Decentralized Feedforward Control of Wind Farms: Prospects and Open Problems

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Decentralized feedforward control of wind farms: prospects and open problems

Maxim Kristalny and Daria Madjidian

Abstract— **The problem of load mitigation in wind turbines located in wind farms is addressed. The benefits of letting turbines communicate with and account for their neighbors are explored. First, the idea of exploiting previewed wind speed measurements is examined. The problem is formulated as an H^2 model matching optimization. The influence of preview length on the performance is analyzed and simulation results are presented. Then, the possibility of cooperation between turbines is studied within a distributed feedforward control scheme. The problem is formulated as a decentralized model matching optimization and several theoretical challenges associated with it are outlined. An approximate frequency domain solution and preliminary simulation results are presented.**

I. INTRODUCTION

Economy of scale makes it attractive to position wind turbines close to each other, forming large-scale wind farms [1]. Although, such a placement may cause various difficulties due to wake effects [2], [3], it may also be beneficial. Most of the research devoted to control of wind turbines for load mitigation focuses on a single turbine, see [4], [5], [1] and the references therein. Yet, it might be advantageous to consider the entire farm. The potential benefits of this could be:

- The ability to share measurements among adjacent turbines. In this setting upwind turbines can provide preview of the upcoming wind speed.
- The possibility for turbines to cooperate in terms of power production. This may introduce more freedom in adjusting individual turbine power and can be exploited for load reduction.

In this paper we examine the possibility of exploiting these benefits by using a distributed feedforward control scheme. A setup of collective-pitch power controlled turbines arranged in a row is considered, as in [6]. The potential of the proposed control strategy is assessed and several theoretical challenges are outlined.

In the first part of the paper we focus on the idea of exploiting previewed wind speed measurements for reducing the loads experienced by an individual turbine. In wind farms such a preview can be obtained from upwind turbines. However, one may also think of other sources, such as LIDAR systems [7]. The idea of using preview in control of wind turbines was discussed in [8]. Yet, so far, only a few results are available on this topic, [9], [10], based on discrete time H^∞ and model predictive control. We adopt a different approach and show that the problem can be conveniently formulated as a continuous time H^2 model matching optimization. We solve it using recent results from [11] and present simulations demonstrating the controller behavior. The simulations indicate that thrust force fluctuations and the associated tower oscillations can be significantly reduced by adjusting turbine power in response to upcoming wind conditions. The influence of preview length on the performance is analyzed.

In the second part of the paper we explore the benefits of cooperation between turbines. Since wind farms are expected to contribute to the stability of the electrical grid, they should be able to receive and maintain power set points. This impedes load mitigation, because it restricts turbines in adjusting their power production. Cooperation adds flexibility by allowing power to be redistributed between the turbines, as shown in [12] and [6]. In [12] the aero-dynamic coupling between turbines is neglected, and a centralized receding horizon control scheme is proposed. In [6] restrictions on communication between turbines are imposed and a distributed state feedback controller is obtained. As in [6], we consider a decentralized setting and allow communication only between adjacent turbines. However, we assume that only wind speed measurements (as opposed to the entire state in [6]) are communicated. We show that under a reasonable assumption, the problem is essentially a feedforward problem and can be formulated as a decentralized H^2 model matching optimization. A number of fundamental challenges associated with it are outlined and possible remedies are discussed. The ideas from [13] are exploited to find an approximate frequency domain solution. This is used to obtain preliminary simulation results, which reveal the nature of cooperation between turbines under the proposed control strategy.

The paper is organized as follows. The models of an individual turbine and a complete farm are described in Section II. Section III focuses on feedforward control of an individual turbine based on previewed wind speed measurements. Section IV is devoted to the distributed feedforward control of an entire farm. Finally, some concluding remarks are available in Section V.

Notation: Given a transfer matrix $G(s)$, its conjugate is denoted by $G^{\sim}(s) := [G(-s)]'$. If $G \in L^2$, $\{G\}_-$ and $\{G\}_+$ refers to the projection of G onto H^2 and $H^2_\perp$ respectively. The Kronecker, Khatri-Rao and Hadamard (entry-wise) products of two matrices are denoted by $A \otimes B$, $A \odot B$ and $A \circ B$, respectively. The vector operator $\mathrm{vec}(A)$ denotes a vector formed by stacking all columns of A. For a diagonal matrix $D = \mathrm{diag}(d_1, \ldots, d_n)$, the notation $\mathrm{dvec}(D) := \begin{bmatrix} d_1 & \ldots & d_n \end{bmatrix}'$ is used.

