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# An Analysis of Two-Dimensional Pilot-Symbol Assisted Modulation for OFDM

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*Abstract*— In this paper we analyze two-dimensional (2D) pilot-symbol assisted modulation (PSAM) for wireless orthogonal frequency-division multiplexing (OFDM). 2D-PSAM has been suggested for wireless OFDM by several authors and is included in a preliminary draft of the standard for European digital video broadcast. We generalize the analysis of single-carrier PSAM to the 2D time-frequency lattice of OFDM. We analyze 2D-PSAM for two different channel estimators: one with good performance and high complexity and one suboptimal with low complexity. We verify that a good rule of thumb is to place the pilots at least as twice as close, in time and frequency, as required by the 2D sampling theorem.

## I. INTRODUCTION

Pilot-symbol assisted modulation (PSAM) was first introduced in the late 80's for single-carrier systems [1]. The use of PSAM requires a flat-fading channel and is based on transmission of known symbols in the data sequence, allowing channel estimation and thus, the use of efficient coherent multi-amplitude constellations. PSAM for the single-carrier case was thoroughly analyzed in [2]. Since the effective SNR is reduced by transmitting pilots, the spacing of the pilots is a trade-off between good channel estimation and high effective SNR. Since each subchannel in orthogonal frequency division multiplexing (OFDM) is flat fading, the good performance of PSAM in the single-carrier case has prompted its use in wireless OFDM systems as well. In OFDM the pilot symbols of PSAM are scattered in the two-dimensional (2D) time-frequency lattice. This modulation technique has been proposed for wireless OFDM by several authors, see *e.g.* [3], and is also included in a standard draft for European digital video broadcast (DVB) [6].

In this paper we analyze 2D-PSAM for an OFDM system. We investigate how the pilot spacing affects performance by using two estimators under different conditions. In one case we use a high-complexity channel estimator under almost ideal conditions. Under more practical conditions we use a sub-optimal estimator of low complexity. As a performance measure we use the

uncoded symbol error rate (SER) for 16QAM.

We show that the SER is almost constant for the high complexity channel estimator, as long as the pilots are placed closer than the 2D sampling theorem requires. To avoid a drastic increase in SER when using the low complexity channel estimator, however, one must ensure that the pilot spacing is well below the requirements set by the 2D sampling theorem.

## II. SYSTEM DESCRIPTION

### A. OFDM-system model

We analyze an OFDM system with  $N$  subcarriers on a bandwidth of  $W$  Hz. Further, to avoid inter-carrier and inter-symbol interference (ICI and ISI) the system uses a cyclic prefix (CP) of length  $T_{cp}$  seconds as guard space. This results in a total symbol length of  $T = N/W + T_{cp}$ . We assume the CP to yield perfect orthogonality. Under these assumptions the input/output relation of the system is [7]

$$y_{k,l} = h_{k,l}x_{k,l} + n_{k,l}, \quad (1)$$

where  $k$  is the subcarrier index,  $l$  is the OFDM symbol index,  $h_{k,l}$  are the subcarrier attenuations,  $x_{k,l}$  are the transmitted signal constellation points, and  $n_{k,l}$  are independent and identically distributed complex Gaussian noise variables.

We use the wide-sense stationary uncorrelated scattering (WSSUS) channel model introduced in [8]. By considering the channel to be constant over one OFDM symbol, the frequency response of the  $M$ -path channel is [9]

$$h(f;t) = \frac{1}{\sqrt{M}} \sum_{n=1}^M e^{j(\theta_n + 2\pi f D_n t - 2\pi f \tau_n)}, \quad (2)$$

where  $\theta_n$  is the phase,  $f D_n$  the Doppler frequency and  $\tau_n$  the delay of the  $n$ th path. All these parameters are independent random variables. To obtain Rayleigh fading with the so called Jakes' spectrum and an exponentially decaying power delay profile with Root Mean

Square (RMS) width  $\tau_{\text{rms}}$ , we choose the probability density functions as [8]

$$\begin{aligned} p_{\theta}(\theta) &= 1/2\pi & 0 \leq \theta < 2\pi, \\ p_{f_D}(f_D) &= \frac{1}{\pi f_{D,\text{max}} \sqrt{1-(f_D/f_{D,\text{max}})^2}} & |f_D| < f_{D,\text{max}}, \\ p_{\tau}(\tau) &= \frac{e^{-\tau/\tau_{\text{rms}}}}{\tau_{\text{rms}}(1-e^{-T_{cp}/\tau_{\text{rms}}})} & 0 \leq \tau \leq T_{cp}. \end{aligned}$$

The random variables  $f_D$  and  $\tau$  are easily obtained from a uniformly distributed random generator with outputs  $\in [0, 1]$  by using the inverse of the desired cumulative distribution function [8].

