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# EVENT-BASED CONTROL OVER NETWORKS: SOME RESEARCH QUESTIONS AND PRELIMINARY RESULTS

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## Abstract

The paper discusses some research questions related to event-based control over networks and presents preliminary results regarding event-based minimum-variance control of first-order systems with specified minimum inter-event times.

## 1. INTRODUCTION

Modern control systems are often built from several smaller, standardized units that are interconnected by a communication network. Local sensing and control is typically performed by mechatronic devices that have embedded micro-controllers, while the overall system of actuators, sensors and control nodes is connected by a wired or wireless network. In these systems, the network bandwidth tends to be the bottleneck resource that limits the application performance. Hence, ways to utilize the bandwidth as efficiently as possible need to be researched.

One promising approach to more efficient bandwidth usage in networked control loops is event-based control. The basic idea is to transmit sensor and command data only when needed. This should be contrasted to traditional sampled-data control (Åström and Wittenmark, 1997), where information is transmitted at regular intervals, regardless of whether anything significant has happened since last time.

Event-based control as a technology is of course not new. It has been used for a long time in such diverse areas as engine control (Hendricks

*et al.*, 1994), robot path planning (Tarn *et al.*, 1996), and control of industrial processes (Kwon *et al.*, 1999). Mostly, however, it has been applied in an ad-hoc way. This can be attributed to the lack of a comprehensive theory, which in turn can be explained by the mathematical difficulties involved. In particular, event-based control schemes always lead to nonlinear system descriptions.

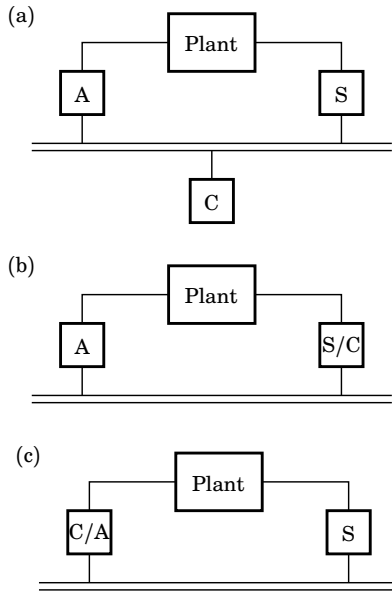
In (Åström and Bernhardsson, 1999) event-based minimum-variance control of first-order systems was studied. It was shown that the output variance could be significantly reduced (by two-thirds for an integrator process) by using event-based rather than periodic control. This shows that event-based sampling has a great potential for decreasing the bandwidth requirements of networked controllers.

In this paper, we outline some of the research questions that arise in the area of event-based control over networks (Section 2). We also study event-based minimum-variance controllers with minimum inter-sample times and give some preliminary results (Section 3).

## 2. RESEARCH QUESTIONS

We here raise a number of issues that deserve further investigation in the area of event-based control over networks.

**What are the possible topologies?** In networked control systems, the sensors (S), actuators (A), and controllers (C) may



**Fig. 1** Some possible topologies for networked SISO control loops.

reside on various nodes in the network. Even for SISO control loops, many different topologies can be imagined, see Fig. 1. The controller could reside on a separate node or be co-located with either the sensor or the actuator. The sensor and actuator nodes could be very simplistic or contain sophisticated observers, depending on how much processing power is available. Hence, the processing power of each node could also be considered to be part of the topology. Further, one might ask under what conditions the topologies in Fig. 1 are equivalent.

**How to define the events?** A key question in event-based control is how the events should be generated. For first-order systems, setting thresholds on the output as in (Åström and Bernhardsson, 1999) may be appropriate. For higher-order systems, thresholds on the state vector will be both hard to define and to monitor. Is there a way to define the state thresholds optimally? Further, care must be taken that events are not generated arbitrarily often.

**How to handle the servo problem?** For event-based control systems, is it possible to combine feedforward reference tracking with feedback disturbance rejection in the same straightforward way as for periodic control systems?

