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Papers, presented at Douglas Mc Henry International Symposium
on Concrete and Concrete Structures in Mexico City, October 1976.
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An Analytical Approach to Fire Engineering Design of Concrete Structures

By Y. Anderberg, S.E. Magnusson, O. Pettersson, S. Thelandersson, and U. Wickstrom

Synopsis: The principles are presented for the main types of the differentiated, structural fire engineering design systems, in practice at present or anticipated to be applied in the future. Such design systems are generally based on real fire exposure characteristics, given by the gastemperature-time curves of the complete fire process and specified in detail with respect to the influence of fire load and the geometrical, ventilation and thermal properties of the fire compartment. The design procedure can be in its entirety analytical or combined analytical and experimental. In the latter case, real fire exposure conditions can be transferred to the heating conditions according to the standard fire resistance test via the concept equivalent time of fire duration. Starting from the present state of knowledge, the possibilities are discussed for a practical application of a complete analytical, differentiated design in regard to fire exposed, reinforced and prestressed concrete structures. Finally, the various sources and kinds of uncertainty in the differentiated design procedure are briefly dealt with within the framework of the structural fire safety problem.

Keywords: fire ratings; fire resistance; fire tests; heat transfer; prestressed concrete; reinforced concrete; structural design; thermal properties

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INTRODUCTION

The internationally prevalent fire engineering design of buildings and elements of building construction is characterized by a schematic procedure, based on classification system and a connected standardized fire test with fixed heating conditions. In the practical design, the results of such classification tests directly are to be compared with the corresponding requirements, specified in the building codes and regulations.

Fig. 1 illustrates the design procedure. For different applications, the codes and regulations are giving the required time of fire duration t_{fd}, for which the structure has to fulfil its load-bearing function. The fire duration time ordinarily depends on the occupation, the height and volume of the building, and the importance of the structure or the structural memeber. The design comprises a proof that the structure has a fire resistance time t_{fr}, determined in the standard fire resistance test /1/, which exceeds the fire duration required. At the test, the test load conventionally is put equal to the design load at service state.

Internationally, the standard fire resistance test /1/ according to ISO 834 is considered to be one of the fire test methods most thoroughly dealt with. In spite of this, the fire resistance test can be seriously criticized. In its present form, the test procedure is insufficiently specified in several respects, for instance concerning the heating and restraint characteristics, the environment of the furnace, and the thermocouples for measuring and regulating the furnace temperature /2, 3, 4, 5, 6, 7, 8, 9/. Consequently, a considerable variation can arise in fire resistance for one and the same structure or structural member, when tested in different fire engineering laboratories with varying furnace characteristics and varying practice, as concerns the support and restraint conditions of the test specimens.

Some countries now are permitting a classification of load-bearing structures with respect to fire exposure to be carried out analytically. This leads to a design procedure according to Fig. 2. The theoretical determination of the fire resistance time of the structure t_{fr} then is to be based on the gastemperature-time curve, specified for the standard fire resistance test and given by the formula

$$T_t - T_o = 345\log_{10}(8t+1) \qquad (1)$$

where

t = time, in minutes,
T_t = furnace temperature at time t, in ^oC,
T_o = furnace temperature at time t = 0, in ^oC.

With the gastemperature-time curve as basic information, the temperature-time fields of the fire exposed structure can be calculated, using

(1) the structural characteristics of the proposed structure,
(2) the thermal properties of the structural materials, and
(3) the coefficients of heat transfer for the various surfaces of the structure

as further input data. Introducing

(4) the mechanical properties of the structural materials, and
(5) the load characteristics

then the time variation of the restraint forces and moments, thermal stresses and load-carrying capacity can be determined. The time, at which the load-carrying capacity has decreased to the level of the design load at service state, defines the time of failure or the fire resistance time t_{fr} and the design criterion to be satisfied is, that $t_{fr} > t_{fd}$.

The schematic character of the prevalent design system together with its great simplification of the fire exposure frequently prevents structural solutions satisfying reasonable requirements on economy and well-defined safety levels. A successive replacement of the system with logically built, differentiated design systems, based on real fire exposure characteristics, then has a high priority. A derivation of such fire engineering systems is also in agreement with the present trend of development of the building codes and regulations in many countries towards an increased extent of functionally based requirements and performance criteria.

In the last ten years, several functionally based, differentiated design methods have been published, as concerns fire exposed loadbearing structures. Mainly, these methods can be referred to one of two different groups with respect to the use of the basic data of the process of fire development. The methods of the first group are characterized by a design procedure, directly based on differentiated gastemperature-time curves of the complete process of a real fire development, specified in detail with regard to the influence of the fire load and the geometrical, ventilation and thermal properties of the fire compartment. Characteristic for the methods of the second group is a design procedure with the varying properties of a real fire development taken into account over an equivalent time of fire duration, connected to the heating according to the standard temperature-time curve, Eq.(1).

In the following, the principles are dealt with for the main types of the differentiated, structural fire engineering design system, in practice at present or anticipated to be applied in the future. The possibilities are discussed for a practical application of a direct, differentiated, analytical design in respect to fire exposed, reinforced and prestressed concrete structures. Finally, some comments are given on probabilistic methods and the structural fire safety.

PRINCIPLES OF A DIRECT, ANALYTICAL, DIFFERENTIATED STRUCTURAL FIRE ENGINEERING DESIGN

For load-bearing structures and structural members, inside a fire compartment, a direct differentiated fire engineering design comprises the following steps, Fig. 3 /10, 11, 12, 13, 14/.

The basis is given by a fully developed compartment fire exposure. Decisive entrance quantities are

(1) the nominal load and load factor for the fire load density,
(2) the combustion properties of this design fire load,
(3) the size and geometry of the fire compartment,
(4) the ventilation characteristics of the fire compartment, and
(5) the thermal properties of the structures enclosing the fire compartment.

Jointly, these quantities determine the rate of burning, the rate of heat release, and the design gastemperature-time curve of the complete fire process. Together with

(6) the structural data of the proposed structure,
(7) the thermal properties of the structural materials, and
(8) the coefficients of heat transfer for the various surfaces of the structure

this design gastemperature-time curve gives the requisite information for a determination of the transient temperature fields of the fire exposed structure. With

(9) the mechanical properties of the structural materials, and
(10) the load characteristics

as further entrance quantities, then a determination can be carried out of the time variation of the restraint forces and moments, thermal stresses, and load-carrying capacity R. The lowest value of this load-carrying capacity of the structure during the complete fire process defines the design load-carrying capacity R_d.

Over nominal loads and load factors for dead load, live load, etc, statistically representative of a fire occasion, a design load effect at fire S_d is defined, interdependent on non-fire design procedure.

A direct comparison between the design load-carrying capacity R_d and the design load effect at fire S_d decides whether the structure can fulfil its required function or not at a fire exposure.

For buildings containing activities, which are particularly important from, for instance, an economical point of view, there can be the motive for requiring that the building can be used again after a fire, almost immediately or very soon, for the current activities in a full extent. If the fire engineering design also comprises such a requirement on re-serviceability of the structure after fire, the design procedure is to be expanded as follows.

From the time curve of the load-carrying capacity R, the design residual load-carrying capacity R_{rd} of the structure after fire is obtained as an end information. This quantity R_{rd} has to be compared with the design load effect at service, non-fire, state on the structure S_{rd}, given by the corresponding nominal loads factors for dead load, live load, etc.

Generally, as concerns the load factors applied to the nominal values of fire load density, live load and dead load, etc, these ought to be derived in a statistically consistent way to match a given safety level, defined by, for instance, a safety index /13/.

A direct, analytical, differentiated design according to Fig. 3 can be carried through in practice today in a comparatively general extent for fire exposed steel structures. The practical application then is facilitated by the availability of a manual /12/, comprising a comprehensive design basis in the form of tables and diagrams which directly are giving the maximum steel temperature for a differentiated, complete process of fire development and the corresponding design load-bearing capacity. The manual has been approved for a general practical use in Sweden by the National Board of Physical Planning and Building.

In comparison with steel structures, fire exposed reinforced and prestressed concrete structures generally are characterized by an essentially more complicated thermal and mechanical behaviour. In consequence, the basis of an analytical, differentiated, structural fire engineering analysis and design is considerably more inclomplete for concrete structures - cf, for instance, /14, 15/. This will be commented on in greater detail in a subsequent chapter.

COMBINED ANALYTICAL AND EXPERIMENTAL DESIGN METHODS, BASED ON REAL FIRE EXPOSURE. EQUIVALENT TIME OF FIRE DURATION

In those cases, at which the present state of knowledge does not enable a complete analytical, differentiated structural fire engineering design to be carried out, the concept equivalent time of fire duration can a useful implement. Generally, the concept has been introduced as a mean for a direct translation from a real fire exposure to a corresponding heating according to the temperature-time curve of the standard fire resistance test, Eq. (1), and vice versa. Depending on the type of design problem to be dealt with and the level of accuracy intended, the character of the concept will vary.

If the available design basis permits a theoretical determination to be performed, as concerns the transient temperature fields but not the design load-carrying capacity of a fire exposed structure, it can be motivated to use a differentiated form of the equivalent time of fire duration concept /11, 12, 16/. Such a definition of the concept is exemplified in Fig. 4, applicable to, for instance, such types of reinforced concrete structural members for which the strength and deformation properties of the reinforcement at elevated temperatures form the decisive failure criterion. In the figure, the full-line curves are showing the time variation of the gastemperature T_t^r and the tension reinforcement temperature T_s^r corresponding to a real fire exposure, determined by the fire load density, and the geometrical, ventilation and thermal properties of the fire compartment. The dash-line curves are giving the standard furnace temperature-time variation according to Eq. (1) T_t^s and the corresponding time curve of the temperature of the tension reinforcement T_s^s. A transfer of the maximum reinforcement temperature for the real fire exposure $T_{s\,max}^r$ to the curve T_s^s determines the equivalent time of fire duration t_e.

Determined in this way, the equivalent time of fire duration t_e depends on the parameters, influencing the process of fully developed fires, as well as on a number of structural parameters - for a reinforced concrete beam of rectangular cross section: the height and the width of the cross section, the distance from the layer of reinforcement to the fire exposed surface, and the resultant emissivity; Fig. 5 /11/.

The way of applying the concept equivalent time of fire duration, principally defined in conformity to Fig. 4, in a differentiated, structural fire engineering design is shown summarily in Fig. 6. As for a complete analytical design, the design procedure starts by a theoretical determination of the rate of burning and heat release, the design gastemperature-time curve of the complete fire process, and the transient temperature fields of the structure or structural member at real fire exposure conditions. Using the gastemperature-time curve according to the standard fire resistance test as an input information, then the connected transient temperature fields of the structure or structural member are determined theoretically. A transfer of the design temperature state for the real fire exposure - e.g. the maximum temperature of the tension reinforcement - to the same temperature state for a thermal exposure according to the standard fire resistance test, gives the equivalent time of fire duration t_e. The last part of the design is to be carried out experimentally. It comprises a determination in a standard fire resistance test of the fire resistance time t_{fr} for the structure, acted upon by the design load effect at fire. Finally, a direct comparison between t_{fr} and t_e decides whether the structure can fulfil its required function or not at a fire.

If the existing state of knowledge does not permit a theoretical determination of either the transient temperature fields or the design load-carrying capacity of the fire exposed structure, the translation from a real fire exposure to the thermal exposure according to the standard fire resistance test must be based on a more rough form of the concept equivalent time of fire duration. At such circumstances, the equivalent time of fire duration t_e can be differentiated only with respect to the characteristics of the fire process and not with respect to the structural parameters. This way of applying the equivalent time of fire duration has been introduced by Law /17/, and Thomas Heselden /18/. A similar, somewhat more generalized approach is presented in /11, 16/. Fig. 7 illustrates the structural fire engineering design connected to this form of the equivalent fire duration concept.

In a long-term perspective, the direct, analytical, differentiated fire engineering design according to Fig. 3 can be seen as the final goal of the current development. The design procedure according to Fig. 6, using an accurate form of the equivalent fire duration concept, represents an indirect method with an equivalent degree of differentiation as the direct method, defined by Fig. 3. For a practical application, both methods require the availability of design diagrams and tables in about the same extent. An advantage of the design method according to Fig. 6 is a better adaption for a direct use of data from standard fire resistance tests. In favour of the design method according to Fig. 3 are speaking better possibilities of dealing with the detailed functional behaviour of the structure with regard to different types of fracture and varying load level and degree of restraint. This method also is more suitable for taking into account such influences as temperature-time dependent basic characteristics of the structural materials and of the details of the structure - for instance, effect of the disintegration of the materials, enlarged short-time effect of creep and shrinkage, effect of crack formation and spalling, strength of fastening devices for different types of insulation, rate of increase in the depth of the charred layer at timber structures. The direct analytical design procedure, as described in Fig. 3, also is in better agreement with the development in progress of the building codes and regulations towards an increased extent of functionally based requirements and performance criteria. Indirect design methods of the type given in Fig. 7 and based on a less accurate equivalent fire duration concept are now introduced in serveral countries. Undoubtedly, this implies an essential step forward in comparison with the schematic fire engineering design according to Fig. 1 and 2, prevalent at present. In spite of this, indirect design methods according to Fig. 7, related to real fire exposure characteristics in a very rough way, should be restricted to be only temporary solutions in the long-term development.

COMMENTS ON A DIRECT, ANALYTICAL, DIFFERENTIATED DESIGN OF FIRE EXPOSED CONCRETE STRUCTURES

In what follows, the main steps of a direct, analytical, differentiated fire engineering design according to Fig. 3 will be discussed and commented on, primarily as concerns the possibilities of a practical application to fire exposed, reinforced and prestressed concrete structures. The treatment will be limited to the thermal and structural behaviour during the fire exposure, i.e. the residual state and post fire behaviour of the structures will be left out. A state of art report on post fire behaviour of concrete structures is given in /19/.

Fire Load Density and Process of Fire Development in a Compartment

At known combustion characteristics, the gastemperature-time curve of a fully developed compartment fire can be calculated in the individual practical application from the heat and mass balance equations of the fire compartment with regard taken to the size, geometry and ventilation of the compartment, and to the thermal properties of the structures enclosing the compartment - Fig. 8 /12, 20, 21, 22, 23, 24/.

Provisionally, an analytical, differentiated fire engineering design of load-bearing structures can be based on the gastemperature-time curves T_t-t according to Fig. 9 /12, 14, 16, 20/, which applies to a fire compartment with surrounding structures made of a material with a thermal conductivity $\lambda = 0.81$ W/m^{o}C and a heat capacity $\rho c_p = 1.67$ MJ/m^{3o}C (fire compartment, type A). Entrance parameters of the diagrams are the fire load density q, defined by the formula

$$q = \frac{1}{A_t} \Sigma m_\nu H_\nu \qquad (MJ/m^2) \tag{2}$$

and the ventilation characteristics of the fire compartment, expressed by the opening factor $A\sqrt{h}/A_t$ ($m^{1/2}$), where

A = total area of window and door openings (m^2),
h = mean value of the heights of window and door openings, weighed with respect to each individual opening area (m),
A_t= total interior area of the surfaces bounding the compartment, opening areas included (m^2),
m_ν= total weight of combustible material ν (kg), and
H_ν= effective heat value of combustible material ν of the fire load (MJ/kg).

Fire compartments with surrounding structures of deviating thermal properties can be transferred to fire compartment, type A, via fictitious values of the fire load density q_f and the opening factor $(A\sqrt{h}/A_t)_f$, see /12, 14, 16/.

For a determination of the opening factor, when the fire compartment also comprises horizontal openings, reference is given to /12, 20/.

It should be stressed that the gastemperature-time curves according to Fig. 9 generally have been determined on the assumption of ventilation controlled fires. As a consequence, the curves are not intended to be used directly for theoretical comparisons with, for instance, experimentally obtained results from wooden crib compartment fires of strongly marked fuel bed controlled type. One principal reason for chosing ventilation controlled fire characteristics as a general assumption for the determination of the design curves in Fig. 9 is dictated by the great difficulty in finding representative values of the free surface area and the porosity properties of real fire loads of furniture, textiles, and other interior decorations, which are essential parameters for a combustion description of a fuel bed controlled fire but of minor importance for the development of ventilation controlled fires. Another principal reason is related to the fact that the gastemperature-time curves themselves do not constitute the primary interest of the problem in this connection but an intermediate part of a determination of the decisive quantity, viz. the minimum load-bearing capacity of the fire exposed structure during a complete fire process. For fuel bed controlled fires, the assumption of ventilation control leads to a structural fire engineering design which will be on the safe side in practically every case, giving an overestimation of the maximum gastemperature and a simultaneous, partly balancing underestimation of the fire duration. For the minimum load-bearing capacity, the gastemperature-time curves according to Fig. 9 give reasonably correct results, verified in /12, 13, 22/.

