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New Multi-User OFDM Scheme: Braided Code Division Multiple Access

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Abstract—In this paper, we present a new multiple access scheme using OFDM modulation which we call braided code division multiple access (BCDMA). The principle behind this novel multiple access technique is to combine in one single scheme a very efficient error correction code, interleaving for channel diversity exploitation and, simultaneously, a multiple access method for a large number of users. BCDMA is based on braided codes, which are iteratively decodable codes defined on graphs and provide excellent performance with relatively low decoding complexity.

I. INTRODUCTION

There is a huge demand for higher data rates in next generation wireless systems. The typical multi-user scenario, where several users access the same medium and the same resources to perform communication, requires powerful and efficient broadband transmission techniques that enable the utilization of the spectrum resources with very high efficiency. Orthogonal frequency division multiplexing (OFDM) is a transmission technique that elegantly addresses this problem and is currently being considered as base technology for current and future wireless communication systems. Moreover, error correcting codes are indispensable elements to improve the overall capacity of communication systems.

With the discovery of turbo codes [1], coding theorists and practitioners have learned how to approach Shannon’s capacity with practical codes. Later, Gallager’s low-density parity-check (LDPC) codes [2] have been also recognized to be capacity-approaching [3]. In [4], the convolutional counterparts of Gallager’s LDPC block codes, i.e., LDPC convolutional codes (LDPC-CCs), were introduced. The LDPC-CCs have the advantage that they are not limited to a fixed block length as in the case of the LDPC block codes. Furthermore, the Tanner graphs underlying the LDPC-CCs have some interesting properties that enable the so-called pipeline decoding [4] and low-complexity encoding via shift-register operations. In [5]–[9], the LDPC-CCs were generalized to the case when the variable nodes and check nodes of the underlying Tanner graph are substituted by some simple component codes (e.g., Hamming codes and convolutional codes). It was shown that these generalized LDPC-CCs – which were named braided codes (BCs) – have excellent performance, good distance properties, and also that they can be continuously decoded with high speeds and low complexity using pipeline decoders.

In this paper, we present a novel multiple access technique that is based on OFDM and BCs. We have denominated this new scheme braided code division multiple access (BCDMA).1 As we will discuss throughout this paper, the BCDMA scheme results in a conceptual change in the organization of modems for multiple access systems. This is due to the fact that error correction coding, interleaving for diversity exploitation, modulation and multiple access – which are generally processed by separate elements of a modem – are now concentrated in one single entity that is derived from the typical two-dimensional array representation of the BCs.

II. MULTIPLE CONVOLUTIONAL PERMUTORS

Multiple convolutional permutors (MCPs) [11] are the fundamental elements in the constructions presented in the following sections. We define an MCP as a diagonal-type semi-infinite binary matrix P of the form

\[
P = \begin{pmatrix}
P_{0, \delta} & P_{0, \delta+1} & \cdots & P_{0, \Delta} & 0 & 0 & 0 & \cdots \\
0 & P_{1, \delta+1} & \cdots & P_{1, \Delta} & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots \\
0 & \cdots & 0 & P_{t, t+\delta} & \cdots & P_{t, t+\Delta} & 0 & \cdots
\end{pmatrix}
\]  

(1)

In (1), \(0 \leq \delta \leq \Delta\) and each row has exactly \(\Gamma\)'s and 0's elsewhere. Moreover, each of the first \(\Delta\) columns have at most \(\Gamma\)'s and all other columns have exactly \(\Gamma\)'s. We also assume that there exist time instants \(t = t_1\) and \(t = t_2\) such that \(p_{t_1, t_1+\delta} = 1\) and \(p_{t_2, t_2+\Delta} = 1\).

In this context, \(\Gamma\) is called multiplicity of the MCP. The parameters \(\delta\) and \(\Delta\) are called minimum and maximum permutor delays, respectively. The width of the permutor is defined as \(w = \Delta - \delta + 1\). Furthermore, we say that an MCP is periodic if and only if there exists a \(T\) such that \(p_{t, t'} = p_{t+T, t'+T}, \forall t, t' \in \mathbb{Z}_T\). The minimal value of \(T\) for which this last equation still holds is called period of the MCP. An MCP with period \(T\) can be constructed from a \(T \times T\) multiple block permutor using the unwrapping procedure presented in [4].