**Sensitivity towards delay and jitter?** If not carefully designed, event-based schemes may actually be more sensitive towards delays than periodic schemes. An unexpected delay between an event-based

sensor and an actuator may render the system in state where no more events will be generated. On the other hand, event-based actuators (so called generalized hold circuits) are known to be able to increase the robustness towards jitter in networked control loops (Sala, 2005).

**What information to transmit?** This question is related to that of which events to generate. Often, more information than the event itself could be transmitted. For instance, a sensor node could use fast sampling of a noisy measurement signal to get a good estimate of the plant state vector, and then send this estimate once an event is generated. For sensor nodes with less computational power, it may be reasonable to instead transmit a vector of recorded measurement values to the controller. Similarly, the controller might transmit a vector of precomputed control actions to a simple actuator node.

**Implications from scheduling theory?** In scheduling theory, the utilization factor that an aperiodic activity exerts on a resource is computed as  $U = C/T$ , where  $C$  is the worst-case usage time, and  $T$  is the minimum inter-event time. By this definition, the event-based controller designed in (Åström and Bernhardsson, 1999) has infinite utilization and cannot be scheduled on any network. Similarly, event-based transmissions increase the release jitter in the receiving nodes, increasing the CPU utilization factor as well. Hence, for practical applications, it is necessary to put a lower bound on the inter-event times. (The question of how this bound can be selected and how it affects the performance is investigated in the next section.)

**How to design event-based observers?** Having access to fewer (on average) and irregularly spaced measurement and command signals, another key question in event-based control is how to design good observers. Optimal event-based observers were explored in the Master's thesis (Henningsson, 2005). In general, obtaining the optimal state estimate (expressed as a probability density function) involves solving a set of nonlinear PDEs. The thesis proposes ways design suboptimal observers by the use of logarithmic concave functions.

### 3. PRELIMINARY RESULTS

In (Åström and Bernhardsson, 1999) periodic sampling and event-based sampling are compared for control of first-order stochastic systems. Event-based control is realized by applying an impulse control action whenever the magnitude of the system state exceeds a certain threshold. The event-based scheme is found to yield only a fraction of the quadratic cost as compared to periodic sampling, with the same mean time between events.

In many instances it is not reasonable to assume that two events may occur in an arbitrarily short time. Here, we investigate the situation when there is a specified minimum inter-arrival time between the events.

#### 3.1 Control Problem

Consider the first-order system described by the stochastic differential equation

$$dx = ax dt + du + \sigma dw,$$

where  $x$  is the state,  $u$  the control signal,  $w$  is a Wiener process with  $E(dw) = 0$ ,  $E(dw^2) = dt$ ,  $a$  is the pole of the system and  $\sigma$  is the intensity of the process noise.

The state is assumed to be available to the controller at all times. The control signal  $u$  is zero, except at events where it is allowed to be a Dirac pulse of any magnitude. After an event, there must be a time delay of at least  $T$  before the next event.

Given the quadratic cost function

$$J(t_0, t_1) = E \left( \int_{t_0}^{t_1} c(x(t)) dt + q_e N_e(t_0, t_1) \right),$$

where  $c(x) = qx^2$  is the state-dependent cost,  $N_e(t_0, t_1)$  is the number of events in the interval  $(t_0, t_1)$ , and  $q$  and  $q_e$  are weighting factors. It is desired to find a causal control strategy that minimizes

$$\lambda = \lim_{t \rightarrow \infty} \frac{J(0, t)}{t}$$

subject to the minimum inter-event time  $T$ .

For problem described above, it is easy to see that the optimal controller must satisfy the following:

- At any event,  $u$  is chosen to bring  $x$  to the origin.
- When the time elapsed since the last event is less than  $T$ , the controller is in the *inactive state* and no event is generated.

- When the time elapsed since the last event is greater than  $T$ , the controller is in the *active state*. Whether to generate an event or not is decided as a function of  $|x|$ .
- If in the active state an event should be triggered when  $x = x_1$ , it should also be triggered whenever  $|x| \geq |x_1|$ .

