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Shackling the footloose firm? Factor interests and majority voting

Carl-Johan Belfrage and Fredrik Gallo

August 11, 2006

Abstract

Adding majority voting to a simple new economic geography model, we analyse under which circumstances politically determined barriers to international firm relocation exist. Two countries, differing in market size, consider abolishing restrictions on firm mobility. Eliminating these restrictions will fully or partially de-industrialize the small country as firms relocate to the larger market.

We show that there is unanimous support for (resistance against) the removal of obstacles to firm relocation in the large (small) country if the country size difference is small, while a large difference in size gives rise to domestic conflicts of interest and international cross-factor alignments of interests. Furthermore, trade liberalisation may have facilitated the removal of barriers to firm relocation in large countries. Finally, political integration between trading countries is likely to contribute to the removal of barriers to firm relocation, and support for (resistance against) such a development comes primarily from the immobile factor in the large (small) country.

1 Introduction

In an increasingly integrated world, small peripheral countries run a risk of becoming deindustrialised as firms relocate to larger markets. For instance, a survey conducted by SIF (Sweden’s second largest trade union) concludes that more than 20 per cent of the firms located in the south of Sweden relocated to other countries between 2000 and 2004 (SIF, 2006).

Fears of such a development were expressed by the Swedish Minister of Industry, Employment and Communications in an interview in the Swedish national daily Svenska Dagbladet (2002, our translation):
"We can never accept that Swedish companies close domestic plants and move to other EU countries as a result of our regulatory framework making it too easy and unexpensive. We suspect that this is presently the case. The rules need to be more consistent within the EU. Or else we need to consider making our own rules more restrictive."

In an overview of legal conditions in nine EU countries, the Swedish National Labour Market Board and the National Institute for Working Life find that it is easier for firms to relocate from Sweden than from other countries in Europe (Swedish National Labour Market Board, 2002). According to the same study, the authorities in Spain can stop a firm from dismantling its Spanish operations if it is not considered motivated. The Swedish Trade Union Confederation is also concerned with how easy it is for firms to move their activity out of the country. In a case study of how German tyre manufacturer Continental AG closed its plant in the Swedish municipality of Gislaved in 2002, laying off 774 employees in a town with about 10,000 inhabitants, the organisation calls for a number of measures to govern (and cushion the effects of) plant closure (The Swedish Trade Union Confederation, 2004). Such calls are echoed by the European Trade Union Confederation, which stresses the importance of using political, legislative and financial instruments at the European level in developing a "strategic and pro-active approach" regarding restructuring and delocalisation.

At the same time, concern about fiercer competition and increased foreign control over domestic industry flourishes in larger countries. Measures to restrict foreign firm and capital inflows include bans or limits on foreign ownership, and requirements that the majority of board of directors must be nationals or residents (Golub, 2003).

While the last twenty years have in general been marked by increased liberalisation regarding firm and capital flows among OECD countries, there are significant variations across countries and industry sectors. At the end of the 1990s Canada, Iceland, Mexico
and Turkey were overall the most restrictive countries, while many other OECD countries had quite a few barriers to foreign capital inflows in sectors such as telecommunications, airline transport, electricity and banking (Golub, 2003).

It seems apparent that political incentives to inhibit firm relocation in both small and large countries may arise, and some governments have indeed made efforts to make it difficult for resident firms to relocate some or all of their activities abroad. In this paper we add majority voting to a simple new economic geography model to gain some insight into why politically determined barriers to international firm relocation are erected and removed. Two countries, differing in market size, consider abolishing restrictions on firm mobility. Eliminating these restrictions will deindustrialise the small country as firms relocate to the larger market. We analyse how various groups of residents in each country are affected by the resulting change in industrial structure, and examine voting outcomes under alternative jurisdictional configurations. In the first, the two countries are politically independent and each country’s position on policy reflects the wish of the majority of its own population only. In the second, the two countries have undertaken political integration and form a single political jurisdiction. The policy choice is then the will of the majority of the countries’ populations combined. We show that there is unanimous support for (resistance against) the removal of obstacles to firm relocation in the large (small) country if the country size difference is small, while a large difference in size gives rise to domestic conflicts of interest and international cross-factor alignments of interests. Furthermore, trade liberalisation may have facilitated the removal of barriers to firm relocation in large countries. Finally, political integration between trading countries is likely to contribute to the removal of barriers to firm relocation, and support for (resistance against) such a development comes primarily from the immobile factor in the large (small) country.

The rest of the paper is organised as follows. The next section reviews previous literature dealing with public policy in a new economic geography framework, while
section 3 introduces the economic model. Section 4 presents and analyses the outcomes of the political model, and section 5 concludes.

2 Related Literature

New economic geography models have been used to analyse public policy of various kinds. One strand of research focuses on tax competition among countries. Traditional tax competition literature (see Wilson, 1999, for a survey) stresses that when the mobility of factors of production increases as a result of economic integration, countries might engage in a race to the bottom to avoid eroding tax bases. However, according to Andersson and Forslid (2003), Kind et al. (2000), and Ludema and Wooton (2000), this need not be the case. They show that when economic integration causes industry to concentrate in certain countries, agglomeration rents emerge. These rents are taxable and allows a country to impose higher taxes without running the risk of losing its industry. So while economic integration may increase the mobility of countries’ tax bases it may also create counteracting forces, enabling countries to set different tax levels.

Another vein of research analyses regional policies such as public infrastructure investment and subsidies as instruments to attract industry. Martin and Rogers (1995) examine how improving a country’s infrastructure affects industry location. If the investment is undertaken to improve the domestic transportation system, then it induces an inflow of firms to that country. The reason is a relatively higher demand for the domestically produced goods, which allows firms to better exploit economies of scale. On the other hand, if it is trade with the rest of the world that is facilitated, then it might lose its industry. In Robert-Nicoud and Sbergami (2004) an economically strong region may lose its industry to a smaller region. Voters have preferences over policy (a subsidy to capital) and over a political dimension (ideology). They show that the big region will attract the industrial core only if its relative economic strength (due to its larger size) overcomes its
relative political weakness (stemming from a higher dispersion of the population along the political dimension).

Baldwin et al. (2003) contains a thorough treatment of trade policy in new economic geography models. They show *inter alia* how countries can attract industry by unilaterally imposing import barriers; that reciprocal trade liberalisation can deindustrialise small countries and that such a development can be avoided if the big country continuously lowers its trade barriers more than the smaller one. The risk of delocation is also reduced if relocation is costly and/or goes against comparative advantage. In addition, they examine how forming preferential trade agreements affects industry location between the emerging trade bloc and the rest of the world as well as within the trade bloc.

We differ from the above mentioned studies in that we add a political economy approach and focus on restrictions on firm mobility.

