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Published in:
Proc. IEEE Vehicular Technology Conference

DOI:
10.1109/VETEC.1999.778362

1999

TURBO CODES: CORRELATED EXTRINSIC INFORMATION AND ITS IMPACT ON ITERATIVE DECODING PERFORMANCE

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Abstract – The performance of a turbo code is dependent on two properties of the code: its distance spectrum and its suitability to be iteratively decoded. The performance of iterative decoding depends on the quality of the extrinsic inputs; badly correlated extrinsic inputs can deteriorate the performance. While most turbo coding literature assumes that the extrinsic information is uncorrelated, we investigate these correlation properties. An *iterative decoding suitability* measure is presented, intended to serve as an indication on the degree of correlation between extrinsic inputs. The suitability measure can be used as a complement to the weight distribution when ranking interleavers.

I. INTRODUCTION

The interleaver used in turbo codes [1] has two major tasks. The first is to ensure a good distance spectrum of the code by breaking up so called self-terminating input sequences, see e.g. [2]. In the case of maximum likelihood decoding this would be the natural and single task of the interleaver. However, turbo codes are decoded iteratively which is not optimal in the sense of making maximum likelihood decisions. The performance of iterative decoding compared to maximum likelihood decoding is dependent on the quality of the extrinsic information, which is the information being exchanged between the constituent decoders in the iterative decoding scheme. The choice of interleaver affects the degree of correlation between extrinsic inputs, and thereby the performance of iterative decoding. In this paper we present and investigate an *iterative decoding suitability* (IDS) measure, which indicates the degree of correlation between nearby extrinsic inputs.

The outline of the paper is as follows. In Section IIthe correlation properties of the extrinsic informa-

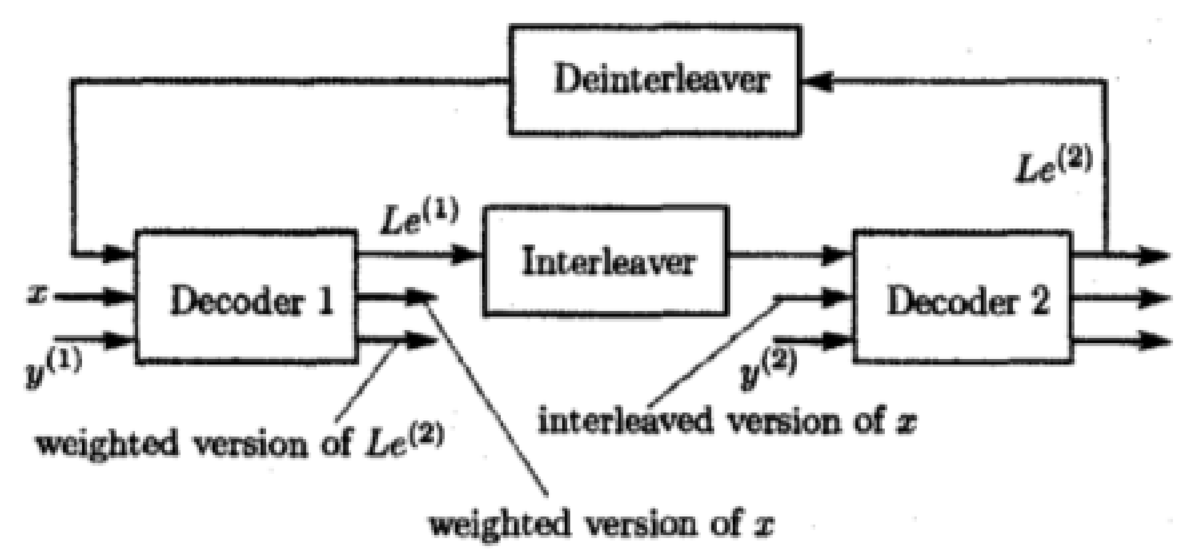

Figure 1: Structure of the iterative decoder.

tion are studied. Based upon this investigation, an iterative decoding suitability measure is proposed. In Section III, this measure is applied on a large number of interleavers, in order to evaluate its usefulness.

