ceases to be an axiom on the object level). Thus, we have just surpassed the limits on well–formedness, set by the axiom of relative comprehension. The corresponding interpretability is too wide to permit describability of set comprehension within the language.

As we have seen, attempts at describing comprehension in a theory in a set language disclose a fundamental problem that naturally leads to a complementaristic resolution. First, to assess the problem it is necessary to comprehend language as a complementaristic phenomenon with its description and interpretation processes. Second, we may well utilize the particular knowledge which is available in particular set theories to elaborate on the tension between describability and interpretability within a set language.

3.3 *Inaccessibility and Complementarity*

Another way of revealing within a set language itself its complementaristic nature is via a notion of (in)explicability and its set theoretic associate, (in)accessibility. For the notion of (in)explicability (Löfgren 1966), all peculiarities of a set are abstracted away, except its cardinal number which is characterized by the minimal number of monadic predicates needed to ensure that the elements of the set are all distinct. With P for the minimal set of monadic predicates needed to explain a set S in this cardinal sense, we say that S is inexplicable when $\mathrm{card}(P) \geq \mathrm{card}(S)$, because then we need for the explanation a cardinal which is already as great as the cardinal to be explained, and a circularity is revealed.

Considering explicability of explicability and so forth, i.e., explicability chains, together with a reverse process of production chains, we were led to a notion of explicability (Löfgren 1966) whereby a set is (in)explicable if and only if (in)accessible in a set theoretic sense developed in particular by Tarski (1938).

> *Inaccessibility.* A cardinal $\mathbf{m} \neq \mathbf{0}$ is inaccessible if and only if:
>
> (i) $\mathbf{m}$ cannot be expressed as the sum of fewer than $\mathbf{m}$ cardinals each less than $\mathbf{m}$, and
>
> (ii) $\mathbf{m}$ cannot be expressed as an exponentiation of cardinals each less than $\mathbf{m}$:
>
> $$\mathbf{p} < \mathbf{m} \quad \& \quad \mathbf{q} < \mathbf{m} \quad \Rightarrow \quad \mathbf{p}^{\mathbf{q}} < \mathbf{m}.$$

The first infinite cardinal, $\aleph_o$, is inaccessible. It cannot be expressed as a finite sum of finite cardinals, nor as an exponentiation of finite cardinals. $\aleph_c = 2^{\aleph_o}$ (the cardinal of the continuum of real numbers) on the other hand is not inaccessible, being expressed as an exponentiation of smaller cardinals. For the same reason no cardinal in the whole Cantor hierarchy of infinite cardinals except $\aleph_o$ is inaccessible.

Thus, not all sets that are naturally conceived as sets — like the set of natural numbers with its cardinal number $\aleph_o$— are accessible, or explicable. Just like some forms of circularity, and some forms of (partial) self–reference, are naturally acceptable, so are some inaccessible sets and in particular the smallest, with cardinal number $\aleph_o$. On a metalevel, it is possible to provide relative proofs of the consistency of inaccessibility, and thus to establish a weaker explanation or modelling than the classically noncircular. Namely, a complementaristic understanding of inaccessibility (inexplicability), allowing sets to exist with a set language although they cannot be completely described there beyond their mere existence.

3.4 *Interpretability of Axioms of Infinity — Complementarity Uncovered*

The existence of inaccessible cardinals greater that $\aleph_o$ cannot be proved from the ordinary axioms of set theory (even when including the axiom of choice), but may be added as an independent axiom — which can be considered as a powerful axiom of infinity.

The ordinary axiom of infinity, which is included in most set theories, only aims at the existence of a denumerably infinite set (with cardinal number $\aleph_o$). The point that this is inaccessible, has interesting complementaristic consequences concerning the *interpretability of the axiom.* A usual form of the axiom is the following.

> *Axiom of Infinity.* There exists a set w, with the empty set $\emptyset$ among its elements, such that with every contained element x, also $x \cup \{x\}$ is a contained element:
>
> $$\exists w\ (\emptyset \in w\ \ \&\ \ \forall x\ (x \in w\ \Rightarrow\ x \cup \{x\}\ \in w)).$$

The problem which interests us is: how can this *finite string of symbols* be understood, or interpreted, as describing *an infinite set?*

The problem is usually essayed as follows. To understand that w contains an infinity of distinct elements, we observe that w contains the elements $\emptyset$, $\{\emptyset\}$, $\{\emptyset, \{\emptyset\}\}$, etc, which are all distinct (according to the axiom of extensionality). For convenience we use the von Neumann set model for the ordinals, namely with 0 for $\emptyset$ and $n = \{0, 1, 2, ..., n-1\}$ for $n > 0$. Then $0 \in w$, and $n \in w \;\Rightarrow\; n+1 \in w$, because $n \in w$ implies according to the axiom that $n + 1 = n \cup \{n\}$ belongs to w. By induction on the base clause $0 \in w$, and the generating clause $n \in w \;\Rightarrow\; n+1 \in w$, we obtain $\forall n \in N\colon\; n \in w$. Thus, the set of numbers N, which is infinite, is a subset of w, meaning that w is an infinite set.

We see how the interpretation of the axiom, namely that it describes an infinite set, utilizes an induction principle — which *presupposes* an already given infinite induction structure, the infinite well ordered set N.

We may of course ask for other possible ways of carrying out the interpretation. However, the fact that the denumerable infinity aimed at is inaccessible reveals an inescapable circularity, indicating that any attempt at describing the interpretation must remain incomplete. This is, of course, what the linguistic complementarity says. Yet the complementarity says more, namely that the infinite set can be conceived, and communicated about, in a *shared* formal language. Should we be called upon to describe this shared language, we would say, in a metalanguage, that it has an inductive interpretation process. But as soon as we try to describe the inductive interpretation process in the language in which it occurs, we not only run into circularity problems as illustrated, but also into difficulties concerning uniqueness of descriptions.

An induction principle should not only have base and generating clauses (like the ones we have used above), but also an extremal clause ensuring that every set that satisfies the inductive description does not contain other elements than those inductively defined. The extremal clause is gravely difficult because of the Löwenheim–Skolem metatheorem. Namely, that if a set theory has denumerably infinite models, then it also has infinite models of any cardinality. Set theories cannot, by however elaborate axiomatization, be made categorical.

