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ESTIMATING THE COMPUTATIONAL CUT-OFF RATE FOR THE GILBERT-ELLIOT CHANNEL WITH SHORT INTERLEAVING

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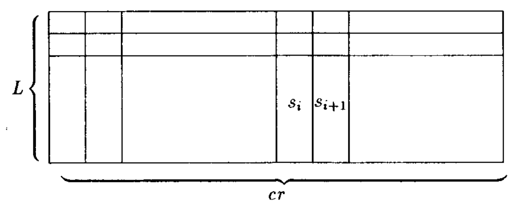
Abstract—In this paper we estimate the computational cut-off rate for sequential decoding when used together with short interleaving to communicate over the Gilbert-Elliot channel.

Although the curse of sequential decoding is that its computational performance deteriorates drastically when errors occur in clusters it has been shown [1] that it is feasible to use sequential decoding together with a short interleaver to exploit the memory of the Gilbert-Elliot channel. In this paper we estimate the computational cut-off rate R_{comp} for our sequential decoder.

The Gilbert-Elliot channel model has two states: 'Good' (G) and 'Bad' (B). Let P and Q denote the transition probabilities $G \to B$ and $B \to G$, respectively. The binary channel crossover probability is ε in G and $\frac{1}{2}$ in B. A rate $R = b/c$ convolutional encoder is followed by an $L \times cr$ interleaver:



The code symbols are written rowwise into the interleaver and read columnwise. After transmission over the Gilbert-Elliot channel and deinterleaving we use a stack algorithm-like sequential decoder.

Suppose that the ith symbol read from the deinterleaver corresponds to state s_i and that the $(i+1)$th symbol corresponds to state s_{i+1}. The conditional probabilities $P(s_{i+1} \mid s_i)$, where $s_i, s_{i+1} \in \{G, B\}$, are given by the following

Lemma 1 *Let $\Phi = P(B \mid G) = 1 - P(G \mid G)$ and $\Psi = P(G \mid B) = 1 - P(B \mid B)$. The transition probabilities $\Phi \overset{\text{def}}{=} \Phi_L$ and $\Psi \overset{\text{def}}{=} \Psi_L$ are the solutions for l=L of*

$$\begin{aligned} \Phi_l &= (1 - \Psi_{l-1})P + \Phi_{l-1}(1 - P) \\ \Psi_l &= (1 - \Phi_{l-1})Q + \Psi_{l-1}(1 - Q), \end{aligned}$$

where $l = 2, 3, \ldots, L$ and $\Phi_1 = P, \Psi_1 = Q$. □

Our sequential decoder should estimate the channel state corresponding to each received symbol. In the ideal case it knows exactly this state (optimistic approach); in worst case it knows nothing about the state (pessimistic approach). We estimate the computational cut-off rate in both cases, viz., $R^{(o)}_{\text{comp}}$ and $R^{(p)}_{\text{comp}}$, respectively. Clearly, $R^{(p)}_{\text{comp}} < R^{(o)}_{\text{comp}}$.

The optimal metric increments in the optimistic case does not depend directly from Φ and Ψ:

State transition	Received symbol equal to code symbol?	Metric increment
$G \to G$	Yes	$a_G = \log 2(1 - \varepsilon) - R$
$G \to G$	No	$c_G = \log 2\varepsilon - R$
$G \to B$	-	$b_G = -R$
$B \to B$	-	$a_B = -R$
$B \to G$	Yes	$b_B = \log 2(1 - \varepsilon) - R$
$B \to G$	No	$c_B = \log 2\varepsilon - R$

Then we have

Lemma 2 *Let μ denote the cumulative metric along the correct path. In the optimistic case we have $P(\eta \le x) \le A2^{-hx}$, where $A > 1$ does not depend on x and h is the smallest root of*

$$\det\begin{pmatrix} ((1-\Phi)(1-\varepsilon)2^{ha_G} + (1-\Phi)\varepsilon 2^{hc_G} - 1) & \Phi 2^{hb_G} \\ (\Psi(1-\varepsilon)2^{hb_B} + \Psi\varepsilon 2^{hc_B}) & ((1-\Psi)2^{ha_B} - 1) \end{pmatrix} = 1.$$

□

We can prove that $R^{(o)}_{\text{comp}}$ corresponds to the root $h = -1/2$.

Theorem 3

$$R^{(o)}_{\text{comp}} = 2\log\frac{(1-\Phi)2^{-\frac{R_0}{2}} + 1 - \Psi - \sqrt{((1-\Phi)2^{-\frac{R_0}{2}} - 1 + \Psi)^2 + 4\Phi\Psi 2^{-\frac{R_0}{2}}}}{2(1 - \Phi - \Psi)2^{-\frac{R_0}{2}}},$$

where $R_0 = 1 - \log(1 + 2\sqrt{\varepsilon(1-\varepsilon)})$. □

In the pessimistic case we have the following metric increments:

State transition (hypothetical)	Received symbol equal to code symbol?	Metric increment
$G \to G$	Yes	$a_G = \log 2(1 - \Phi)(1 - \varepsilon) - R$
$G \to G$	No	$c_G = \log 2(1 - \Phi)\varepsilon - R$
$G \to B$	-	$b_G = \log \Phi - R$
$B \to B$	-	$a_B = \log(1 - \Psi) - R$
$B \to G$	Yes	$b_B = \log 2\Psi(1 - \varepsilon) - R$
$B \to G$	No	$c_B = \log 2\Psi\varepsilon - R$

If we use these metric increments Lemma 2 is valid also in the pessimistic case. Finally, we have

Theorem 4

$$\begin{aligned} R^{(p)}_{\text{comp}} &= 2\log((1-\Phi)^{\frac{1}{2}}2^{-\frac{R_0}{2}} + (1-\Psi)^{\frac{1}{2}} \\ &- \sqrt{((1-\Phi)^{\frac{1}{2}}2^{-\frac{R_0}{2}} + (1-\Psi)^{\frac{1}{2}})^2 - 4((1-\Phi)^{\frac{1}{2}}(1-\Psi)^{\frac{1}{2}} - \Phi^{\frac{1}{2}}\Psi^{\frac{1}{2}})2^{-\frac{R_0}{2}}}) \\ &- 2\log(2((1-\Phi)^{\frac{1}{2}}(1-\Psi)^{\frac{1}{2}} - \Phi^{\frac{1}{2}}\Psi^{\frac{1}{2}})2^{-\frac{R_0}{2}}), \end{aligned}$$

where $R_0 = 1 - \log(1 + 2\sqrt{\varepsilon(1-\varepsilon)})$. □

References

[1] G. Bratt, R. Johannesson, and K. Sh. Zigangirov, "On sequential decoding for the Gilbert-Elliot channel", presented at the IEEE International Symposium on Information Theory, June 19–24, 1988, Kobe, Japan.