Thus, the only parameter left to specify the optimal controller is the threshold  $r$ , such that an event is triggered whenever the controller is in the active state and  $|x| \geq r$ . The threshold should be chosen to minimize  $\lambda$ .

#### 3.2 Performance as a Function of Threshold

To find the optimal threshold  $r$ , the closed-loop system will be characterized as a function of  $r$ . Introduce the storage function  $V(x)$  such that

$$E(c(x) dt + dV(x)) = \lambda dt, \quad (1)$$

when the controller is in the active state and that

$$E \left( \int_{t_0}^{t_0+T} c(x) dt + V(x(t_0+T)) - V(x^-(t_0)) \right) + q_e = \lambda T \quad (2)$$

when there is an event at time  $t_0$ .

It follows from (1) that

$$\begin{aligned} \lambda &= c(x)dt + V'(x)E(dx) + \frac{1}{2}V''(x)E(dx^2) \\ &= c(x)dt + axV'(x)dt + \frac{1}{2}\sigma^2V''(x)dt \end{aligned}$$

so that

$$\frac{1}{2}\sigma^2V''(x) + axV'(x) + c(x) - \lambda = 0.$$

The solution of the equation

$$\frac{1}{2}\sigma^2f''(x) + axf'(x) = \kappa(x) \quad (3)$$

with  $f(0) = f'(0) = 0$  can be found from

$$\frac{1}{2}\sigma^2f'(x) = e^{-\frac{a}{\sigma^2}x^2} \int_0^x e^{\frac{a}{\sigma^2}y^2} \kappa(y) dy.$$

The storage function can now be written as

$$V(x) = \lambda V_\lambda(x) + V_c(x),$$

where  $V_\lambda$  is found by inserting  $\kappa(x) = 1$  in (3) and  $V_c$  by inserting  $\kappa(x) = -c(x)$ .

To apply (2) the following partial results are needed. The expected state cost during one period of inactive state is

$$J_T = E \left( \int_{t_0}^{t_0+T} c(x) dt \mid x(t_0) = 0, u(t) = 0 \right) \\ = q\sigma^2 \frac{e^{2aT} - (1 + 2aT)}{4a^2}.$$

After one period of inactive state  $x$  has a Gaussian distribution with zero mean and variance

$$V_T = E(x(t_0 + T)^2 \mid x(t_0) = 0, u(t) = 0) \\ = \sigma^2 \frac{e^{2aT} - 1}{2a}.$$

Let  $\varphi(x)$  be the Gaussian probability density with zero mean and variance  $V_T$ . Then (2) can be written as

$$\lambda T = J_T + \int \varphi(x) V(x) dx - V(r) + q_e,$$

or

$$\lambda \left( T - \int_{-r}^r \varphi(x) (V_\lambda(x) - V_\lambda(r)) dx \right) \\ = J_T + q_e + \int_{-r}^r \varphi(x) (V_c(x) - V_c(r)) dx, \quad (4)$$

from which  $\lambda$  can be solved for.

The optimal controller can be found by minimizing  $\lambda$  as a function of  $r$ . Although the equation (4) may seem cumbersome, the integrals can be easily and closely approximated by substituting 10–30 degree series expansions of the Gaussian functions involved.

### 3.3 Probability Distribution of the State

The probability density  $f(x)$  of the state in the active mode can be obtained from the diffusion equation

$$\frac{1}{2} \sigma^2 f''(x) - ax f'(x) - af(x) + \varphi(x)/T_m = 0,$$

where  $T_m$  is the mean time between events and  $f(x)$  is normalized so that  $\int f(x) dx$  is the probability that the controller is in the active state.