Thermal Properties and Transient Temperature Fields at Fire Exposure

A theoretical determination for the transient temperature fields of a fire exposed concrete structure requires a thorough knowledge of the relevant thermal properties - the thermal conductivity λ and the specific heat c_p, alternatively the enthalpy I, connected to the specific heat c_p through the relation

$$I = \int_0^T c_p dT \quad (3)$$

where

T = temperature

For normal weight concrete the thermal conductivity λ decreases with increasing temperature. This is illustrated for a granite aggregate concrete in Fig. 10 /25/ which also shows the λ variation under cooling from different maximum temperature levels. The curves are demonstrating the difference in temperature dependence of the thermal conductivity for an initial heating process and a subsequent cooling process. This difference has to be taken into account in a theoretical fire engineering design, especially in calculating the residual state of a concrete structure after a fire exposure.

The influence of moisture on the thermal conductivity of concrete presents special difficulties. This is relevant for temperatures within the range up to 200°C. Well-defined measurement of λ for moist material in this temperature range are difficult to undertake due to the complicated interaction between moisture and heat flow.

As concerns the enthalpy of concrete, available methods of measurement only can give this quantity versus temperature under cooling. The latent heat of various reactions taking place under the initial heating then is not included. Curve 1 in Fig. 11 shows the enthalpy I_v per unit volume in this way /26/. Curve 2 gives that variation of the enthalpy which can be expected during heating of concrete without free moisture. The curve has been determined theoretically on the basis of stochiometric calculations and simplified assumptions on the chemical reactions /27/. A significant difference between the two curves exists for temperature above 500°C

The most important modification of the enthalpy curve measured under cooling, however, is due to the presence of evaporable water. As long as experimental evidence is lacking, the influence of moisture on the enthalpy has to be included in a simplified way in calculating the temperature-time fields at fire exposure. Usually, then it is assumed that all the moisture "boils" at the temperature 100°C with the required heat of evaporation giving a discontinuous step in the enthalpy curve at this temperature. Such a simplification also gives acceptable results for most practical purposes.

In reality, the evaporation of moisture in fire exposed concrete is not comparable to that of a free water surface. Capillary forces, adhesive forces, and interior steam pressure will increase the temperature, when the evaporation takes place. In a fire exposed concrete structure, the moisture distribution is changing continuously during the heating. Principally, it is then not correct to include the effect of free moisture into the thermal properties.

Available methods for a calculation of the transient heat flow within a fire exposed structure are based on the Fourier equation of heat conduction in non transparent, non porous materials.

In application to concrete structures, this equation constitutes an approximation of the problem. Concrete is classed as a porous material which implies that a heat transfer occurs also by convection and radiation in the pores of the material. Furthermore, the heat transfer is connected to a simultaneous moisture transport and from a strict thermodynamical point of view, these two transport mechanisms have to be analyzed parallelly over a system of partial differential equations.

For a practical determination of the temperature-time fields in fire exposed structures, numerical methods have been developed and arranged for computer calculations. Such numerical methods are based either on finite difference /15, 26, 28/ or on finite element approximations /29/. The methods have to start out from approximations of the thermal properties at elevated temperatures according to above. The methods are opening the possibilities for systematic determinations of the temperature-time fields for varying conditions of fire exposure and varying structural characteristics, giving a basis in the form of diagrams and tables for facilitating a differentiated fire engineering design in practice. The temperature in different points of the cross section of a fire exposed concrete structure, then can be calculated with sufficient accuracy without modeling the reinforcement of the cross section, if the percentage of the reinforcing steel is less than about 4 per cent /28, 29/.

A systematized design basis of the described type is now successively produced. A fragmentary example is shown in Fig. 12, giving the maximum temperature in different points of a concrete beam of rectangular cross section fire exposed from below on three surfaces*. The fire exposure is characterized by complete gastemperature-time curves according

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*From a design basis, computed by Ulf Wickström, for a manual, to be edited by the National Board of Physical Planning and Building in Sweden.

to Fig. 9, differentiated with respect to the fire load density q and the opening factor $A\sqrt{h}/A_t$ of the fire compartment.

Mechanical Properties and Structural Behaviour at Fire Exposure

A transfer of the transient temperature fields of a fire exposed concrete structure to data on the structural behaviour and load-bearing capacity requires an advanced knowledge on the strength and deformation properties of concrete and reinforcing steels in the temperature range associated with fires.

Comparatively detailed information then is available for some types of reinforcing steels, as concerns stress-strain relation, short-time creep, and residual str ngth /30, 31, 32, 33/.

For concrete, the deformation behaviour at elevated temperatures is much more complicted than for the reinforcement /15, 34/. The various sources of deformation are controlled by a large number of variables and and the different types of deformation are not independent of each other. The strain increment in a certain moment depends on the preceding stress and temperature histories.

The possibility of applying an ultimate load approach on those types of fire exposed concrete structures, for which the concrete component has a decisive influence, depends on whether the deformability of heated concrete is sufficient for the redistribution of stresses to take place. Another essential aspect in this connection is the definition of the ultimate stress, since this quantity depends on the previous stress history. In /34/ it is suggested that for ordinary application the ultimate stress might be determined from tests, where the specimens are first loaded to certain stress levels and then heated until failure occurs.

An accurate analysis of the mechanical behaviour of a fire exposed concrete structure implies that the constitutive relations between stresses and strains are known, the time-dependent behaviour included. In comparison with metallic or ceramic materials, stressed concrete then presents special difficulties in that respect that during the first heating considerable deformations develop which do not occur at stabilized temperature.

The first formulation of a realistic constitutive equation for concrete under transient, high-temperature conditions was published by Thelandersson /35/ for concrete in pure torsion. The constitutive equation was derived in terms of the strain components: instantaneous stress-related strain, constant temperature creep strain, and transient strain. The instantaneous stress-related strain is based on stress-strain relations obtained under constant, stabilized temperature. The constant temperature creep is the time dependent strain measured under constant stress and temperature. The third component, the transient strain, is developed only if the temperature increases in the concrete under load. Ordinarily, then the transient strain constitutes the major part of the total deformation.

A corresponding, computer-oriented, constitutive model for fire exposed concrete in uniaxial compression is derived by Anderberg and Thelandersson in /36/. The total deformation is expressed in terms of thermal strain, including shrinkage, instantaneous stress-related strain, and transient strain and parameter formulations are made for each of the strain components on the basis of test results. This model is more complex than that corresponding to torsion due to the influence of stress and temperature history on the behaviour. Partly, this is expressed in the model by the interdependence between the instantaneous stress-related and the transient strain components.

From the present state of knowledge, as concerns the mechanical properties of concrete and reinforcing steels at transient temperature conditions, it follows, that such phenomena easily can be predicted for fire exposed concrete structures, for which the strength and deformation properties of the reinforcement constitute the decisive failure criterion. This applies to the ultimate moment capacity of simply supported beams and slabs of reinforced and prestressed concrete. The transfer from temperature to load-bearing capacity in the hot state then can be done via Fig. 13 /37/, giving the decrease in strength, caused by heating, in some typical reinforcement and prestressing steels. Other types of failure - as shear, bond and anchorage failures - have not been the subject of any systematic studies in connection with fire and little is known about them at present.

For fire exposed, continuous beams and slabs it seems justified to assume that the limit state theory can be applied in many cases /38, 39/. It should be noted, however, that the rotations induced by thermal gradients are considerable and the rotation capacity required for a complete redistribution of moments therefore can be greater than at ambient conditions. The influence of thermal exposure on the rotation capacity of concrete structures has not yet been systematically studied. In continuous beams, exposed to fire from below, portions with negative moments will be affected by the fire mainly in the compression zone. Here the possibility, that concrete failure occurs before the reinforcement yields, must be considered. A comprehensive, combined theoretical and experimental study of fire exposed concrete beams or plate strips with rotational and axial restraint is reported in /39/. The study comprises a detailed analysis of the structural behaviour and load-carrying capacity during a complete fire process, varied with respect to the fire load density q and the opening factor of the fire compartment $A\sqrt{h}/A_t$. The theoretical determination of the structural behaviour is based on the constitutive models for concrete in compression and reinforcing steel in tension at transient, high-temperature conditions, derived in /35/ and /33, 39/, respectively.

For non-slender, centrically loaded columns and walls, the failure occurs when the compressive strength of the concrete is exceeded. If sufficient plastic deformations can develop at fire exposure, then the ultimate state can be analyzed according to the plastic theory. At the present state of knowledge, it is difficult to say whether such an assumption is generally justified or not. Studies, made by Bengtsson /40/, indicate the validity of the assumption, as concerns a theoretical determination of the residual, load-bearing capacity of concrete columns after fire, Fig. 14.

Also for more complicated applications, for instance a theoretical analysis of the structural behaviour of fire exposed concrete frames, mathematical models and connected computer programs are available /15, 41, 42/. The most comprehensive program is that presented in /43/, which is capable of providing a broad spectrum of response data, including the time history of displacements, internal forces and moments, stresses and strains in concrete and in steel reinforcement, as well as the current states of concrete with respect to cracking or crushing and steel reinforcement with respect to yielding. Instability phenomena and second order effects are not included in the program.

Spalling

An additional factor of uncertainty in an analysis of a fire exposed concrete structure is the spalling phenomenon. When the spalling occurs, the geometry of the structure is changed and the temperature will increase more than expected from the calculations, based on the original geometry. The spalling may also directly influence the structural behaviour. Hence, a special estimate must be made, as regards the risk of spalling, which constitutes an additional problem in the application of an analytical differentiated design. It should be noted, however, that the same problem also is inherent in the conventional schematic design procedure, related to classification systems.

By experience it is known that the disposition of concrete to spalling increases
at high moisture content,
at presence of compressive stresses from exterior loading or prestress,
at high rate of temperature increase,
at highly unsymmetrical temperature distribution,
at thin walled cross sections, and
at high percentage of reinforcement.

The risk of spalling is greater for concrete with wuartz aggregate than with, for instance, limestone aggregate. An increase of the air content of the concrete gives an improved resistance against spalling. Primarily, spalling is caused by one or several of the following mechanisms /43, 44, 45/:

(1) Vapour pressure due to vaporization of moisture in the material,
(2) thermal stresses due to restrained temperature deformations, including restraint stresses from difference in thermal elongation of concrete and reinforcement,
(3) structural disintegration of the aggregate.

In order to prevent the occurence of spalling, the diagram in Fig. 15 can be used as a simple guidance in the design /45/. The diagram is based on extensive experimental studies covering a wide region of variations with respect to concrete quality and temperature exposure. The diagrams gives a borderline, determined by the maximum stress σ_0

from exterior loading and prestress and by the cross section width b or web thickness t. Above this borderline a destruction by spalling probably will occur at a fire exposure, and below, the structure will be safe with regard to spalling. The results are directly valid for concrete structures with a low percentage of reinforcement. An increase of the percentage of reinforcement results in an increased risk of spalling.

STRUCTURAL SAFETY OF FIRE EXPOSED CONCRETE STRUCTURES

Recent developments in the theory and practical application of probabilistic methods and structural safety have been very rapid. As one example on this, it can be mentioned that the new "Draft Code for Loading Regulations", issued by the Nordic Committee for Building Regulations (NKB), explicitly permits structural design to be based on measurements and concepts evaluated directly within a probabilistic framework. On the other hand, many areas exist where the complexity of the situation has impeded any attempt of non-deterministic analysis. One prominent example concerns the field of fire exposed structures or structural members. To the authors knowledge, the only paper discussing the application of reliability based design methods in this connection is /13/. This paper is exclusively concerned with insulated steel structural components, but the structure of the developed methodology for a systematized safety analysis is quite general and applicable to a wide class of load-bearing structures or structural members. The procedure is connected to the basic probabilistic concepts used in normal structural design, as explained and derived in /46/.

A safety analysis of fire exposed, load-bearing concrete structures may have several different basic aims /13, 47/:

(1) to provide a systematized scheme for coordinating reported information about uncertainty and information about structural behaviour into a stochastic model, describing the resistance function R and the load effect function S,

(2) to evaluate from this model quantitative measures, e.g. in form of a safety index β, of the structural safety of the building component exposed to a fully developed compartment fire, and

(3) to demonstrate that for a proposed reliability-based design the code parameters, e.g. the safety indices, may be selected to match the safety level of current design.

A rational attempt to calculate the structural reliability of a loading situation with so many stochastic parameters, many of whom are interdependent in a complex manner, must have one fundamental basis: an analytical or mathematical deterministic model, yielding at least a rough description of the different physical phenomena. This analytical or design theory forms the skeleton, in relation to which all available statistical information should be evaluated. The analytical design scheme according to Fig. 3 fulfils this requirement. In addition, such a design scheme also is giving an indication of the different component variabilities lumped together in the load-carrying capacity R_d and the load effect term S_d.

Broadly speaking, the uncertainty in R_d may be grouped into the following categories /13, 47/:

- scatter in material properties,
- variability due to fabrication and workmanship error,
- uncertainties in the mathematical modeling of the physical phenomena, as measured by laboratory tests with well known test specimen characteristics (material properties, member geometry and size, structural restraint conditions, heat transfer conditions), and
- uncertainty due to difference between idealized laboratory conditions and actual service, "in situ", condition.

Spalling, bond, anchorage, and shear failure at fire exposure are examples of failure modes where at present the non-deterministic characteristics have to be estimated by non-analytical methods, for example by coordinating tests results from the substantial amount of experiments already carried out.

In /13/, the reliability levels are compared for load-bearing steel structures between the standard fire design procedure and the differentiated fire engineering design method, as described in Fig. 3. The comparison demonstrates how the flexibility of the differentiated design method results in a drastically improved consistency for the failure probability. The studies also emphasize that the differentiated design method - in contrast to the standard design procedure - has the capability of being systematically improved as knowledge increases.

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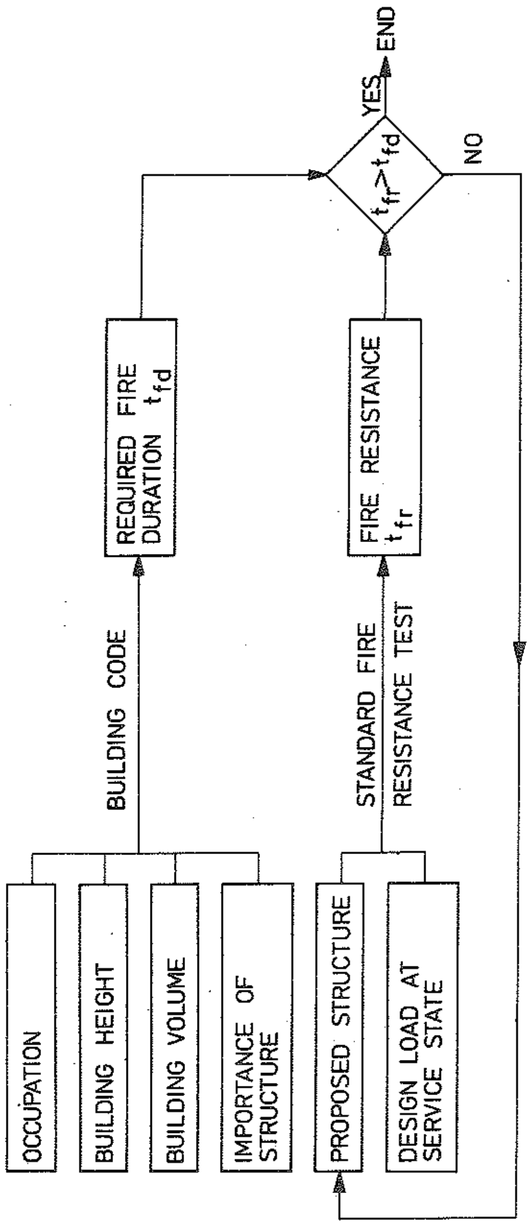

Fig. 1--Conventional fire engineering design of load-bearing structures, based on classification and results of standard fire resistance tests