Considerations of this sort led von Neumann, in his pioneering work on set theory (1925), to conclude:

> "after all, a new difficulty appears here, one that is essentially different from those pointed out by Russell and Brouwer. The denumerable infinite as such is beyond dispute; indeed, it is nothing more than the general notion of the positive integer, on which mathematics rests and of which even Kronecker and Brouwer admit that it was "created by God". But its boundaries seem to be quite blurred and to lack intuitive, substantive meaning. Upward, in the "nondenumerable", this is quite certain in view of Löwenheim's and Skolem's investigations. Downward, in the "finite", it is at least very plausible, for categoricity is lacking, as is any foothold that would enable us to make the definition of "finite" determinate. Moreover, *even Hilbert's approach is powerless here,* since this objection does not concern consistency but the univocality (categoricity) of set theory.
>
> At present we can do no more than note that we have one more reason here to entertain reservations about set theory and that for the time being no way of rehabilitating this theory is known."

Indeed, according to the linguistic complementarity, there will never be a set language which allows complete, categorical formal descriptions of sets. As long as sets are naturally conceived as comprehensions (interpretations) of descriptions, sets are given, or exist, with their set languages — for which the linguistic complementarity prevails.

Before language, or rather before language was objectified in its complementaristic conception, it may have been natural to admit, with Kronecker and Brouwer, that N, the set of the positive integers, was created by God. Now, it suffices to say that N exists, as an inaccessible set, together with a set language which has evolved, as all languages do, without possibilities to completely describe themselves. Yet, they are conceivable as complementaristic phenomena.

4 *Complementarity and Processual Conceptions*

Although the interpretation process of a language cannot be completely described in the language itself, a language may admit partial descriptions of its interpretations. We want to relate a further understanding of the complementarity for particular languages with the partiality of their self–description.

For this study, we turn to languages where the interpretation processes can be modelled as effective processes. In this domain, we have a *processual* conception of functions developed which, unlike the classical function concept, lends itself nicely for developing self–reference and its partialities.

Also, the processual view of functions is instrumental for the very conception of complementarity in its transcendable cases. Indeed, such a further study of complementarity, like that in degrees of self–reference, do require access to a metalangauge.

It is no accident that our very conception of language, which respects the autological predicament (to conceive of language in language), builds upon description and interpretation *processes*.

4.1 *The Processual Conception of Functions*

Although sets and functions are in many respects interrelated, there is a difference revealed in the corresponding basic symbolisms. As we have elaborated upon in the previous section, in the set symbolism $S = \{x\colon\ Px\}$ we have a distinction between the set itself and its description. By contrast, such a distinction is lacking in the classical function symbolism $f\colon\ A \to B$, which simply says that f is a function, or mapping, from a domain A to a range B — with no indication of any description of the function, not even if f is to be considered in extension or in intension. Even when we talk of particular functions, like $sin\ x$, it is usually not clear whether "sin" refers to a description of the function (like how to compute it), or to the function itself (in whatever sense).

Not until we proceed from the classical mathematical symbolism to that of lambda calculus, or of recursive function theory, will we find symbolisms that reflect the fundamental distinction between a mathematical object and its mathematical description.

Lambda calculus (cf Barendregt 1981) studies functions and their applicative behaviour (and not, as in category theory, functions under composition). *Application* is a primitive operation in lambda calculus, denoted by juxtapposition such that fa symbolizes the application of the function f to the argument a. Complementary to application there is *abstraction*. Let $f(x)$ be an expression describing the determination of the value of the function at the argument x. Then $\lambda x\ f(x)$ symbolizes the function f as abstracted out from its expression, thus with a distinction between description (in terms of expression) and interpretation (abstraction of the mathematical function object from its expression).

In recursive function theory, particularly in its development by Kleene (1952), a partial recursive function is symbolized $\{e\}$, where e is a description (Gödel number) for a program that makes a universal Turing machine compute the function. Again, in this symbolism we have a distinction between the description e of a function and the function itself $\{e\}$.

The *processual conception of functions* is related with the above conceptions in lambda calculus and in recursive function theory. It takes one further step, however, in focusing on a (computation) process as having both structure (Turing machine) and behaviour (computation). We assume a process to be deterministic such that its behaviour is determined by its structure. If this determinism is appealed to in a description of the behaviour, but not itself described in the description, then we can describe phenomena that otherwise would not be possible. This is a consequence of a complexity thesis of von Neumann (1966), namely that for very complex systems it is easier to describe their structure than to describe their behaviour. This thesis is actually derivable form the linguistic complementarity; cf Löfgren (1987b).

In (Löfgren 1990a), we use a clarifying symbolism as follows. A function f in its processual conception is symbolized $[f]$. Here f is a description number of the (computation) process — in terms of its structure (Turing machine). Instead of symbolizing application like $fa = b$, we write:

$$[f]a \downarrow b,$$

with the explanation

$$[f]a \quad = \quad y\colon \ \ (T(f,a,y) \ \ \& \ \ Uy = b),$$

meaning that the application of $[f]$ to a converges to b. Or, more explicitly, the application of $[f]$ to a results in a terminal computation process, with Gödel number y, whose last segment, Uy, is b. As usual, $T(z,x,y)$ is the application predicate: "when applied to the argument a, a structure Z, with Gödel number z, performs a process Y, with Gödel number y".

4.2 *Gödel's Finite Procedure*

An early appeal to processes in a complementaristic sense can be found in Gödel (1931, 1965), where he recognizes the necessity of including the concept of finite procedure, in terms of a Turing machine computation process, in the very conception of a formal system.

In his fundamental papers from the early thirties, Gödel conceives a formal mathematical system such that its formulas, rules of inference, and axioms are "constructive". He explains this as follows (Gödel 1934, p 41):

> "That is, for each rule of inference there shall be a finite procedure for determining whether a given formula B is an immediate consequence (by that rule) of given formulas $A_1, ..., A_n$, and there shall be a finite procedure for determining whether a given formula A is a meaningful formula or an axiom."

Although the involved "finite procedure" may have been sufficiently clear from an intuitive stand point, the question remained whether this concept, too, could be formally defined, or else how to explain the involved intuition.