III. BRAIDED CODES

An uncomplicated way to understand the BCs is by taking a look at their encoders. Fig. 1 shows a generic encoder structure for BCs. In this picture, \(\mathbf{P}^{(0)}, \mathbf{P}^{(1)}\) and \(\mathbf{P}^{(2)}\) are MCPs with multiplicities \(\Gamma_0, \Gamma_1\) and \(\Gamma_2\), and widths \(w_0, w_1\) and \(w_2\), respectively. Moreover, \(U_t\) is a vector of information symbols that enter the systematic encoder at time instant \(t\), and \(V_t^{(0)} = U_t, V_t^{(1)}\) and \(V_t^{(2)}\) are the vectors of coded symbols

1 Some basic principles of BCDMA were formulated in [10].
that leave the encoder at time instant $t$. The feedback of parity-bits of the horizontal encoder into the vertical one and vice versa, which can also be seen in Fig. 1, is characteristic of the BCs and results in very good error correction capabilities [5], [6].

In the remainder of this paper, we will focus on BCs that are constructed using binary rate $R = 2/3$ conventional convolutional codes as component codes. The BCs resulting from this construction are called braided convolutional codes (BCCs) and they will have an overall rate $R = 1/3$. Moreover, we will assume symmetric MCPs, i.e., $\Gamma_0 = \Gamma_1 = \Gamma_2 = \Gamma$ and $w_0 = w_1 = w_2 = w$. In this sense, an implementation for a BCC encoder is exemplarily shown in Fig. 3, where the MCPs $P^{(0)}$, $P^{(1)}$, and $P^{(2)}$ with $\Gamma = 5$ and $w = 10$ correspond to the white cells of the array regions indicated by $P^{(0)}$, $P^{(1)}$, and $P^{(2)}^T$, respectively. As we can note, this implementation is based on a two-dimensional memory array, which is processed in a sliding window fashion. The encoding of the information bits is then performed as follows. At the time instant $t$, the vector of information symbols $U_t = (u_{t,0}, \cdots, u_{t,-1})$ enters the BCC encoder and its elements are stored in their corresponding positions in $P^{(0)}$. Then, the pairs of symbols $(u_{t,j}, \tilde{v}_{t,j}^{(1)})$, $j = 0, \cdots, \Gamma - 1$, are read from the memory array and given as inputs to the horizontal component code that produce as output the vector of symbols $V_t^{(1)} = (v_{t,0}^{(1)}, \cdots, v_{t,-1}^{(1)})$ whose elements are stored in their corresponding positions in $P^{(1)}$. A similar procedure is done for the vertical encoding. In this case, the pairs of symbols $(\mathbf{u}_{t,j}, \tilde{v}_{t,j}^{(2)})$, $j = 0, \cdots, \Gamma - 1$, are given as inputs to the vertical component code to produce $V_t^{(2)} = (v_{t,0}^{(2)}, \cdots, v_{t,-1}^{(2)})$ whose elements are stored in $P^{(2)}$. By doing so, in each time instant $t$ the output of the BCC encoder is the vector of symbols $V_t = (V_t^{(1)}, U_t, V_t^{(2)}) = (v_{t,0}^{(1)}, v_{t,1}^{(1)}, v_{t,2}^{(1)}, v_{t,3}^{(1)}, v_{t,4}^{(1)}, U_t, v_{t,0}^{(2)}, U_t, v_{t,1}^{(2)}, U_t, v_{t,2}^{(2)}, v_{t,3}^{(2)}, v_{t,4}^{(2)}, v_{t,0}^{(1)}, v_{t,1}^{(1)}, v_{t,2}^{(1)}, v_{t,3}^{(1)}, v_{t,4}^{(1)})$, as it is shown by the dashed box in Fig. 3.