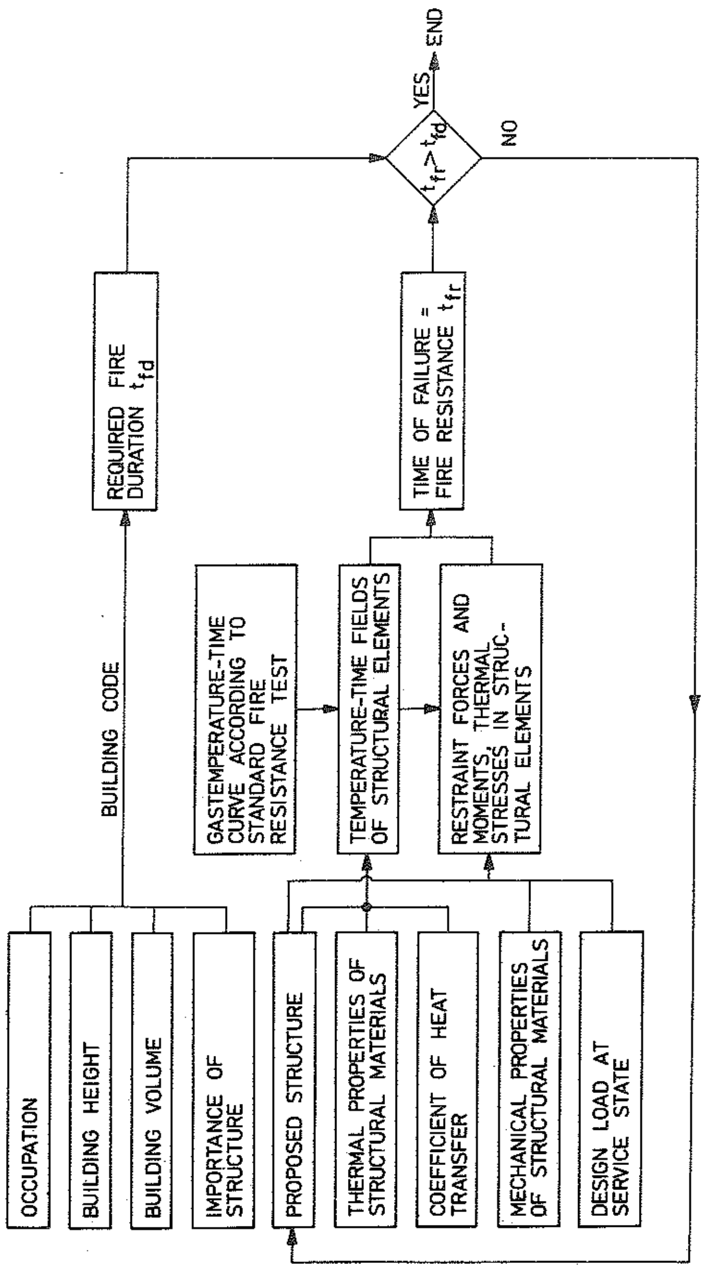

Fig. 2--Theoretical procedure of a fire engineering design of load-bearing structures, based on classification and thermal exposure according to standard fire resistance tests

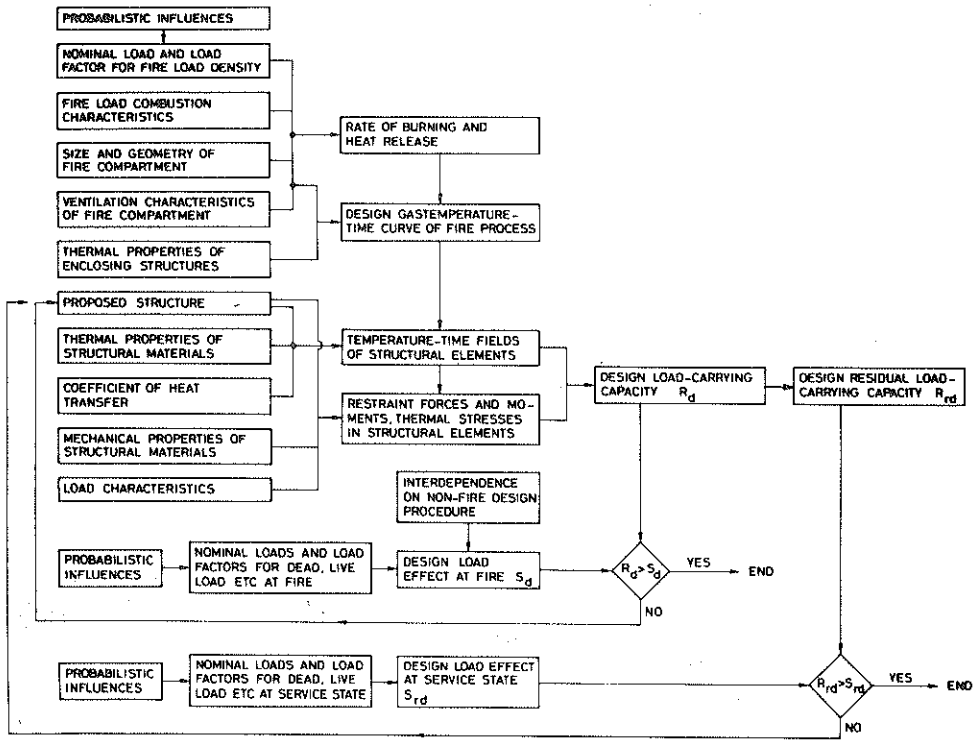

Fig. 3--Procedure of a differentiated fire engineering design of load-bearing structures with additional requirement on reserviceability after fire

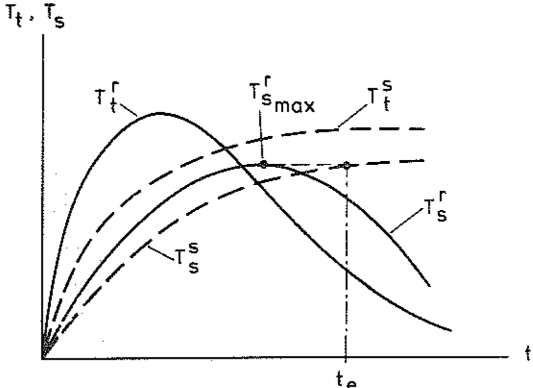

Fig. 4--The principle of a differentiated definition of the equivalent time of fire duration t_e, exemplified for a fire exposed reinforced concrete structural member

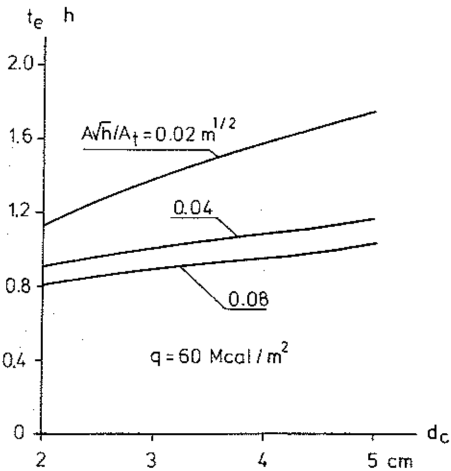

Fig. 5--Reinforced concrete beam of rectangular cross section with height 44.7 cm and width 22.4 cm, exposed to a fire on three sides. Equivalent time of fire duration t_e at varying opening factor $A\sqrt{h}/A_t$ and distance d_c from the layer of the reinforcement to the underneath side of the beam. Fire load density $q = 60\ Mcal/m^2$, Eq. (2)

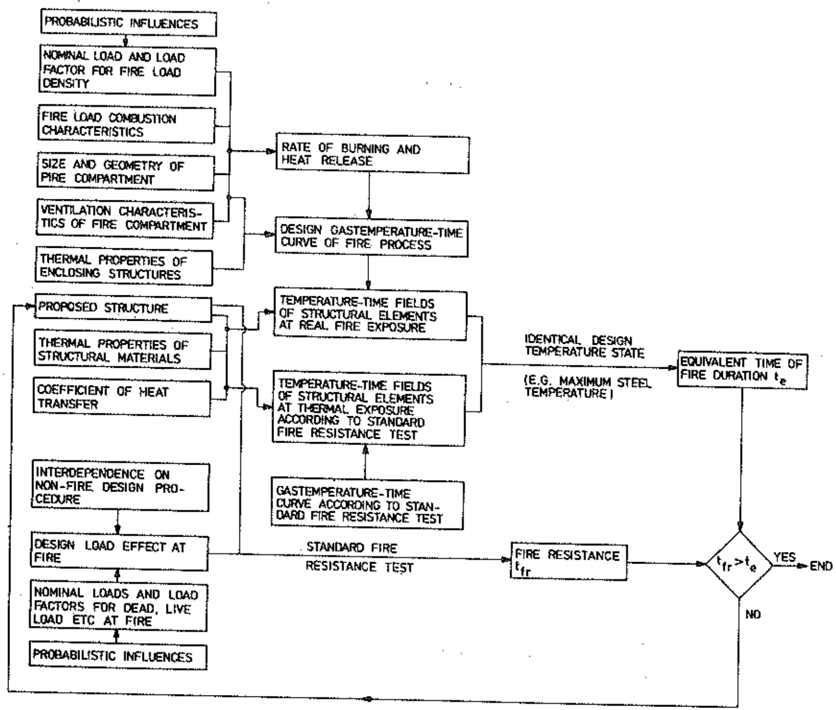

Fig. 6--Procedure of a differentiated fire engineering design of load-bearing structures, based on a theoretical determination of the equivalent time of fire duration t_e and an experimental determination of the fire resistance t_{fr}

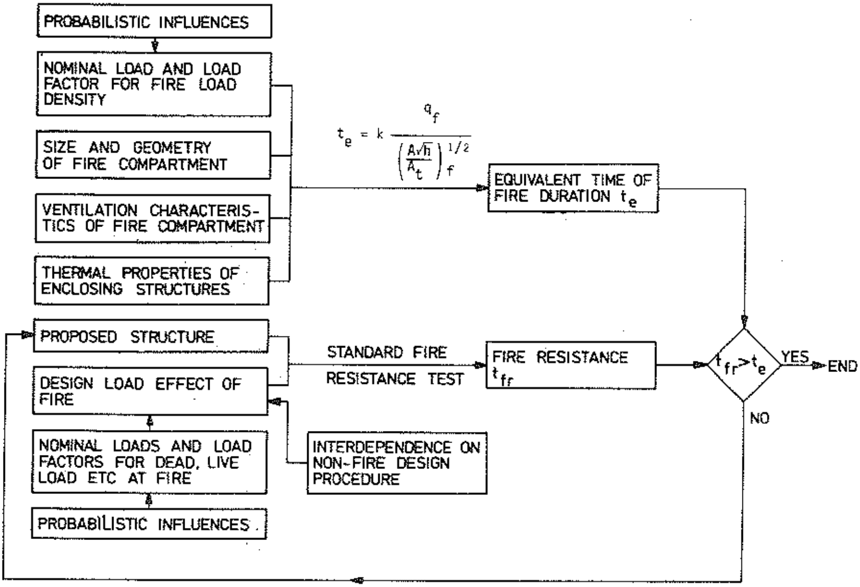

Fig. 7--Structural fire engineering design, based on real fire exposure characteristics and using a roughly estimated, equivalent time of fire duration t_e and an experimentally determined fire resistance t_{fr} as the main quantities to be compared

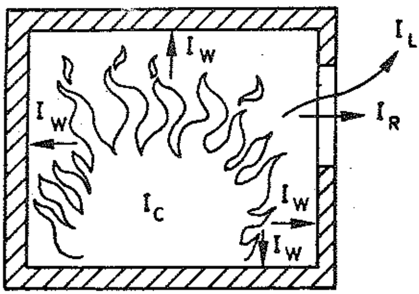

Fig. 8--Energy balance equation $I_C = I_L + I_W + I_R$ of a fire compartment. I_C it the heat release per unit time from the combustion of the fuel, and I_L, I_W and I_R the quantities of energy removed per unit time by change of hot gases against cold air, by heat transfer to the surrounding structures, and by radiation through the openings of the compartment, respectively

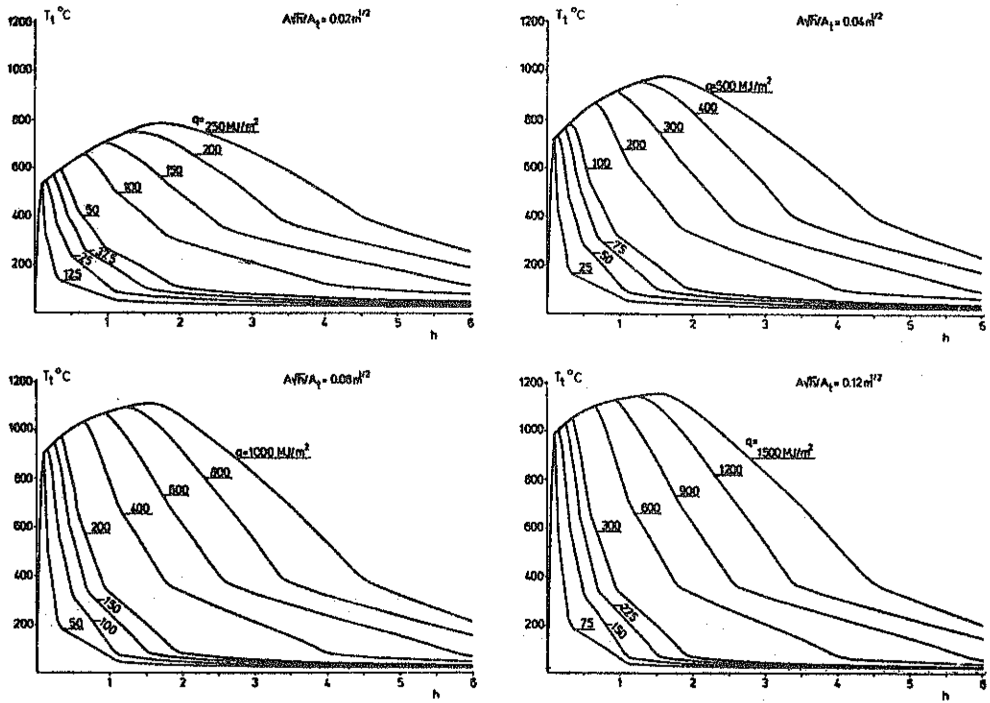

Fig. 9--Gastemperature-time curves T_t-t of the complete process of fire development for different values of the fire load density q and the opening factor $A\sqrt{h}/A_t$. Fire compartment, type A

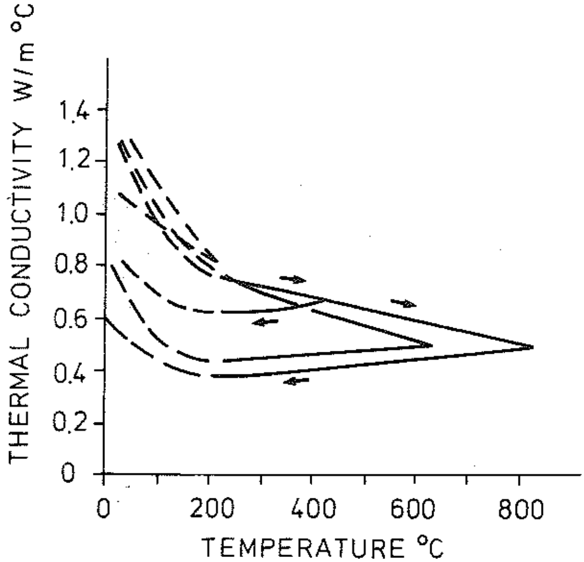

Fig. 10--Thermal conductivity λ for concrete with granite aggregate as a function of temperature under heating and subsequent cooling. Cement: aggregate 1:6, w/c = 0.7

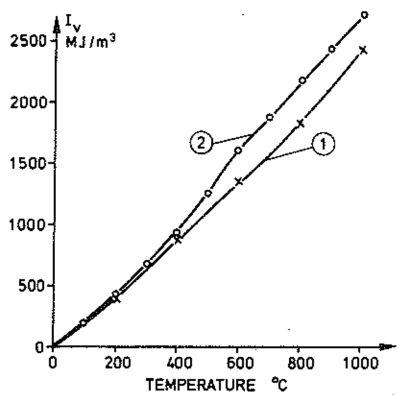

Fig. 11--Enthalpy Iv per unit volume as a function of temperature for concrete with granite aggregate. 1 Measured curve under cooling /26/, 2 theoretical curve /27/

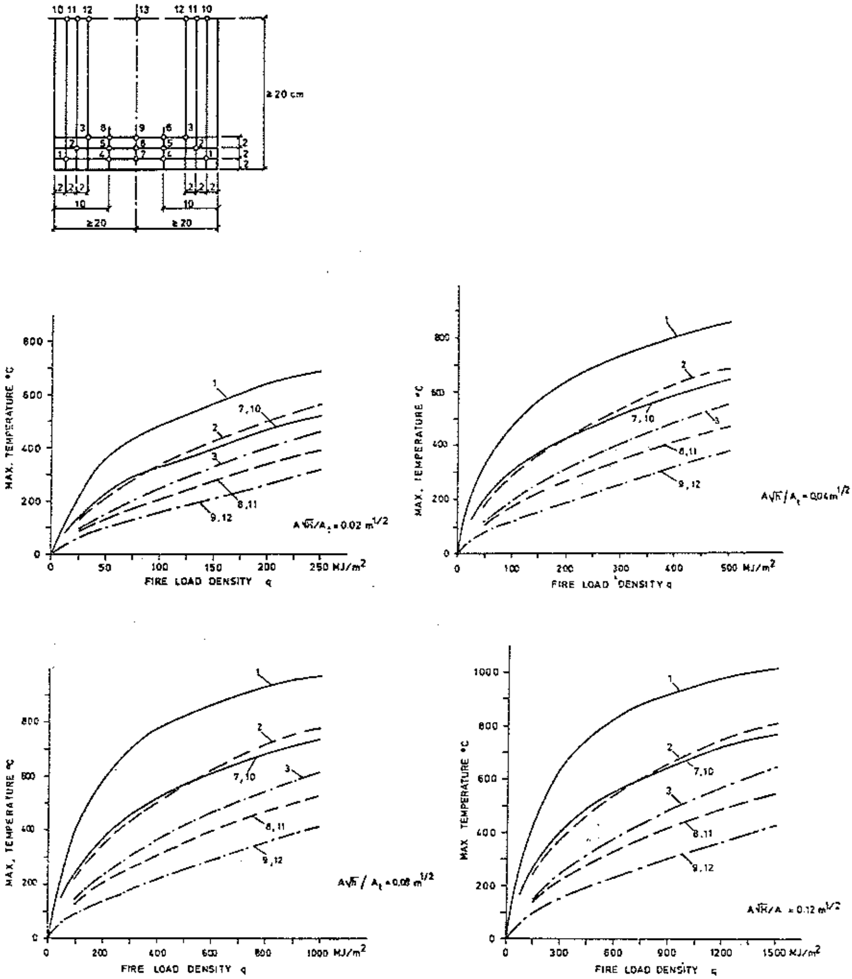

Fig. 12--Maximum temperature in different points of a concrete beam of rectangular cross section, fire exposed from below on three sides. Fire exposure according to Fig. 9

