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Abstract:

Large review costs lead to time-dependent price setting rules. State-dependent rules become more likely when there is an increase in: set-up costs, the variability of the equilibrium price or the efficiency loss associated with being away from equilibrium.

Keywords: time-dependent price rule; state-dependent price rule

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1. Introduction

Since the quest for finding micro foundations for price- and wage-setting behaviour began in the 1970s, macroeconomic models have been developed using either time-dependent or state-dependent price and wage setting rules. Recently Klenow and Kryvtsov (2008), however, presented micro-evidence of price changes which is difficult to square with either time- or state-dependent models. One reason may be the simultaneous existence of both rules, or the use of combined rules, as indicated by survey evidence (Álvarez 2007 and Fabiani et al. 2005). Thus the empirical findings point to more complicated macromodels.

As a first step towards understanding the empirical findings, I present as simple model of the choice between the two pricing rules. I build on the insight of Blanchard and Fischer (1989, p. 413) that time-dependent rules should arise when the costs of reviewing the state are large. I show that large review costs favour time-dependent rules and that large set-up costs favour state-dependent rules. In addition, I show that an increase in the variability of the underlying equilibrium price process favours state-dependent rules as does an increase in the efficiency loss associated with being away from equilibrium. The model is a synthesis of the seminal models of the optimal state-dependent rule by Barro (1972) and the optimal time-dependent rule by Gray (1978).

Section 2 describes the equilibrium price process and the loss functions. Section 3 derives the optimal length and loss of the time-dependent rule and section 4 derives the optimal threshold, the average length, and the loss of the state-dependent rule. Section 5 analyzes the choice between the rules. Section 6 concludes with some supporting recent survey evidence of pricing behavior.

2. The equilibrium price process and the loss functions

Following Barro (1972), define a discrete random walk in unit steps per draw with no drift for the equilibrium nominal price, u_i , as

$$u_i = u_{i-1} + \varepsilon_i \tag{1}$$

where ε_i is a random variable that takes either the value +1 with probability 0.5 or the value -1 with probability 0.5. The forecast error variance of u between two draws is then equal to one. The forecast variance over one period of calendar time, say a day, will be the number of draws per day, say *k*. Since the random walk implies that the forecast variance increases linearly with the number of draws, the daily forecast variance is *k*. Denote the daily variance σ^2 . The forecast variance of the equilibrium price *t* days ahead is then equal to $t\sigma^2$.

With no drift (inflation) in equilibrium prices, it is optimal for both the state and the time-dependent rules (contract) to set the price equal to the equilibrium price and keep it fixed until the next setting.[1] The time-dependent rule price runs for a fixed length of time, while the state-dependent rule resets the price to the equilibrium price when the deviation between the actual and the equilibrium price reaches a pre-set threshold. Both rules are assumed to pay the same fixed set-up cost when the price is reset.

The expected loss function used by Gray and Barro is made up of two parts: 1) the fixed set-up (menu) cost, γ, averaged per day and 2) the expected efficiency cost per day associated with being away from the equilibrium price. To these two costs, I add a fixed review cost, ρ. The state-dependent rule pays it each day, while the time-dependent rule pays it when a new price is set.

[1] Letting the price change to reflect known drift (inflation) in the underlying price process, does not affect the optimal length decision for the time-dependent rule as shown by Gray (1978). The optimal state-dependent rule with drift (inflation) is analyzed by Sheshinski and Weiss (1983).

The efficiency loss at a point in time is proportional to the squared deviation of the actual (contract) price from the equilibrium price, since the efficiency loss can be measured as the dead-weight triangle arising from the discrepancy between the actual and the equilibrium price. The expected efficiency loss is then proportional to the unconditional variance of the price deviation.

Denote the time between new prices D (length of contracts). We can write the expected loss as the sum of the average per period set-up cost, the average review cost and the unconditional variance of the price deviation multiplied by an efficiency parameter θ.[2] The expected loss for the time-dependent (TD) rule is then:

$$E\left[Loss_{TD}\right] = (\gamma + \rho) / D_{TD} + \theta E\left[du_t\right]_{TD}^2 \qquad (2)$$

and the expected loss for the state-dependent (SD) rule is:

$$E\left[Loss_{SD}\right] = \gamma / E\left[D_{SD}\right] + \theta E\left[du_t\right]_{SD}^2 + \rho \qquad (3)$$

3. The optimal time-dependent rule

In Gray's model of the optimal time-dependent rule, the length between resetting the price is the decision variable.[3] The length determines the expected loss.

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[2] θ summarizes the underlying demand and supply elasticities, which determine the dead-weight loss.

[3] The following is a reformulation of the Gray (1978) continuous-time model into discrete time to make it comparable to the Barro (1972) model.