II. CORRELATED EXTRINSIC INPUTS

The iterative decoding scheme studied in this paper is depicted in Figure 1. Each soft-input/soft-output constituent decoder has three inputs and three outputs, according to e.g. [3]. The three inputs at decoding step k are the systematic input, x, the parity input, $y^{(1)}$ or $y^{(2)}$, and the extrinsic input from the previous decoder. The three outputs are a weighted version of the systematic input, a weighted version of the extrinsic input, and finally the new extrinsic output. These extrinsic outputs are used as input *a priori* probabilities in the next decoding step. The decision variables after each decoder is achieved as the sum of the three outputs, for each time instant. All input and output sequences are N bits long, which is also the size of the interleaver. Each constituent decoder employs the maximum *a poste-*

riori probability (MAP) algorithm as described in [3].

The systematic and parity inputs for each decoding step depend only on the received values for each specific bit. The extrinsic inputs, however, are dependent on a range of symbols in the received systematic and parity sequences. The following subsections investigate the nature of these dependencies.

The first decoding step

We start by investigating the dependencies between the outputs from the first constituent decoder and the input sequences. Since the extrinsic inputs to the first decoder are all zero, the decoder outputs are only dependent on the systematic and parity input sequences. Consider, for example, the decision variable at position i. This output is naturally dependent on the decoder inputs at position i, i.e. x_i and $y_i^{(1)}$. Furthermore, as a result of the trellis code, it is also (decreasingly) dependent on the inputs at time $i \pm 1$, $i \pm 2$, $i \pm 3$ and so on. In the following, these dependencies are investigated by the corresponding correlation coefficients. Examples of such correlation coefficients are depicted in Figure 2, showing the correlation between the systematic inputs and both a decision variable (solid) and an extrinsic output (dashed). These coefficients were empirically obtained by simulating the decoding of a recursive convolutional code with feedback and parity polynomials 15_{oct} and 17_{oct}, respectively, repeatedly transmitting the all-zero code word.

The correlation coefficients between the extrinsic outputs from the first decoder and the sequence of systematic inputs are in the following represented by the matrix $\rho_{Le,x}^{(1)}$. Element (i,j) of $\rho_{Le,x}^{(1)}$, denoted $\rho_{Le_i,x_j}^{(1)}$, is the correlation coefficient between extrinsic output i after the first decoding step and systematic input j. Note that $\rho_{Le_i,x_j}^{(1)}$ is primarily dependent on the distance between i and j, and not on their absolute values, except for edge effects near the beginning and end of the sequences.

The second decoding step

The difference between the first and second decoding step is the presence of extrinsic inputs. Output i from the second decoder is not only dependent on the sequence of systematic and parity inputs in the vicinity of position i, but also on the extrinsic inputs in the same vicinity. Each of these extrinsic inputs are in turn correlated to the channel inputs in the vicinity of its origin, before interleaving. Therefore, these extrinsic inputs provide output i from the second decoder with correlation to various parts of the systematic and parity sequences. Provided that the interleaver is suitably chosen, output i from the second decoder will thus be correlated to a wide range of channel inputs, not only to those in the vicinity of i. Intuitively, the decoding performance will be positively influenced by an increase in the number of channel inputs that affect each decoder output.