Using the channel model in (2), the channel attenuations in (1) are given by

$$h_{k,l} = h\left(\frac{k}{W}; lT\right).$$

### B. Scenario

In the analysis we consider an OFDM system with a bandwidth of  $W = 5$  MHz operating in the 2.2 GHz frequency band. The number of subcarriers is  $N = 1024$  which makes the effective symbol length  $205 \mu\text{s}$ . The environment is a macrocell and is assumed to have a maximum delay spread of  $10 \mu\text{s}$  and a maximum Doppler frequency of  $240$  Hz. Thus, the Doppler frequency relative to the inter-tone spacing is  $5\%$  and corresponds to a vehicle speed of  $120$  km/h. The power delay profile is exponentially decaying with RMS-width  $\tau_{\text{rms}} = 2.2 \mu\text{s}$ . To eliminate ISI, we use a guard space of  $T_{cp} = 10 \mu\text{s}$ . The length of the OFDM symbol is  $T = 205 + 10 = 215 \mu\text{s}$  which makes the relative size of the guard space  $5\%$ . With a maximum Doppler rate of  $5\%$  and  $1024$  subcarriers, the signal-to-ICI ratio is  $24$  dB [10]. This is negligible in the SNR ranges we investigate and consequently, we ignore the ICI.

### C. 2D-PSAM

We analyze 2D-PSAM where pilot symbols are transmitted on every  $M_f$ th subcarrier in every  $M_t$ th OFDM symbol. Figure 1 shows a schematic picture of the OFDM time-frequency lattice, where the relative number of pilots is  $\varepsilon = 1/M_t M_f$ . The analysis is performed with a rectangular pilot-pattern.

The 2D sampling theorem puts fundamental limits on the required density of the pilot pattern. With  $N = 1024$  subcarriers on a  $W = 5$  MHz bandwidth, a cyclic prefix of length  $T_{cp} = 10 \mu\text{s}$ , and a maximal relative Doppler frequency of  $f_{D,\text{rel}} = 5\%$ , we need

$$M_t < \frac{1}{2\left(1 + \frac{WT_{cp}}{N}\right) f_{D,\text{rel}}} \approx 9.5 \quad (3)$$

and

$$M_f < \frac{N}{WT_{cp}} \approx 20.5 \quad (4)$$

to avoid undersampling of the channel [9].

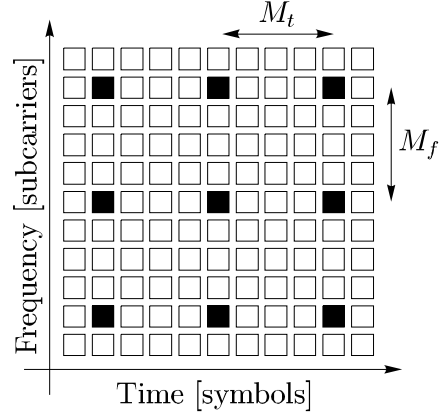


Figure 1 – A neighbourhood of the OFDM time-frequency lattice, containing both data ( $\square$ ) and pilots ( $\blacksquare$ ).

We define the per-symbol SNR as

$$\text{SNR} = \frac{E\{|x_{k,l}|^2\} E\{|h_{k,l}|^2\}}{E\{|n_{k,l}|^2\}},$$

which for a relative number of pilots  $\varepsilon$  is reduced by

$$\text{SNR}_{\text{loss}} = -10 \log_{10} (1 - \varepsilon) \text{ dB}.$$

Thus, we include only pilots carrying data in the calculations of SNR. Therefore, we loose more of the per-symbol SNR if we choose a denser pilot pattern. The pilot pattern design problem is a trade-off between good channel estimation (closely spaced pilots) and high SNR (sparsely spaced pilots). It is notable that the data rate is  $N(1 - \varepsilon)/T$  symbols/sec., i.e., depends on  $\varepsilon$ .