### 3 The Economic Model

We employ the footloose capital model developed in chapter three in Baldwin et al. (2003), but with a quasilinear utility specification for tractability. There are two countries (Home and Foreign), two sectors ($A$ and $M$), and two factors of production (capital, $K$, and labour, $L$). We use an asterisk to denote Foreign variables. Labour is assumed to be internationally immobile, whereas capital can flow freely between the countries (although these flows may be subject to policy). Although capital itself is mobile, its owners are not: capitalists live and spend in their countries of origin. Home and Foreign are identical with respect to tastes, technology and openness to trade. However, they differ in factor endowments. While the ownership of the world’s capital stock is equally divided between the countries, Home is assumed to have a larger work force. The $A$ sector produces a homogeneous good under constant returns to scale and perfect competition, using labour only. Firms in the monopolistically competitive $M$ sector produce differentiated goods.
under increasing returns to scale, and employ both capital and labour. Trade in the $M$ sector’s goods are subject to iceberg trade costs, whereas the $A$ sector’s goods are shipped freely.

Consumers maximise the utility function

$$U = \alpha \ln C_M + C_A; \quad C_M = \left( \int_{i=0}^{n^w} c_i^{\frac{\sigma-1}{\sigma}} \, di \right)^{\frac{\sigma}{\sigma-1}}; \quad 0 < \alpha < 1 < \sigma;$$

where $C_A$ is consumption of the homogeneous good, $C_M$ is the aggregate of differentiated goods, $\sigma$ denotes the constant elasticity of substitution between any pair of the differentiated goods (it also equals the price elasticity of demand due to the Chamberlinian large group assumption).

In the $A$ sector firms employ labour only and production is given by $X_A = L_A$. Perfect competition enforces marginal cost pricing, i.e. $p_A = w_L$ and $p_A^* = w_L^*$. Taking the homogeneous good as the numéraire and assuming costless trade in $A$ we have $p_A = p_A^* = 1$ and thus $w_L = w_L^* = 1$.\(^1\) In the $M$ sector, production of each variety involves a fixed cost of one unit of $K$ and $a_m$ units of $L$ per unit of output. The assumed unit input requirement of capital implies that the worldwide mass of firms ($n^w$) equals the world’s capital stock ($K^w = K + K^*$). Denoting the reward to capital $\pi$ and each firm’s level of output $x$, a $M$ sector firm’s cost function is

$$TC = \pi + w_L a_m x.$$  

Profit-maximising firms set prices equating marginal revenue to marginal cost, $p_j(1 - \frac{1}{\sigma}) = w_L a_m$. Since preferences, technology and wages are equal across countries, the same pricing condition applies in Foreign so that

$$p_j = p_j^* = \left( \frac{\sigma}{\sigma - 1} \right) w_L a_m.$$  

\(^1\)For both countries to have an active $A$ sector after trade, and thus for the useful equalisation of wages across countries, we need to assume that Home’s $M$ sector employment of labour is less than its labour supply even if all $M$ sector firms locate there. The required condition is $s_L > \frac{2}{\sigma} \alpha \left( \frac{1}{n^w} + \frac{1}{K^w} \right)$. 
Consumers maximise the utility function in equation (1) with respect to the budget constraint $Y = PC_M + p_A C_A$, where $Y$ represents income and $P = \left[ np_j^{1-\sigma} + n^* \left( \tau p_j^* \right)^{1-\sigma} \right]^{1/\sigma}$ is the price index of the differentiated goods aggregate $M$. Their budgets will then be allocated between $M$ and $A$ sector consumption according to $C_M = \alpha P^{-1}$ and $C_A = Y - \alpha$, where $\alpha$ is total spending per consumer on the $M$ sector’s goods. A Home consumer’s demand for variety $j$ is

$$c_j = \begin{cases} \frac{\alpha p_j^{1-\sigma}}{p_j^{1-\sigma}} & \text{if } j \text{ is produced in Home} \\ \frac{\alpha (\tau p_j^*)^{1-\sigma}}{p_j^{1-\sigma}} & \text{if } j \text{ is produced in Foreign} \end{cases},$$

where $\tau > 1$ is the iceberg trade cost. Total demand will simply be a matter of summing over the number of individuals involved. Assuming that each worker owns (and inelastically supplies) one unit of labour and owns no capital, whereas each capitalist owns one unit of capital and is not part of the labour force, the overall demand for a variety $j$ of $M$ produced in Home becomes $(L + K) c_j + \tau (L^* + K^*) c_j^*$. It will hence yield a profit of

$$\Psi_j = (p_j - w_L a_m) (L + K) c_j + (p_j - w_L a_m) (L^* + K^*) \tau c_j^* - \pi.$$ 

Competition for the fixed stock of capital in the $M$ sector bids up the return to capital until pure profits are zero ($\Psi_j = 0$). Using the demand functions, the pricing condition and the price index, the return to capital employed in Home can be written $\pi = \frac{\alpha}{\sigma} \left( \frac{(L+K)}{(n+n^*)} + \phi \frac{(L^*+K^*)}{(n+n^*)} \right)$ and for capital employed in Foreign $\pi^* = \frac{\alpha}{\sigma} \left( \phi \frac{(L+K)}{(n+n^*)} + \phi \frac{(L^*+K^*)}{(n+n^*)} \right)$, where $\phi \equiv \tau^{1-\sigma}$ is the usual measure of trade freeness, ranging from 0 (autarky) to 1 (free trade).

Next we introduce some commonly used definitions and normalisations (see Baldwin et al. (2003), ch. 3) to facilitate exposition. First, we let $n + n^* = n^w = K^w = 1$. We define the share of capital owned by Homers as $s_K \equiv \frac{K}{K^w}$, the share of firms located in Home (and hence the share of capital employed in Home) as $s_n \equiv \frac{n}{n^w}$, and the share of labour in Home as $s_L \equiv \frac{L^w}{K^w}$. It is also useful to express the global labour-capital ratio as $\gamma \equiv \frac{L^w}{K^w} = L^w$. This permits us to write

$$\pi = \frac{\alpha}{\sigma} \left( \frac{s_L \gamma + s_K}{s_n + \phi (1-s_n)} + \phi \frac{(1-s_L) \gamma + 1-s_K}{\phi s_n + 1-s_n} \right).$$

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Our assumptions of equal ownership of capital and a larger Home market translate into \( s_K = \frac{1}{2} \) and \( s_L > \frac{1}{2} \). With preferences represented by the utility function in equation (1), the indirect utility function is

\[
V = -\alpha \ln P + Y + k,
\]

where \( P = [s_n + \phi (1 - s_n)]^{1/\sigma} \) is the price index, \( k \equiv \alpha \left( \ln \left( \frac{\alpha(\sigma-1)}{\alpha \sigma} \right) - 1 \right) \) and

\[
Y = \begin{cases} 
1 & \text{for workers} \\
\pi & \text{for capitalists} 
\end{cases}
\]

We are now ready to investigate how industry structure and factor owners (capitalists and workers) in each country are affected by an elimination of barriers to firm relocation. We will analyse two types of scenarios (eventually to be considered policy alternatives) concerning the location of M sector production. Our point of departure is that prohibitive barriers to firm relocation exist and that Home and Foreign consider eliminating them. In this initial situation, which we will refer to as the restricted case (R), capital and hence firms are prevented from moving. In the second one, called the unrestricted case (U), capital and firms are allowed to flow freely between the countries. Two points are worth noting about the latter case. First, since capitalists live and consume in their country of origin and their income earned abroad is repatriated, only nominal differences in the return to capital govern capital flows. Second, we can have either full or partial agglomeration in Home when capital is free to move. The actual degree of firm concentration in the larger country depends on the strength of the home market effect, as is explained in more detail below.