The solution is

$$\frac{1}{2} \sigma^2 f(x) = \frac{1}{T_m} e^{\frac{ax^2}{\sigma^2}} \int_{-r}^x e^{-\frac{ay^2}{\sigma^2}} \int_y^0 \varphi(z) dz dy.$$

The mean time between events can now be solved for from the fact that

$$T_m = T + T_m \int f(x) dx.$$

### 3.4 Results for First-Order Systems

All results are presented for the case  $T = 1$ ,  $\sigma = 1$ ,  $q = 3$ ,  $q_e = 0$ . With this choice  $r^2 = \lambda$  for the integrator case. Except for  $q_e$ , the results apply for arbitrary values of the constants when scaled properly.

*The Integrator Case.* In the integrator case ( $a = 0$ ) the computations are considerably simplified since all Gaussian functions except  $\varphi(x)$  become unity. The results when  $|aT|$  is small (i.e. reasonably fast sampling) are similar to the integrator case.

Fig. 2 shows  $\lambda$  as a function of  $r$ . Periodic sampling corresponds to  $r = 0$ . The loss rate  $\lambda$  has a minimum for some  $r > 0$  and then grows toward infinity (since the process is not asymptotically stable).

The storage function  $V(x)$  is composed of two parts, a constant value for  $|x| \geq r$  and a varying part for  $|x| < r$ . The boundary is where an event is triggered when the controller is in the active mode.

Reasonably  $V(x)$  must be nondecreasing at  $x = r$ , for as soon as  $V(x) \geq V(r)$  the optimal controller would trigger an event, yielding the storage cost  $V(r)$ . Thus  $V'(r)$  must be  $\geq 0$ .

From the figure it is actually seen that the optimal  $r$  is obtained when  $V'(r) = 0$ , that is for continuously differentiable  $V(x)$ . This property of continuous  $V'(x)$  seems to hold generally for the system considered.

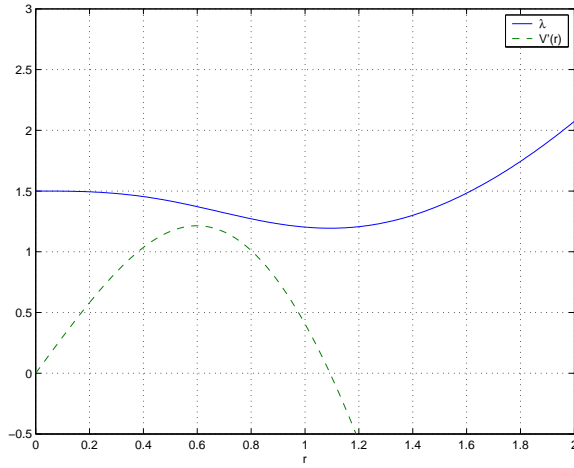
Fig. 3 shows  $V(x)$  and the active mode density  $f(x)$  for the optimal controller obtained from the minimizing  $r$  above.

*General First-Order Systems.* Fig. 4 shows  $r$ ,  $\lambda$  and  $T_m$  for the optimal controller as function of  $a$ . For  $a$  outside the range in the plot, faster sampling should probably be considered.

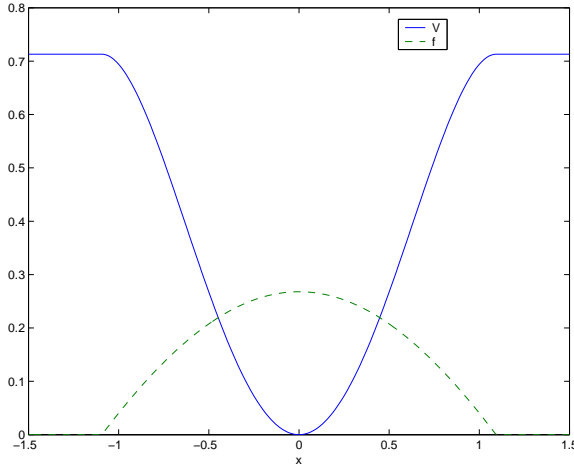
The threshold increases with  $a$ , which is probably because the inactive periods when the system runs open loop become much more costly as  $a$  increases. Still, the variation is within a factor of two in the wide range plotted.