The unconditional price deviation variance is equal to the average of the price variances $t\sigma^2$ occurring t days after the last resetting of the price averaged over the period with a constant price, such that t goes from 0 to D_{TD}. The average loss is then:

$$E[Loss_{TD}] = \frac{\gamma + \rho}{D_{TD}} + \theta \frac{1}{D_{TD}} \sum_{t=0}^{D_{TD}} t\sigma^2 = \frac{\gamma + \rho}{D_{TD}} + \theta \frac{D_{TD}}{2} \sigma^2 \qquad (4)$$

Minimizing the loss in (4) with respect to length gives the optimal length D_{TD}^* in days:

$$D_{TD}^* = \sqrt{2(\gamma + \rho) / \theta\sigma^2} \qquad (5)$$

as shown by Gray (1978, eq. 11), here with ρ added to γ.

4. The optimal state-dependent rule

In Barro's model of the optimal state-dependent rule, the threshold for changing the price is the decision variable. The threshold determines the expected length between price resets and the expected loss.

Two results from the theory of random walks with "reflecting barriers" were used by Barro (1972, pp. 22-23) to solve for the unconditional expected length $E[D_{SD}]$ and unconditional price deviation $E[du_t]_{SD}^2$ as functions of the threshold.[4] Denote the threshold h, measured in price deviation units du_t .The reflecting barrier process is defined as a random walk of the deviation which returns to zero when it hits one of the thresholds h or -h, and starts over

[4] The results were adapted from Feller (1968, Ch 14). A presentation of the Barro model is given in Blanchard and Fischer (1989, pp. 402-5).

again. The first result states that the expected length, $E[D_{SD}]$, in days between barrier contacts - here renegotiations - is approximately:[5]

$$\mathrm{E}[D_{\mathrm{SD}}] = h^2 / \sigma^2 \tag{6}$$

The second result is that the unconditional price variance is approximately

$$E[du_t]_{SD}^2 = h^2 / 6 \tag{7}$$

Plug in (6) and (7) into the loss (3):[6]

$$E[Loss_{SD}] = \gamma / E[D_{SD}] + \theta E[du_t]_{SD}^2 + \rho = \gamma\sigma^2 / h^2 + \theta h^2 / 6 + \rho \tag{8}$$

Minimize the loss in (8) with respect to *h* to find the optimal threshold level *h**:

$$h^* = (6\gamma\sigma^2 / \theta)^{1/4} \tag{9}$$

as shown by Barro (1972, eq. 19). Insert optimal *h** in (6) to find expected length when the threshold is optimal:

$$E[D_{SD}^*] = h^{*2} / \sigma^2 = \sqrt{6\gamma / \theta\sigma^2} \tag{10}$$

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[5] Implied by the *D* and σ^2 equations (not numbered) in Barro (1972, p. 22).

[6] Corresponding to Barro (1972, eq. 18) with ρ added.

5. The choice of rule

To find the determinants of the choice of price rule, I look at the gain, G, of switching from a time-dependent to a state-dependent rule, defined as the difference between the expected loss of the time-dependent and the state-dependent rule.[7] Subtract the loss (8) from the loss (4), and evaluate at the optimal settings:

$$\begin{aligned} G(D_{TD}^{*}, D_{SD}^{*}, h^{*}) &= E\left[Loss_{TD}^{*}\right] - E\left[Loss_{SD}^{*}\right] \\ &= \left[(\gamma+\rho)/D_{TD}^{*} + \theta D_{TD}^{*}\sigma^{2}/2\right] - \left[\gamma\sigma^{2}/h^{*2} + \theta h^{*2}/6 + \rho\right] \end{aligned} \tag{11}$$

We find the determinants of the choice of rule by taking the derivative with respect to the exogenous variables ρ, γ, σ^2, and θ. By the envelope theorem, we can ignore the effect of changes in D_{TD}, D_{SD}, and h.[8]

An increase in the review costs, ρ, decreases the likelihood of a state-dependent rule as

$$\partial G(D_{TD}^{*}, D_{SD}^{*}, h^{*})/\partial\rho = 1/D_{TD}^{*} - 1 < 0 \tag{12}$$

, since $D_{TD} > 1$ by the choice of short calendar units. An increase in the review cost increases the likelihood of time-dependent rule, since the loss of the state-dependent rule increases in direct proportion to the increase in the review cost, while the time-dependent rule loss is proportional to the frequency of adjustment.

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[7] This is the same strategy used by Blanchard (1979) who analyzed the choice between fully contingent one-period contracts and restricted one-period price-indexed contracts, when there is a fixed cost of writing a fully contingent contract.