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2 We have investigated firm relocation in the presence of relocation costs. However, the analysis proved algebraically unrevealing even in the simplest case of an additive, per firm relocation cost. The R and U cases in the text correspond to the polar cases of prohibitive relocation costs and free relocation.
3.1 Firm location equilibria and welfare effects

In the restricted case each country is self-sufficient in capital so \( n = K \) and \( n^* = K^* \) or, in terms of shares, \( s_n = s_K = \frac{1}{2} \) (since capital ownership is assumed to be equally divided between the countries). The return to capital in each country becomes

\[
\pi^R = \frac{\alpha \sigma \left( 2s_L \gamma + 1 + \phi(2(1-s_L) \gamma + 1) \right)}{1 + \phi},
\]

and

\[
\pi^{*R} = \frac{\alpha \sigma \left( 2s_L \gamma + 1 + 2(1-s_L) \gamma + 1 \right)}{1 + \phi},
\]

where \( \pi^R > \pi^{*R} \) since \( s_L > \frac{1}{2} \). Abolishing the restrictions on firm mobility would thus lead to a flow of firms from Foreign to Home. Whether all firms or just a part of them move to Home depends on the strength of the home market effect. For interior long-run equilibria (i.e. partial agglomeration) we can find the share of firms located in Home by solving \( \pi = \pi^* \) for \( s_n \) using equations (5) and (6), yielding

\[
s_n = \frac{1}{2} + \frac{\gamma + \frac{1 + \phi}{1 + \gamma - \frac{1}{2}} (s_L - \frac{1}{2})}{1 - \frac{1}{2}}.
\]

The following figure is a plot of this solution against \( s_L \) (setting \( \phi = 0.5 \) and \( \gamma = 2 \)).

Figure 1: The allocation of \( M \) sector firms in the \( U \) case

The thick lines illustrate the equilibrium allocation of \( M \) sector firms in the \( U \) case (strictly speaking we are only interested in values of \( s_L > \frac{1}{2} \), but for the sake of completeness we plot over the whole interval \((0, 1)\)). The thick line with a positive slope displays the cases of partial agglomeration; the upper flat part shows full agglomeration in Home \((s_n = 1)\). The thin line illustrates \( s_n = s_L \). The home market effect is evident as a
given increase in $s_L$ leads to a more than proportional increase in Home’s share of firms. When the difference in market size becomes large enough all firms will locate in Home: the critical level is $s_L \geq \frac{2\gamma+1-\phi}{2(1+\phi)}$, which we will refer to as the sustain condition.\(^3\) An increase in $\gamma$ or $\phi$ would increase the slope of the thick line displaying interior equilibria and rotate it counterclockwise around the midpoint ($\frac{1}{2}, \frac{1}{2}$). It would also decrease the critical level of $s_L$ above which we get full agglomeration in Home (thus increasing the length of the upper flat thick line).

We next analyse how the factor owners in each country are affected by the firm location outcomes described above. The analysis is based on a comparison of indirect utilities in the $R$ and $U$ cases for the four different categories of individuals. In the $R$ case indirect utility for people in Home and Foreign are

$$V^R_L = V^*_L = \frac{\alpha}{\sigma-1} \ln \left( \frac{1+\phi}{2} \right) + 1 + k$$

$$V^R_K = \frac{\alpha}{\sigma-1} \ln \left( \frac{1+\phi}{2} \right) + \pi^R + k$$

$$V^*_K = \frac{\alpha}{\sigma-1} \ln \left( \frac{1+\phi}{2} \right) + \pi^*_R + k,$$

where the first term in the right-hand sides is the utility of the price index in the $R$ case, $\pi^R$ and $\pi^*_R$ are given above and $L$ ($K$) refers to workers (capitalists). Turning to the $U$

\(^3\)This critical level of $s_L$ can be derived in (at least) two ways. Either we use the expression for $s_n$ in the main text and solve $s_n \geq 1$ for $s_L$. Or we can take full agglomeration in Home as given and check that the capital income a deviating firm would earn in Foreign is less than in Home; that is, solve $(\pi - \pi^*)|_{s_n=1, s_K=\frac{1}{2}} \geq 0$ for $s_L$. 

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case, we have

\[ \frac{1}{2} < s_L < \frac{2\gamma+1-\phi}{2\gamma(1+\phi)} \]
\[ \text{Partial agglomeration} \]

\[ \frac{2\gamma+1-\phi}{2\gamma(1+\phi)} \leq s_L < 1 \]
\[ \text{Full agglomeration} \]

<table>
<thead>
<tr>
<th>( V^U_L )</th>
<th>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{(s_L\gamma + \frac{1}{2})(1+\phi)}{1+\gamma} \right) + 1 + k )</th>
<th>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{\alpha}{\sigma} (1 + \gamma) \right) )</th>
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<tbody>
<tr>
<td>( V^U_K )</td>
<td>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{(s_L\gamma + \frac{1}{2})(1+\phi)}{1+\gamma} \right) + \pi^U + k )</td>
<td>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{\alpha}{\sigma} (1 + \gamma) \right) + \pi^U + k )</td>
</tr>
<tr>
<td>( V^*_U )</td>
<td>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{(1-s_L\gamma + \frac{1}{2})(1+\phi)}{1+\gamma} \right) + 1 + k )</td>
<td>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{\alpha}{\sigma} (1 + \gamma) \right) + \pi^*_U + k )</td>
</tr>
<tr>
<td>( V^*_K )</td>
<td>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{(1-s_L\gamma + \frac{1}{2})(1+\phi)}{1+\gamma} \right) + \pi^*_U + k )</td>
<td>( \frac{\sigma}{\sigma - 1} \ln \left( \frac{\alpha}{\sigma} (1 + \gamma) \right) + \pi^*_U + k )</td>
</tr>
</tbody>
</table>

where \( \pi^U = \pi\big|_{s_n=1, s_K-\frac{1}{2}} = \frac{\alpha}{\sigma} (1 + \gamma) \) and \( \pi^*_U = \pi^U \) (due to the free movement of capital).