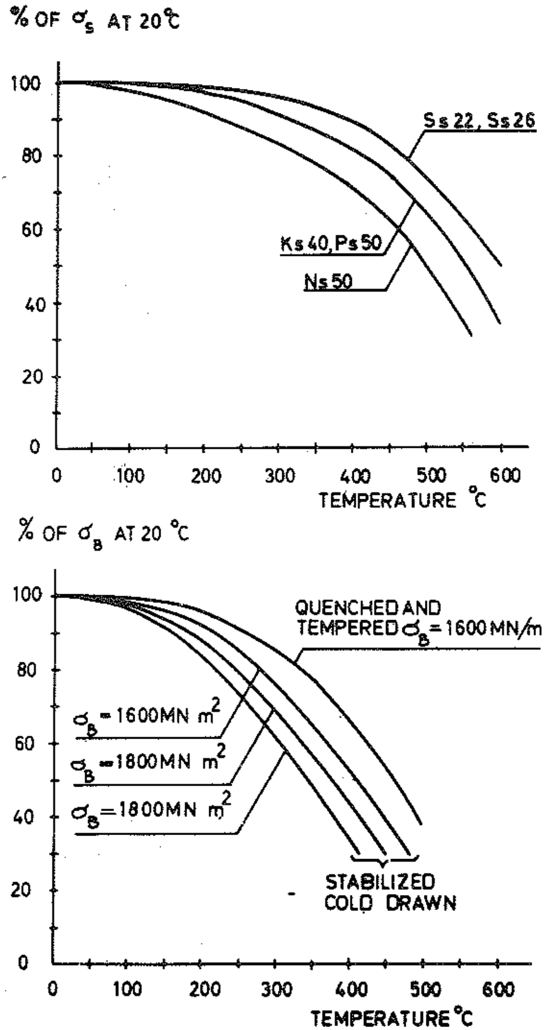

Fig. 13--Decrease in strength, caused by heating, of reinforcing steels (a), and prestressing steels (b), respectively

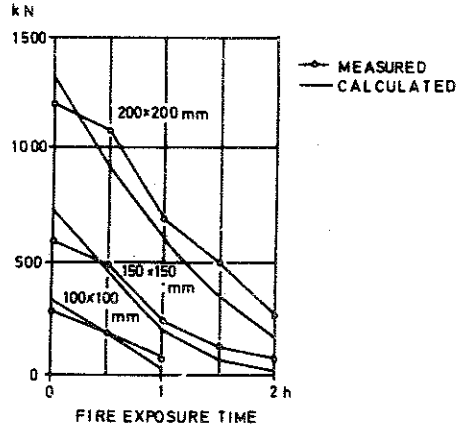

Fig. 14--Measured and calculated values of the residual load-bearing capacity of non-slender, concrete columns as function of fire exposure time at standard heating conditions

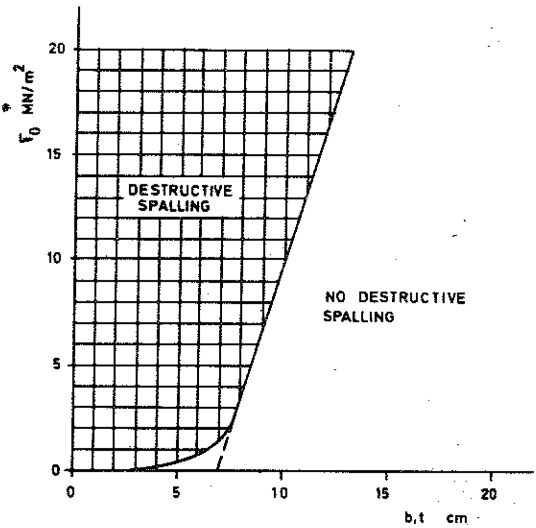

Fig. 15--Borderline between destructive and nondestructive spalling of fire exposed concrete structures with a low percentage of reinforcement. σ_0 is the maximum compressive stress from exterior loading and prestress, b width of cross section, and t web thickness

SP 55-8

A Constitutive Law for Concrete at Transient High Temperature Conditions

By Y. Anderberg and S. Thelandersson

Synopsis: A computer-oriented constitutive model for concrete in compression, valid at first heating of concrete up to 800°C is described. The total deformation is expressed in terms of thermal, instantaneous stress-related, creep and transient strain components, where the transient strain is a concept introduced to describe the particular behaviour under changing temperature. Comparisons with independent tests demonstrate that the material behaviour is described in a very appropriate way. The model is applied in a simple example calculation, showing that thermal stresses due to non-uniform temperature distribution are very insignificant or even non-existant. Stresses due to restrained thermal expansion cannot in themselves contribute to compression failure of concrete.

Keywords: compressive strength; concretes; creep properties; deformation; high temperature; strains; stress-strain relationships; temperature rise (in concrete); thermal expansion; thermal gradient; thermal stresses

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INTRODUCTION

The recent development in fields like structural fire protection has created a need for a better understanding of the mechanical behaviour of concrete at high temperature (up to 600-800^{o}C). Most experimental investigations up to this date have been performed under steady state temperature conditions, showing the effect of temperature on strength, elastic modulus, stress-strain relation and in some cases creep (at constant temperature). In the recent years, however, it has become obvious /1. 2. 3. 4, 5, 6/ that the behaviour under transient conditions cannot be predicted from information gained in such tests.

The purpose of this paper is to formulate a computer-oriented model for the mechanical behaviour of concrete based on tests at transient as well as steady-state temperature conditions. The model is developed on a purely phenomenological level, but its validity is checked against independent tests representing arbitrary stress and temperature histories. Its intended application is for conditions typical for fire exposure i.e. relatively rapid heating in the temperature range up to 800^{o}C. This means that the conditions at temperatures below 200^{o}C, where the influence of moisture is important, are less emphasized. A more appropriate modelling in this range requires that the complex interaction between heat and moisture flow and its influence on the mechanical behaviour should be considered. The model is also mainly intended to describe the behaviour when the concrete is heated for the first time - as is the case for fire exposure - and is not generally applicable for subsequent temperature cycles, where the behaviour is essentially different.

TESTS

The material model is mainly based on tests performed by the authors reported in detail in /6/. These tests were performed on concrete cylinders having a diameter of 75 mm and a length of 150 mm. The concrete was made of standard Portland cement and quartzite aggregate in the following proportions (weight units)

Cement	1
Water	0.6
Sand (< 8 mm)	2.88
Aggregate	1.92

The specimens were tested in a small furnace placed in a testing machine producing axial compressive load. The deformations were measured continuously with a linear motion potentiometer connected to the specimen with quartz tubes. The concrete cylinders were heated axisymmetrically at different constant rates and the temperature was measured with thermocouples in the furnace and in certain points in the specimen.

The test data referred to in the following discussion originate from this investigation unless otherwise stated. When the temperature in the specimen is specified under transient conditions - i.e. when the temperature distribution is non-uniform - it refers to the temperature 25 mm from the central axis.

MECHANICAL BEHAVIOUR OF STRESSED CONCRETE DURING HEATING

When ordinary concrete is heated for the first time above 150-$-200^{o}C$, chemical decomposition - mainly dehydration - of the cement paste is gradually induced. This is the most significant reason why mechanical properties such as strength and elastic modulus decrease with the temperature. The decomposition of the cement paste due to heating is more or less an irreversible process, which permanently brings the material into a new state different from the original state. If a piece of concrete is heated to a certain temperature level, which is then sustained, the material soon attains a state of equilibrium corresponding to the temperature in question. Most measurements of high-temperature mechanical properties have been made under such a state of equilibrium.

A temperature rise in the material will produce an instability and activate the reactions responsible for the decomposition, and we can expect that the mechanical response of the material under stress is considerably affected by this kind of instability. This may explain the fact that a temperature rise accelerates the deformations under load, as has been shown in several investigations /1, 2, 3, 4, 5, 6/. This "temperature change effect" has been confirmed in flexural, torsional and compressive testing and for moderate as well as high temperatures. An example is given in figure 1, which shows a comparison between two torsional tests /4/ performed under identical temperature conditions, heating at a constant rate of $2^{o}C \cdot min^{-1}$ from $20^{o}C$ to $400^{o}C$, followed by stabilized temperature of about $400^{o}C$. The specimens were cylindrical bars (diameter 150 mm), loaded with a constant torque equal to 30% of the ultimate torque at ambient conditions. In the first case, curve 1, the load was applied after the maximum temperature had been attained, while in the second case (curve 2) the load was applied before the heating commenced. In the second case where the specimen was subjected to load while heated, very large deformations occured, mainly during the period when the temperature was increasing.

This effect is very pronounced at elevated temperature both in compression and torsion, and explains why a realistic material model cannot be based on tests performed at steady-state conditions.

The behaviour in compression under heating is illustrated in Fig. 2, /6/, showing measured strains (full lines) for specimens being stressed at different levels and heated to failure at a constant rate of $5^{o}C \cdot min^{-1}$. The figure shows that the thermal expansion is strongly reduced under stress and for a stress equal to about 40% of the ambient temperature strength, the thermal expansion is fully compensated by the stress induced deformation. As the temperature approaches a critical value the compressive strain increases rapidly and finally failure occurs. Similar results have been reported in /7/ and /8/.

Test results of the type shown in Fig. 2 are an important key to the understanding of the material behaviour at transient conditions and they are very fundamental in the formulation of the constitutive law.

CONSTITUTIVE LAW

Generally, the constitutive law for concrete under transient, high-temperature conditions may be expressed as follows

$$\varepsilon = \varepsilon(\sigma(t),\ T(t),\ \tilde{\sigma}) \qquad (1)$$

where

ε = total strain at time t
σ = stress
T = temperature
$\tilde{\sigma}$ = stress history

Phenomenologically, an adequate formulation is obtained if the total strain is seen as the sum of four different strain components each of which is connected to and correlated with a specified type of test. Eq. (1) can thus be rewritten

$$\varepsilon = \varepsilon_{th}\,(T) + \varepsilon_{\sigma}(\tilde{\sigma},\ \sigma,\ T) + \varepsilon_{cr}\,(\sigma,\ T,\ t) + \varepsilon_{tr}\,(\sigma,\ T) \qquad (2)$$

where

ε_{th} = thermal strain, including shrinkage, measured on unstressed specimens under variable temperature

ε_{σ} = instantaneous, stress-related strain, based on stress-strain relations obtained under constant, stabilized temperature

ε_{cr} = creep strain or time-dependent strain measured under constant, stabilized temperature

ε_{tr} = transient strain, accounting for the effect of temperature increase under stress, derived from tests under constant stress and variable temperature

The ususal sign convention is used here, positive for tensile and negative for compressive stresses and strains.

Parameter formulations for each of the strain components can be made on the basis of test results reported by the authors and others /5, 6, 7, 8/. The description of the material model in this general context is made for the case when the stresses are negative during the whole history. When applying the model in a structural analysis the behaviour in tension or in cases where the stresses change signs may be described by introducing relatively simple assumptions, see /9/, which is justified in many cases by the fact that the tensile stresses in the concrete are comparatively insignificant for the structural behaviour.

Thermal Strain, ε_{tr}

The thermal strain during heating is a simple function of temperature and is directly given by the measured thermal expansion curve, see for instance Fig. 2 (0%).

Stress-related Strain, ε_{σ}

The calculation of the stress-related strain is based on the concept that at every state a specified stress-strain relation is valid for the material. The stress-strain relation should be such that it approximately reflects the response of the material to a change in stress. We know from experimental evidence that two main factors affect the stress-strain behaviour, the current temperature and the prehistory of stress, cf. eq. (2).

The influence of temperature is illustrated in Fig. 3, showing measured stress-strain relations at different temperatures. The specimens were unstressed during heating. It is further a well established fact that if the concrete is subjected to stress during the period of heating the high-temperature stress-strain relationship will be considerably different. Most important is that the deformability is smaller but a slight increase in strength is also observed.

The general description of the stress-strain envelope selected here consists of a parabolic branch starting from the origin followed by a linear descending branch /10/. The following three parameters are used to describe the envelope

σ_u = compressive strength at the current state

ε_u = strain at maximum stress $(\frac{d\sigma}{d\varepsilon_{\sigma}} = 0)$, termed ultimate strain

E^* = slope of descending branch

Although the high-temperature compressive strength to a certain degree depends on the prehistory of stress it is reasonable to neglect this and assume that σ_u is a unique function of temperature which can be determined from ordinary tests of the type shown in Fig. 3. The relation between compressive strength and temperature obtained in the tests performed by the authors is shown in Fig. 4.

The ultimate strain ε_u, on the other hand, is significantly affected by the prehistory of stress. This is clearly illustrated in Fig. 5 from /5/, which shows that for specimens being unstressed during heating, ε_u (its absolute value) increases monotonously with temperature, while if the specimens are stressed under heating with 30% of the strength at ambient conditions, ε_u is almost unaffected by the temperature rise. Accordingly, in predicting ε_u, the influence of the previous stress history must be taken into account.

The prehistory of stress at a given time is best expressed by the accumulated transient strain, ε_{tr}, see the discussion below, and we can write

$$\varepsilon_u = \varepsilon_u\ (T,\ \varepsilon_{tr}) \tag{3}$$

The experimental data indicate that this relationship can be reduced to the following simple form (note that compressive strains are negative)

$$\varepsilon_u = \min\ (\varepsilon_{uo},\ \bar{\varepsilon}_u - \varepsilon_{tr}) \tag{4}$$

where

ε_{uo} = the ultimate strain at ambient conditions

$\bar{\varepsilon}_u$ = $\bar{\varepsilon}_u(T)$, the ultimate strain corresponding to $\varepsilon_{tr} = 0$, i.e. a prehistory of stress equal to zero.

$\bar{\varepsilon}_u(T)$ corresponds to the curve labelled 0% in Fig. 5. The implication of eq. (4) is that (ε_u) is reduced by an amount equal to $|\varepsilon_{tr}|$, due to the prehistory of stress, but is always greater than or equal to the ultimate strain at ambient conditions, (ε_{uo}).

The slope of the descending branch E* is taken to -880 MPa independent of temperature.

In a time step calculation the stress-related strain is given by, cf /10/,

$$\varepsilon_{\sigma,i} = \min\left[f_i(\sigma_i)\quad,\qquad \varepsilon_{0,i-1} + \frac{\sigma_i}{E_{c,i}}\right] \tag{5}$$

where

$\varepsilon_{\sigma,i}$ = stress-related strain at the time t_i

σ_i = stress at time t_i

$f_i(\sigma_i)$ = stress-strain envelope valid at time t_i

$\varepsilon_{o,i-1}$ = permanent inelastic strain at previous time t_{i-1}

$E_{c,i}$ = elastic modulus at time t_i given by $E_c = 2\cdot(\sigma_u/\varepsilon_u)$

Eq. (5) implies that the incremental change between two time steps is either a loading or an unloading process, depending on the change in stress and temperature. The permanent inelastic strain $\varepsilon_{o,i-1}$ is used to determine whether unloading or loading is prevalent for the time step.

Creep Strain, ε_{cr}

The creep strain ε_{cr} at constant temperature and constant stress is given in /6/

$$\varepsilon_{cr} = \beta_o \cdot \frac{\sigma}{\sigma_u(T)} \cdot \left(\frac{t}{3}\right)^p \cdot \exp\left[k_1 \cdot (T - 20)\right] \tag{6}$$

where

β_o = constant

σ = stress

$\sigma_u(T)$ = ultimate stress at current temperature

t = time

p = constant

k_1 = constant

T = temperature

Eq. (6) expresses the creep v.s. time for any given combination of stress and temperature. In a general application with variable stress and temperature the evaluation of ε_{cr} is based on the strain hardening principle, according to a procedure described in /6/.

Creep data reported in /6/ were correlated according to eq (6) giving $\beta_o = -0.53\cdot10^{-3}$, $p = 0.5$ and $k_1 = 3.04\cdot10^{-3}\ {}^oC^{-1}$. The creep strain component is the least important one in the model and might be neglected if a simpler version of the model is preferred.

