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# ESTIMATING THE COMPUTATIONAL CUT-OFF RATE FOR THE GILBERT-ELLIOT CHANNEL WITH SHORT INTERLEAVING

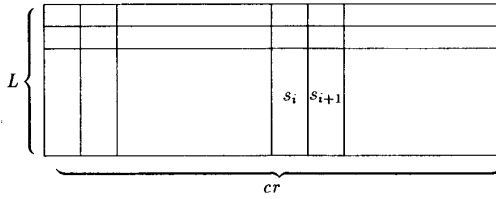
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**Abstract**—In this paper we estimate the computational cut-off rate for sequential decoding when used together with short interleaving to communicate over the Gilbert-Elliot channel.

Although the curse of sequential decoding is that its computational performance deteriorates drastically when errors occur in clusters it has been shown [1] that it is feasible to use sequential decoding together with a short interleaver to exploit the memory of the Gilbert-Elliot channel. In this paper we estimate the computational cut-off rate  $R_{\text{comp}}$  for our sequential decoder.

The Gilbert-Elliot channel model has two states: 'Good' ( $G$ ) and 'Bad' ( $B$ ). Let  $P$  and  $Q$  denote the transition probabilities  $G \rightarrow B$  and  $B \rightarrow G$ , respectively. The binary channel crossover probability is  $\varepsilon$  in  $G$  and  $\frac{1}{2}$  in  $B$ . A rate  $R = b/c$  convolutional encoder is followed by an  $L \times cr$  interleaver:



The code symbols are written rowwise into the interleaver and read columnwise. After transmission over the Gilbert-Elliot channel and deinterleaving we use a stack algorithm-like sequential decoder. Suppose that the  $i$ th symbol read from the deinterleaver corresponds to state  $s_i$  and that the  $(i+1)$ th symbol corresponds to state  $s_{i+1}$ . The conditional probabilities  $P(s_{i+1} | s_i)$ , where  $s_i, s_{i+1} \in \{G, B\}$ , are given by the following

**Lemma 1** Let  $\Phi = P(B | G) = 1 - P(G | G)$  and  $\Psi = P(G | B) = 1 - P(B | B)$ . The transition probabilities  $\Phi \stackrel{\text{def}}{=} \Phi_L$  and  $\Psi \stackrel{\text{def}}{=} \Psi_L$  are the solutions for  $l=L$  of

$$\begin{aligned} \Phi_l &= (1 - \Psi_{l-1})P + \Phi_{l-1}(1 - P) \\ \Psi_l &= (1 - \Phi_{l-1})Q + \Psi_{l-1}(1 - Q), \end{aligned}$$

where  $l = 2, 3, \dots, L$  and  $\Phi_1 = P, \Psi_1 = Q$ . □

Our sequential decoder should estimate the channel state corresponding to each received symbol. In the ideal case it knows exactly this state (optimistic approach); in worst case it knows nothing about the state (pessimistic approach). We estimate the computational cut-off rate in both cases, viz.,  $R_{\text{comp}}^{(o)}$  and  $R_{\text{comp}}^{(p)}$ , respectively. Clearly,  $R_{\text{comp}}^{(p)} < R_{\text{comp}}^{(o)}$ .

The optimal metric increments in the optimistic case does not depend directly from  $\Phi$  and  $\Psi$ :

| State transition  | Received symbol equal to code symbol? | Metric increment                    |
|-------------------|---------------------------------------|-------------------------------------|
| $G \rightarrow G$ | Yes                                   | $a_G = \log 2(1 - \varepsilon) - R$ |
| $G \rightarrow G$ | No                                    | $c_G = \log 2\varepsilon - R$       |
| $G \rightarrow B$ | -                                     | $b_G = -R$                          |
| $B \rightarrow B$ | -                                     | $a_B = -R$                          |
| $B \rightarrow G$ | Yes                                   | $b_B = \log 2(1 - \varepsilon) - R$ |
| $B \rightarrow G$ | No                                    | $c_B = \log 2\varepsilon - R$       |

Then we have

**Lemma 2** Let  $\mu$  denote the cumulative metric along the correct path. In the optimistic case we have  $P(\eta \leq x) \leq A2^{-hx}$ , where  $A > 1$  does not depend on  $x$  and  $h$  is the smallest root of

$$\det \begin{pmatrix} ((1 - \Phi)(1 - \varepsilon)2^{ha_G} + (1 - \Phi)\varepsilon 2^{hc_G} - 1) & \Phi 2^{hb_G} \\ (\Psi(1 - \varepsilon)2^{hb_B} + \Psi\varepsilon 2^{hc_B}) & ((1 - \Psi)2^{ha_B} - 1) \end{pmatrix} = 1. \quad \square$$

We can prove that  $R_{\text{comp}}^{(o)}$  corresponds to the root  $h = -1/2$ .

**Theorem 3**

$$R_{\text{comp}}^{(o)} = 2 \log \frac{(1 - \Phi)2^{-\frac{R_0}{2}} + 1 - \Psi - \sqrt{((1 - \Phi)2^{-\frac{R_0}{2}} - 1 + \Psi)^2 + 4\Phi\Psi 2^{-\frac{R_0}{2}}}}{2(1 - \Phi - \Psi)2^{-\frac{R_0}{2}}},$$

where  $R_0 = 1 - \log(1 + 2\sqrt{\varepsilon(1 - \varepsilon)})$ . □

In the pessimistic case we have the following metric increments:

| State transition (hypothetical) | Received symbol equal to code symbol? | Metric increment                              |
|---------------------------------|---------------------------------------|---|
| $G \rightarrow G$               | Yes                                   | $a_G = \log 2(1 - \Phi)(1 - \varepsilon) - R$ |
| $G \rightarrow G$               | No                                    | $c_G = \log 2(1 - \Phi)\varepsilon - R$       |
| $G \rightarrow B$               | -                                     | $b_G = \log \Phi - R$                         |
| $B \rightarrow B$               | -                                     | $a_B = \log(1 - \Psi) - R$                    |
| $B \rightarrow G$               | Yes                                   | $b_B = \log 2\Psi(1 - \varepsilon) - R$       |
| $B \rightarrow G$               | No                                    | $c_B = \log 2\Psi\varepsilon - R$             |

If we use these metric increments Lemma 2 is valid also in the pessimistic case. Finally, we have

**Theorem 4**

$$R_{\text{comp}}^{(p)} = 2 \log \left( (1 - \Phi)^{\frac{1}{2}} 2^{-\frac{R_0}{2}} + (1 - \Psi)^{\frac{1}{2}} - \sqrt{((1 - \Phi)^{\frac{1}{2}} 2^{-\frac{R_0}{2}} + (1 - \Psi)^{\frac{1}{2}})^2 - 4((1 - \Phi)^{\frac{1}{2}}(1 - \Psi)^{\frac{1}{2}} - \Phi^{\frac{1}{2}}\Psi^{\frac{1}{2}})2^{-\frac{R_0}{2}}} \right) - 2 \log \left( 2((1 - \Phi)^{\frac{1}{2}}(1 - \Psi)^{\frac{1}{2}} - \Phi^{\frac{1}{2}}\Psi^{\frac{1}{2}})2^{-\frac{R_0}{2}} \right),$$

where  $R_0 = 1 - \log(1 + 2\sqrt{\varepsilon(1 - \varepsilon)})$ . □

## References

- [1] G. Bratt, R. Johannesson, and K. Sh. Zigangirov, "On sequential decoding for the Gilbert-Elliot channel", presented at the IEEE International Symposium on Information Theory, June 19-24, 1988, Kobe, Japan.