In a postscript to the 1934 paper, Gödel (1965) writes:

> "In consequence of later advances, in particular of the fact that, due to A M Turing's work, a precise and unquestionably adequate definition of the general concept of formal system can now be given, the existence of undecidable arithmetical propositions and the non–demonstrability of the consistency of a system in the same system can now be proved rigorously for *every* consistent formal system containing a certain amount of finitary number theory.
>
> Turing's work gives an analysis of the concept of 'mechanical procedure' (alias 'algorithm' or 'computation procedure' or 'finite combinatorial procedure'). This concept is shown to be equivalent with that of a 'Turing machine'. A formal system can simply be defined to be any mechanical procedure for producing formulas, called provable formulas. For any formal system in this sense there exists one in the sense of page 41 above that has the same provable formulas (and likewise vice versa), provided that the term 'finite procedure' occurring on page 41 is understood to mean 'mechanical procedure'. This meaning, however, is required by the concept of formal system, whose essence it is that reasoning is completely replaced by mechanical operations on formulas."

Thus, it is by including the processual concept of computation into the concept of formal system, that Gödel is able to demonstrate his incompleteness results for formal systems.

A formal system, used to develop formal theories, is thus a system that utilizes procedures (processes) — which cannot themselves be completely described by some theory in the system.

We understand this as an early complementaristic conception of formal systems or formal languages. This is the more interesting since the object of Gödel's study is purely syntactical formal systems, whereas the linguistic complementarity is about the tension between description (syntax) and interpretation (semantics). However, as pointed out in Löfgren (1989), not even an artificial language can be totally without semantics, and Gödel's analysis implies an unavoidable interpretation also for artificial languages. A rule of inference, even in an artificial language (where it is often looked upon as a production rule), is to be interpreted as a real act of producing another sentence (an act that can be performed by a Turing machine).

4.3 *The Partiality of Self–Reference*

With reference as a primary function of language, we may look at self–reference as follows. Let us say that $f(a) = b$ symbolizes that a refers to b through the reference function f. Then various forms of self–reference are: $f(a) = a$ (a refers to itself through f), $f(a) = f$ (a refers, through the reference function f, to f itself), etc, and finally, $f(f) = f$ (f refers to itself through itself).

Indeed such "formalizations" of self–reference are puzzling and will remain so until conceptualization of the f–symbols is attempted. The formalism itself does not give away any indication of how to conceive of the various f occurences, or whether the self–reference is intended as a reference by interpretation or as a reference by description. Just like a distinction between description and interpretation is necessary for the conception of language, such a distinction is equally necessary for the conception of self–reference.

In his paper on lambda calculus, Church (1941) writes as follows with a clarifying distinction between conceptualizations of functions:

> "In particular it is not excluded that one of the elements of the range of arguments of a function f should be the function f itself. This possibility has been frequently denied, and indeed, if a function is defined as a correspondence between two previously given ranges, the reason for this denial is clear. Here, however, we regard the operation, or rule of correspondence, which constitutes the function, as being first given, and the range of arguments then determined as consisting of the things to which the operation is applicable."

In recursive function theory, the other mathematical domain with a symbolization which does distinguish between a function and its description, we have the recursion theorems of Kleene (1952), from which we can derive partial self-descriptions (see Löfgren 1972, 1979, 1990a) like:

$$[f]\, x \downarrow f.$$

In other words, there is a function process that computes a description f of itself. This self–reference is partial in that it does not describe how the description f is interpreted into $[f]$.

The excluded interpretation, namely the process of going from f to $[f]$, may be cut in parts in order to help understanding how much of it can possibly be included within the self-description:

$$f \;\rightarrow\; q \;\rightarrow\; F \;\rightarrow\; [f].$$

(The reader is invited to compare with section 6.3, where we outline von Neumann's cutting of the measuring process in his discussion of the self-reference problem for quantum mechanical measurement.)

f is a description in the form of a Gödel number, and q is a first interpretation (decoding) of f into a finite sequence of quadruples. This quadruple sequence q characterizes a Turing machine structure F, such that F is a structural interpretation of q. Finally, $[f]$ is the beaviour of F, i.e., the function in its processual conception.

Here, also the first part of the interpretation steps, that of going from f to q, is within the domain of the recursion theorem. In other words, there is a function $[f']$, which describes itself better than $[f]$ describes itself:

$$[f']\, x \downarrow q'.$$

The next interpretation step, going from q to the structure F, may be looked at as a "generalized" computation (cf von Neumann 1966). It is similar to an ordinary computation process in that it can be viewed as a programmed process — but with objects that are not in themselves neutral elements from an alphabet of letters, but symbols which are real elements taken from an alphabet of machine elements such that, for example, a juxtapposition of two such elements "automatically" enforces a certain behaviour.

von Neumann (1966) suggests an extended recursion theorem, with reference to a "universal constructor" as an extension of a "universal computer", whereby the degree of self–description can be extended to encompass also the second interpretation step:

$$[f''] \, x \downarrow F'',$$

thus saying that there is a function $[f'']$, in its processual conception, which produces its own process structure F''.

The final step, going from F to $[f]$, however, cannot be entirely included within the domain of self–description but has to appeal to an undescribed complementary access to a reality which "automatically" realizes the behaviour of a structure.

In the biological domain of genetic language we have a wealth of examples of how interpretation processes in the end must utilize (environmental) behaviours which cannot themselves be described (in genotype). Compare Löfgren (1990a).

5 *Complementarity in Quantum Mechanics*

In classical physics, like in Newtonian mechanics, the fundamental formalism describes physical phenomena without objectifying the measuring processes that are subsumed in the conceptualization of the phenomena. In other words, it is assumed that the physical states can be objectively observed and measured outside the domain of classical physics not to interfere with the physical phenomena. Accordingly, classical physics can be formulated without any reference to language (such as observation language), and in the language needed for the formalism there is no trace of self–reference.

In quantum physics, on the other hand, the formalism does describe the process of measurement, together with the evolution of an unobserved state, as a physical phenomenon. Recognizing measurement as part of the interpretation process in the physical language, we understand that a self–reference problem is traceable in the foundations of quantum physics. Namely, to describe (partially) the interpretation process of a language in the language itself. That is a problem for which the linguistic complementarity provides a fundamental understanding.