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II. MODELING

A. Turbine model

In this work we use a third order model of an NREL 5MW turbine, equipped with a standard internal controller, see [14] and [12]. The internal controller manipulates the generator torque and the blade pitch angle in order to meet a prescribed power demand. We will assume that the demand does not exceed the power available from the wind and, as a result, the internal controller operates in the third mode.

Denote the nominal mean wind speed and power demand by V_{nom} and p_{nom}. A linearized model of a turbine near its operating point will be denoted by $P = \begin{bmatrix} P_V & P_u \end{bmatrix}$. The partitioning is with respect to the two inputs V and $u = p_{\text{ref}}$ that stand for deviations in the wind speed and the power demand from their nominal values. Note that in the considered setting the second input is the only available control signal. Neglecting the generator dynamics, we will assume the actual deviation in power production equals p_{ref}. The three outputs of P will be denoted by F, w, and β and stand for the deviations in the thrust force, rotor speed, and pitch angle, respectively. The vector containing all outputs of the system will be denoted by $z := \begin{bmatrix} F & w & \beta \end{bmatrix}'$.

We will consider tower oscillations as an external dynamical mode induced by the thrust force. It will be approximated by a second order system with natural frequency $\omega_{\text{twr}} = 2$ rad/sec and damping coefficient $\zeta_{\text{twr}} = 0.08$, which is consistent with [14].

Remark 1: Considering a turbine equipped with a standard internal controller is restrictive, as it rules out direct access to the pitch and the generator torque. At the same time, it simplifies the problem and facilitates experiments in existing wind farms by eliminating the need in hardware replacement. Note that the ideas, the problem formulations and the open challenges raised below can naturally be extended to more general situations with no internal controller and/or individual pitch capabilities.

B. Wind farm model

We consider a row of N equidistant turbines and assume that the wind direction is parallel to the row. Although coupling between turbines due to wake effects plays an important role in quasi-static analysis, [2], it is less relevant once the dynamic behavior is considered in the vicinity of an operating point. Practical studies suggest that variations in wind speed caused by upwind turbines pitching are small compared to the natural variation in the wind [15]. Motivated by this we will neglect the influence of pitch activity on the wind flow and model the propagation of wind deviations between adjacent turbines by a delay and additive noise.

Following the discussion above, a wind farm can be described by the block diagram depicted in Figure 1 with solid lines. Systems P_i in this diagram stand for turbine models, linearized around their operating points. As in the previous subsection, turbine models will be partitioned with respect to their inputs $P_i = \begin{bmatrix} P_{i,V} & P_{i,u} \end{bmatrix}$. Signals u_i represent the control signals and constitute variations in turbines power

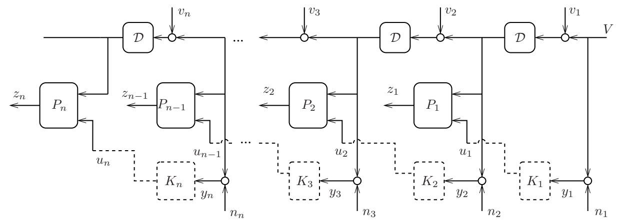

Fig. 1. Block diagram of a wind farm (solid) and distributed feedforward control scheme (dashed)

production. The output of the turbine i is denoted by $z_i := \begin{bmatrix} F_i & w_i & \beta_i \end{bmatrix}'$, where the notations are consistent with those in the previous subsection. The deviation of the incoming ambient wind speed from its nominal value is denoted by V. The time delays associated with wind propagation are represented by $\mathcal{D} = \mathrm{e}^{-sh}$. The additive wind noises are denoted by v_i. Signals y_i represent wind speed measurements and n_i stand for the corresponding measurement noises. The stacked vectors of measured signals, turbine outputs, and controls will be denoted by

$$\bar{y} := \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, \quad \bar{z} := \begin{bmatrix} z_1 \\ \vdots \\ z_N \end{bmatrix}, \quad \bar{u} := \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}.$$