A. BCCs and $M$-ary Modulation Schemes

BCCs are essentially binary codes. However, we can use the array representation of the BCCs also to encode symbols of $M$-ary modulation schemes. In this case, each used cell of the memory array will store $q = \log_2(M)$ bits. In other words, the symbols $u_{t,j}$, $\mathbf{u}_{t,j}$, $\tilde{v}_{t,j}^{(1)}$, $\tilde{v}_{t,j}^{(1)}$, $\tilde{v}_{t,j}^{(2)}$, and $\tilde{v}_{t,j}^{(2)}$ will be binary vectors of length $q$. The encoding is then performed similar to the binary case, however, each parity symbol is obtained by encoding the $q$ bits of each input symbol sequentially. For instance, in order to obtain the parity symbols $v_{t,j}^{(1)} = (v_{t,j,0}^{(1)}, \cdots, v_{t,j,q-1})$ from the symbols $u_{t,j} = (u_{t,j,0}, \cdots, u_{t,j,q-1})$ and $\tilde{v}_{t,j}^{(2)} = (\tilde{v}_{t,j,0}, \cdots, \tilde{v}_{t,j,q-1})$, the bits $u_{t,j,l}$ and $\tilde{v}_{t,j,l}$ enter the rate $R = 2/3$ component convolutional encoder to produce the bit $v_{t,j,l}^{(1)}$ for $l = 0$ to $q - 1$.

B. Decoders for BCCs

The BCs can be decoded with a pipelined decoder architecture that enables continuous, high-speed decoding. As it can be observed in Fig. 2, the main characteristic of this kind of decoder is the fact that all $I$ decoding iterations can be performed in parallel by $2I$ concatenated identical processors $D_i^e$, where $i = 1, \ldots, I$ and $e = 1, 2$. Thus, after an initial decoding delay, the estimated values for the code symbols are output by the pipeline decoder at each processing cycle in a continuous manner. For the particular case of BCs, the $2I$ parallel processors are implementing an windowed version of the BCJR algorithm [12], [13]. In this case, each processor operates in a finite window of length $W$ and calculates the a posteriori probabilities (APP) of all code symbols (i.e., information and parity symbols).

More specifically, the decoding of BCs using the pipeline decoder occurs as follows. At the $t$-th time instant, a vector of log-likelihood ratios (LLRs) $R_t$ corresponding to the vector of code symbols $V_t$ enters the memory pipeline as a priori information. Then, the read-logic $I_1$ transfers the necessary data from the memory pipeline to the horizontal component code decoder $D_1^{(1)}$. On its turn, the processor $D_1^{(1)}$ performs the windowed BCJR decoding on the data and transfer the obtained extrinsic LLRs to the memory pipeline through the write-logic $O_1$. After this, the processor $D_1^{(2)}$ starts processing new data. Observe in Fig. 2 that the current memory region used by $D_1^{(2)}$ is separated from the vertical decoder $D_2^{(2)}$ by $\tau$ positions. This is to avoid that different decoders operate in overlapping sets of coded bits at the same time. When the extrinsic LLRs produced by $D_1^{(1)}$ reaches the decoder $D_1^{(2)}$, vertical decoding is performed using the extrinsic LLRs produced by $D_1^{(2)}$ as a priori information. The new extrinsic LLRs obtained by $D_1^{(2)}$ are written into the memory pipeline and $D_1^{(1)}$ starts to process new data. This process repeats
white cells are associated with User 2 would need as basis for its realization an OFDM system (i.e., cells inside the dashed box) to a subcarrier of an OFDM users. In this case, we map each of the array cells at time 

\[ t \]

example shown in Fig. 3 can also be interpreted as the

values of the decoded bits.

reaches the vertical decoder through all horizontal and vertical processors until the data 

the final LLRs and hard decision is done to determine the

of a BCDMA system for two users. In Fig. 3, we denote

the symbols transmitted by User 1 at time instant 

\[ t \]

\[ \{ v^{(1)}_{t,0}, v^{(1)}_{t,1}, v^{(1)}_{t,2}, v^{(1)}_{t,3}, v^{(1)}_{t,4}, u_{t,0}, u_{t,1}, u_{t,2}, u_{t,3}, u_{t,4}, v^{(1)}_{t,0}, v^{(1)}_{t,1}, v^{(1)}_{t,2}, v^{(1)}_{t,3}, v^{(1)}_{t,4} \} \]

will be mapped to the OFDM subcarriers

\[ C_1(t) = \{ 0, 5, 7, 8, 9, 10, 11, 12, 13, 18, 20, 21, 23, 26, 27 \} \]

respectively. Accordingly, the symbols of User 2 at time 

\[ t \]

\[ \{ 1, 2, 3, 4, 6, 14, 15, 16, 17, 19, 22, 24, 25, 28, 29 \} \]