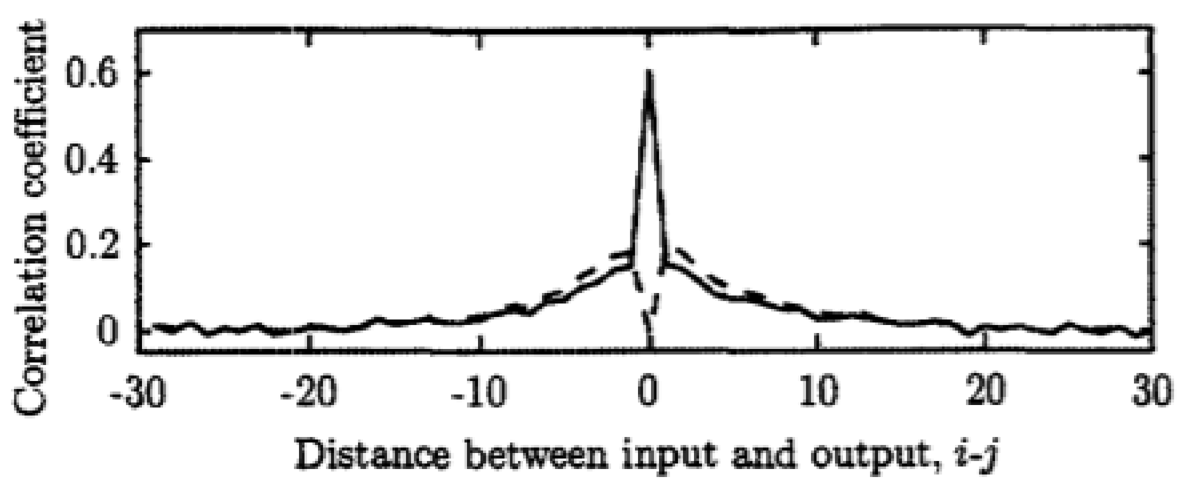

Figure 2: Correlation coefficients between the inputs and outputs of the first decoder. Solid: correlation between decision variable i and systematic bit j. Dashed: correlation between extrinsic output $Le_i^{(1)}$ and systematic bit j.

The dashed line in Figure 3 shows an example of the correlation between extrinsic output 50 from the second decoder and the entire sequence of systematic inputs, for a turbo code constructed with a 105-bit interleaver. The solid line shows the corresponding correlation coefficients between decision variable 50 and the same sequence of systematic inputs. The peak around position 92 stems from the fact that extrinsic output 92 from the first decoder, for this particular interleaver, is interleaved to position 50 in the input sequence to the second decoder.

When it comes to investigating the performance of iterative decoding, we are primarily interested in the correlation coefficients between the extrinsic outputs and the input sequences, since it is the extrinsic outputs that are passed on to the next decoder in the iterative decoder. As in the case for the first decoder, the correlation coefficients between the extrinsic output from the second decoder and the systematic input sequence are represented by the matrix $\rho_{Le,x}^{(2)}$.

Iterative decoding suitability

Above we have discussed the correlation between the outputs of the constituent decoders and the received sequences, exemplified by the correlation coefficients

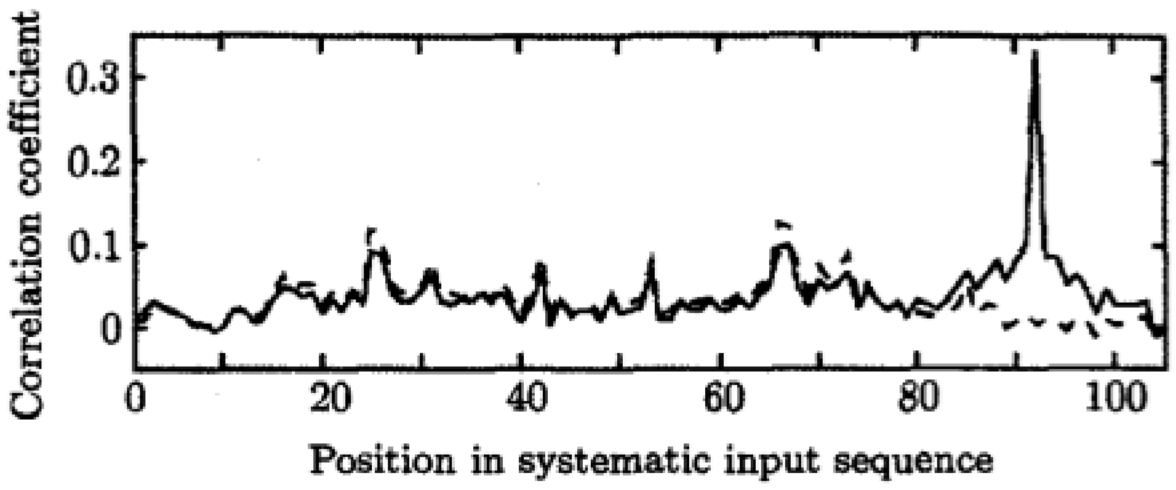

Figure 3: Correlation coefficients between the inputs and outputs of the second decoder. Solid: correlation between decision variable 50 and the sequence of systematic inputs. Dashed: correlation between extrinsic output 50 and the sequence of systematic inputs.

to the systematic sequence. These correlation coefficients after the second decoder are dependent on the specific interleaver choice; by changing the interleaver, we change the correlation properties of the extrinsic information. Especially, interleavers with short cycles[1] result in high correlation to some parts of the received sequence, and lower correlation to other parts. Intuitively, we would like to spread this correlation as evenly as possible, since an output highly correlated to a few positions is very sensitive to channel noise at these positions.