Note that the return to capital is the same no matter whether the equilibrium that materialises in the \( U \) case involves partial or full agglomeration (see the Appendix for an explanation). We next compare the individual indirect utilities in the \( R \) case to those in the \( U \) case, first when the lack of restrictions on firm relocation leads to full agglomeration and then when it leads to partial agglomeration in Home.

### 3.1.1 Welfare effects when \( U \) leads to full agglomeration

When the unrestricted case involves full agglomeration, it is straightforward to show that \( V^U_L > V^R_L \) and \( V^*_U > V^*_L \) (the required condition is that there is some positive level of trade costs, \( \phi < 1 \)). Workers in Home thus unambiguously prefer \( U \), while Foreign workers prefer \( R \). The reason is that workers’ nominal incomes are unaffected by firm relocation, so they care only about the price index effect. Home workers gain from agglomeration as they no longer have to pay any trade costs on \( M \) goods, while Foreign workers are worse off since all varieties now have to be imported from Home. For capital owners a second effect comes into play. For Home capitalists we have \( V^*_K - V^R_K = \pi^U - \pi^R + (-\alpha) \ln \left( \frac{P^U}{P^R} \right) \), where \( P^i \) is the price index in case \( i = R, U \). They gain from a lower price index when capital is allowed to move freely \( \left( \frac{P^U}{P^R} < 1 \right. \) hence \( \ln \left( \frac{P^U}{P^R} \right) < 0 \), increasing \( V^*_K - V^R_K \), but their income also falls as a result of the inflow of firms from Foreign \( (\pi^U < \pi^R) \).
The opposite is true for Foreign capitalists: \( V^U_K - V^R_K = \pi^U - \pi^R + (-\alpha) \ln \left( \frac{P^U}{P^R} \right) \), where \( \pi^U > \pi^R \) and \( \ln \left( \frac{P^U}{P^R} \right) > 0 \). More varieties are subject to trade costs increasing their price index, but on the other hand they get a higher income when their capital is employed in Home. Whether capitalists gain or lose in the unrestricted case depends on the size difference between Home and Foreign. For Home we have that \( V^U_K - V^R_K > 0 \) if
\[
s_L < \frac{1}{2} + \frac{A}{2\gamma} \equiv RHS_H, \tag{9}
\]
where \( A \equiv -\frac{\sigma(1+\phi)}{(1-\sigma)(1-\phi)} \ln(\frac{1+\phi}{2\phi}) \), and for Foreign \( V^*_U - V^*_R > 0 \) if
\[
s_L > \frac{1}{2} + \frac{B}{2\gamma} \equiv RHS_F, \tag{10}
\]
where \( B \equiv \frac{\sigma(1+\phi)}{(1-\sigma)(1-\phi)} \ln(\frac{1+\phi}{2\phi}) \). It is readily established that, for any \( \phi < 1 \), the following holds: \( \frac{1}{2} < RHS_H < RHS_F < 1 \). Capitalists’ preferences over \( R \) and \( U \) thus depend on how much bigger Home is than Foreign. The conflicts and alignments of interest among capital owners and workers, in Home and Foreign, can be summarised as follows:

\footnote{The last inequality does however require the added proviso that \( \gamma > B \), i.e. that the world’s labour-to-capital ratio is large enough. The economic intuition behind this condition is explained in the text under Figure 3.}

\footnote{Conditions (9) and (10) could also be expressed as \((2s_L - 1) \gamma < A \) and \((2s_L - 1) \gamma > B \), respectively, where the left-hand-sides represent the absolute difference in the number of workers (and, with equal numbers of capitalists therefore simply the total difference in the number of individuals) and hence (when multiplied by \( \alpha \)) the absolute cross-country difference in nominal spending on \( M \).

An alternative way of interpreting the contents of the restrictions is to focus on the elasticity parameter \( \sigma \) which exerts a strong influence over \( A \) and \( B \). Higher values of \( \sigma \) (indicating a higher elasticity of substitution between varieties of \( M \), and a higher price elasticity of demand for each variety), and hence lower values of \( A \) and \( B \), imply a stronger relative impact of the price index effect on welfare and thus expands (reduces) the range of values of \( s_L \) for which Home (Foreign) capitalists prefer \( U \).}
Table 1. Preferences over $R$ and $U$ (full agglomeration)

<table>
<thead>
<tr>
<th></th>
<th>$\frac{1}{2} &lt; s_L &lt; RHS_H$</th>
<th>$RHS_H &lt; s_L &lt; RHS_F$</th>
<th>$RHS_F &lt; s_L &lt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home workers</td>
<td>$U$</td>
<td>$U$</td>
<td>$U$</td>
</tr>
<tr>
<td>Home capitalists</td>
<td>$U$</td>
<td>$R$</td>
<td>$R$</td>
</tr>
<tr>
<td>Foreign workers</td>
<td>$R$</td>
<td>$R$</td>
<td>$R$</td>
</tr>
<tr>
<td>Foreign capitalists</td>
<td>$R$</td>
<td>$R$</td>
<td>$U$</td>
</tr>
</tbody>
</table>

We see that when the difference in country size is moderate (first column), then the conflict of interest is between countries. Home residents want unlimited capital movement and Foreigners want barriers to firm relocation. As Home becomes increasingly larger (second column) capitalists in Home lose and want to restrict capital movement. For a really large difference in size (third column) we have cross-country and cross-factor alignment of interests. Home capitalists and Foreign workers prefer restrictions, whereas Foreign capitalists and Home workers would like to see deregulation. The reason why Home capitalists turn from $U$ to $R$ when $s_L$ *ceteris paribus* increases is explained by the derivative $\frac{\partial (V^U_K - V^R_K)}{\partial s_L} = -\frac{\partial \pi^R}{\partial s_L} = -\frac{\alpha^2 (1-\phi)}{\sigma (1+\phi)} < 0$. That is, the equilibrium price indices and $\pi^U$ are all unaffected by an increase in Home’s share of labour, while $\pi^R$ rises with it, increasing the loss of going from $R$ to $U$ for Home capitalists. For a high enough level of $s_L$ the loss dominates the gain from a lower price index and Home capitalists prefer $R$ to $U$. For capitalists in Foreign $\frac{\partial (V^{*U}_K - V^{*R}_K)}{\partial s_L} = -\frac{\partial \pi^{*R}}{\partial s_L} = \frac{\alpha^2 (1-\phi)}{\sigma (1+\phi)} > 0$. A higher $s_L$ decreases $\pi^{*R}$, making the $U$ alternative more attractive. For high levels of $s_L$ the positive effect of a higher return to capital dominates the negative price index effect and Foreign capitalists prefer $U$ instead of $R$.

