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Nonequivalent Cascaded Convolutional Codes Obtained from Equivalent Constituent Convolutional Encoders[1]

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Abstract — **Cascaded convolutional codes with conventional convolutional codes as constituent codes are powerful and attractive to use in communication systems where very low error probabilities are needed. This paper clearly demonstrates the dramatic effect the replacement of the inner convolutional encoder by an equivalent one could have on the cascaded convolutional code.**

I. Introduction

The cascade of two convolutional encoders without an interleaver but with matched rates is a cascade of a rate $R_o = b_o/c_o$ outer encoder of memory m_o and a rate $R_i = b_i/c_i$ inner encoder of memory m_i, where $b_i = c_o$. The *cascaded convolutional code* C_c is encoded by the rate $R_c = R_o R_i = b_o/c_i \stackrel{\text{def}}{=} b_c/c_c$ convolutional encoder of memory $m_c \leq m_o + m_i$.

Let the information sequence $\boldsymbol{u}^c(D)$ of the cascaded rational encoder $G^c(D)$ be encoded as the cascaded codeword $\boldsymbol{v}^c(D)$,

$$\boldsymbol{v}^c(D) = \boldsymbol{u}^c(D)G^c(D) = \boldsymbol{u}^c(D)G^o(D)G^i(D), \qquad (1)$$

where $G^o(D)$ and $G^i(D)$ are the rational generator matrices of the outer and inner encoders, respectively.

II. Nonequivalent Cascaded Encoders from Equivalent Constituent Encoders

Let

$$G^{c\prime}(D) = G^{o\prime}(D)G^{i\prime}(D) \qquad (2)$$

be the generator matrix of the cascaded convolutional code, where $G^{o\prime}(D) = T^o(D)G^o(D)$ and $G^{i\prime}(D) = T^i(D)G^i(D)$. We show that $G^{c\prime}(D)$ and $G^c(D) = G^o(D)G^i(D)$ are, in general, not equivalent.

If only the outer generator matrix $G^o(D)$ is replaced by an equivalent generator matrix $G^{o\prime}(D)$, then the new cascaded generator matrix $G^{c\prime}(D)$ will be equivalent to the cascaded generator matrix $G^c(D)$.

Since it is the code sequences from the outer encoder that serve as information sequences for the inner encoder, the cascaded convolutional code is a proper (assuming $R_o < 1$) subset of the inner convolutional code, $\mathcal{C}^c \subset \mathcal{C}^i$. Replacing the inner encoder with an equivalent inner encoder changes the mapping from the inner information sequences to the inner code sequences and, therefore, also the subset of the inner convolutional code. In general, we will obtain a different cascaded convolutional code when we replace the inner encoder by an equivalent one. This fact will be illustrated by examples. We also notice that since $\mathcal{C}^c \subset \mathcal{C}^i$, catastrophicity of the inner generator matrix does not imply catastrophicity of the generator matrix for the cascade.

Can the inner generator matrix be chosen such that we obtain the same cascaded convolutional code, i.e., such that $G^{c\prime}(D)$ is equivalent to $G^c(D)$? Indeed it can, which will be illustrated by a simple example. If $G^{c\prime}(D)$ and $G^c(D)$ are equivalent generator matrices, then for some invertible $b_o \times b_o$ matrix $S(D)$

$$G^{c\prime}(D) = G^o(D)T(D)G^i(D) = S(D)G^c(D). \qquad (3)$$

III. Systematic Cascaded Encoders from Systematic Constituent Encoders

From basic encoding matrices we can easily obtain equivalent rational systematic encoding matrices,

$$G^c_{\text{sys}}(D) = G^o_{\text{sys}}(D)G^i_{\text{sys}}(D), \qquad (4)$$

where $G^o_{\text{sys}}(D)$, and $G^i_{\text{sys}}(D)$ are systematic equivalents to $G^o(D)$ and $G^i(D)$, respectively. In general, the systematic encoding matrix $G^c_{\text{sys}}(D)$ is not equivalent to $G^c(D) = G^o(D)G^i(D)$.

If $G^o(D)$ and $G^i(D)$ are minimal, then a minimal realization of $G^c_{\text{sys}}(D)$ has at most the same number of delay elements as the cascaded encoder $G^c(D)$ has when realized as a cascade of the two minimal encoders $G^o(D)$ and $G^i(D)$.

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[2] S. Höst, R. Johannesson, and V. V. Zyablov, "Nonequivalent Cascaded Convolutional Codes Obtained from Equivalent Constituent Convolutional Encoders", submitted to *Problemy Peredachi Inform.*