Accordingly, it is not surprizing to find ideas on complementarity being developed in quantum mechanics even though, at the surface, a physical discipline may appear nonlinguistic. In fact, quantum mechanics has turned out to provide a real stimulus for an independent development of ideas around complementarity, in particular as a means of understanding the self–referential measurement problem — which remains an outstanding problem in quantum philosophy.

It should be noticed that *within* quantum mechanics the term complementarity was being used in different senses already from the start. Compare Bernays (1948, p. 66):

> "Schon in der Quantenmechanik tritt der Terminus 'komplementär' in verschiedener Art der Anwendung auf."

After exposing various lines of development of complementarity in quantum mechanics, we will in the next section argue that Bohr's primary view of complementarity, which started the developments in quantum mechanics, is related to the linguistic complementarity.

5.1 *Bohr's Como Formulation of Complementarity; The Primary View*

Although the use of the term "complementarity" in quantum mechanics is traceable back to a unique event, the Como Congress in 1927 where Bohr introduced it, his underlying philosophy may have been in development over several previous years (cf Mehra 1974). Perhaps was Bohr familiar with Bergson's earlier use of the term complementarity in philosophy, but I have not seen him refer to Bergson and, after all, connections between their uses of the term are not very obvious (compare, however, Section 6). After 1927, Bohr continued to develop his understanding of complementarity, in particular in terms of "phenomena".

Throughout the development, however, he sustained one view which is clearly seen, and formulated, in the written account, Bohr (1928), of the Como lecture. From the first page of this paper we quote:

> "This [quantum] postulate implies a renunciation as regards the causal space–time co-ordination of atomic processes. Indeed, our usual description of physical phenomena is based entirely on the idea that the phenomena concerned may be observed without disturbing them appreciably.
>
> "Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed.
>
> "This situation has far–reaching consequences. On the one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible, and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there can be no question of causality in the ordinary sense of the word. The very nature of the quantum theory thus forces us to regard the space–time co-ordination

> and the claim of causality, the union of which characterises the classical theories, as complementary but exclusive features of the description, symbolising the idealisation of observation and definition respectively.
>
> "Indeed, in the description of atomic phenomena, the quantum postulate presents us with the task of developing a 'complementarity' theory the consistency of which can be judged only by weighing the possibilities of definition and observation."

In the following, I will refer to this view of complementarity as a binary relation between definability and observability, in terms of "Bohr's primary view of complementarity".

As suggested, in our usual encounters with classical physics, definability and observability are each complete. What can be defined in physical theory can be physically observed. And, conversely, what can be physically observed can be described and defined in physical theory. But in quantum mechanics there is, due to the quantum postulate and the systemic enclosure of observation (by measurement) within the physical domain, a tension between definability (possibilities of definition) and observability (possibilities of observation). It is this relation, for which Bohr suggests a possibly forthcoming complementarity theory, that I am referring to as Bohr's primary view of complementarity.

In particular, we notice that Bohr emphasizes "the space–time co-ordination" (like in a symbolization $q(x,t)$) and "the claim of causality" (like in saying that the state q' follows causally the state q) as complementary but exclusive features of quantum mechanics. It seems important at this point to distinguish between $q(x,t)$ as a description and "the state q" as interpretation of the description, i.e., the observed state. The distinction will be explained further in subsection 5.4.

Bohr's suggestion for a possible "theory" of complementarity will be commented upon in section 6 where we compare Bohr's primary view of complementarity with the linguistic complementarity.

5.2 *Bohr's View of Complementarity in Terms of Phenomena*

In the Como paper, Bohr uses the term "phenomenon" in somewhat varied meanings but mostly for an atomic process that may be the object of observation by a distinct agency (cf the second paragraph in the quotation in section 5.1). In other words, he distinguishes between the atomic phenomenon and the agency used for its observation — even though he recognizes that "an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation".

In his later writings, Bohr changes the meaning of *phenomenon* to refer to a *systemic whole* of an atomic process in interaction with a measuring apparatus. This is for example seen in the following quotation from Bohr (1963; page 4).

> "While, within the scope of classical physics, the interaction between object and apparatus can be neglected or, if necessary, compensated for, in quantum physics this interaction thus forms an inseparable part of the phenomenon. Accordingly, the unambiguous account of proper quantum phenomena must, in principle, include a description of all relevant features of the experimental arrangement."

For example, contrary to classical physics, in quantum physics an electron *per se* is not an appropriate object. But an "electron investigated in a bubble chamber" is, as a phenomenon, an appropriate quantum physical object. Again, an "electron investigated in an interference experiment" is to be regarded *another* phenomenon — with the consequence that the wave–particle problem for electrons is resolved. This systemic way of understanding an otherwise inexplicable experience, i.e., a resolution in terms of the indivisibility of a phenomenon, has been called a *complementaristic resolution.*

Beside his primary view of complementarity (connected with the relation between observability and definability), Bohr looked upon complementarity as a relation obtaining between two phenomena. It is my impression that he did not then only refer to the indivisibility of the phenomena (a consequence of the primary view), but also to a subsumed completeness, namely that two complementary phenomena together in some sense form a complete picture.

5.3 *On the Completeness of Complementary Phenomena*

Let us compare Bohr's two complementarity relations, that between definability and observability on the one hand, and that between two phenomena on the other, with respect to completeness.

In a natural physical sense, the first relation, that in Bohr's primary view of complementarity, is complete. Within physics, i.e., within a language for physical theories, there are no further modes of activity than observation by measurement and description by definition. Hence complementarity, as a binary relation between observability and definability (within the physical language), is complete (within the language).

Complementarity between phenomena, on the other hand, which Bohr also conceives as a binary relation, may not represent completeness in the same natural sense. For example, how can we know that the two phenomena, 'electron investigated in a bubble chamber' and 'electron investigated in an interference experiment', representing the particle and wave appearances of the electron respectively, together give a complete picture. As long as we look upon physics as occupied with the wave and particle aspects of matter (or with energy and time as another possible pair), we have an impression of completeness. But these aspects are classical aspects, and we know that when passing on to the domain of quantum mechanics we must also be prepared to objectify observability which is, basically, a linguistic activity. Indeed, recent attempts at resolving the measurement problem in quantum mechanics do appeal also to the phenomenon of consciousness of an observer. In other words, in order to reach completeness for a complementarity relation between phenomena, it may be necessary to conceive of it not as a binary, but a ternary (or further many placed) relation.