The vector of all exogenous signals will be denoted by $\bar{w} := [\, V \mid v_1 \; \cdots \; v_N \mid n_1 \; \cdots \; n_N \,]'$. Let us define the systems $G_c := \text{diag}(P_{1,u} \; \ldots \; P_{N,u})$,

$$G_m := \left[\begin{array}{c|ccccc|cccc} 1 & 0 & \cdots & 0 & 0 & & 1 & 0 & \cdots & 0 \\ 1 & 1 & \ddots & \vdots & \vdots & & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 & 0 & & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 & 0 & & 0 & 0 & \cdots & 1 \end{array}\right],$$

$$G_w := \left[\begin{array}{c|ccccc|cccc} P_{1,V} & P_{1,V} & 0 & \cdots & 0 & & 0 & 0 & \cdots & 0 \\ P_{2,V} & P_{2,V} & P_{2,V} & \cdots & 0 & & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 & & \vdots & \vdots & \ddots & \vdots \\ P_{N,V} & P_{N,V} & P_{N,V} & \cdots & P_{N,V} & & 0 & 0 & \cdots & 0 \end{array}\right],$$

where G_m has the dimension $N \times 2N+1$ and the multichannel delay $\Lambda := \text{diag}\{\mathcal{D}, \mathcal{D}^2, \ldots, \mathcal{D}^N\}$. The relations between the signals in the block diagram in Figure 1 are then given by $\bar{y} = \mathcal{D}^{-1}\Lambda\bar{G}_m\bar{w}$ and $\bar{z} = G_c\bar{u} + \Lambda G_w\bar{w}$, which will be used later on in Section IV.

III. FEEDFORWARD CONTROL OF AN INDIVIDUAL TURBINE

In this section we focus on the individual turbine behavior and address the following questions:

- Is it possible to reduce turbine loads by short term power adjustments with respect to the measured wind speed?
- To what extent is preview of the wind speed beneficial?

A. Problem formulation and solution

The problem naturally falls into the open-loop measured disturbance attenuation scheme, depicted in Figure 2. The wind speed deviation V acts as a disturbance measured with the noise n. The delay h in the first plant input corresponds to the length of preview available to the controller K. The

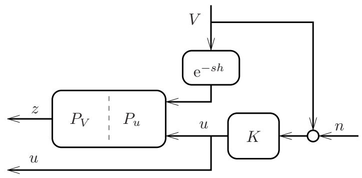

Fig. 2. Individual turbine control scheme

aim of the controller is to keep the components of z small. Pitch activity should be kept low to reduce wear in the pitch mechanism. Deviations of rotor speed from its rated value should not be large due to mechanical design constraints. Finally, fluctuations in thrust force should be alleviated, since they introduce oscillations, which cause damage to the tower and other mechanical components [2]. To prevent large fluctuations in the produced power, the control signal u should also be penalized.

The relation between the input and output signals in Figure 2 is given by:

$$\begin{bmatrix} z/V & z/n \\ u/V & u/n \end{bmatrix} = \begin{bmatrix} e^{-sh}P_V & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} P_u \\ 1 \end{bmatrix} K \begin{bmatrix} 1 & 1 \end{bmatrix}.$$

Define the cost transfer function for the optimization as

$$H := \begin{bmatrix} W_z & 0 \\ 0 & W_u \end{bmatrix} \begin{bmatrix} z/V & z/n \\ u/V & u/n \end{bmatrix} \begin{bmatrix} W_V & 0 \\ 0 & W_n \end{bmatrix},$$

where W_u, W_V and W_n are the weights for u, V and n, respectively, and $W_z = \mathrm{diag}\{W_F, W_w, W_\beta\}$ contains weights for all the components of z. For simplicity, we let the input weights be static $W_V = k_V$, $W_n = k_n$. The weight of the thrust force is chosen as $W_F = k_F \frac{s+\omega_{\mathrm{twr}}}{s^2+\omega_{\mathrm{twr}}^2}$. Note that it contains unstable poles on the imaginary axis, which correspond to the natural frequency of the tower. Together with input-output stability requirement on H, this imposes a constraint not to awake tower oscillations. The weights for the pitch angle and rotor speed are chosen to be static $W_\beta = k_\beta$, $W_w = k_w$. Finally, we choose $W_u = k_p \frac{(0.1s+1)}{s} \cdot \frac{(s+\omega_{\mathrm{twr}})^2}{s^2+0.02\ \omega_{\mathrm{twr}}+\omega_{\mathrm{twr}}^2}$. The integrator prevents shifting the power set point for permanent changes in wind conditions and the resonant peak damping tower oscillations by means of oscillations in power production.

Defining the transfer matrices

$$\left[\begin{array}{c:c} G_1 & G_3 \\ \hdashline G_2 & 0 \end{array}\right] := \left[\begin{array}{cc:c} e^{-sh}W_z P_V W_V & 0 & W_z P_u \\ 0 & 0 & W_u \\ \hdashline W_V & W_n & 0 \end{array}\right] \tag{1}$$

and choosing $||H||_2$ as the performance criterion, the problem can now be formulated as model matching optimization.