Following the above discussion, we use the standard deviation of the correlation coefficients between extrinsic outputs and the sequence of systematic inputs as a quality measure. Denoting this standard deviation for extrinsic output i with V_i, we get

$$V_i = \frac{1}{N-1}\sum_{j=1}^{N}\left(\rho^{(2)}_{Le_i,x_j} - \overline{\rho^{(2)}_{Le_i,x}}\right)^2, \qquad (1)$$

where $\overline{\rho^{(2)}_{Le_i,x}} = \frac{1}{N}\sum_{j=1}^{N}\rho^{(2)}_{Le_i,x_j}$, i.e. the average value of the correlation coefficients on row i of $\rho^{(2)}_{Le,x}$. A low value on V_i indicates a good quality of extrinsic output $Le^{(2)}_i$, since it is then evenly correlated to the sequence of systematic inputs. An interleaver well suited for iterative decoding should thus have low values on all V_i:s.

The above standard deviations indicate the quality of the extrinsic inputs to the second decoder. However, they do not reveal anything about the quality of the extrinsic information used as inputs in the third decoding step. The correlation properties of these extrinsic inputs are instead influenced by the deinterleaving rule. Therefore, the above calculations are performed for both the interleaver and the deinterleaver[2]. The corresponding correlation matrix for the deinterleaver is denoted $\rho^{(2)'}_{Le,x}$, which is substituted for $\rho^{(2)}_{Le,x}$ in (1) to calculate the standard deviations V'_i. We now have two sets of measures, the V_i:s and the V'_i:s, representing the quality of the extrinsic inputs to each of the two constituent decoders. To obtain a single measure on the iterative decoding suitability for an interleaver, we calculate the average of all the V_i:s and V'_i:s[3]. The *iterative decoding suitability* (IDS) measure is thus achieved as

$$\text{IDS} = \frac{1}{2N}\left(\sum_{i=1}^{N} V_i + \sum_{i=1}^{N} V'_i\right). \qquad (2)$$

Analytical approximation of correlation coefficients

It is not straightforward to derive analytical expressions of the correlation matrices $\rho^{(2)}_{Le,x}$ and $\rho^{(2)'}_{Le,x}$, which are needed to form the iterative decoding suitability measure. However, we have investigated a simple model in which the desired coefficients are expressed as a linear combination of the correlation after the first decoder, $\rho^{(1)}_{Le,x}$. The correlation coefficients after the first decoder are in turn approximated by an exponentially decaying function given by

$$\hat{\rho}^{(1)}_{Le_i,x_j} = \begin{cases} ae^{-c|j-i|} & \text{if } j \neq i, \\ 0 & \text{otherwise.} \end{cases} \qquad (3)$$

The constants a and c are adjusted so that the height and rate of decay of the approximation matches the empirically found coefficients. Figure 4a shows the approximated values (dotted line) with $a = 0.23$ and $c = 0.18$, together with the empirically found coefficients (solid line). The correlation coefficients between the extrinsic outputs of the second decoder and the received sequence x is in turn approximated as

$$\hat{\rho}^{(2)}_{Le,x} = \frac{1}{2}\hat{\rho}^{(1)}_{Le,x}P\left(I + \hat{\rho}^{(1)}_{Le,x}\right), \qquad (4)$$

where P is a permutation matrix representing the interleaver. The interleaved sequence is obtained by multiplying the original sequence by P, i.e. as

[1] A short cycle occurs when two bits in a sequence are close to each other both before and after interleaving.

[2] A block interleaver with a large unbalance in the number of rows and columns is an example of an interleaver that result in large differences in the quality of the extrinsic inputs to the constituent decoders.

[3] In in earlier investigation [4], IDS was defined as $\max(\overline{V}, \overline{V'})$. Later, we have found that the average is more conceptually appealing.