### 3.1.2 Welfare effects when $U$ leads to partial agglomeration

Given that Home is the larger country ($s_L > \frac{1}{2}$), Home workers want $U$ and Foreign workers always prefer $R$ even when the unrestricted alternative leads to partial agglomeration.
in Home. For capitalists in Home we have $V^U_K - V^R_K > 0$ provided that

$$\phi > \frac{\gamma (2s_L - 1) - \left(\frac{\sigma}{\sigma-1}\right) \ln \left(\frac{2\gamma s_L + 1}{1+\gamma}\right)}{\gamma (2s_L - 1) + \left(\frac{\sigma}{\sigma-1}\right) \ln \left(\frac{2\gamma s_L + 1}{1+\gamma}\right)} \equiv \phi^H.$$  \hfill (11)

For Foreign capitalists $V^*U_K - V^*R_K > 0$ if

$$\phi < \frac{\gamma (2s_L - 1) + \left(\frac{\sigma}{\sigma-1}\right) \ln \left(\frac{2\gamma (1-s_L) + 1}{1+\gamma}\right)}{\gamma (2s_L - 1) - \left(\frac{\sigma}{\sigma-1}\right) \ln \left(\frac{2\gamma (1-s_L) + 1}{1+\gamma}\right)} \equiv \phi^F.$$  \hfill (12)

As $s_L$ and $\gamma$ appear both in levels and in logs in $\phi^H$ and $\phi^F$, we cannot solve these inequalities in terms of $s_L$ to ease the comparison with the inequalities (9) and (10). Instead we solve (11) and (12) numerically as equalities and make use of how $\phi^H$ and $\phi^F$ vary with $s_L$ to draw the conclusions displayed in figure 2 below.

Figure 2: Capitalists’ preferences over $R$ and $U$ ($\sigma = 1.5, \phi = 0.5$)

The thick line is the sustain condition $s_L = \frac{2\gamma + 1 - \phi}{2\gamma(1+\phi)}$; only combinations of $s_L$ and $\gamma$ below it are consistent with partial agglomeration in Home. The curve separating areas $I$ 

Note, however, that workers and capitalists in Home are worse off under partial agglomeration compared to full agglomeration. They earn the same income in either case, but less varieties are produced locally and trade costs have to be paid for Foreign imports. The opposite is true for Foreign workers and capitalists: they are better off under partial agglomeration.

It can be shown, though, that $\phi^F < \phi^H$ always holds.
and $II$ gives all the combinations of $s_L$ and $\gamma$ for which (11) holds with equality. It thus shows combinations of $s_L$ and $\gamma$ where Home capitalists are indifferent between $R$ and $U$, and we will refer to it as their "indifference curve". The curve separating areas $II$ and $III$ is (12) holding with equality, conveying the same information for Foreign capitalists. Now, start anywhere on Home’s indifference curve and decrease $s_L$ for a given $\gamma$. This would decrease $\phi^H$ as $\frac{\partial \phi^H}{\partial s_L} > 0$ (see the Appendix for a proof) and hence $\phi > \phi^H$ must hold below the curve. That is, inequality (11) holds and $U$ is the preferred alternative for Home capitalists (this is true anywhere in area $I$). In areas $II$ and $III$ the opposite is true: Home capitalists want $R$. Due to a similar logic (now with $\frac{\partial \phi^F}{\partial s_L} < 0$), Foreign capitalists want $R$ if we are above/to the left of Foreign’s indifference curve (areas $I$ and $II$) and $U$ if we are below/to the right of it (area $III$). The labels in the various areas of the figure are a summary of capitalists’ preferences over $R$ and $U$. They highlight Home capitalists’ preferred alternative first and Foreign capitalists’ preference within parentheses: ”$U(R)$” thus means that capital owners in Home prefer $U$, whereas their Foreign counterparts want $R$.

The major difference compared to full agglomeration is that changes in $s_L$ (and $\gamma$) do affect the price indices in the $U$ case featuring partial agglomeration. This is most easily seen by taking

$$\frac{\partial (V^U_K - V^K_R)}{\partial s_L} = - \frac{\partial \pi^R}{\partial s_L} + (-\alpha) \frac{\partial (\ln \frac{p^U}{p^R})}{\partial s_L} = - \frac{\alpha 2\gamma (1-\phi)}{\sigma (1+\phi)} + (-\alpha) \left[ - \frac{1}{(\sigma-1)} \frac{2\gamma}{2\gamma^2 s_L + 1} \right].$$

The first term is the same negative effect on the return to Home-owned capital as in the full agglomeration case; the second term is new and it is positive. As figure 1 shows, an increase in Home’s share of labour increases Home’s share of firms, which in turn lowers Home’s price index in the $U$ case and increases $V^U_K - V^K_R$. The opposite happens for Foreign capital owners:

$$\frac{\partial (V^U_K - V^K_R)}{\partial s_L} = - \frac{\partial \pi^*}{\partial s_L} + (-\alpha) \frac{\partial (\ln \frac{p^U}{p^*})}{\partial s_L} = \frac{\alpha 2\gamma (1-\phi)}{\sigma (1+\phi)} + (-\alpha) \left[ \frac{1}{(\sigma-1)} \frac{2\gamma}{2\gamma^2 (1-s_L) + 1} \right].$$

A given increase in $s_L$ decreases the capital income in the $R$ case relative to what it would be in the $U$ case, but the adverse price index effect is strengthened as more varieties have to be imported from Home. Once we arrive at full agglomeration in Home, the price index effect remains constant leaving only the capital income effect to be influenced by
changes in $s_L$.

3.1.3 Summary of preferences and comparative statics

We know that Home workers always prefer $U$ to $R$, while Foreign workers always want $R$. To get a full characterisation of capitalists’ preferences over $R$ and $U$ under both full and partial agglomeration outcomes, we now combine Figure 2 with Table 1 in the following figure.

Figure 3: Summary of capitalists’ preferences ($\sigma = 1.5$, $\phi = 0.5$)

Again, the thick curve is the sustain condition $s_L = \frac{2\gamma + 1 - \phi}{2\gamma (1 + \phi)}$. It partitions the relevant $\gamma/s_L$-space into two subspaces. Below it we find all the combinations of $s_L$ and $\gamma$ that yield partial agglomeration (this area is in turn partitioned as in figure 2). Above the thick curve we have the combinations of $s_L$ and $\gamma$ yielding full agglomeration in Home. The curve separating areas $IV$ and $V$ is $RHS_H (= \frac{1}{2} + \frac{A}{2\gamma})$; the one separating areas $V$ and $VI$ is $RHS_F (= \frac{1}{2} + \frac{B}{2\gamma})$. The summary of capital owners’ preferences in each area uses the same labeling convention that we explained under figure 2.

The effects of changing Home’s share of labour ($s_L$) on capitalists’ preferences were explained in relation to Table 1 and Figure 2 above. We now turn to the effects of
changing the world’s ratio of labour to capital ($\gamma$) and the level of trade freeness ($\phi$).