## III. PERFORMANCE ANALYSIS

We divide the performance analysis into two different categories. First, we analyze a situation where the channel estimator is of very high complexity and has high performance. In this case, the symbol-error rate is mainly determined by the pilot pattern. Second, we analyze a realistic situation where the channel estimator is of low complexity and also suffers from design mismatch. This estimator is designed according to a generalization of the worst-case recommendations for single-carrier PSAM in [2] and applied to the channel described in II.B. Comparing the two estimators reveals the impact of non-ideal channel estimation in combination with 2D-PSAM.

First we calculate the mean-squared error (MSE) of these linear estimators from the statistical properties of the channel and the noise [4, 9]. In both cases we proceed by calculating the uncoded 16QAM SER from the MSE according to the analytical expressions given in [11] for  $2 \leq M_t \leq 13$  and  $2 \leq M_f \leq 35$ .

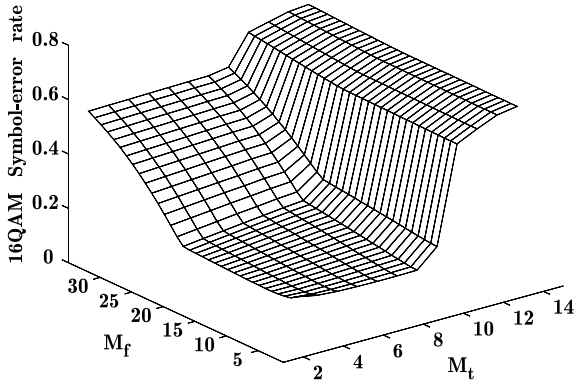


Figure 2 – Uncoded 16QAM SER at 15 dB SNR as a function of pilot pattern ( $M_t$  and  $M_f$ ) with size  $31 \times 11$  2D- Wiener filter estimator.

#### A. The ideal case

To get an analysis of the pilot pattern that shows SER performance under (almost) ideal conditions, we apply a 2D-Wiener filter that uses 341 pilot symbols, 31 in frequency  $\times$  11 in time, to estimate the channel attenuations. The linear 2D-Wiener filter can be expressed [9] as

$$\hat{h}_{k,l} = \mathbf{W}\mathbf{p} = \mathbf{R}_{hp}\mathbf{R}_{pp}^{-1}\mathbf{p}, \quad (5)$$

where  $\mathbf{W}$  is the linear minimum mean-squared estimator (LMMSE),  $\mathbf{p}$  is a vector of back-rotated observations ( $p_{k,l} = y_{k,l}/x_{k,l}$ ) at the 341 pilot positions,  $\mathbf{R}_{hp}$  is the cross-covariance between the estimated channel attenuation  $h_{k,l}$  and the observations  $\mathbf{p}$ , and  $\mathbf{R}_{pp}$  is the auto-covariance matrix of the observations  $\mathbf{p}$ . This estimator requires 341 multiplications per estimated attenuation and is therefore of little practical interest.

With no design mismatch, the MSE becomes [12]

$$\sigma_{k,l}^2 = E \left\{ \left| h_{k,l} - \hat{h}_{k,l} \right|^2 \right\} = \sigma_h^2 - \mathbf{W}\mathbf{R}_{hp}^H, \quad (6)$$

where  $\sigma_h^2 = E\{|h_{k,l}|^2\}$  and  $(\cdot)^H$  denotes the Hermitian transpose. By using the MSE we get the uncoded 16QAM SER from expressions given in [11].

The SER at 15 dB SNR is displayed in Figure 2. According to the calculations, the minimal SER is 0.19 at  $M_t = 3$  and  $M_f = 5$ . This yields  $\varepsilon = 1/15 \approx 7\%$  pilots, 93% data and a corresponding SNR loss of 0.30 dB. Since the optimum is relatively flat,  $M_t$  and  $M_f$  may be increased to allow higher data rate.

Noteworthy is that the SER starts to increase rapidly when the channel becomes undersampled for  $M_t > 9.5$  and  $M_f > 20.5$ , see expressions (3) and (4).