$\mathbf{OP}_1$: Given G_1, G_2 and G_3 as defined in (1) find $K \in H^\infty$, which guarantees

$$H = G_1 - G_3 K G_2 \in H^2 \cap H^\infty \tag{2}$$

and minimizes $||H||_2$.

Availability of preview, captured by the delay element in the definition of G_1, renders $\mathbf{OP}_1$ infinite dimensional. One of the ways to address this problem is by discretizing the time axes and treating the delay on equal footing with the rest of problem dynamics, [6]. This approach, however, leads to a substantial increase of computational burden and is subject to numerical difficulties. Efficient methods for the solution of a one-side version of $\mathbf{OP}_1$ are well studied, [16], [17]. Recently, a solution of a two-side problem was obtained in [11]. Applying this result to our problem, the optimal feedforward controller can be constructed. Its behavior is illustrated by simulations in the following subsection.

B. Simulation results

The results presented below are obtained for a turbine operating at $V_{\mathrm{nom}} = 15$ m/sec, $p_{\mathrm{nom}} = 4$ MW and for the following weight parameters: $k_V = 1$, $k_n = 1 \times 10^{-3}$, $k_u = 1.8 \times 10^{-2}$, $k_F = 0.5$, $k_w = 2 \times 10^6$, $k_\beta = 1 \times 10^{-3}$.

The natural questions when using preview are whether it can yield a noticeable performance improvement, and if so, what is its relation to the preview length. To address these questions, a curve of minimal achievable $||H||_2$ as a function of preview length is presented in Figure 3. It shows that the reasonable scale of preview length in our application is a number of seconds. In fact, 98% of possible improvement is achieved with a preview of 1.7 sec.

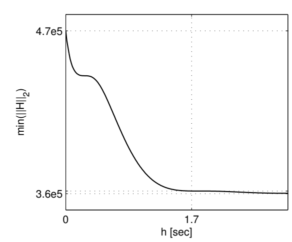

Fig. 3. Performance vs. h

Below we will compare the behavior of three systems: 1. without feedforward control; 2. with feedforward control based on local measurements; 3. with feedforward control based on measurements with preview of 1.7 sec. To illustrate the behavior of these systems, we analyze their response to a rectangular pulse in the wind velocity. The pulse starts at the time zero, lasts 1 sec and has an amplitude of 0.5 m/sec.

Plots of the thrust force, nacelle displacement (tower deflection), pitch angle and power deviation are presented in Figure 4. We see that feedforward control with and without

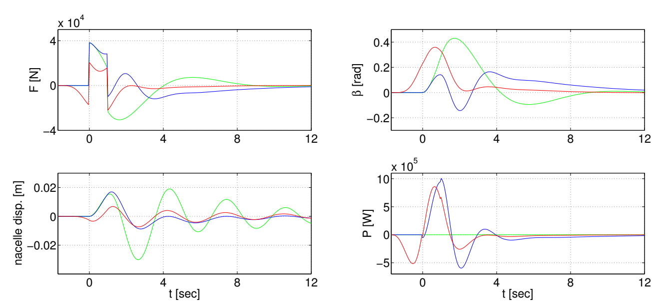

Fig. 4. Simulation results: green — without feedforward; blue — feedforward without preview; red — feedforward with 1.7 sec preview.

preview substantially damps tower oscillations. Note, however, that once no preview is available, the first peak in the thrust force can not be reduced. This is not a surprise taking into account that the relation between the thrust force and wind speed has a feedthrough term. The resulting immediate force changes can not be compensated by relatively slow pitch dynamics. The benefit of preview is that it offers the controller time to prepare the system prior to the wind gust arrival. Indeed, we see that once preview is available, pitch activity starts before the gust hits the turbine. As a result, the gust arrival is preceded by a drop in the thrust force. This decreases the peak value of the force and also the first peak in tower oscillations. It is worth noting that the reduction in thrust force and tower oscillations is obtained without increasing pitch activity. The feedforward controller changes pitching behavior, yet, the magnitude of pitch angle deviations remains smaller than in the original system. The same observation is true also for rotor speed, whose plot is not presented due to space limitations.

In summary of the first part of this paper, simulation results demonstrate that feedforward control based on previewed wind speed measurements can be beneficial. In particular, short term adjustments in power production can lead to a substantial reduction of the thrust and tower fluctuations.