Transient Strain, ε_{tr}

The transient strain ε_{tr} develops under stress when the temperature increases and is a very important component in the strain behaviour. Tests indicate that this strain is essentially irrecoverable and occurs only under the first heating, cf /4/.

It is impossible to design tests where the transient strain can be directly measured, but it may be evaluated from tests of the type shown in figure 2, as the difference between the measured total strain ε and the other three components

$$\varepsilon_{tr} = \varepsilon - \varepsilon_{th} - \varepsilon_{\sigma} - \varepsilon_{cr} \tag{7}$$

The order of their importance can be studied from figure 6, which is based on the curve representing 35% load in figure 2. The predominance of the transient strain component is obvious.

An analysis of the test data suitable for this purpose /4, 5, 6, 7/ shows that ε_{tr} is approximately time invariant and can be treated as an instantaneous response to a temperature rise and also that it is reasonably linear with stress. The accumulated transient strain for concrete heated under constant stress σ can be expressed in the simple form

$$\varepsilon_{tr} = \frac{\sigma}{\sigma_u} \cdot f(T) \tag{8}$$

where f(T) is a function of temperature. It was furthermore found that the temperature dependence f(T) is very similar to that of the thermal strain. We can therefore write

$$\varepsilon_{tr} = - k_2 \cdot \frac{\sigma}{\sigma_u} \cdot \varepsilon_{th} \tag{9}$$

Eq. (9) correlates very well with the test results shown in Fig. 2 (full lines) and by plotting $\varepsilon_{tr}/(\sigma/\sigma_u)$ against ε_{th} the constant k_2 is obtained by regression to 2.35. The degree of accuracy in the correlation is illustrated in Fig. 2, where the broken lines correspond to the results obtained if the material model is applied on these tests. The tests reported in /7/ and /8/ were correlated in the same way, leading to the conclusion that the form suggested in eq. (9) is fully acceptable.

In a general application with variable stress we can use the incremental form

$$\Delta\varepsilon_{tr} = -k_2 \cdot \frac{\sigma}{\sigma_u} \cdot \Delta\varepsilon_{th} \tag{10}$$

Eq. (10) postulates that the increment of transient strain is directly proportional to the current stress.

MATERIAL BEHAVIOUR UNDER COOLING

The discussion up till now has been limited to the case when the concrete is heated for the first time. In fire exposure situations we are often interested in analysing the behaviour during the subsequent cooling period. The knowledge of the behaviour under cooling is somewhat limited, but a relatively accurate description may still be made. A reasonable assumption is that the prevalent stress-strain relation during cooling is defined by the state corresponding to the previous maximum temperature and it also seems that the transient strain increment should be equal to zero as soon as a decrease in temperature takes place. The creep strain may in view of its unimportance be evaluated in the same way under cooling as under heating even if there is no evidence confirming this behaviour. The only significant problem arises with the thermal strain as regards its degree of reversibility.

It is clear that the main part of the thermal expansion is reversible, but the irreversible component is not insignificant for quartz aggregate concrete heated above the quartz inversion limit /7/. However, as a first approximation, pending more complete experimental information, it may be justified to assume that the thermal expansion is fully reversible.

VALIDITY OF THE MODEL

The constitutive model described above is quantitatively developed from four different types of experimental data each giving information of a specific strain component. The general validity of the model can be checked independently against other types of experiments wiht arbitrary load and temperature histories.

Fig. 7 shows the behaviour of initially unloaded specimens, being heated at $1^{o}C \cdot min^{-1}$ and $5^{o}C \cdot min^{-1}$ under fully restrained deformation ($\varepsilon \equiv 0$). The measured restraint load (full lines) obtained for this case is compared with the results from the material model described above (broken lines). The agreement is strikingly good for booth rates of heating. It could be noted that the rate of heating has no significant influence on the results, thus confirming that the main part of the total deformation is time invariant.

Another example is shown in Fig. 8 for a test where the specimen was heated with $5^{o}C \cdot min^{-1}$ and the load was changed stepwise according to figure 8b. Again the model proves very reliable in predicting the behaviour.

EXAMPLE CALCULATIONS

To illustrate the significance of the model in a simple example, it has been applied in calculating the stress distribution and deformations for a circular cross section (diameter 150 mm) under sustained axial load exposed to axisymmetrical heating. Example results of this exercise are shown in Fig. 9 for a rapid heating characterised by an external temperature rise of approximately $8^{o}C \cdot min^{-1}$. The temperature distributions at selected times are shown in Fig. 8d and the corresponding stress distributions are reproduced in Fig. 8a, b and c for the three load levels 0, 20 and 40% respectively, referred to the strength at ambient conditions.

In view of the large thermal gradients, a traditional stress analysis would lead to a high concentration of stress in the outer parts of the cylinder and a corresponding unloading of the inner core. But this will not be the case when the real material behaviour is taken into account. In fact, the response of the concrete is such that the so called thermal stresses will be very insignificant or even non-existent. It can be concluded, which furthermore is indicated in Fig. 8a, that stresses due to restrained thermal expansion cannot in themselves contribute to compression failure of the concrete. It is quite obvious that an elastic stress distribution bears little resemblance with reality.

The stress distributions shown in Fig. 8b and c are significant for the behaviour. At the external load level 40%, practically no redistribution of stresses takes place during the first hour, in spite of the large temperature gradients. After about 1 h, when the temperature in the outer parts exceeds 400°C the stresses decrease in the outer zone and increase markedly in the central core. This is simply due to the fact that the strength falls markedly above 400°C.

At the load-level 20% stresses are redistributed during the first 0.5 h resulting in an increase in stress in the outer zone and a decrease in the central core. However, after about 1 hour the behaviour in principle is similar to the one discribed at the stress-level 40%.

CONCLUSIONS

1. The mechanical behaviour of concrete is considerably different at transient temperature conditions from that observed under steady-state temperature conditions.

2. A constitutive law valid at transient conditions can be formulated in terms of four strain components, thermal strain, instantaneous stress-related strain, creep strain and transient strain.

3. The transient strain is an important concept in the model, accounting for the deformations occurring under stress upon a temperature rise.

4. Thermal stresses created due to restrained thermal expansion are very significant in concrete upon first heating and cannot in themselves contribute to compression failure.

5. The constitutive model developed here reflects the actual behaviour in a very appropriate way and can easily be applied in a computer analysis of thermally exposed structures.

6. The suggested model is especially important for problems like statically indeterminate concrete structures and slender columns, where an accurate estimate of the deformations is necessary.

7. The most important input of material properties in the model is strength v.s. temperature, thermal expansion and possibly ultimate strain.

8. Further experimental investigations of concrete at high temperatures should be of the type shown in Fig. 2, i.e. heating the specimens to failure under sustained stress.

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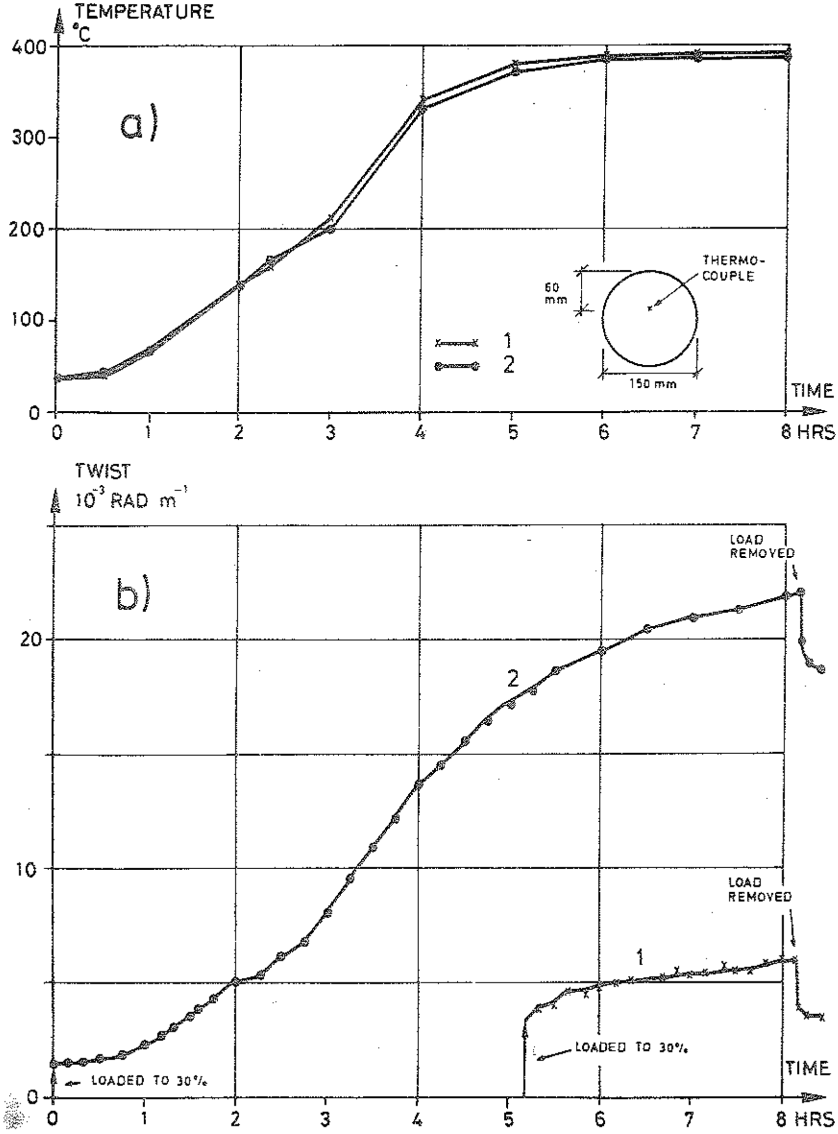

Fig. 1--Measured twist as a function of time (Fig. b), for a cylindrical specimen subjected to torsional loading. Fig. a shows the measured temperature in a specified point inside the specimen. Curve 1: torque applied after the temperature is stabilized. Curve 2: torque applied prior to heating

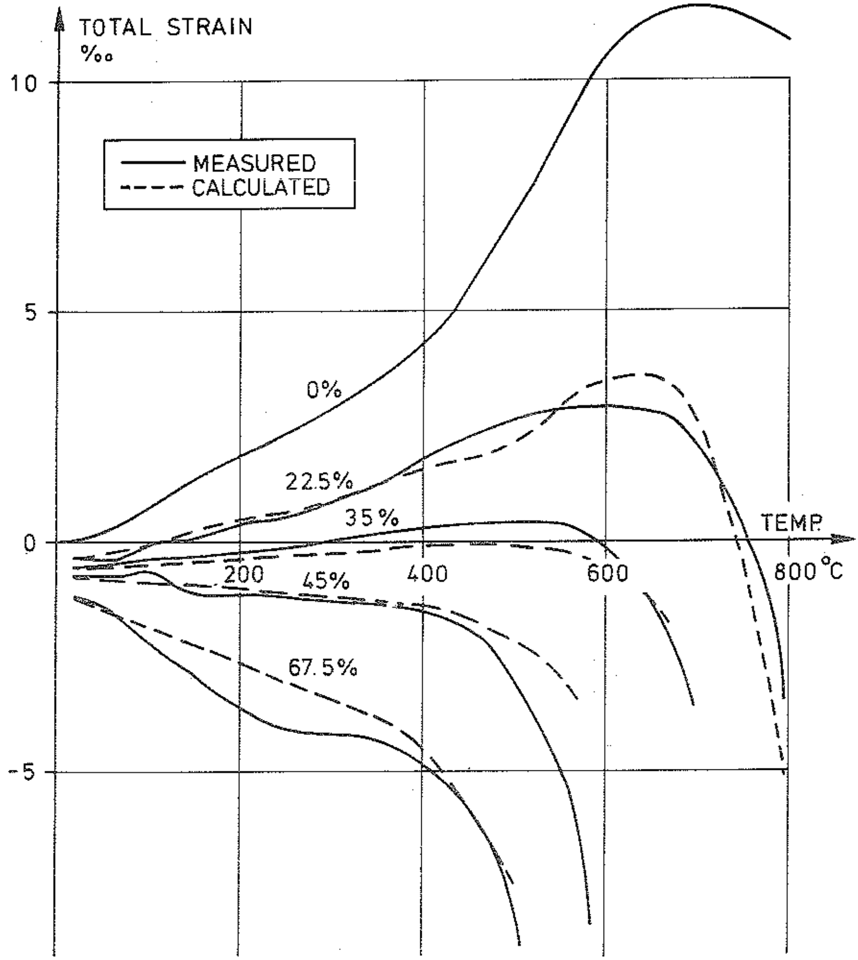

Fig. 2--Deformation upon heating (5 C·min^{-1}) for different levels of compressive stress (per cent of strength at ambient conditions) /6/. Full lines indicate testing and the dashed lines are the results obtained from the material behavior model

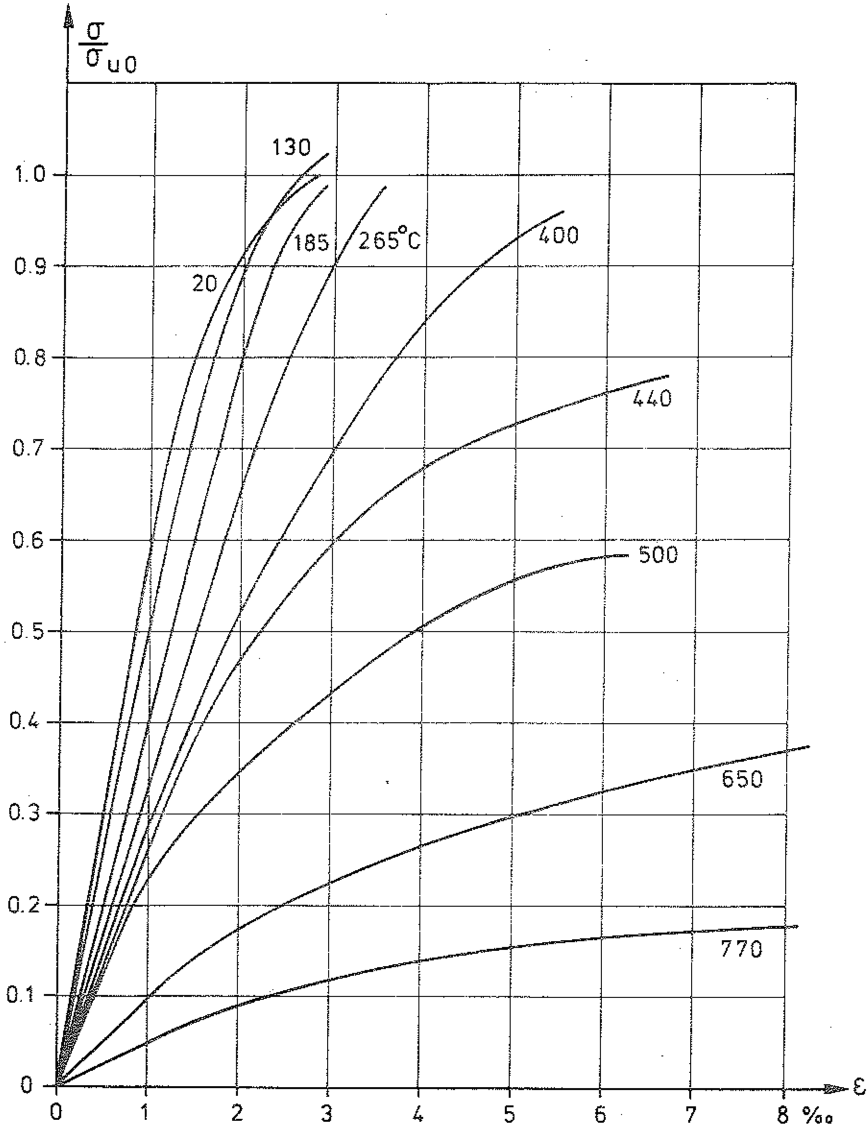

Fig. 3--Stress-strain relations at different temperatures. The stress is given relative to the strength at ambient conditions, σ_{uo}. Rate of heating 5 $C \cdot min^{-1}$. Rate of loading 14 $Mpa \cdot min^{-1}$