In this connection, it may be appropriate to recall the following historical account, provided by Jammer, on the reactions upon Bohr's Como paper. Jammer (1966; page 354) writes:

> "According to Wigner, von Neumann had this to say on Bohr's [Como] lecture: 'Well, there are many things which do not commute and you can easily find three operators which do not commute'."

What von Neumann, as a mathematician, found important may have been Bohr's explanation of Heisenberg's uncertainty relations (see the following subsection) in terms of complementarity as a binary relation between phenomena. In suggesting that noncommutability may be a ternary and not necessarily binary relation, von Neumann may have raised doubts about a subsumed completeness.

Further insights into this type of incompleteness, leading to complementarity as a ternary relation, are suggested in Bernays (1948) and Raven (1949).

5.4 *Pauli's Conception of Complementarity Versus Bohr's*

Pauli (1928), perhaps in response to Bohr's quest for a theory of complementarity, developed a smooth mathematical theory based on a the following conception of complementarity (Pauli 1928; page 89):

> "Wenn aus diesem Grunde die Benutzbarkeit *eines* klassischen Begriffes in einem ausschliessenden Verhältnis zu der eines *anderen* steht, nennen wir diese beiden Begriffe (z.B. Orts– und Impulskoordinaten eines Teilchens) mit Bohr *komplementär*. In Analogie zum Terminus 'Relativitätstheorie' könnte man die moderne Quantentheorie daher auch 'Komplementaritätstheorie' nennen."

However, to my knowledge, Bohr himself did never use his concept of complementarity this way as a relation between two classical concepts like position and momentum. In the mathematical framework of quantum mechanics, Pauli's suggestion led to the use of the term complementarity for noncommutability of operators, or noncompatibility of observables (like position and momentum) i.e., with complementarity a well defined abstract mathematical relation. In such an abstraction, the essence of Bohr's concept of complementarity will be lost.

In order to understand this difference between Bohr's and Pauli's conceptions of complementarity, let us recall the Heisenberg uncertainty relations, which Pauli refers to, formulated in the mathematical framework for quantum mechanics. Here, operators, $\hat{A}$, $\hat{B}$, .. describe observables, A, B, .., i.e., physical quantities which can be measured by experiment. The possible results are eigenvalues of the eigenstates of the operators. When the system is in a "state", represented by the mathematical object $|\psi\rangle$, the value obtained in a measurement of an observable A is a random variable with a probability distribution. The mean of this distribution, i.e., the average value obtained in a large number of measurements on identical systems in this state is $\langle A\rangle = \langle\psi|\hat{A}|\psi\rangle$. The standard deviation, which is a measure of the spread of the results, is the *uncertainty* in A, denoted ΔA, such that: $(\Delta A)^2 = \langle\psi \mid \hat{A}^2 \mid \psi\rangle - \langle\psi \mid \hat{A} \mid \psi\rangle^2$. The Heisenberg *uncertainty relations* takes the general form:

$$\Delta A \cdot \Delta B \geq \tfrac{1}{2} \mid \langle i \, [A, B] \rangle \mid .$$

Here, $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ is the *commutator* of the hermitian operators $\hat{A}$ and $\hat{B}$ which describe the observables A and B. ($i \, [\hat{A}, \hat{B}]$ is itself hermitian, and taken to describe an observable, it is denoted $i \, [A, B]$).

Thus, if the operators $\hat{A}$ and $\hat{B}$ do not commute, i.e, their commutator $[\hat{A}, \hat{B}]$ is different from zero, the uncertainty product $\Delta A \cdot \Delta B$ is greater than or equal to a definite positive number, with the interpretation that if the precision in measuring A is increased, the precision in a simultaneous measurement of B is decreased.

The point we want to stress is that the uncertainty relation is a *consequence* of the mathematical formulation of quantum mechanics and its statistical interpretation. For a proof, we refer to Sudbery (1986, page 59). Thus, to use the concept of complementarity for noncommutability of operators (noncompatibility of observables), and to establish in this sense pairs of complementary observables — this represents a *consequence* of the quantum mechanical conception. Bohr's views of complementarity, however, are epistemological rather than mathematical, and as such *foundational* for quantum mechanics rather than a consequence.

It is my opinion that clarity is to be gained if the term complementarity in quantum mechanics were to be used for Bohr's conceptions and not for noncommutability (noncompatibility). I see no reason why these latter concepts could not be referred to by their mathematical names. And, after all, this is what seems to happen anyway. Most modern texts on quantum mechanics talk of noncommutability (noncompatibility) in these very terms, and not in terms of complementarity.

It should be mentioned that Pauli, notably in his correspondence, also used the term complementarity as a binary relation between phenomena, much like Bohr, in contexts outside of quantum mechanics. Compare Laurikainen (1988), as well as a forthcoming volume III to Pauli (1919-1939).

6 *Reducibility of Complementarities to the Linguistic Complementarity*

As surmised, all forms of complementarity which can naturally be regarded as such, i.e., as autologically nonfragmentable wholes, are reducible to the linguistic complementarity. For the involved concept of reduction, we refer to Löfgren (1987c). In this section, we will support the reducibility with specific demonstrations for the concepts of Bergson and Bohr. Furthermore, we will argue that the self–reference problem for quantum mechanical measurement can be naturally resolved in terms of the linguistic complementarity, thereby pointing at a possibility of *linguistic models* for quantum mechanics, for which *mechanistic* models do not suffice.

As outlined, complementarity refers to *wholistic* situations where fragmentation does *not* succeed. In general, a failure of an attempted fragmentation may refer to either a difficulty in conceptual experience, or to an unsuccessful description. With description the forceful link in communicating *conceptual* experience, we may have situations with a successful *conceptual* fragmentation of wholes, for which attempts at *description* is unsuccessful. The linguistic complementarity is a paradigm case. The interpretation processes of a language are not fully describable in the language, although at play there and conceivable as interpretation processes (and describable in a metalanguage, provided one such exits). Compare, from section 3.4, the *conceivability* of an infinite set of cardinal number $\aleph_o$ and the impossibility of *describing* the concept as an interpretation. That a conception is possible is due to the existence of a set language. That a complete description is impossible in the language is due to the inaccessibility of the set.