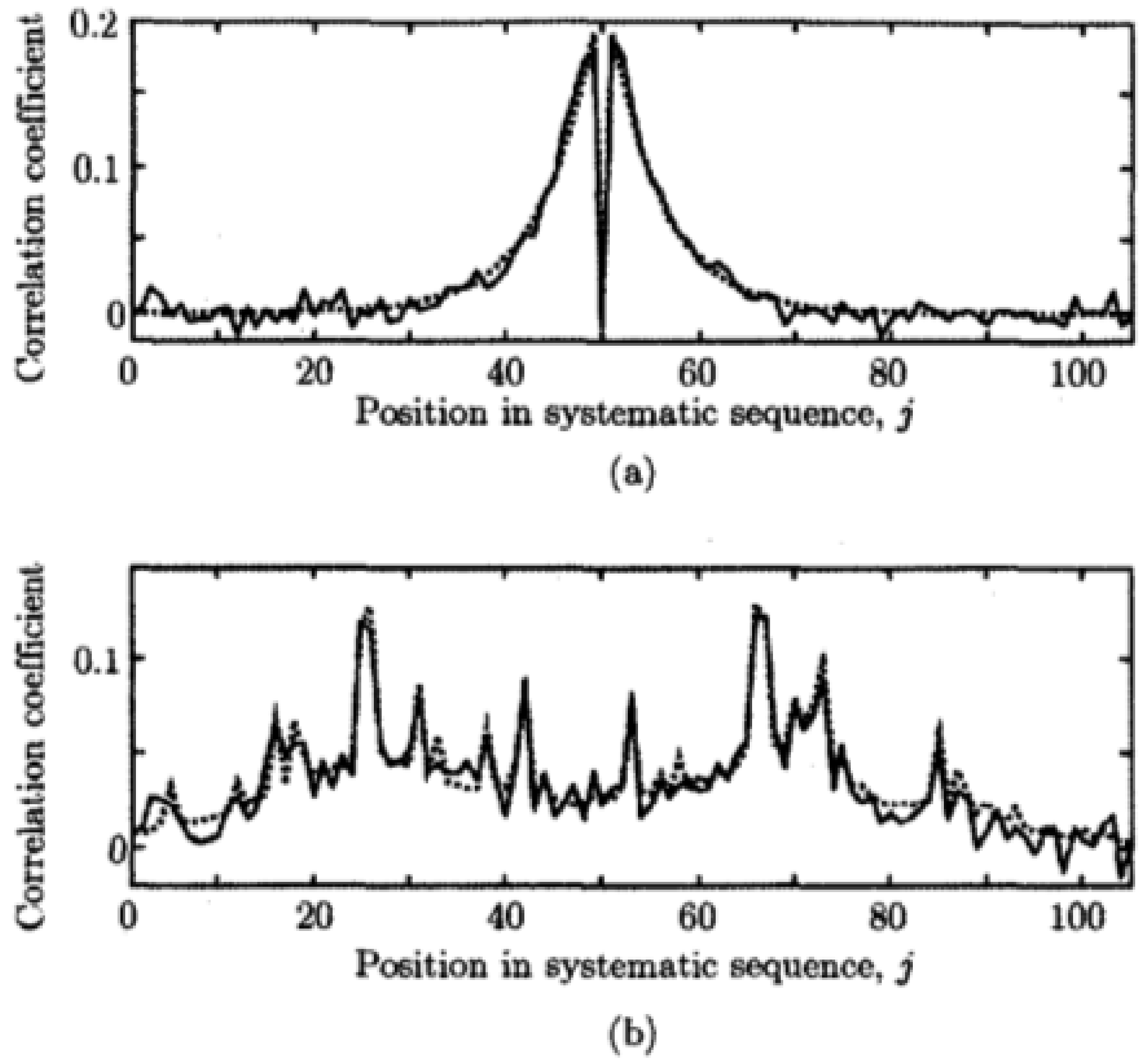

Figure 4: Comparison of approximated (dotted) and empirically found (solid) correlation coefficients between extrinsic output 50 and the sequence of systematic inputs, after a) the first decoder and b) the second decoder.

xP. The two terms in (4) stems from the two input sequences to the second decoder that result in correlation between the new extrinsic outputs and the systematic inputs, i.e. the systematic and extrinsic input sequences. The relevance of (4) is evaluated by comparing the approximated values with empirically found correlation coefficients, both shown in Figure 4b. The plot depicts the correlation coefficients between extrinsic output 50 from the second decoder and the sequence of systematic inputs. The proposed approximation is naturally not an exact description of the decoding process, but it captures the main features of the empirically found correlation coefficients.