**The influence of the world’s labour-capital ratio ($\gamma$)** An increase in $\gamma$ means that both countries’ markets increase in absolute terms. This raises the return to capital in both $R$ and $U$ and, given that $U$ involves full agglomeration, leaves the two price indices unaffected (since it has no influence over how the overall mass of firms is distributed between the countries in either case, and since it does not influence price-setting by firms, see equation (3)). The net effect on Home capitalists’ utility is

$$\frac{\partial (V^U_K - V^R_K)}{\partial \gamma} = \frac{\partial \pi^U}{\partial \gamma} - \frac{\partial \pi^R}{\partial \gamma} = \frac{\alpha}{\sigma} - \frac{2\alpha(\phi s_L + (1-s_L))}{\sigma(1+\phi)} < 0 \text{ since } s_L > \frac{1}{2}.$$  

While an increase in the world’s labour stock increases both $\pi^U$ and $\pi^R$, it increases the latter more. This makes the $R$ case more attractive and for a high enough level of $\gamma$ the capital income loss of going from $R$ to $U$ dominates the price index gain. Hence, as we move to the right above the sustain condition (i.e. for any given $s_L$), Home capitalists change from wanting $U$ (in area IV) to preferring $R$ (areas V and VI). The opposite is true for Foreign capitalists. An increase in $\gamma$ increases $\pi^U$ more than $\pi^R$; the net effect on utility is

$$\frac{\partial (V^U_K - V^R_K)}{\partial \gamma} = \frac{\partial \pi^U}{\partial \gamma} - \frac{\partial \pi^R}{\partial \gamma} = \frac{\alpha}{\sigma} - \frac{2\alpha(\phi s_L + (1-s_L))}{\sigma(1+\phi)} > 0 \text{ if } s_L > \frac{1}{2}.$$  

If Home’s market is large enough in absolute terms ($\gamma > B$), then it becomes possible for the capital income gain to outweigh the price index loss and Foreign capitalists to turn from $R$ to $U$ (area VI). The major difference when $U$ implies partial agglomeration, is an additional effect on the $U$ case price index. Since an increase in $\gamma$ implies an increase in the absolute market size difference, we get home market magnification and hence an increase in the share of industry located in Home. This lowers (increases) the price index in Home (Foreign), making the $U$ ($R$) case more attractive to Home (Foreign) capitalists.

**The influence of trade costs** An increase in trade freeness ($\phi$) reduces the absolute country size difference necessary for full agglomeration to arise. This is exhibited in a counterclockwise rotation of the upward-sloping line in figure 1, and a downward shift of
the thick line representing the sustain condition in figures 2 and 3.

Lowered trade costs yield increased access to the larger Home market for firms located in Foreign and therefore reduce (increase) the income of Home (Foreign) capitalists in the $R$ case ($\frac{\partial \pi^R}{\partial \phi} = -\frac{2\alpha\gamma}{\sigma(1+\phi)} = -\frac{\partial \pi^*R}{\partial \phi} < 0$). Under partial agglomeration, the price index improvements at $R$ and $U$ are equally large in terms of utility and the only effect of increased trade freeness on $V^U_K - V^R_K$ ($V^{*U}_K - V^{*R}_K$) comes through a decrease (increase) in capital income under $R$. Hence, freer trade increases the relative attraction of the $U$ ($R$) alternative to Home (Foreign) capitalists. This implies a rightward shift of the lines separating areas $I$ and $II$ ($II$ and $III$) in figure 3.

If $U$ instead leads to full agglomeration, an increase in trade freeness has no influence on the $U$ price index in Home (since its residents have access to all varieties free of trade costs under full agglomeration), but lowers the $U$ price index in Foreign (while still lowering the price indices in $R$ in both countries). Using (9) it is readily established that $\frac{\partial \text{RHS}^H}{\partial \phi} > 0$. In terms of figure 3, freer trade therefore means an expansion of area $IV$ where $U$ is preferred by Home capitalists. Similarly, from (10) we have $\frac{\partial \text{RHS}^F}{\partial \phi} < 0$. For Foreign capitalists, increased trade freeness leads to an expansion of area $VI$, where $U$ is the preferred outcome. The area above the full agglomeration condition, where both Home and Foreign capitalists prefer $R$ (area $V$), is hence squeezed from both left and right as $\phi$ rises.

To summarize, Home capitalists are more likely to prefer unlimited firm mobility the freer trade is. Foreign capitalists are also more likely to prefer free firm mobility the freer is trade, except if the country size difference is so small that $U$ involves only partial agglomeration.

We next introduce majority voting to analyse potential political choices.
4 The Political Model

We will use a simple majority voting approach to investigate two politically different scenarios. In the first one we view Home and Foreign as two independent political jurisdictions. In the second one they form a single political entity. We analyse each case in turn.

4.1 Independent jurisdictions

For purposes of exposition that will be explained in relation to table 2 below, we will in this section and the next focus on a possible decision to allow $U$ under the assumption that this yields full agglomeration in Home. With two politically independent entities and majority voting in each country, only the parameter restriction in the third column in table 1 above yields non-trivial political results. The reason is that both countries must individually decide to allow free capital movements or not, effectively giving each country a veto. According to the first column, everybody in Home wants $U$ and everybody in Foreign wants $R$. Even if Home abolishes its barriers to firm relocation, Foreign will not and the outcome will be $R$. A similar reasoning applies for the case displayed in the second column: all Foreign residents want $R$ and the country’s restrictions on firm relocation will not be lifted. In the third column, there are $s_L \gamma$ Home votes in favour of the $U$ outcome and $s_K \left(= \frac{1}{2}\right)$ votes against. The unrestricted alternative will thus win a majority in Home provided that $s_L \gamma > \frac{1}{2\gamma}$. In Foreign, the $U$ outcome prevails if $s_L \gamma > 1 - \frac{1}{2\gamma}$. Which of these inequalities that is the strictest depends on the value of $\gamma$. If $\gamma > 1$, then $s_L \gamma > 1 - \frac{1}{2\gamma}$ needs to hold for both countries to vote in favour of $U$.