As demonstrated, the linguistic complementarity is well documented for formal languages, namely as a difficulty with descriptions of integral experiences, such as comprehension of sets and conceptions of functions.

In languages for biology and physics, it is primarily via problems of self–reference that a necessity of objectifying the phenomenon of language arizes. With language itself a paradigm case of complementarity, it is natural that independently proposed notions of complementarity in these areas of biology and physics turn out reducible to the linguistic complementarity.

6.1 *Reducibility of Bergson Complementarity*

Bergson (1907, 1911), the first to use the term complementarity, exhibited, particularly in his ideas on evolution, a marked wholistic way of thinking. In this direction, he came across concepts of complementarity, even though he did not go all the way to objectify language, as done here. Rather, he was interested in knowledge and its evolution, and talked freely of how we perceive of it and of our intuitions for it, and this with a brilliant mastery of the natural language.

In our present view, we conceive of knowledge as existing and evolving in some language. Language, in our general conception, has come into existence (by evolution) before knowledge, which is a particular form of conscious experience possible only in highly developed languages. True, that such a commitment represents a form of knowledge of language, but to be comprehensible it presupposes language with its interpretation processes which cannot be fully reduced to descriptive knowledge in the language.

Thus, when comparing Bergson's notions of complementarity with the linguistic complementarity, we have to be careful and concentrate on the introspective view in Bergson's thinking and on his intuition for the limitations of introspection — which is what he talks of in terms of complementarity.

Let us see how Bergson looks upon *time*, in its reality and in its scientific description, in terms of complementarity. This, for comparison with an independently suggested explanation of time in terms of the linguistic complementarity in Löfgren (1984).

Bergson (1911; page 342) contrasts two types of knowledge of time. One is the physical knowledge, namely with time described in a physical theory in terms of "moments of time, which are only arrests of our attention". The other refers to the flow of time, "the very flux of the real".

> "The first kind of knowledge has the advantage of enabling us to foresee the future and of making us in some measure masters of events; in return, it retains of the moving reality only eventual immobilities, that is to say, views taken of it by our mind. It symbolizes the real and transposes it into the human rather than expresses it."

I will return with comments, here only making the remark that we have before us an unmistakable linguistic situation. Namely, a description of time with an interpretation, physical time, which is but a fragment of real time. In other words, the description of physical time is but a partial description of real time.

Bergson continues (page 343):

> "The other knowledge, if it is possible, is practically useless, it will not extend our empire over nature, it will even go against certain natural aspirations of the intellect; but, if it succeeds, it is reality itself that it will hold in in a firm and final embrace. Not only may we thus complete the intellect and its knowledge of matter by accustoming it to install itself within the moving, but by developing also another faculty, complementary to the intellect, we may open a perspective on the other half of the real. ...
>
> To intellect, in short, there will be added intuition."

Further (page 344), Bergson writes:

> "The flux of time is the reality itself, and the things which we study are the things which flow. It is true that of this flowing reality we are limited to taking instantaneous views. But, just because of this, scientific knowledge must appeal to another knowledge to complete it.
>
> "In our hypothesis, on the contrary [to ancient science and metaphysics], science and metaphysics are two opposed although complementary ways of knowing, the first retaining only moments, that is to say, that which does not endure, the second bearing on duration itself."

For comparison, we refer to Löfgren (1984) where we explain the describability problem for time in terms of an autological argument for the linguistic complementarity. We first observe that it is not possible to give a complete description of time, conceived as a real dynamic flux. In short, the argument is as follows (page 10).

Assume that there is a theory T in a language L, such that T describes time completely. By this we mean that every true L–sentence about time is provable in T. It must then be possible to interpret T autologically, such that a length as well as time is associated with the proof of every true L–sentence about time. As we know from section 4.2 on Gödel's early complementaristic conception of formal systems, this is true even if T is a purely formal theory in a purely formal language L. Therefore, we may associate with a true

Gödel sentence G (saying of itself that it is not provable in T; cf Löfgren 1987a) another true sentence G^* of L with the interpretation "there is no time associated with a proof of this sentence". Here G^* is a true L–sentence about time, yet not provable in T. This contradiction against the assumed completeness of T is thus an autological argument for the impossibility of describing time completely.

On page 11 of Löfgren (1984), we suggest a distinction between *dynamic time*, referring to our intuition of real time, and *static time*, referring to a part of real time that can be captured in a formal theory of time. Concerning the origin of our time conceptions, we proposed (page 11) the following thesis.

> "*Time thesis.* Our intuitive notion of time, notably dynamic time, originates with our abilities to conceptualize changes associated with autological situations, like when describing description, when observing observation, etc."

In support we argued as follows:

> "Faced with conceptions that cannot be completely described, we tend to account for them in terms of internal cerebral processes that go beyond description in the actual language. Nevertheless, the conceptions may be seen in this language as autological projections, i.e., as fundamentals that can only be accounted for in terms of themselves. But they cannot be justified in the language where they are seen, because their explanation, or unfoldment, requires that we transcend the language in a metalanguage of a higher type.
>
> By contrast, in a non–autological world we only consider phenomena that are sufficiently regular to be completely describable, for example static time. The world–lines in an Einsteinean space–time would be an example. Here the curvature may account for gravitational forces and corresponding movements in the static way. In space–time itself nothing happens."

The two views provide very similar results. Bergson suggests that real time cannot be grasped by science without reference to a complementary metaphysics. With science referring to that which can be formally described, and metaphysics to knowledge or intuition of the real, Bergson's suggestion is almost identical with our autological result. Namely that real time cannot be grasped in any formal language *by description* but well by *complementaristic conception* (autological projection; cf the conceivability of the inaccessible $\aleph_o$ in a set language in section 3.3).