The loss rate  $\lambda$  follows quite closely below the loss rate of the optimal periodic controller  $\lambda_p$  and represents a loss reduction of about 20% for  $aT < 0.5$ . For greater  $aT$ , the gain from event-based control decreases. The ratio  $\frac{\lambda}{\lambda_p}$  actually has a minimum for  $aT \approx -0.5$ .

The mean time between events seems to settle at about  $1.8T$  for negative enough  $aT$ , corresponding to 80% longer time between events than with periodic control. For the integrator case the figure is 62% longer, and for unstable



**Fig. 2** Loss rate  $\lambda$  as a function of threshold  $r$  for the integrator case. Also shown is the storage function derivative at the edge of the no-event region. The derivative has a zero crossing for the  $r$  when  $\lambda$  is minimized, corresponding to a continuously differentiable storage function.



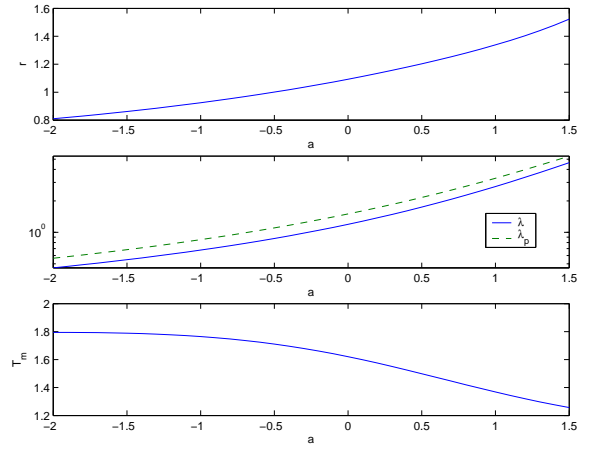
**Fig. 3** Storage function and probability density over the active mode for the optimal controller when  $a = 0$ . The density is zero for  $|x| > r$  since the system never dwells in the active mode under this condition.

systems the mean time decreases quite quickly, because of the short dwell time of the state within  $|x| < r$ .

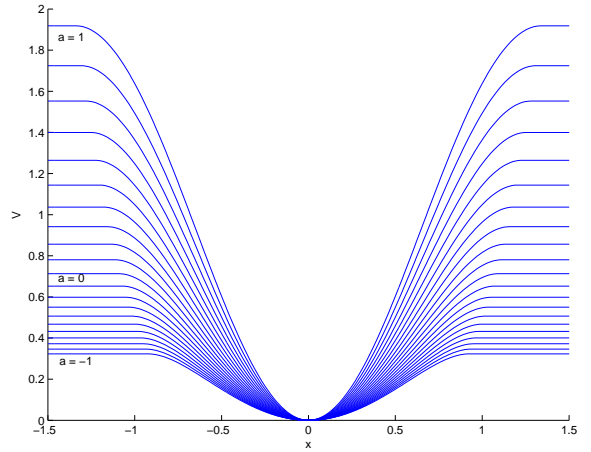
For negative enough  $aT$ , the decrease in  $r$  makes the controller more similar to the periodic case, as does the decrease in  $T_m$  for positive enough  $aT$ . Thus it should be expected that the event-based control differs most from the periodic case for modest  $|aT|$ .

Fig. 5 shows the storage function for different values of  $a$ . Beside the effect of increasing threshold values and increasing cost in general, the curves look very similar.

Fig. 6 shows the probability density over the active mode for different values of  $a$ . For negative  $a$  the curve becomes quite sharp, whereas for



**Fig. 4** Threshold, loss rate and mean time between events as a function of  $a$  for first order systems controlled by the optimal controller. The rate  $\lambda_p$  is the loss rate of the optimal periodic controller. Notice the log-scale in the middle plot.



**Fig. 5** Storage functions for the optimal controller with  $a$  stepped from  $-1$  to  $1$  in increments of  $0.1$ .