In the Appendix we show that $B > 1$ and since $\gamma > B$ must hold for this case to exist, we know that $\gamma > 1$ holds too. Then the strictest of the voting restrictions is Foreign’s: $s_L \gamma > 1 - \frac{1}{2\gamma}$. An overall $U$ win then requires i) $s_L \gamma > 1 - \frac{1}{2\gamma}$ (Foreign’s voting restriction) and ii) $s_L \gamma > \frac{1}{2} + \frac{B}{2\gamma}$ (from inequality (10)). It is easily demonstrated that the right-
hand side in i) is smaller (greater) than the right-hand side in ii) whenever \( \gamma < B + 1 \) \((\gamma > B + 1)\). We thus have the following sub-cases of voting outcomes (where all the required parameter restrictions are mutually consistent):

Table 2. Summary: voting results (independent jurisdictions, full agglomeration)

<table>
<thead>
<tr>
<th>Voting outcome</th>
<th>( \gamma \leq B )</th>
<th>( B &lt; \gamma &lt; B + 1 )</th>
<th>( B + 1 &lt; \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>Always</td>
<td>if ( \frac{1}{2} &lt; s_L &lt; \frac{1}{2} + \frac{B}{2\gamma} )</td>
<td>if ( \frac{1}{2} &lt; s_L &lt; 1 - \frac{1}{2\gamma} )</td>
</tr>
<tr>
<td>( U )</td>
<td>Never</td>
<td>if ( \frac{1}{2} + \frac{B}{2\gamma} &lt; s_L &lt; 1 )</td>
<td>if ( 1 - \frac{1}{2\gamma} &lt; s_L &lt; 1 )</td>
</tr>
</tbody>
</table>

As the discussion above has made clear, it is straightforward to relate capitalists’ preferences over \( R \) and \( U \) under full agglomeration to the voting restrictions since they all can be solved for in terms of \( s_L \). As that is not possible under partial agglomeration (due to the expressions in inequalities (11) and (12)), we relegate the discussion about the voting outcomes in that case to the text under the figures 4 and 5 below.

4.2 A unified political entity

In the case of a single political jurisdiction the total number of votes is \( L + L^* + K + K^* = L^W + K^W = \gamma + 1 \). The major difference compared to the previous section is that the de facto national veto right disappears and that all three columns in table 1 above become politically interesting. From a votecounting point of view, however, the cases in columns 1 and 3 are quantitatively identical. The reason is that the number of capitalists in each country is the same \((K = K^* = \frac{1}{2})\). Hence it does not matter for the voting outcome whether it is Home or Foreign capitalists that side with the workers of a country, and we can collapse those two cases into a single one.

For columns 1 and 3 in table 1 \((s_L \in \left( \frac{1}{2}, \text{RHS}_H \right) \cup \left( \text{RHS}_F, 1 \right))\), the number of votes in favour of \( U \) is \( s_L \gamma + \frac{1}{2} \), whereas \((1 - s_L) \gamma + \frac{1}{2} \) persons vote for \( R \). The \( U \) outcome always prevails as \( s_L > \frac{1}{2} \). The capitalists’ votes "net out" and the opinion of the larger country’s
workers is decisive. The conclusion is that in the case of a unified political entity, the barriers to firm relocation will be eliminated for all \( s_L \in \left( \frac{1}{2}, RHS_H \right) \cup \left( RHS_F, 1 \right) \).

In the interval represented by column 2 \((RHS_H < s_L < RHS_F)\), only Home workers gain from \(U\). Now, \(U\) can only win the election if \( s_L/\gamma > (1 - s_L/\gamma) + 1 \Longleftrightarrow s_L > \frac{\gamma + 1}{2\gamma} \), which thus is the relevant voting restriction under a single jurisdiction. For \( A \geq 1 \) it is readily verified that the voting restriction is not binding as it is automatically fulfilled for every \( s_L \) in the column 2 interval (i.e. \( \frac{\gamma + 1}{2\gamma} < RHS_H \) holds). So if \( A \geq 1 \), then we know that \(U\) always wins. On the other hand, for \( A < 1 \), then \( RHS_H < s_L < \frac{\gamma + 1}{2\gamma} \) yields \(R\) and \( \frac{\gamma + 1}{2\gamma} < s_L < RHS_F \) results in \(U\). We summarise the reasoning above in table 3:

Table 3. Summary of voting results (single jurisdiction, full agglomeration)

<table>
<thead>
<tr>
<th>Voting outcome</th>
<th>1 &lt; A &lt; B &lt; \gamma</th>
<th>A &lt; 1 &lt; B &lt; \gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R)</td>
<td>Never</td>
<td>if ( \frac{1}{2} + \frac{A}{2\gamma} &lt; s_L &lt; \frac{\gamma + 1}{2\gamma} )</td>
</tr>
<tr>
<td>(U)</td>
<td>Always</td>
<td>if ( s_L \in \left( \frac{1}{2}, \frac{1}{2} + \frac{A}{2\gamma} \right) \cup \left( \frac{\gamma + 1}{2\gamma}, 1 \right) )</td>
</tr>
</tbody>
</table>

We next illustrate the two political scenarios’ potential voting outcomes.

### 4.3 Comparing potential voting outcomes under all agglomeration scenarios

Figure 4 below presents the potential outcomes when there is a majority vote in each country.
The lowest of the thickest curves in figure 4 is the sustain condition. Hence the areas $R1 - R3$ correspond to the case of partial agglomeration, while $R4, R5$ and $U$ correspond to full agglomeration in Home. The highest of the thickest curves in figure 4 is the "efficient front" of $RHS_F (= \frac{1}{2} + \frac{B}{2\gamma}$; the negatively sloping part) and Foreign's voting restriction ($= 1 - \frac{1}{2\gamma}$; the positively sloping part): it shows which of the two that is the strictest. The curve separating $R1$ and $R2$ is the Home capitalists' indifference curve (where (11) holds with equality). The curve separating $R2$ and $R3$ is Foreign's equivalent. In the areas $R1$ and $R2$ all Foreign residents want $R$ and an elimination of barriers to firm relocation will be blocked by a Foreign veto. In area $R3$ Foreign capitalists want $U$, but we are below Foreign's voting restriction. In other words, Foreign capitalists are not numerous enough to win the domestic vote and $R$ is the outcome here too. In the areas $R4$ and $R5$ we are either in the two first columns of table 1 in which a Foreign veto yields $R$ (area $R4$), or we are in the third column (where Foreign capitalists want $U$), but below the voting restriction (area $R5$). Only in area $U$ are there enough capitalists relative to workers in Foreign to ensure a $U$ win in Foreign's vote. The corresponding potential outcomes under a single political jurisdiction are illustrated in figure 5 below.
In figure 5 we always get $U$ above the sustain condition (area $U_1$), since $A > 1$ holds under the chosen parameter values. Below the sustain condition, we get $U$ to the left of Home’s indifference curve (area $U_2$). The reason is that all of Home’s residents are in favour of $U$ there, whereas all Foreigners want $R$. Since both countries have the same number of capitalists, Home’s larger labour force tips the balance in favour of $U$. To the right of Foreign’s indifference curve (area $U_3$), a coalition of Foreign capitalists and Home workers form a majority and defeat $R$ voters. In the area between the two indifference curves, only Home workers are in favour of $U$. The curve separating $R$ and $U_4$ is the voting restriction $\frac{\gamma+1}{2\gamma}$ and $U$ results if we are above it. The reason is that the lone factor favouring $U$ is then large enough to defeat the three groups of factor owners preferring $R$.