The ways of argument are different, however. Bergson is playing extensively on various mental experiences whereas I am attempting to localize the *source* of complementarity in the general phenomenon of language with its complementary descriptions and interpretations. It is this general complementaristic phenomenon which allows studies of reduction of various instances of complementarity impressions. Just like languages can be compared with respect to reducibility or translatability (see Löfgren 1987c), so can complementarities. But without taking steps towards identifying the language in which a specific complementarity occurs, it may remain in a state of isolated comprehension.

Bergson uses the term complementarity in various contexts. For the conception of time as we have just seen, for the conception of space as we have hinted at in section 1. In his account of Leibniz' space conception, Bergson (1911, page 351) continues (cf the quotation in section 1) as follows:

> "In just the same way, the visible relief of an object is equivalent to the whole set of stereoscopic views taken of it from all points, so that, instead of seeing in the relief a juxtapposition of solid parts, we might quite as well look upon it as made of the *reciprocal complementarity* of these whole views, each given in block, each indivisible, each different from all the others and yet representative of the same thing. The Whole, that is to say, God, is this very relief for Leibniz, and the monads are these complementary plane views; for that reason he defines God as 'the substance that has no point of view', or, again, as 'the universal harmony,' that is to say, the reciprocal complementarity of monads."

Here, Bergson looks upon *views* as possibly complementary with respect to a whole, chosen such that we do have a (meta) understanding of it (as a juxtapposition of solid parts). Notice, that it is only with a suppression of this metaunderstanding that the complementarity of the views arize (cf the the linguistic complementarity for a case where there is a metalanguage, i.e., when the complementarity is transcendable).

From a limited aspect of completeness, the example may be compared with Bohr's use of complementarity for a relation between phenomena, where it is also subsumed that complementary phenomena together do form a *complete* representation. Just as there is a problem here (cf section 5.3) concerning a natural conception of completeness, we see it reflected in the composition of Bergson's example. Views of an object may be taken not only from all points in external space and time, but also from internal points — which requires a language with introspective capacity. Here, and not before, there is a natural completeness, namely to describe language in language. This is what Leibniz' God is supposed to be able to do, and without him complementarity arizes: we have to accept that there are phenomena which cannot be completely described but for which we can have a complementaristic understanding.

To avoid a possible misunderstanding, I want to emphasize that languages themselves — in their complementaristic understandings — are related to each other in terms of reducibility or translatability and *not* in terms of complementarity. We will return to this point in the next subsection.

6.2 *Reducibility of Bohr Complementarity*

Bohr's primary view of complementarity as a relation between definability and observability (measurability) is reducible to the linguistic complementarity as a relation between describability and interpretability. To understand this, we first observe that definability is a special case of describability and observability (measurability) a special case of interpretability. The latter point may be further developed as follows.

Sentences are *interpreted* by the rules (formulated in a metalanguage) for their *truth condition*, i.e., the sufficient and necessary condition for their truth (cf Carnap 1968; page 22). Thus, in a physical context, with truth conditions in terms of measurement, we can identify an interpretation process as a measurement process, or a process of observation by measurement. In Löfgren (1989), we have argued this point further with reference to Margenau (1966).

To describe a physical system in terms of measurements is thus a task which is connected with a *criterion of verification* of the description — which is much more broadly dealt with for languages in general, where also inductive inferences are at play both in the description and interpretation processes. In these we have *criteria of acceptance*, like that of internal consistency of theses which may be inductive generalizations from observations by measurement (see Löfgren 1982).

The linguistic complementarity concerns description–interpretation processes that constitute a language in general, and we ask if this complementarity will contain any nontrivial meaning when specializing the acceptance criterion to a verification criterion of quantum mechanical observation by measurement. The answer is yes. It is in recognizing the quantum mechanical measurement problem as self–referential, with the measurement process, as interpretation process, itself belonging to the quantum physical domain, that we encounter a phenomenon of language for which the linguistic complementarity takes the form of Bohr's primary view. The point is further explained in section 6.3.

We next turn to Bohr's view of complementarity in terms of phenomena. This view encompasses partly the idea of a *complementaristic resolution*, and partly an idea of completeness, namely that a collection of two complementary phenomena is complete in some natural sense (as explained in section 5.3).

Bohr's idea of *a complementaristic resolution* has been analyzed in Lindenberg and Oppenheim (1974). They find that, in general, a complementaristic resolution is justified in situations where an insoluble *intra*–domain problem exists, and where an *inter*–domain resolution is to be found. In quantum mechanics, the intra–domain problem would correspond to for example an uncontrollable interaction between an electron *per se* and the experimental arrangement used to investigate the electron. But if we don't remain inside the conceptual sphere of classical physics, where the insoluble intra–domain dilemma exits, but open up an inter–domain of classical and quantum conceptions, a resolution may be found. Namely, where classical physical properties (particle–like, wave–like), are not applied to entities like electron *per se*, but to phenomena, i.e., indecomposables such as electron *with* the experimental arrangement of its investigation (cf section 5.2).

Such a complementaristic resolution is what we have in the complementaristic conception of language. Here "domain" and "problem", as well as "unsolvability", refer to a language as object of investigation. The "intra–domain problem" is the problem of describing the language in its domain, i.e., in itself. That this really is "insoluble", we know from the impossibility of describing the interpretation process of a language within the language itself (cf the arguments for the linguistic complementarity). The complementaristic resolution is to conceive of the language as an indecomposable whole of descriptions *with* interpretations. That a full understanding of such a complementaristic resolution requires access to a metalanguage, is what we have argued throughout section 3.

Finally, in Bohr's case we have beside the idea of a complementaristic resolution, also a use of the term complementarity in expressing *completeness* of collections of two (perhaps more) phenomena. Two (or more) phenomena are said complementary if they together provide a complete picture.

It may be thought that this latter use of complementarity would correspond, in the linguistic case, to saying that two (or more) languages might be complementary — in a sense of providing more useful information when taken together. Surely, mastery of two languages may represent a more complete picture than that of one. But information can be said additive only if conceivable, and conceivability requires *one* language. Thus, for two languages to contribute towards a more complete information, there must be a third language into which the other two can be reduced (or translated). If so, there is no need, or reason, for calling the two primary languages complementary (or reciprocally complementary; cf section 6.1), simply because a unifying language must be in existence for the increase in completeness to be realized, and we then have a simple case of a whole with well defined parts. This would correspond more to the well defined term "complementary" in elementary set theory with a well defined universe, than to the term "complementarity" with connotations to real tensions, or oppositions that occur in attempts to conceive of real wholes.