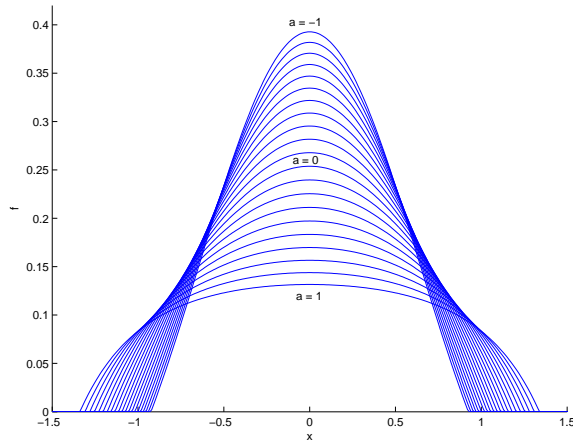
positive  $a$  it becomes more and more rectangular because of the inflating effect of the unstable dynamics. Also noticeable is the decrease in dwell time in the active mode as seen as a drop in the area under the curve when  $a$  increases.

### 3.5 Event-Based Control for Less Events

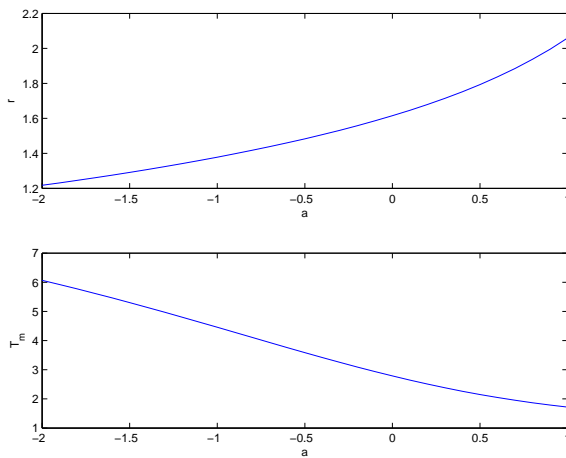
The main advantage of the event-based control scheme seems to be the increase in mean time between events. It could be well justified to trade the modest reduction in loss rate for a greater reduction in rate of events.

Fig. 7 shows  $r$  and  $T_m$  for event-based controllers tuned to give the same  $\lambda$  as in the periodic case, but maximum  $T_m$ . The  $r$  curve is similar to the previous case, but a bit higher in value.

For  $a = 0$ ,  $T_m \approx 2.8$  corresponding to 180% longer time between events or a 64% reduction



**Fig. 6** Probability density over the state in active mode for the optimal controller with  $a$  stepped from  $-1$  to  $1$  in increments of  $0.1$ .



**Fig. 7** Threshold and mean time between events as a function of  $a$  for first order systems controlled by an event-based controller tuned to give the same loss rate as the optimal periodic one but less events.

in mean event rate. For negative  $a$  the curve grows quickly and almost linearly, with for instance  $T_m \approx 3.6$  for  $a = -0.5$ . For positive  $a$ ,  $T_m$  falls toward 1.

### 3.6 Comparison Between Periodic and Event-Based Control

In the framework considered, it seems that minimum-variance control with minimum inter-event time  $T$  compares favourably to periodic control with sampling period  $T$ . For reasonable  $T$ , about 20% reduction in output variance and 60% longer mean time between events can be expected, or 180% longer mean time between events for the same output variance.

It is interesting to note that unlike periodic controllers, the design of the event-based controller depends on the process noise intensity  $\sigma$  in that  $r$  scales with it. If the actual process noise is

much greater than designed for, the controller approaches a periodic controller. If the process noise is much smaller, the controller approaches the case of arbitrary delay between events as treated in (Åström and Bernhardsson, 1999). Thus the interpretation of  $r$  as a threshold of tolerable state error should not be overlooked.

## 4. CONCLUSION

The preliminary results show that event-based control can perform better than periodic control even when considering the same network utilization factor in the comparison. This and many other issues in event-based control over networks deserve further research.

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