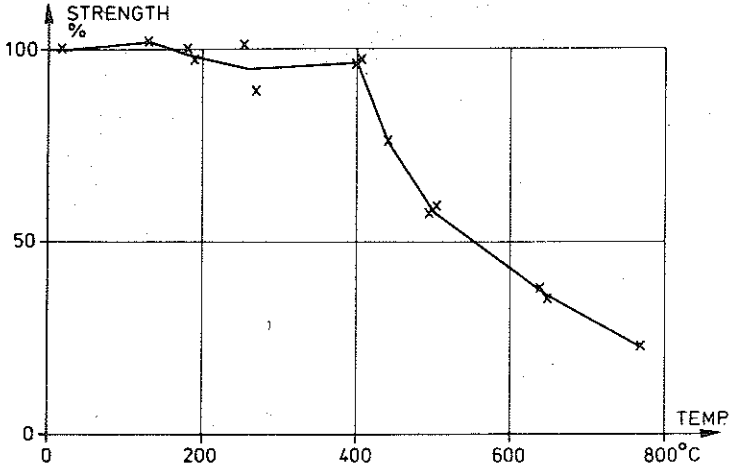

Fig. 4--Effect of temperature on the strength of concrete /6/, cf. Fig. 3

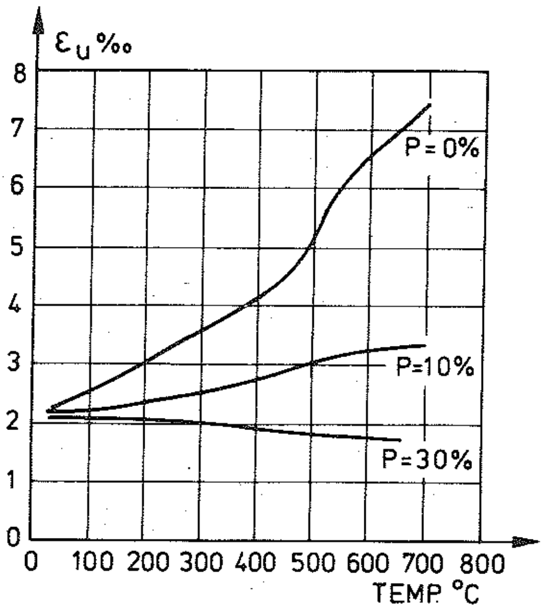

Fig. 5--Ultimate strain ε_u as a function of temperature for specimens stressed at different levels during the heating period /5/

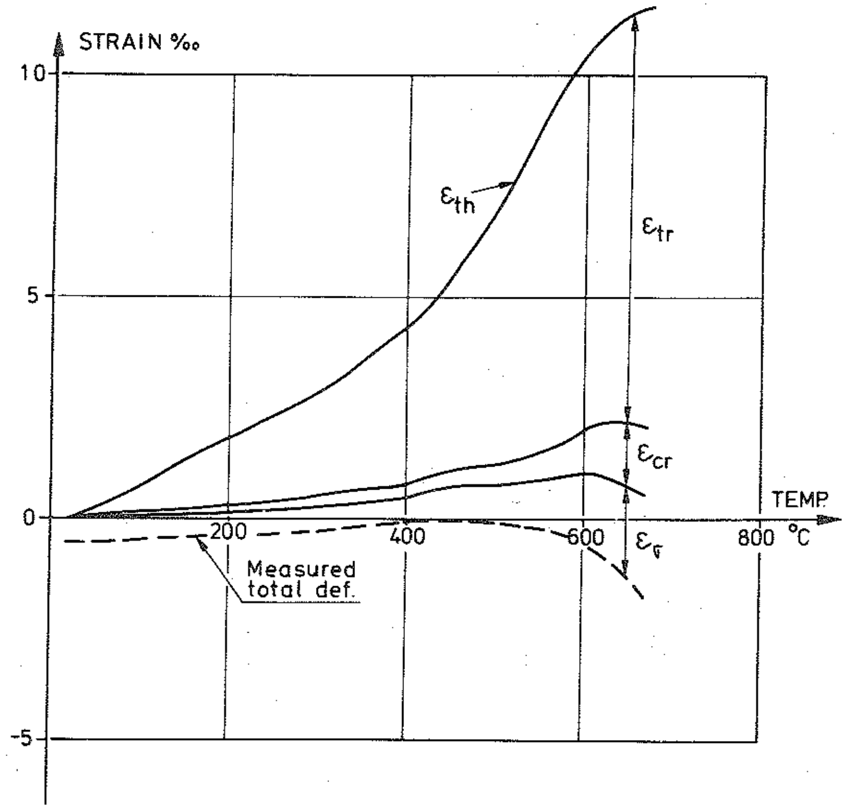

Fig. 6--Relation between different strain components for a specimen stressed at 35 percent and heated to failure (cf. Fig. 2)

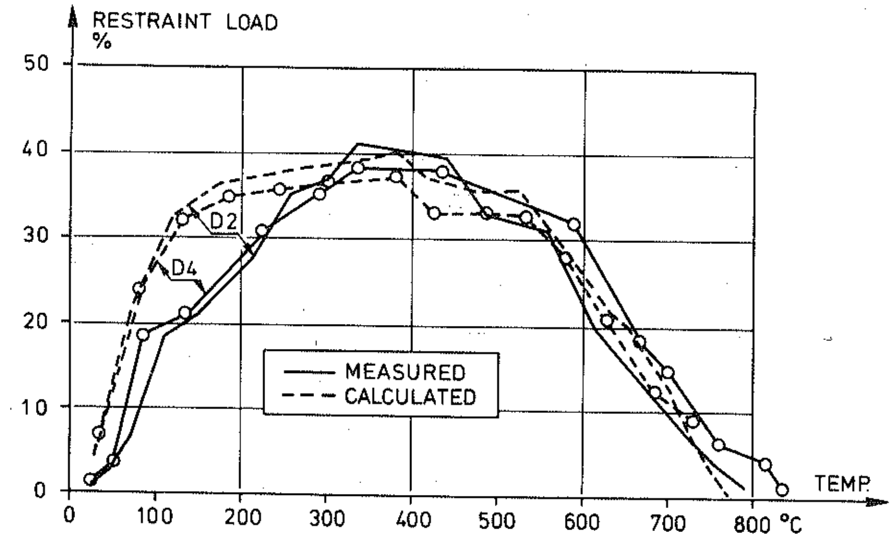

Fig. 7--Measured and calculated restraint load for specimens with restrained deformation while being heated. The specimens were unstressed from the beginning. Rate of heating: 5 $C\cdot min^{-1}$ (A) and 1 $C\cdot min^{-1}$ (B)

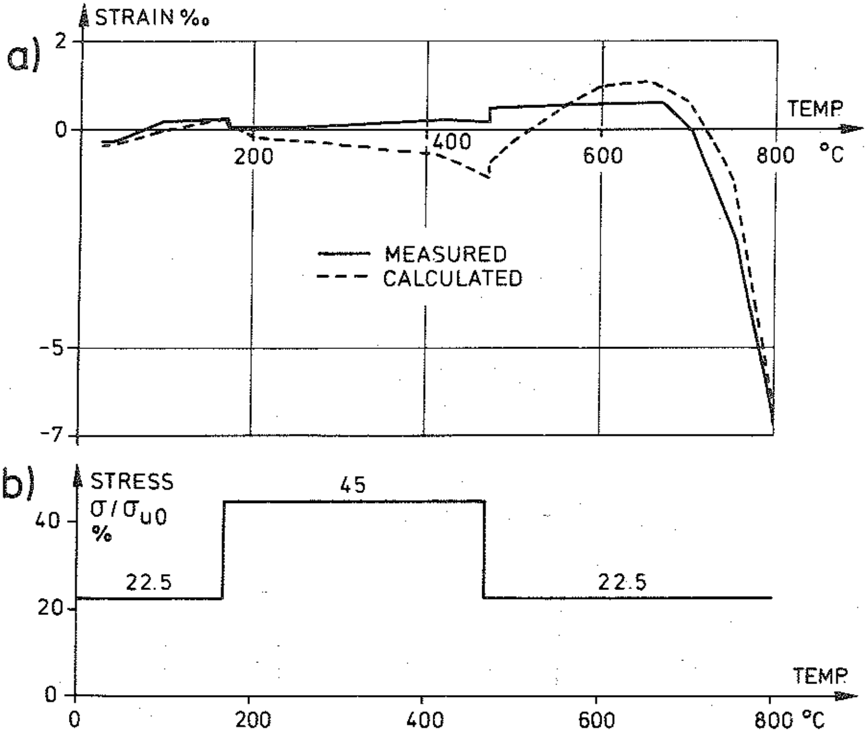

Fig. 8--Measured and calculated strain as a function of temperature (Fig. a), for a specimen heated with 5 $C \cdot min^{-1}$ and subjected to stepwise change in stress (Fig. b)

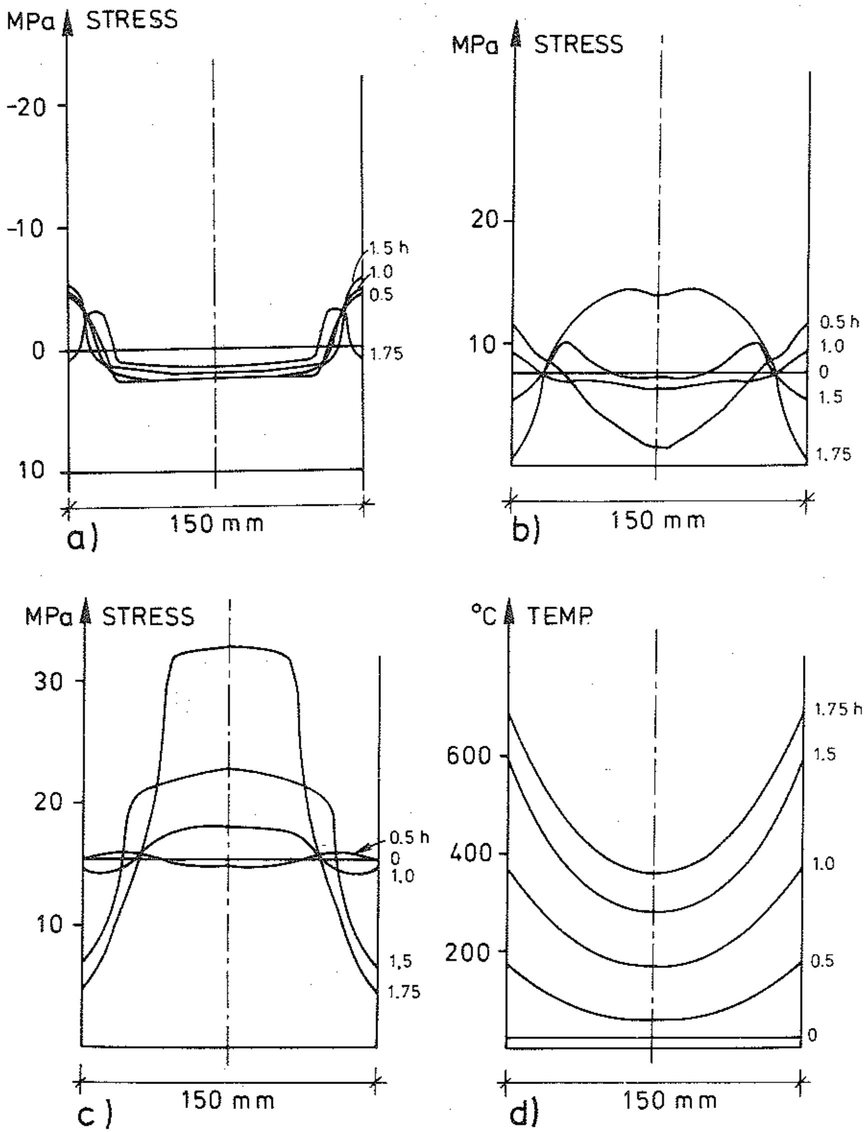

Fig. 9--Calculated stress distributions for axially loaded (compression) cylindrical cross section under transient thermal exposure. Axial load 0, 20, and 40 percent of strength at ambient conditions (Fig. a, b and c respectively). Fig. d shows the transient temperature fields at selected times

SP 55-17

Fire-Exposed Hyperstatic Concrete Structures: An Experimental and Theoretical Study

By Y. Anderberg

Synopsis: Analytical predictions of thermal and mechanical behaviour of reinforced concrete structures exposed to differentiated complete fire processes including the cooling phase are presented and verified by tests. The modelling of the fire response comprises a heat flow analysis in the first step and a structural analysis in the second step, based on two separate computer programs. The evaluated structural fire response is compared with the measured behaviour in a great number of experimental tests in which, the fire process and the external load level are widely varied. The experimental investigation refers to a well-defined hyperstatic structure, viz. a reinforced concrete plate strip fire-exposed on one side and completely fixed against rotation at both ends while axial movement is free to develop. The outline of the project is built on the philosophy of a functionally based, differentiated design procedure for fire exposed, load-carrying and separating structures. Such a design procedure refers to performance criteria and postulates that the real physical processes with respect to fire exposure, heat transfer and structural behaviour are predicted as far as possible.

Keywords: computer programs; fire tests; heat transfer; limit state design; loads (forces); moments; plates (structural members); reinforced concrete; restraints; structural analysis; thermal properties

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INTRODUCTION

Most experimental investigations, published in literature of fire-exposed, reinforced and prestressed concrete structures have been performed under statically determinate conditions and by the use of an internationally standardized fire process. Additionally, the effect of cooling, which follows a heating phase, is ignored and the objective in most cases has been limited to determining the fire resistance time. Furthermore published laboratory tests on fire-exposed structures have not hitherto been combined with a complete analytical behaviour study. One reason for that is the lack of knowledge about the mechanical behaviour of different structural materials at transient, high temperature conditions. However, valuable contributions within the fire research field concerning concrete structures can be found in /1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13/.

This paper presents a detailed analysis of the complex structural behaviour and load-carrying capacity of hyperstatic concrete members, exposed to a complete process of a fire development. The analysis consists of a theoretical study, combined with laboratory tests performed under well-defined conditions. The theoretical study comprises a calculation of temperature distribution of the reinforced concrete structure, used in the experiment, and based on these temperature-time fields, an evaluation of the structural response. The calculation of structural behaviour at fire is based on recently developed material behaviour models of concrete in compression /14/ and tension /15/ valid at transient as well as steady state high-temperature conditions. A correspondant constitutive law is also used for the reinforcing steel. A more thorough presentation of the whole study is given in /15/.

The experimental results on the measured behaviour verify the analytical study for a great number of cases i.e. the fire response of hyperstatic concrete structures is realistically predicted.

The present study can be employed for theoretical design purposes and be systematically used for computing tables and diagrams, facilitating the practical design. Furthermore expensive fire testings of structural members can be replaced by analytical predictions of thermal and structural response.

EXPERIMENTAL STUDY

The experimental part of the project is mainly focused on a fundamental study of the behaviour of a well-defined type of hyperstatic structure, namely a reinforced concrete plate strip exposed to fire on one side and completely fixed against rotation at both ends but free to move longitudinally (Fig. 1).

The plate strip has a span length of 2.5 m and a total length of 3.5 m, while the cross-sectional area bxh is 0.3x0.15 m^2. Furthermore the plate strip is reinforced with 5ø10 KS 40 bars placed symmetrically at top and bottom along the strip with a concrete layer of 0.025 m.

The experimental investigation embraced 81 tests performed for varying types of fire exposure i.e. different combinations of opening factor (0.01 - 0.08 $m^{1/2}$) and fire load (31 - 2010 $MJ \cdot m^{-2}$). The differentiated fire exposure applied is in accordance with a real fire situation and determined by the opening factor, $A\sqrt{H}/A_t$ ($m^{1/2}$), the fire load density q (MJ/m^2 of the total surrounding surface area of the compartment), and the thermal properties of the surrounding structures. A = opening area (m^2), H = opening height (m), and A_t = total interior surface, bounding the compartment, opening area included (m^2).

One part of the investigation was devoted to a profound study of the structural response for a pure thermal exposure. The other part comprised a study of the simultaneous effect of an external transverse load consisting of two symmetrically placed concentrated loads 0.76 m apart, and a fire exposure. The loading levels 1/4, 1/2, 3/4 and 1/1 of P_{all} were used, where P_{all} = 16 kN denotes the allowable load at ambient conditions according to Swedish Concrete Standards. Furthermore the flexural rigidity during fire exposure was determined for nil-loaded plate strips. As a complementary study on residual state, the residual flexural rigidity and load-carrying capacity was also measured.

In considering the influence of differentiated fire processes and in following the structural behaviour during a complete fire, until the residual state of stresses and deformations is reached, the investigation reported in /15, 16/ constitutes the first systematic one of its kind.

THEORETICAL STUDY

Thermal Analysis

The transient temperature-time fields of the fire-exposed concrete structure are calculated in the actual application by a finite element computer program, developed from a program library constructed by

U. Wickström at Lund Institute of Technology. The program library consists of a supply of permanent system routines and problem adapted routines, which must be constructed by the user. One-dimensional, two-dimensional and axi-symmetrical problems with various boundary conditions as prescribed heat flow, temperature and adiabatic state can be solved.

The current program, which solves the one-dimensional case, considers the temperature dependence of thermal properties, viz. thermal conductivity and enthalpy even during a cooling phase, but disregards the simultaneous moisture transport within the structure. In the experimental investigation of the plate strip the fire exposure was unilateral and approximately symmetrical but varied longitudinally. Therefore, the simulation of thermal exposure was divided into four fire zones each with a characteristic temperature-time curve and owing to symmetry only half the structure was studied. Due to the discretization of the structure into segments, where each segment has a mean thermal exposure the heat flow in axial direction is of secondary importance and is therefore neglected in the calculations. The measured temperature distribution along the plate strip on surface and 3 cm inside the structure is given in Fig. 2 at a fire exposure, characterized by the opening factor 0.04 $m^{1/2}$ and the fire load 502 $MJ \cdot m^{-2}$. This diagram illustrates how the temperature outside a mid-region is decreasing towards the supports.