6.3 *Complementaristic Understanding of the Self–Referential Measurement Problem in Quantum Mechanics*

In quantum mechanics, also the measurement process is part of the physical domain, and a self–referential problem arizes, whether the measuring apparatus is considered as a classical physical device as in Bohr's conception, or as a quantum mechanical device as in von Neumann's conception of quantum mechanics.

Bohr's requirement that a measuring device must be an apparatus obeying classical physical laws stems from his insight that physical measurements must be unambiguously communicable to function as a basis for an objectively erected physical theory. The requirement leads into a problem of self–reference. Landau and Lifshitz (1958; pages 2–3) express this as follows:

> "By *measurement*, in quantum mechanics, we understand any process of interaction between classical and quantum objects, occurring apart from and independently of any observer. The importance of the concept of measurement in quantum mechanics was elucidated by N. Bohr.
>
> "Thus, quantum mechanics occupies a very unusual place among physical theories: it contains classical mechanics as a limiting case, yet at the same time it requires this limiting case for its own formulation."

von Neumann (1932, 1955) adheres to what he calls Bohr's "dual" conception of quantum mechanics, and suggests a formalization within the domain of a single quantum theory without invoking a classical domain, namely by splitting the theory in two parts as follows.

On the one hand there is a description, the Schrödinger equation, of how the state of the unobserved system is "automatically" changed under the action of an energy operator. This evolution process of the system, referred to as "process **2**", is strictly causal.

To this description is joined a description of how a state undergoes a non–causal change during a measurement, referred to as "process **1**", resulting in a mixture with determined probabilities.

Now, with the measuring apparatus belonging to the quantum mechanical domain, the composition of a system S and an apparatus A is another quantum mechanical system S' which evolves according to process **2** — unless itself measured upon by a second type apparatus A' aiming at inferring the state of A and therefrom the state of S. If comparison with experiment is to be possible, there must somewhere in such a chain of apparatus be an observer that actually perceives a definite meter reading — or else we get into an infinite regress.

von Neumann tries to resolve the difficulty by arguing that the complete measuring process (including a final meter reading which is not described by process **1**) can be cut in parts as follows. The first part of the chain belongs entirely to the quantum mechanical domain, whereas the second part of the chain contains an observer performing a final meter reading. And, by further argument, that this cut can be placed along the chain, without hampering the final result, with such a liberty that it justifies the established truncation of the measurement regress.

Let us now look at von Neumann's theory, with its two parts, from the point of view of the linguistic complementarity. Recall the observation from section 6.2 that observation by measurement is how to interpret the state symbols of the formalism. We then immediately see that von Neumann captures the self–referential character of the problem, in attempting to describe the interpretation process for the quantum mechanical theory in the theory itself. From the linguistic complementarity we now understand that this must result in but partial success. For in no language can its interpretation process be completely described in the language itself.

This resolves, in a qualitative way, the self–referential measurement problem for quantum mechanics. Namely, by understanding that its solution is in terms of a complementaristic conception — of language — which cannot be reduced to descriptive theories only. And this, without invoking reference to phenomena of cognition which occur only in highly evolved languages, whereas every language is a complementaristic phenomenon.

At first it may seem out of context to talk of linguistic models for quantum mechanics. If such an impression prevails, it may be due to an habit of perceiving language only in its colloquial sense of spoken natural language objectified primarily in terms of grammar. This is *not* what we refer to. We are referring to the general concept of language in its complementaristic conception.

In this sense it is by no means unnatural to conceive of linguistic models for quantum mechanics. On the contrary. As soon as the self–referential nature in quantum mechanics is recognized, for example in terms of the problem of measurement of measurement, linguistic models come into focus because self–reference *is* a linguistic phenomenon (with language in the general sense indicated).

Furthermore, the dominant probabilistic interpretation of quantum mechanics has itself a natural foundational embedding in complementaristic language. Namely, if we try to go beyond probability as a mere calculus, down to the meanings of probability as knowledge of lack of knowledge, then the projection of such metaknowledge on an object level (needed for scientific describability) will be subject to the linguistic complementarity. Such a foundational embedding of probability is developed in Löfgren (1990b).

A related question is whether we can go beyond a qualitative insight into quantum mechanics in terms of linguistic models, and utilize some more detailed knowledge of the linguistic complementarity. This is a large and difficult topic which we will not attempt here, except for hinting at a more detailed study of the complementarity for quantum mechanical languages in terms of degrees of partiality of self–reference.

By way of further progression, we may study phenomena of cognition (in connection with the measurement problem), which are beyond the reach of mechanistic modelling, in terms of linguistic models. Here, we can refer to Löfgren (1977) concerning a particular form of cognition, existential perception.

6.4 *Conclusions*

Naturally, there have been numerous suggestions for formulations of properties of the various concepts of complementarity in attempting to understand them. In particular, there is a large literature concerned with such understandings of Bohr's concepts of complementarity. By way of a selection, we mention Jammer (1966, 1974), Folse (1985), Mehra (1974) — and Bohr's own quest at a "theory" of complementarity should not be forgotten as an attempt in this direction of understanding complementarity in terms of properties.

Our conclusion, based on the concept of the linguistic complementarity, is that attempts at describing complementarity, such as formulating properties of it, will remain essentially incomplete — and difficult to understand for reasons of consistency — as long as our aim is that of providing a description. By contrast, if we try to conceive of complementarity as we conceive of a *paradigm*, i.e., with reference to *both* theory *and* field of application, then we may have taken a step that widens the horizons for further progress. What is a natural paradigm for complementarity? Without much hesitation, I would answer: Language conceived in its generality in terms of description and interpretation processes. In fact, in its complementaristic conception, language is the paradigm of paradigms (obviously so, after identifying "field of application" with "intended interpretation").

In particular, as explained in Löfgren (1987c), the very problems of reducibility, not only concerning concepts of complementarity, become well structured in a context of objectified language.

7 *References*

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