[8] Since we take the derivative around the optimum where $\partial G(D_{TD}^{*}, D_{SD}^{*}, h^{*})/\partial D_{TD}^{*} = 0$ and $\partial G(D_{TD}^{*}, D_{SD}^{*}, h^{*})/\partial h^{*} = 0$.

The effect of an increase in the set-up cost, γ, is given by

$$\partial G(D_{TD}^*, E(D_{SD}^*), h^*)/\partial\gamma = 1/D_{TD}^* - \sigma^2/h^* = 1/D_{TD}^* - 1/E(D_{SD}^*) = \left(1/\sqrt{2(\gamma+\rho)} - 1/\sqrt{6\gamma}\right)/\theta\sigma^2 \tag{13}$$

The direct effect of γ on the gain is proportional to the difference in the frequency (1/*D*) of paying the set-up cost. If review costs are sufficiently low ($\rho < 2\gamma$), state-dependent rules renegotiate less often than time-dependent rules. Hence in this case, higher set-up costs make state-dependent rules more attractive.

An increase in uncertainty, σ^2 , increases the likelihood of state-dependent rule as

$$\partial G(D_{TD}^*, D_{SD}^*, h^*)/\partial\sigma^2 = \theta D_{TD}^*/2 - \gamma/h^{*2} = \frac{\sqrt{\theta}}{\sqrt{2\sigma^2}}\left(\sqrt{\gamma+\rho} - \sqrt{\gamma}/\sqrt{3}\right) > 0 \tag{14}$$

The greater flexibility afforded by constant review increases the gain from a state-dependent rule.

By the same token, an increase in the efficiency cost, θ, increases the likelihood of a state-dependent rule as,

$$\partial G(D_{TD}^*, E(D_{SD}^*), h^*)/\partial\theta = D_{TD}^*\frac{\sigma^2}{2} - \frac{h^{*2}}{6} = \frac{\sigma}{\sqrt{2\theta}}\left(\sqrt{\gamma+\rho} - \sqrt{\gamma}/\sqrt{3}\right) > 0 \tag{15}$$

6. Conclusion

The results confirm the conjecture of Blanchard and Fischer (1989, p. 413) that time-dependent rules arise when review costs are high and that state-dependent rules arise when set-up costs are high. These straightforward results tallies with the exclusive reliance on set-up costs in macromodels based on state-dependent rules, such as Lucas and Golosov (2007) and the exclusive reliance on review costs in macromodels based on time-dependent rules, such as Reis (2006).

Some empirical support for the results presented here can be found in the survey evidence on firms´ pricing behavior collected by the ECB research group "The Inflation Persistence Network". Álvarez (2007, table 7), summarized and analyzed the evidence on the relative use of state- and time-dependent rules. Two pieces of evidence support the positive effect of uncertainty on the use of state-dependent rules. First, state-dependent rules are most common in countries with the highest and most volatile inflation. Second, time-dependent rules are more common in the service sector, where uncertainty of marginal costs is low. The positive effect of increased efficiency cost on the use of state-dependent rules is supported by the finding that higher competition increases the likelihood of state-dependent rules, since higher competition implies more elastic demand and higher dead-weight losses associated with being away from equilibrium. Finally, Fabiani et al. (2005) report that time-dependent rules are more common in industries with wide-spread use of collective agreements. Typically collective agreements are time-dependent and should decrease the uncertainty in price setting in those industries.

References

Álvarez, L., 2007, What do micro price data tell us on the validity of the New Keynesian Phillips curve? Economics Discussion Papers 2007-46.

Barro, R. J., 1972, A theory of monopolistic price adjustment, Review of Economic Studies 39, 17-26.

Blanchard, O. J., 1979, Wage-indexing rules and the behavior of the economy, Journal of Political Economy 87, pp. 798-815.

Blanchard, O. J., Fischer, S., 1989, Lectures on macroeconomics, (The MIT Press, Cambridge, MA).

Fabiani, S., et al., 2005, The pricing behavior of firms in the euro area. New survey evidence, Working paper No. 535, European Central Bank.

Feller, W., 1968, An introduction to probability theory and its applications, Vol. 3, (Wiley and Sons, New York).

Gray, J. A., 1978, On indexation and contract length, Journal of Political Economy 86, 1-18.

Golosov, M. and R.E. Jr Lucas, 2007, Menu costs and Phillips curves, Journal of Political Economy 115, 171-199.

Klenow, P.J. and O. Kryvtsov, 2008, State-dependent or time-dependent pricing: Does it matter for recent U.S. inflation? Forthcoming Quarterly Journal of Economics.

Reis, R., 2006, Inattentive producers, Review of Economic Studies 73, 791-821.

Sheshinski, E. and Y. Weiss, 1983, Optimum pricing policy under stochastic inflation, Review of Economic Studies 50, 513-529.