We can observe that the MCPs are the key components of our construction. They are responsible for defining the BCCs, as well as, the subcarrier allocation pattern of the users. In Fig. 3, we denote \( \mathbb{P}^{(0)}, \mathbb{P}^{(1)} \) and \( \mathbb{P}^{(2)} \) as being the sets containing the MCPs \( \mathbf{P}^{(i)}_k, i \in \{ 0, 1, 2 \} \), \( k \in \{ 0, \cdots, K - 1 \} \), of the \( K = 2 \) users of the system.\(^3\) The generalization for \( K \geq 3 \) is straightforward. However, a fundamental condition that the sets \( \mathbb{P}^{(i)}, i \in \{ 0, 1, 2 \} \), must fulfill in order to define a BCDMA system is that their elements must be orthogonal to

\[ \mathbb{P}^{(2)} \]

in the set containing the inverse MCPs \( \mathbb{P}^{(2)}_k, k \in \{ 0, \cdots, K - 1 \} \).

IV. BRAIDED CODE DIVISION MULTIPLE ACCESS

In addition to the array representation of a BCC, the example shown in Fig. 3 can also be interpreted as the

subcarrier allocation pattern of a BCDMA system for two users. In this case, we map each of the array cells at time \( t \) (i.e., cells inside the dashed box) to a subcarrier of an OFDM system. For instance, the BCDMA scheme depicted in Fig. 3 would need as basis for its realization an OFDM system with at least \( N_{OFDM} = 30 \) subcarriers. We assume that the white cells are associated with User 1 and gray cells are associated with User 2. The striped cells are dummy cells, they do not contain any information. Moreover, in the Fig. 3, only the code symbols of User 1 are depicted in the array.

If we number the OFDM subcarriers from the left to right, through all horizontal and vertical processors until the data reaches the vertical decoder \( D^{(2)}_t \). This last processor outputs the final LLRs and hard decision is done to determine the values of the decoded bits.

Fig. 3. Array representation of a braided convolutional code (BCC) and of a braided code division multiple access (BCDMA) scheme for two users.
each other. Considering the expression (1), this orthogonality condition can be formulated as
\[ p_{t,i}^{(i,k_1)}, p_{t',i}^{(i,k_2)} = 0, \quad \forall t, t' \in \mathbb{Z}^+ \text{ and } k_1 \neq k_2, \]
where \( p_{t,i}^{(i,k)} \) is an element of the MCP \( P_{i}^{(i,k)} \), \( i \in \{0, 1, 2\} \), associated with the \( k \)-th user of the system. In words, the condition imposed by (2) means that the different users of the BCDMA system do not share OFDM subcarriers to transmit their symbols at the time instant \( t \). Thus, they do not cause interference to each other.

A. Spectrum Management for BCDMA Systems

In spite of having the subcarrier allocation pattern determined by the error control coding scheme, the BCDMA system is still very flexible concerning the spectrum management for the several users. Considering that the available spectrum is given by the number of OFDM subcarriers \( N_{\text{OFDM}} \), some straightforward strategies for spectrum management are listed below:

a) If \( \lfloor N_{\text{OFDM}}/3w \rfloor < 2 \) the throughput of the users can be increased or reduced by increasing or reducing the multiplicities \( \Gamma \), respectively. Alternatively, different rows of the MCPs can be overlapped into one single OFDM symbol to increase the throughput, or the elements of the rows of the MCPs can be spread across different OFDM symbols to decrease the throughput.

b) If \( \lfloor N_{\text{OFDM}}/3w \rfloor \geq 2 \) the above mentioned techniques can also be applied. In addition, different rows of the MCPs can be placed into orthogonal sub-bands of a single OFDM symbol to increase the throughput.