III. EVALUATION OF IDS

To evaluate the iterative decoding suitability measure we compared simulated error rate performances (on an AWGN channel) of a large number of pseudo-random interleavers to their IDS values. In addition, truncated union bounds on the frame-error rate probability for each interleaver was calculated. Two interleaver sizes were used, 105 and 500 bits.

Figure 5 shows a scatter plot of the truncated union bounds[4] and IDS values for 200 105-bit interleavers.

[4] The bound is truncated after Hamming distance 22.

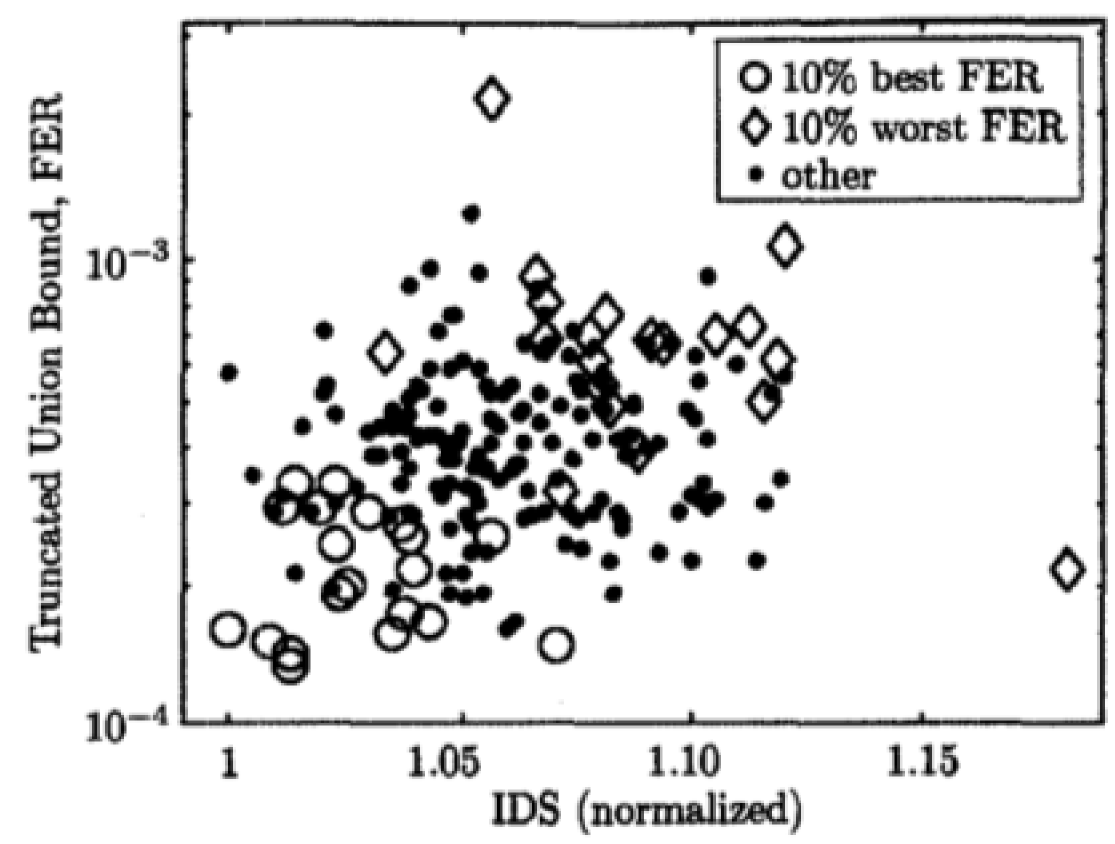

Figure 5: Scatter plot of the IDS values and truncated Union Bounds for 200 105-bit pseudo-random interleavers. The truncated union bound and the simulations are performed at $E_b/N_0 = 2.5$ dB. The 10% best and worst performing interleavers, during simulation, are marked with rings and diamonds respectively.