The prediction of temperature distribution history for fire-tested concrete plate strips was in all cases in a very good agreement with measurements under heating as well as during cooling phase. As the temperature-time fields serve as input in the structural analysis program these have successfully contributed for further progress in calculations of mechanical response.

Structural Analysis

Research contributions /14, 15, 17/ enabling the development of functionally based, realistic behaviour models, valid for concrete at transient thermal exposures have opened for practically reliable computations of structural behaviour. Computer-oriented material behaviour models of concrete and steel valid during heating and a subsequent cooling phase taking into account the history of temperature and stress as well as any process of unloading used in the study are here briefly described.

Material Behaviour Models

The complete constitutive law of concrete in compression as well as tension under transient high temperature conditions is developed on a purely phenomenological level. The behaviour model for concrete in compression is already presented in another symposium paper "A Constitutive Law for Concrete at Transient High-Temperature Conditions", and details behind the formulation are therefore omitted as regards this matter.

The total deformation is seen as the sum of four different components and introducing $\tilde{\sigma}$, which denotes the history of stress the following relation is established

$$\varepsilon = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\tilde{\sigma}, \sigma, T) + \varepsilon_{cr}(\sigma, T, t) + \varepsilon_{tr}(\sigma, T) \quad (1)$$

where

ε^c = total strain of concrete

ε^c_{th} = thermal strain of concrete

ε^c_{σ} = instantaneous, stress-related strain of concrete

ε^c_{cr} = creep strain of concrete

ε^c_{tr} = transient strain of concrete

Furthermore there exists an interrelationship between the stress-related and the transient strain component.

In a time step calculation of the structural response at fire, the total strain is known, which means that the instantaneous stress-related strain is directly assessed from eq. (1). The evaluation of current stress is then based on a stress-strain relationship valid at every state. The complete stress-strain loop can be studied in Fig. 3. On tension side relatively simple assumptions have been made and two linear branches have here been chosen to describe the behaviour. The slope of the second descending branch is evaluated from measured moment-curvature relationship of a segment of the actual structure taking into account the integrated effect of the tensile carrying capacity between the cracks. The unloading branch on tension side always leads to origin or to ε_o (permanent inelastic strain) depending on the prehistory and any unloading process.

When tension stresses are considered the creep strain is evaluated as in compression, but the transient strain is assumed to be zero as experimental data are lacking.

As regards the model on compression side the reader is referred to the aforementioned paper.

The constitutive law of steel is far from that complicated as concerns concrete but has a more traditional definition. Only three components are involved as shown in the following equation.

$$\varepsilon^s = \varepsilon^s_{th}(T) + \varepsilon^s_{\sigma}(\sigma, T) + \varepsilon^s_{cr}(\sigma, T, t) \quad (2)$$

where

ε^{s} = total strain of steel

ε^{s}_{th} = thermal strain of steel

ε^{s}_{σ} = instantaneous, stress-related strain of steel

ε^{s}_{cr} = creep strain of steel

This law is further described in /15, 18/.

Analytical Study

The structural analysis program is a modified version of "FIRES-RC", originally constructed at University of California, Berkeley /18/. For the present application, this version can be characterized as an extension of the Berkeley program, reconstructed for use on a Univac 1108 Computer, which furthermore can be used also for frame analyses. The program is based on a non-linear direct stiffness formulation coupled with a time-step integration and functionally reliable analytical models here briefly described are utilized. The continual change and redistribution of forces and moments in a fire-exposed hyperstatic structure are evaluated for each time step by an iterative approach. This approach is used to find incremental changes in deformed shape, which results in equilibrium between external and internal forces. Secondary order effects and shear forces are not considered in the program.

The computer program is capable of evaluating a detailed structural behaviour and extensive comparisons with measured behaviour for a great number of tests in terms of time-history of bending restraint moments and deflections have verified the analytical predictions. A complementary test carried out on a fire-exposed, simply supported plate strip without external load is also studied theoretically, and a good agreement was attained. In view of the good agreement, which will be demonstrated in this paper a behaviour study of axially and rotationally restrained plate strips at fire under external load is furthermore made.

RESULTS

Predicted and Observed Structural Behaviour

An example of a temperature-time field calculation compared with experimental measurements is given in Fig. 4 at six depths of the mid-section of the fire-exposed plate strip. This fire zone is characterized by the furnace and surface temperatures also illustrated in the figure and the fire exposure corresponds to a fire load, $q = 502\ MJ\cdot m^{-2}$ and an opening factor, $A\sqrt{H}/A_t = 0.04$ (cf. Fig. 2). The calculation agrees very well with the measurements except at temperatures below $100^{o}C$, where moisture transport disturbs the concordance. This example is a typical result for the calculations.

Predicted structural response here presented is pervadingly based on the temperature-time fields illustrated above. The calculation of structural behaviour is in Fig. 5 compared with observations in terms of the restraint bending moment and the mid-section deflection. Comparisons are made at these load levels viz. $P = 0$, $P = 1/2 \cdot P_{all}$ and $P = P_{all}$. For the calculation of the substructured plate strip shown in the figure the concordance in behaviour is quite satisfactory under heating as well as during cooling phase. At the load level $P = P_{all}$ the yield moment is reached.

A more detailed analysis of structural response to fire is described by the distribution of all strain components inherent in the material behaviour model, which together constitute the evaluated behaviour and the stress distribution. Such a strain and stress distribution is in Fig. 6 reproduced for segment 5 for the test illustrated in Fig. 5 and characterized by the load level $P = P_{all}$. Segment 5 is representative for the mid-section and the result is shown at selected times. The total strain distribution is also indicated as a dashed line giving the curvature of the section. Furthermore the stress distribution is accounted both for concrete and steel subslices. Due to the imposed thermal gradient excessive thermal strains, which successively are compensated by the transient strain and the creep strain, are developing in portions close to the fire-exposed surface. A servere degradation due to cracking rapidly takes place inside the cross-section and after 1 hour even subslice 1 is cracked. Consequently the bottom as well as the top reinforcing bars and their thermal behaviour signify a dominating effect on the structural response of the strip. During cooling great stress-related positive strains develop and the curvature continues to increase somewhat, while stresses are redistributed. The redistribution after 4.8 hours indicate that all subslices are cracked except No. 7 which is transferred into compression. The detailed explanation behind the structural response (Fig. 5) can be found by studying the strain and stress distribution of each section during the fire process.

Another example illustrating the significance of the computer model is given in Fig. 7 for a simply supported plate strip solely thermally exposed as in the previous example. The predicted thermal midpoint deflection is here in an extraordinary agreement with the measurements.

Further Calculations

In order to illustrate how an axial restraint added to the rotational restraint on a fire-exposed plate strip changes the structural behaviour two calculations are presented at the load levels $P = 0$ and $P = 1/2\ P_{all}$. Results are reproduced in Fig. 8 as regards time-history of thermal restraint moment, axial compression force and deflection process at midsection. The increase in thermal restraint moment is similar to comparative tests in Fig. 5, but the maximum value is here somewhat higher. However, after 0.7 hours the behaviour is quite different and a sudden decrease in restraint moment is characteristic in both cases and after 5 hours considerable positive values are further attained. This dramatic change is owing to the axial force, which reaches as maximum 40% of the ultimate compressive load at ambient temperature. This thermal force also seems to be almost independent of external loading which to some extent is noticed for the deflection process at mid-section. The calculated time histories of restraint moment for the two tests are in conformity with each other and the difference between the curves is approximately the same as the initial value due to load. The different mode of behaviour can be analyzed from the time-variation of the strain and stress distribution at different cross-sections of the plate strip. An example of the stress-distribution is in Fig. 9 given for the end and the mid-section of the nil-loaded plate strip. The sudden decrease of restraint moment illustrated above is due to excessive transient strains and the decrease in compressive strength above 400°C for concrete layers close to the fire-exposed surface. After 1 hour in subslice 1 at mid-section the decrease in stress is obvious as a result of the aforementioned reasons.

CONCLUSIONS

1. For investigated plate strips the residual bending stiffness and strength after fire were found to have a decrease within the range 0-30% and 0-20% respectively, for fire loads q less than 1000 $MJ \cdot m^{-2}$.

2. Reliable predictions of thermal response of concrete structures at fire, carried out by a finite element computer program, are demonstrated for a great number of tests at varying fire exposure conditions.

3. Based on predictions in 2 and functionally trustworthy analytical models of material behaviour valid at transient, high temperature conditions computerized calculations of fire-exposed, hyperstatic concrete structures have been validated for a great number of cases. The computer program used was originally developed in Berkeley /18/.

4. Material behaviour models based on experimental data obtained at steady state conditions applicated in the prediction of structural response to fire results in unrealistically great thermal restraint forces and moments (if less than yield moment) and sometimes in an erroneous collapse state.

5. Thermal restraint forces never exceed 45% of the load-carrying capacity at ambient conditions, in analysed fire-exposed hyperstatic concrete structures, axially restrained.

6. The influence of restraint forces and moments on the load-carrying capacity of a fire-exposed, hyperstatic concrete structure depends on the deformation capacity of concrete at high temperatures.

7. The current study indicates the limit state design to be applied as a consequence of the fact that in no test or calculation the rotation demand exceeded the rotation capacity. The same indication also has been noted previously by Gustaferro /9/. For a general conclusion, further studies are necessary.

8. From calculations and tests performed it seems to be justified to treat thermal restraint forces and moments separated from those caused by an external load, as long as the yielding moment has not been reached at any cross-section (important condition). The sum of these components is approximately equal to the total forces and moments of the structure at fire.

9. The present study gives a basis for a thorough understanding of the mechanical behaviour of fire-exposed hyperstatic concrete structures. That can be employed for theoretical design purposes and be systematically used for computing tables and diagrams, facilitating the practical design.

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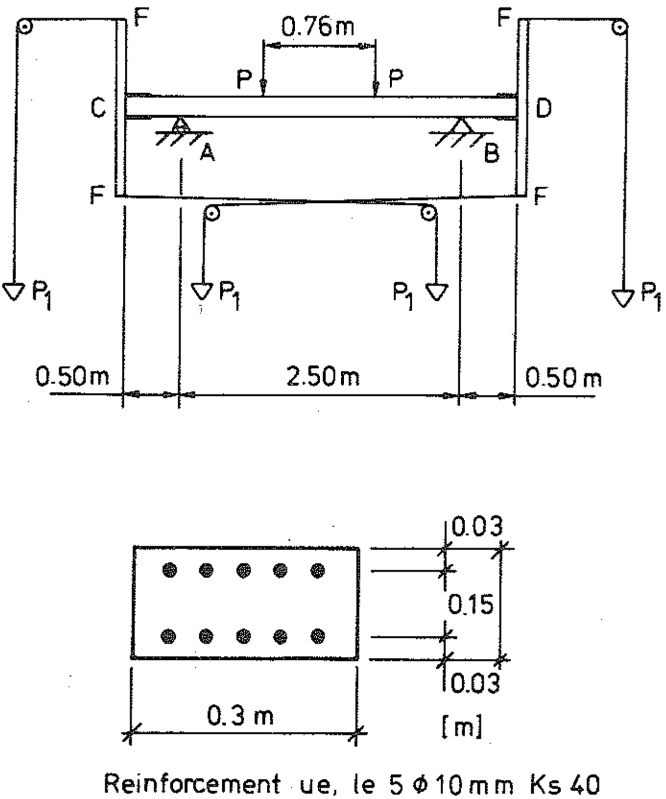

Fig. 1--(a) Principal form of test arrangement for the plate strip restrained against rotation and free to move axially
(b) Cross section of the plate strip

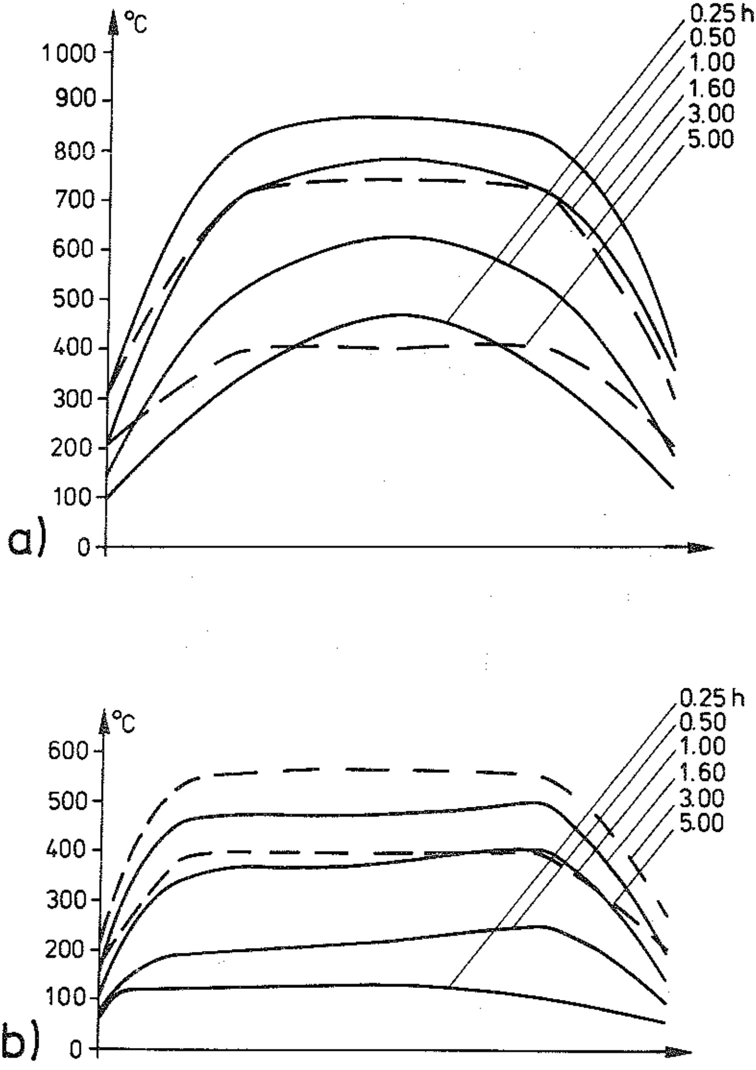

Fig. 2--Temperature distribution along
(a) Fire-exposed surface
(b) Bottom reinforcement (depth 3 cm)
Opening factor = 0.04 $m^{1/2}$
Fire load = 502 $MJ \cdot m^{-2}$
——————— = Heating
- - - - - - - - - - = Cooling

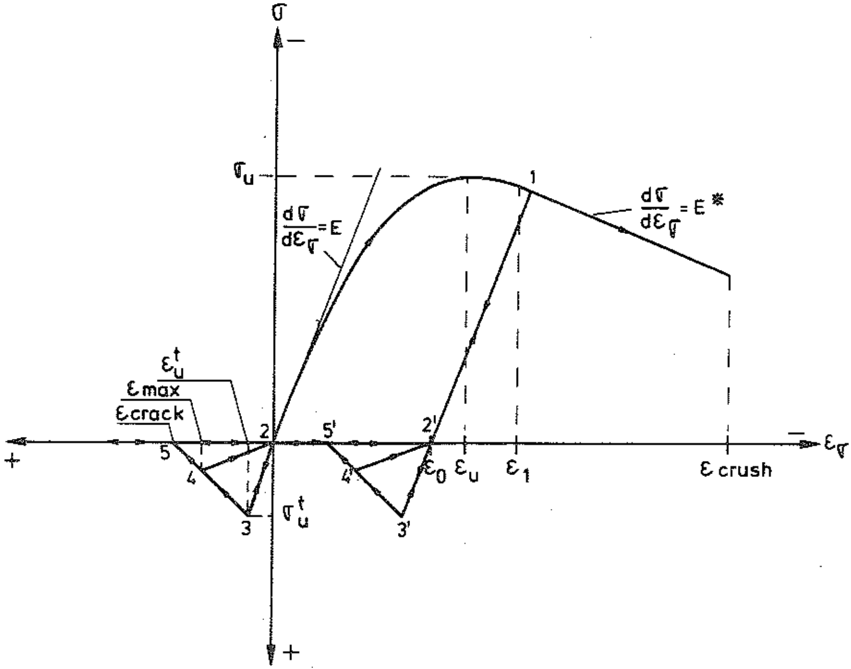

Fig. 3--Complete stress-strain relationship for concrete.

σ_u = compressive ultimate stress

σ_u^t = tensile ultimate stress

E^* = strain hardening modulus

ε_u = ultimate compressive strain

ε_u^t = ultimate tensile strain

ε_1 = strain at transition point

ε_o = inelastic strain

ε_{crack} = formal cracking strain

ε_{max} = maximum tension strain

ε_{crush} = crushing strain

Index c is omitted.