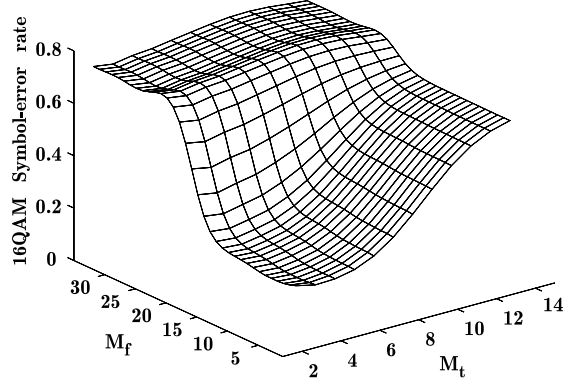


Figure 3 – Uncoded 16QAM SER at 15 dB SNR as a function of pilot pattern ( $M_t$  and  $M_f$ ) with size  $5 \times 2$  separable estimator.

#### B. A practical case

In this practical case, we investigate the pilot pattern when the channel estimator is separable. Since the true channel correlation properties usually are not known at the receiver, we assume that there exist a mismatch between the design correlation and the true correlation. We also get a degradation by using separable filters, which is suboptimal. Then, the MSE becomes

$$\sigma_{k,l}^2 = \sigma_h^2 - \mathbf{R}_{hp}\mathbf{W}^H - \mathbf{W}\mathbf{R}_{hp}^H + \mathbf{W}\mathbf{R}_{pp}\mathbf{W}^H. \quad (7)$$

For the ideal case without a design mismatch the MSE in eq. (7) reduces to eq. (6).

The use of a separable estimator is a good trade-off between complexity and performance [3, 4, 5, 9]. The separable estimator is composed of two one-dimensional FIR-Wiener-filters — one in the frequency-direction and the other in the time-direction. The frequency-direction estimator, which is applied first, has 5 taps and the time-direction estimator has 2 taps. Since the frequency-direction estimator is only applied to OFDM symbols containing pilots, *i.e.* one out of  $M_t$ , this separable estimator only requires  $5/M_t + 2$  multiplications per estimated attenuation. In this case the number of multiplications ranges between 2.3 and 4.5, depending on  $M_t$ , which should be compared with the 341 multiplications in the previous case.

The SER at 15 dB SNR is displayed in Figure 3, which should be compared with the (almost) ideal case in Figure 2. According to the calculations, the minimal SER is 0.22 at  $M_t = 3$  and  $M_f = 5$ . Thus, the optimal pilot spacing is the same as for the case with the high-complexity channel estimator. Although this might not be true for all channel estimators applied in our scenario, we can expect that the optimal pilot spacing is almost the same, *i.e.*,  $\varepsilon \approx 7\%$ .

In the two scenarios we investigate, the optimal pilot spacing is shown to be the same, *i.e.*  $M_t = 3$  and

$M_f = 5$ . This corresponds to a pilot pattern which oversamples the channel 3 times in time-direction and 4 times in frequency-direction.

Noteworthy in the practical case is that the optimum is less flat than for the ideal case and it is therefore important to keep  $M_t$  and  $M_f$  small enough to avoid poor SER due to non-ideal channel estimation. More general we see that by choosing the pilot density, in time and frequency, at least twice as close as required by the sampling theorem we get only a small performance loss. Twice as close as required by the sampling theorem is used as a "rule of thumb" in [4, 5]. By placing the pilots according to these recommendations, a low-complexity estimator design becomes relatively robust against mismatch. This can be of interest when designing an OFDM system in practice.

#### IV. CONCLUSIONS

This analysis of 2D-PSAM for OFDM has shown that pilot spacing is a trade-off between good channel estimation and high effective SNR. Using the SER for 16QAM as a performance measure, the optima, with respect to pilot spacings, were shown to be rather flat. For a 2D channel estimator with a high complexity the 16QAM SER is almost constant as long as the pilots are placed closer than the limits specified by the 2D sampling theorem. When a low-complexity channel estimator is applied, the pilots must be placed closer to avoid poor performance. In this latter case, the effects of channel estimation become visible. Placing the pilots at least twice as close as required by the 2D sampling theorem makes performance less sensitive in terms of model mismatch and choice of estimator structure.

The analysis presented in this paper show only results for an SNR of 15 dB, but investigations beyond this presentation shows that the same type of behavior is experienced for all SNRs.

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