Observe that by applying the spectrum management techniques from above, the orthogonality condition in (2) must be redefined.

B. Further Remarks

There are some points that are worth discussing when considering the constructions presented in this section. As we can observe in Fig. 3, the sub-carriers belonging to a particular user are spread in frequency domain. This is a very important feature that enables the BCDMA system to exploit the frequency diversity of the channel. Another very important property that belongs to the BCDMA system is the frequency hopping experienced by the users in the succession of the time instants, i.e., the subcarriers used by a particular user are different for the succession of time instants. This last property implies that good and also bad parts of the spectrum are equally (or almost equally) distributed among the users. In other words, it means that the BCDMA system is fair.

Additionally, if BCDMA is supposed to be deployed in a cellular system with frequency reuse, the frequency hopping is very important for averaging the inter-cell interference.

V. Simulation Results

In this section, we evaluate the performance of the BCDMA system depicted in Fig. 4(a) by means of computer simulations. As a reference system, we use the arrangement shown in Fig. 4(b), which employs the bit-interleaved coded modulation (BICM) ideas presented in [14] and analyzed in [15] to improve the code diversity of coded modulation on a Rayleigh channel.

For the BCDMA system, the component convolutional codes have rate \( R = 2/3 \) and 4 states. Their generator matrices are given by
\[
G(D) = \begin{pmatrix}
1 & 0 & 1/(1 + D + D^2) \\
0 & 1 & (1 + D^2)/(1 + D + D^2)
\end{pmatrix}. \tag{3}
\]
On the other hand, the component codes of the turbo code of the BICM system have rate \( R = 1/2 \) and also 4 states. Their generator matrices are given by
\[
G(D) = \begin{pmatrix}
1 & (1 + D^2)/(1 + D + D^2)
\end{pmatrix}. \tag{4}
\]
Moreover, all MCPs, interleavers and the subcarrier allocation pattern of the BICM system have been randomly generated.

For both systems, the underlying OFDM system consists of \( N_{\text{OFDM}} = 1024 \) subcarriers and a cyclic prefix of length \( N_{\text{CP}} = 16 \), which is appended to the beginning of each transmitted OFDM symbol. Additionally, we use a Rayleigh flat fading channel in our simulations that is constant over the duration of one OFDM symbol but varies independently from symbol to symbol. Moreover, in the OFDM demodulators of both systems from Fig. 4 a linear zero-forcing equalizer with perfect channel knowledge is used to mitigate the channel distortions.

In our simulations, the BCDMA system was observed in six different configurations, which are listed below:

1. \( w = 50, \Gamma = 5 \), QPSK modulation, block length \( L = 63800 \) bits.
2. \( w = 50, \Gamma = 5 \), 16-QAM modulation, block length \( L = 62800 \) bits.
3. \( w = 100, \Gamma = 10 \), QPSK modulation, block length \( L = 255200 \) bits.
4. \( w = 100, \Gamma = 10 \), 16-QAM modulation, block length \( L = 251200 \) bits.
In this paper, we presented the general principles of a novel multiple access scheme called braided code division multiple access (BCDMA). Besides of providing a flexible and efficient medium access scheme for a large number of users, the application of the BCDMA concept results in a conceptual change in the design of multiple access modems. In this case, processing tasks that are traditionally considered in separate (i.e., error correction coding, interleaving for diversity exploitation, modulation and multiple access) are now combined in one single entity. Moreover, the pipeline decoding of the underlying braided convolutional codes (BCCs) enables very high speed VLSI implementations. Simulation results have also shown that the BCDMA system has a considerable gain in comparison with conventional systems based on bit-interleaved coded modulation.

We consider BCDMA to be a promising research topic. Several points regarding this new technique remain open. For instance, the design of the MCPs to simultaneously maximize the error correction capabilities of the underlying BCC, frequency diversity exploitation, fairness and interference averaging property must be studied. In addition, some other issues like decoding delay reduction, VLSI implementation, rate compatibility, synchronization and channel estimation are also very interesting.

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