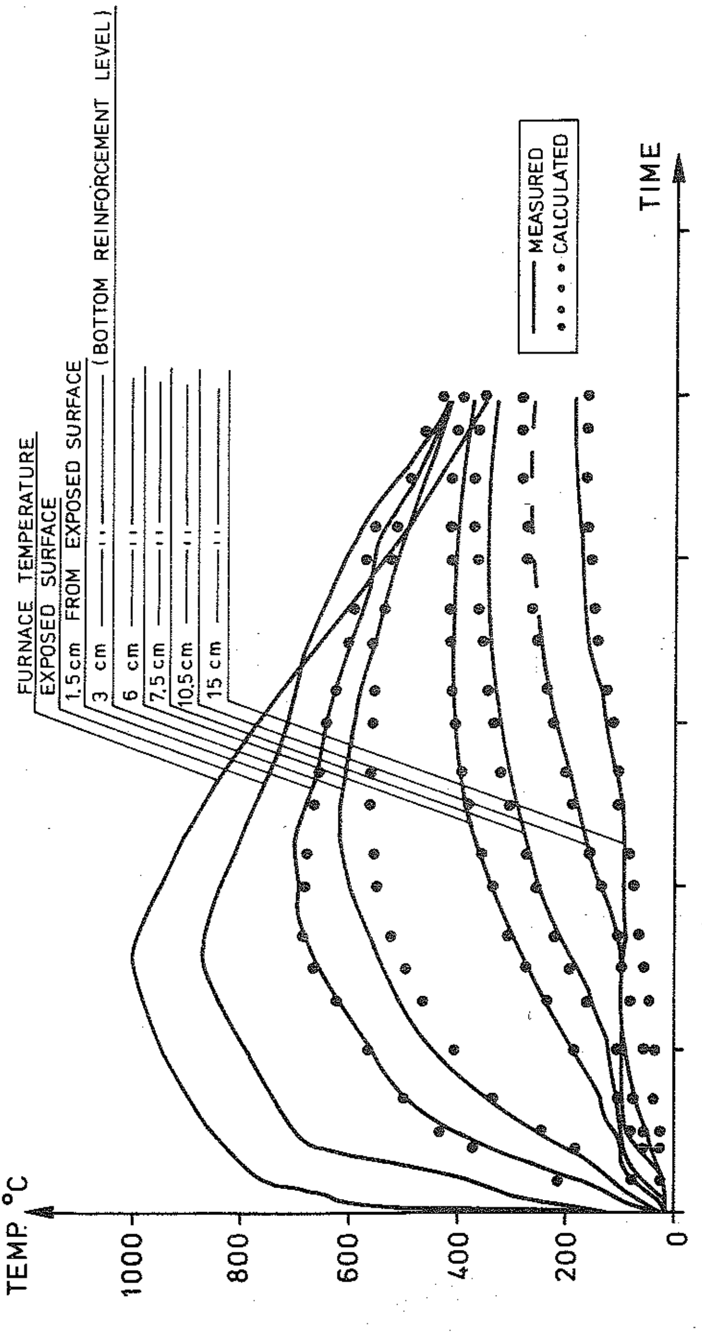

Fig. 4--Temperature-time curves in furnace and at six depths in midsection of the plate strip. Fire exposure in accordance with Fig. 2.

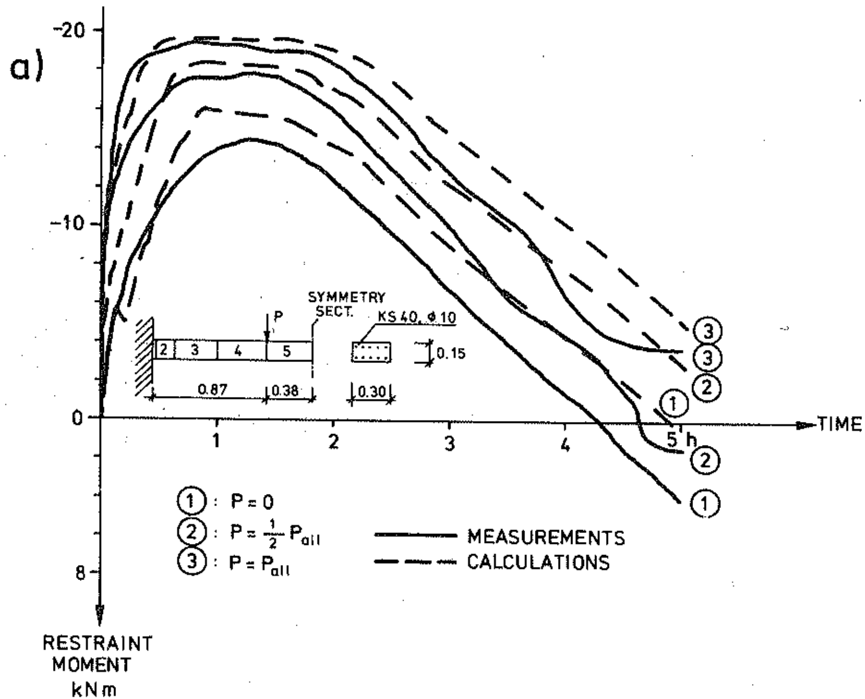

Fig. 5a--Time variation of (a) bending restraint moment and (b) deflection process at midsection. Fire exposure in accordance with Fig. 2

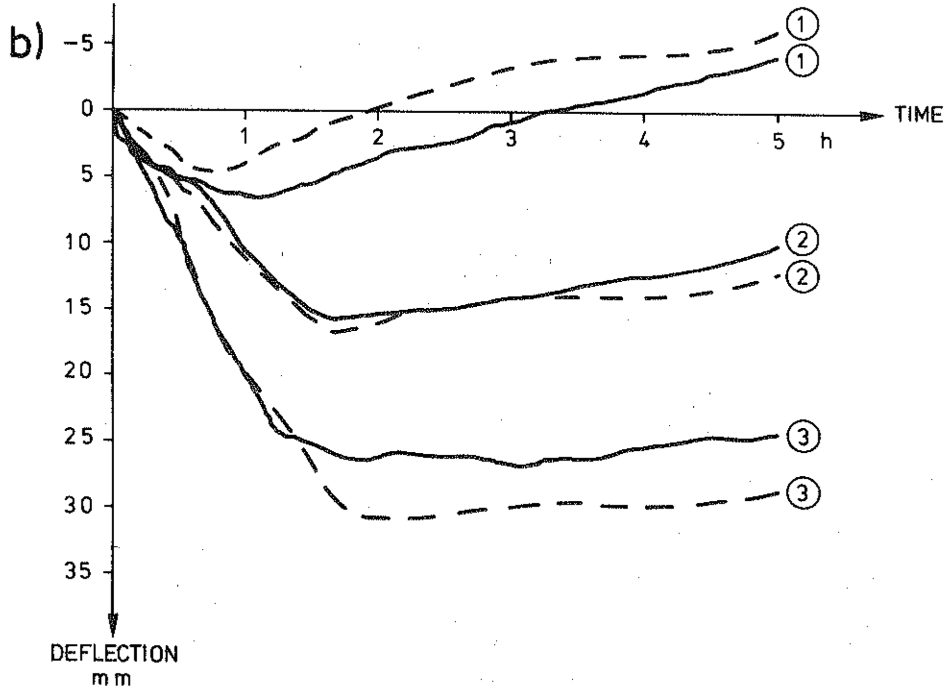

Fig. 5b--Time variation of (a) bending restraint moment and (b) deflection process at midsection. Fire exposure in accordance with Fig. 2

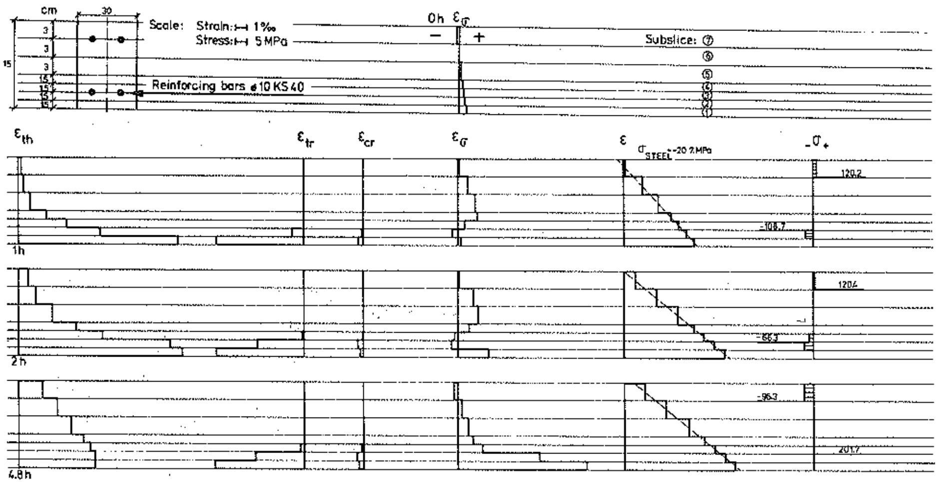

Fig. 6--Strain and stress distribution of the cross-section in midspan at selected times. Fire exposure in accordance with Fig. 4

Load level $P = P_{all}$

ε = total strain $\qquad \varepsilon_{cr}$ = creep strain

ε_{th} = thermal strain $\qquad \varepsilon_{tr}$ = transient strain

ε_{σ} = stress-related strain $\qquad \sigma$ = stress

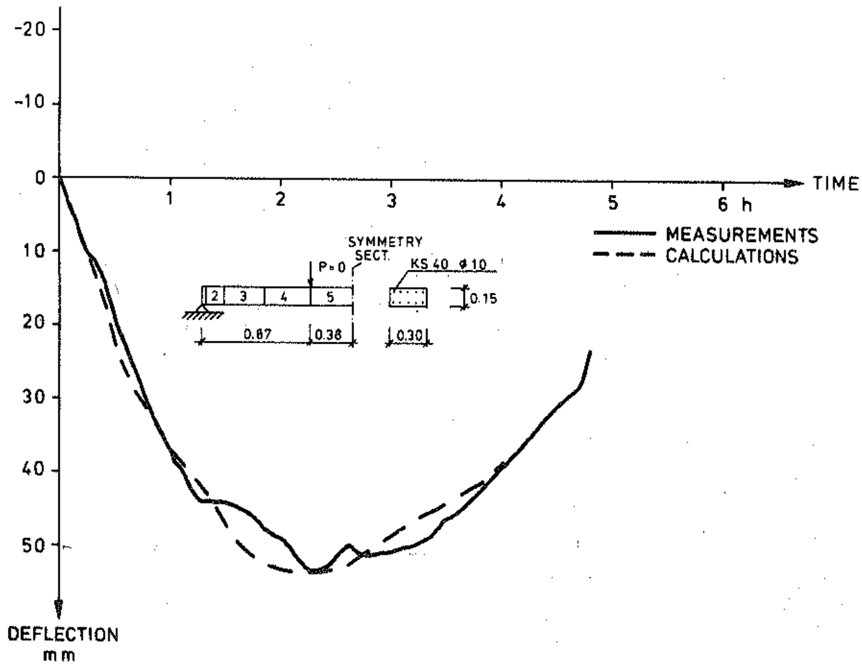

Fig. 7--Deflection process at midsection of nil-loaded simply supported plate strip. Fire exposure in accordance with Fig. 2

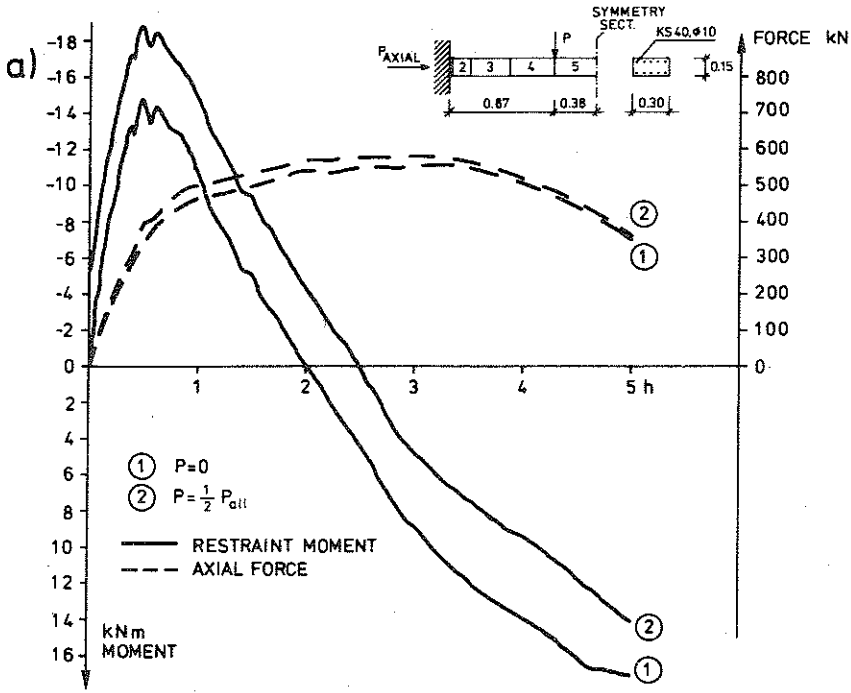

Fig. 8a--Time-variation of (a) bending restraint moment and axial force and (b) deflection process at midsection for a plate strip restrained against axial as well as longitudinal movements.
Load level ① P = 0 and ② P = 1/2 P_{all}
Fire exposure in accordance with Fig. 2

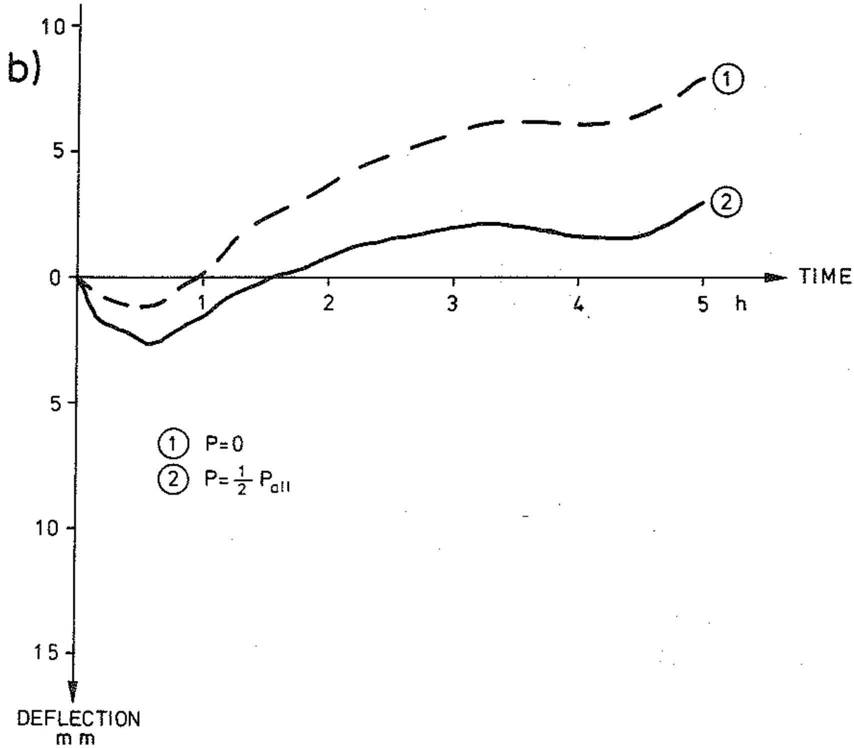

Fig. 8b--Time-variation of (a) bending restraint moment and axial force and (b) deflection process at midsection for a plate strip restrained against axial as well as longitudinal movements.
Load level ① P = 0 and ② P = 1/2 P_{all}
Fire exposure in accordance with Fig. 2

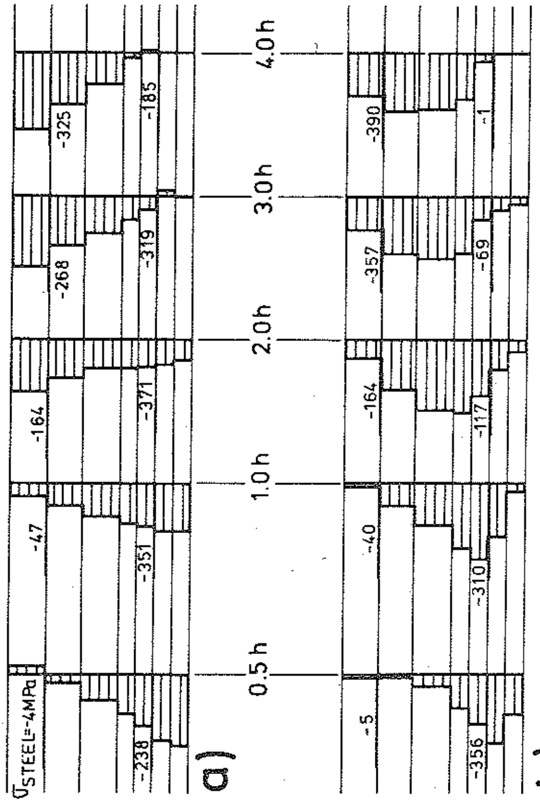

Fig. 9--Stress-distribution of (a) end-section and (b) midsection for identical structure as shown in Fig. 8. Load level P = 0