Appendix: Derivations

Derivation of (4):

$$E\left[Loss_{TD}\right] = \frac{\gamma+\rho}{D_{TD}} + \theta\frac{1}{D_{TD}}\sum_{t=0}^{D_{TD}} t\sigma^2 = \frac{\gamma+\rho}{D_{TD}} + \theta\sigma^2\frac{1}{D_{TD}}\frac{D_{TD}^2}{2} = \frac{\gamma+\rho}{D_{TD}} + \theta\frac{D_{TD}}{2}\sigma^2$$

Derivation of (5) from (4):

$$\partial E(Loss_{TD})/\partial D_{TD} = -\frac{\gamma+\rho}{D_{TD}{}^2} + \frac{\theta\sigma^2}{2} = 0 \Rightarrow \frac{\gamma+\rho}{D_{TD}{}^2} = \frac{\theta\sigma^2}{2} \Rightarrow D_{TD}{}^2 = \frac{2(\gamma+\rho)}{\theta\sigma^2} \Rightarrow D_{TD}^* = \sqrt{\frac{2(\gamma+\rho)}{\theta\sigma^2}}$$

Derivation of (9) from (8):

$$\partial E(Loss_{SD})/\partial h = -2\gamma\sigma^2/h^3 + 2\theta h/6 = 0 \Rightarrow \gamma\sigma^2/h^3 = \theta h/6 \Rightarrow 6\gamma\sigma^2 = \theta h^4 \Rightarrow h^* = \left(6\gamma\sigma^2/\theta\right)^{1/4}$$

Derivation of (10) from (9) in (6):

$$E\left[D_{SD}^*\right] = h^{*2}/\sigma^2 = \left(6\gamma\sigma^2/\theta\right)^{1/2}/\sigma^2 = \left(6\gamma/\theta\right)^{1/2}\sigma/\sigma^2 = \left(6\gamma/\theta\right)^{1/2}/\sigma = \sqrt{6\gamma/\theta\sigma^2}$$

Derivation of (13) from (5) and (9) in (11):

$$\partial G(D_{TD}^*, E(D_{SD}^*), h^*)/\partial\gamma = 1/D_{TD}^* - \sigma^2/h^* = 1/D_{TD}^* - 1/E(D_{SD}^*)$$
$$= 1/\sqrt{2(\gamma+\rho)/\theta\sigma^2} - 1/\sqrt{6\gamma/\theta\sigma^2}$$
$$= \left(1/\sqrt{2(\gamma+\rho)} - 1/\sqrt{6\gamma}\right)/\sqrt{\theta\sigma^2} > 0$$
$$if\, 1/\sqrt{2(\gamma+\rho)} > 1/\sqrt{6\gamma} \Leftrightarrow \sqrt{2(\gamma+\rho)} < \sqrt{6\gamma} \Leftrightarrow 2(\gamma+\rho) < 6\gamma \Leftrightarrow 2\rho < 4\gamma \Leftrightarrow \rho < 2\gamma$$

Derivation of (14) from (5) and (9) in (11):

$$\partial G(D_{TD}^*, D_{SD}^*, h^*)/\partial\sigma^2 = \theta D_{TD}^*/2 - \gamma/h^{*2} = \theta\sqrt{2(\gamma+\rho)/\theta\sigma^2}/2 - \gamma/\sqrt{6\gamma\sigma^2/\theta}$$
$$= \sqrt{\theta}\sqrt{2(\gamma+\rho)/\sigma^2}/2 - \gamma\sqrt{\theta}/\sqrt{6\gamma\sigma^2} = \frac{\sqrt{\theta}}{\sigma^2}\left(\sqrt{(\gamma+\rho)}/\sqrt{2} - \sqrt{\gamma}/\sqrt{6}\right) = \frac{\sqrt{\theta}}{\sqrt{2}\sigma^2}\left(\sqrt{\gamma+\rho} - \sqrt{\gamma}/\sqrt{3}\right) > 0$$

Derivation of (15) from (5) and (9) in (11):

$$\partial G(D_{TD}^*, E(D_{SD}^*), h^*)/\partial\theta = D_{TD}^*\frac{\sigma^2}{2} - \frac{h^{*2}}{6} = \sqrt{\frac{2(\gamma+\rho)}{\theta\sigma^2}}\frac{\sigma^2}{2} - \frac{\sqrt{\left(6\gamma\sigma^2/\theta\right)}}{6}$$
$$= \frac{\sqrt{(\gamma+\rho)\sigma^2}}{\sqrt{\theta 2}} - \frac{\sqrt{\gamma\sigma^2}}{\sqrt{\theta 6}} = \frac{\sigma}{\sqrt{2\theta}}\left(\sqrt{\gamma+\rho} - \sqrt{\gamma}/\sqrt{3}\right) > 0$$