The IDS values are normalized with the lowest found value, since the IDS is a relative comparison. It is observed that the best performing interleavers have low values on *both* the IDS measure *and* the truncated union bound. It is further noted that interleavers that have a low value on one measure but high on the other, perform, in this case, among the 10% worst performing interleavers. This result stress the fact that both the distance spectrum of a turbo code *and* its suitability to be iteratively decoded are important issues when it comes to ranking the performance of interleavers used in turbo codes. The frame-error rates of the 200 turbo codes range from $1 \cdot 10^{-3}$ to $4 \cdot 10^{-3}$ at a signal-to-noise ratio of 2.5 dB, after 20 decoding iterations. The constituent encoders have generator polynomials $(1, 17/15)_{oct}$.

The convergence rate of the iterative decoding is also affected by the correlation properties of the extrinsic information. Figure 6 shows a scatter plot of the normalized IDS values and the required number of decoding iterations for a frame-error rate of $5 \cdot 10^{-3}$.

As pseudo-random interleavers increase in size, the variation in their IDS values tend to diminish. Therefore, the IDS measure is mostly suitable for small pseudo-random interleavers, in the range of 100 to 500 bits. The corresponding scatter plots in Figures 5 and 6 for the 500-bit pseudo-random interleavers are similar to the ones shown for the 105-bit

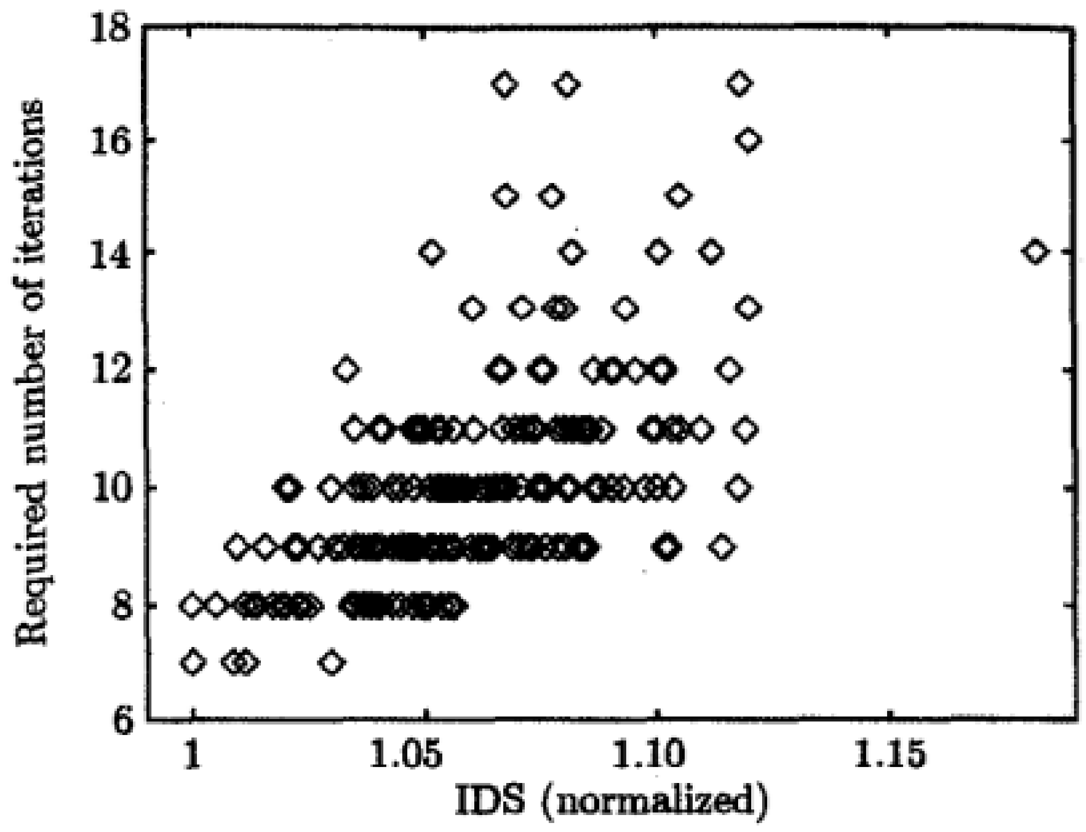

Figure 6: Scatter plot of the IDS values and the required number of decoding iterations for a frame-error rate of $5 \cdot 10^{-3}$, for 200 105-bit pseudo-random interleavers.