From the figures above, it is clear that firm and capital mobility are more likely to be unrestricted if the two countries constitute a single political entity. An obvious explanation is the fact that the countries’ de facto veto right then has been eliminated.
5 Conclusions

Footloose firms raise concerns among small countries’ policy-makers about losing industries, while increased competition and loss of control over domestic industry is a cause of concern in large countries. In this paper we analyse under which circumstances politically determined barriers to international firm relocation will exist. Two countries, differing in market size, consider abolishing restrictions on firm mobility. Eliminating these restrictions will fully or partially deindustrialise the small country as firms relocate to the larger market, giving rise to domestic and international conflicts/alignments of interests among the owners of different factors of production. We examine preferred policies and analyse potential political outcomes with a majority voting rule under two different jurisdictional regimes: when the two countries are politically independent jurisdictions and when they form a single political entity.

We show that there is unanimous support for (resistance against) the removal of obstacles to firm relocation in the large (small) country if the country size difference is small, while a large difference in size gives rise to domestic conflicts of interest and international cross-factor alignments of interests. Furthermore, trade liberalisation may have facilitated the removal of barriers to firm relocation in large countries. Finally, political integration between trading countries is likely to contribute to the removal of barriers to firm relocation, and support for (resistance against) such a development comes primarily from the immobile factor in the large (small) country.

6 References


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Swedish National Labour Market Board (2002): "Kartläggning av det europeiska rättsläget vad gäller arbetsgivarens ansvar vid företagsnedläggningar" (An investigation of the European legal situation concerning employer responsibility in connection to plant closures), report by the legal department of the Swedish National Labour Market Board.


Appendix

Explaining why the return to capital is the same in all $U$ agglomeration outcomes

It is readily established that aggregate world capital income ($s_n \pi + (1 - s_n) \pi^*$) remains constant at $\frac{\alpha}{\sigma} (1 + \gamma)$. The reason is that the pricing condition, a constant markup over a constant marginal cost (see eq. (3)), ensures that the $\alpha$ dollars spent on $M$ by every consumer will always translate into $\frac{\alpha}{p}$ units sold and yield a contribution to capital income of $\frac{\alpha}{p} (p - w_L a_m) = \frac{\alpha}{\sigma}$, regardless of where firms and consumers are located. The logic of the latter claim is as follows. With iceberg transport costs a consumer will receive less for every dollar spent on varieties produced abroad, which will cause consumers to spend a larger share of their income on locally produced goods (if such exist), but a dollar spent on $M$ will nevertheless translate into the same quantity sold by producers. This is perhaps most obvious if one considers migration of a worker from Foreign to Home in a situation where full agglomeration prevails in Home. While still in Foreign, the worker would spend $\alpha$ dollars on $M$ yielding sales of $\frac{\alpha}{p}$ units to firms (all in Home), of which only $\frac{\alpha}{\sigma} > \frac{\alpha}{p}$ units would reach the consumer. After having moved to Home, the worker still spends $\alpha$ dollars on $M$, which also translates into sales by firms of $\frac{\alpha}{p}$ units, although the worker is then better off because all those units can be consumed. If there would still be firms in Foreign, the migrating worker would begin to allocate a larger part of $M$ spending on varieties produced in Home, but since her total $M$ spending (and hence the no. of units commanded) does not change, the increase in Home $M$ quantities sold (and hence Home capital income) will be exactly offset by a reduction in Foreign $M$ quantities sold (and hence Foreign capital income). With the normalised global quantities of capital and labor ($1$ and $\gamma$, respectively), the number of individuals each purchasing a quantity of $\frac{\alpha}{p}$ units of $M$ will be $(1 + \gamma)$ and the constant world capital income will therefore be
\( \frac{\alpha}{\sigma} (1 + \gamma) \).

**Establishing that \( B > 1 \)**

We have \( B > 1 \Leftrightarrow \frac{\sigma}{\sigma - 1} \frac{1 + \phi}{1 - \phi} \ln \left( \frac{1 + \phi}{2 \phi} \right) - 1 > 0 \). Set \( f(\phi) \equiv \frac{\sigma}{\sigma - 1} \frac{1 + \phi}{1 - \phi} \ln \left( \frac{1 + \phi}{2 \phi} \right) - 1 \). We want to show that \( f(\phi) > 0 \) for \( 0 < \phi < 1 \). First, \( f'(\phi) = \frac{\sigma}{(\sigma - 1)(1 - \phi)} \left[ \frac{2}{(1 - \phi)} \ln \left( \frac{1 + \phi}{2 \phi} \right) - \frac{1}{\phi} \right] < 0 \) for \( 0 < \phi < 1 \). Next, we evaluate \( \lim_{\phi \to 0^+} f(\phi) \) by setting \( \phi = \frac{1}{\tau} \to \lim_{\phi \to 0^+} f(\phi) = \lim_{\phi \to 0^+} \frac{\ln \left( \frac{1 + \phi}{2 \phi} \right)}{(\sigma - 1)(1 - \phi)} - 1 = \infty \). Finally, we rewrite \( f(\phi) = \frac{\ln \left( \frac{1 + \phi}{2 \phi} \right)}{(\sigma - 1)(1 - \phi)} - 1 \) and use L'Hôpital's rule to evaluate \( \lim_{\phi \to 1^-} \frac{\ln \left( \frac{1 + \phi}{2 \phi} \right)}{(\sigma - 1)(1 - \phi)} - 1 \Leftrightarrow \lim_{\phi \to 1^-} \frac{\phi}{\sigma - 1} - 1 = \frac{\sigma}{\sigma - 1} - 1 > 0 \). Hence \( f(\phi) \) is a strictly decreasing function bounded from below by the x-axis.

**Establishing that \( \frac{\partial \phi_H}{\partial s_L} > 0 \)**

We have \( \phi_H \equiv \frac{\gamma(2s_L - 1) - \sigma}{\gamma(2s_L - 1) + \sigma} \ln \left( \frac{2s_L + 1}{2s_L - 1} \right) \). Then \( \frac{\partial \phi_H}{\partial s_L} = 4\gamma \frac{\sigma}{(\sigma - 1)} \left[ \frac{\ln \left( \frac{2s_L + 1}{2s_L - 1} \right)}{(2s_L - 1)(\gamma + 1)} - \frac{\gamma(2s_L - 1)\gamma}{2s_L + 1} \right] \), which is positive if \( h(s_L) \equiv \ln \left( \frac{2s_L + 1}{2s_L - 1} \right) - \frac{(2s_L - 1)\gamma}{2s_L + 1} > 0 \), where \( s_L \in \left[ \frac{1}{2}, 1 \right] \). As \( h \left( \frac{1}{2} \right) = 0 \) and \( \frac{\partial h}{\partial s_L} = \frac{2s_L(2s_L - 1)}{(2s_L + 1)^2} > 0 \) for \( s_L > \frac{1}{2} \), we know that \( h(s_L) > 0 \) and hence \( \frac{\partial \phi_H}{\partial s_L} > 0 \). A similar argument can be made to show that \( \frac{\partial \phi_F}{\partial s_L} < 0 \).