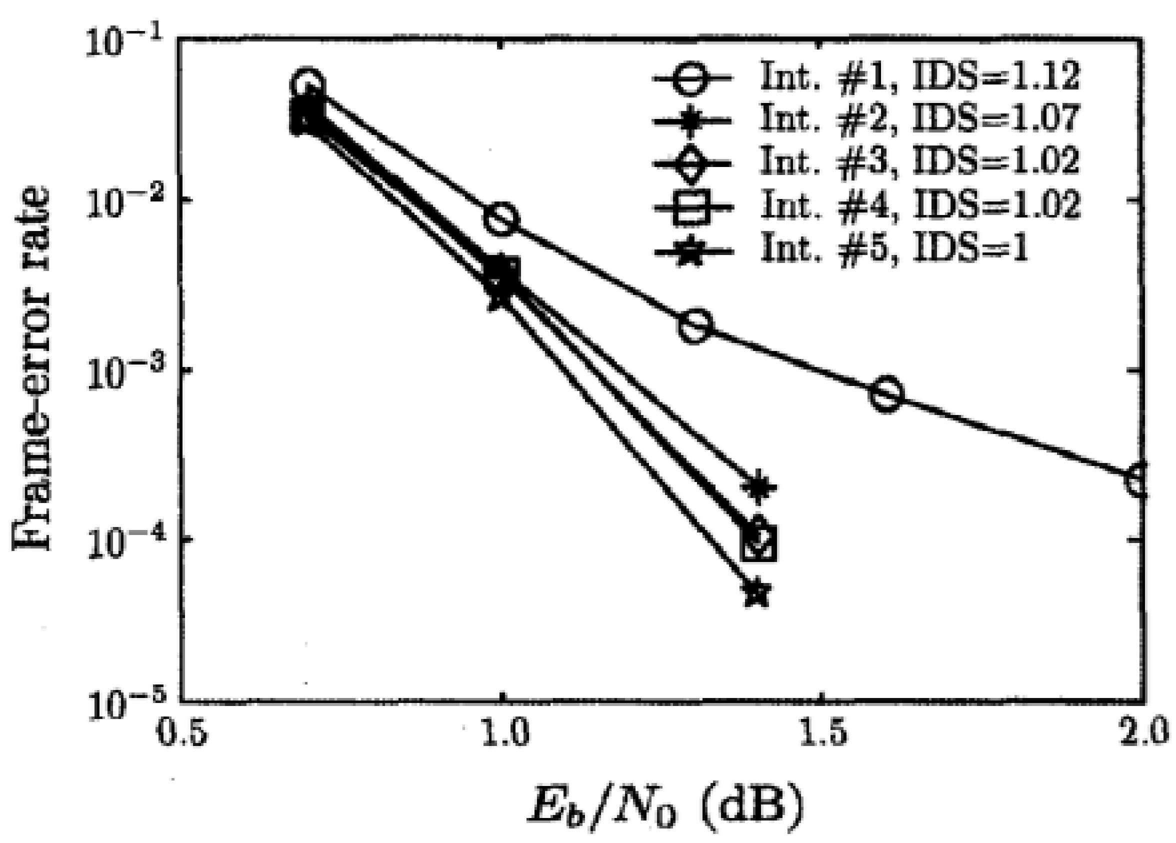

Figure 7: Simulated frame-error rates of designed 640-bit interleavers, and their IDS values.

interleavers, but with a lower significance of the IDS measure.

Figure 7 shows the frame-error rate performances and IDS values of four designed 640-bit interleavers evaluated for the UMTS standardization [5] (Int. #1 is a pseudo-random interleaver, the others are designed). Also in this case, there is a correspondence between low IDS values and good error correcting performances. The turbo codes in this example use 8-state encoders and 8 decoding iterations.

IV. CONCLUSIONS

The correlation properties of the extrinsic information in an iterative decoder have been studied. These correlation properties depends on the particular choice of interleaver, which therefore influence the performance of iterative decoding. An *iterative decoding suitability* measure was presented and investigated, intended as a complement to the distance spectrum when ranking interleaver performances. Simulation results indicate that the IDS is related to both the convergence rate and final frame-error rate of turbo codes. It is given in a closed form expression and requires no simulations which makes it useful for interleaver design and ranking.

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