IV. DISTRIBUTED FEEDFORWARD CONTROL OF THE ENTIRE FARM

Clearly, the control strategy from the previous section can be applied to each of the turbines in the farm. In this setting the i'th turbine controller will generate the control signal u_i based on wind measurements y_i available from the $(i-1)$'th turbine. This situation is depicted in Figure 1 with dashed lines. The downside of this strategy is that adjustments of individual turbine powers result in fluctuations of the overall farm power production. In this section we examine the possibility to alleviate these fluctuations by means of cooperation between the controllers within the control scheme depicted in Figure 1.

A. Problem formulation and the associated challenges

In order to address the issues of cooperation between the controllers, we formulate a problem that accounts for overall power production and, as a result, concerns the entire farm model, described in § II-B. The signals that should be penalized in the optimization are individual turbine power references $\bar{u}$, outputs $\bar{z}$ and deviation of the overall farm power production, which will be denoted by $u_\Sigma := \sum_{i=1}^N u_i$. Using the notations from § II-B, the relation between penalized and exogenous signals can be written as

$$\begin{bmatrix} \bar{z}/\bar{w} \\ \bar{u}/\bar{w} \\ u_\Sigma/\bar{w} \end{bmatrix} = \begin{bmatrix} \Lambda G_w \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} G_c \\ I \\ \bar{s} \end{bmatrix} \bar{K} D^{-1} \Lambda G_m,$$

where $\bar{s} := \begin{bmatrix} 1 \dots 1 \end{bmatrix}$ of an appropriate dimension and $\bar{K} := \text{diag}\{K_1, \dots, K_N\}$. The cost transfer matrix can now be defined as

$$\bar{H} := \begin{bmatrix} W_{\bar{z}} & 0 & 0 \\ 0 & W_{\bar{u}} & 0 \\ 0 & 0 & W_\Sigma \end{bmatrix} \begin{bmatrix} \bar{z}/\bar{w} \\ \bar{u}/\bar{w} \\ u_\Sigma/\bar{w} \end{bmatrix} W_{\bar{w}},$$

where W_Σ is the weight of the overall power deviation,

$$W_{\bar{z}} := \text{diag}\{W_{z_1}, \dots, W_{z_N}\}, \; W_{\bar{u}} := \text{diag}\{W_{u_1}, \dots, W_{u_N}\}$$

contain the weights for all individual turbine outputs and power references, and

$$W_{\bar{w}} := \text{diag}\{W_V, W_{v_1}, \dots, W_{v_N}, W_{n_1}, \dots, W_{n_N}\}$$

contains the weights for all the exogenous input signals. Defining the transfer matrices

$$\left[\begin{array}{c:c} \bar{G}_1 & \bar{G}_3 \\ \hdashline \bar{G}_2 & 0 \end{array}\right] := \left[\begin{array}{c:c} \Lambda W_{\bar{z}} G_w W_{\bar{w}} & W_{\bar{z}} G_c \\ 0 & W_{\bar{u}} \\ 0 & W_\Sigma \bar{s} \\ \hdashline \mathcal{D}^{-1} \Lambda G_m W_{\bar{w}} & 0 \end{array}\right] \tag{3}$$

and choosing the H^2 norm of $\bar{H}$ as the performance criterion, the problem can be formulated as follows.

$\mathbf{OP}_2$: Given $\bar{G}_1$, $\bar{G}_2$, $\bar{G}_3$ as defined in (3) find

$$\bar{K} = \text{diag}\{K_1, \dots, K_N\} \in H^\infty,$$

which guarantee

$$\bar{H} = \bar{G}_1 - \bar{G}_3 \bar{K} \bar{G}_2 \in H^2 \cap H^\infty$$

and minimize $||H||_2$.

The conceptual differences of this problem from $\mathbf{OP}_1$ are:
- the constraint on the design parameter $\bar{K}$ to be diagonal;
- the structure of infinite dimensional elements in definitions of G_1 and G_2.

To the best of our knowledge, there are no ready to use solutions of $\mathbf{OP}_2$ in the literature. The challenge of this problem can be associated with the following issues.

1) *Decentralized model matching stabilization.* It is shown in [18] that in the centralized case one can release stability constraints without changing the problem structure. This serves as a preliminary step in the solution of $\mathbf{OP}_1$ in [11] and the question is whether the result of [18] can be extended to the decentralized setting of $\mathbf{OP}_2$.
2) *Decentralized model matching optimization.* This problem is of a general interest in the context of distributed control, since any quadratically invariant problem can be reduced to model matching with a structural constraint on the Youla parameter, [19]. Explicit state-space formulae for one special case of a structural constraint were derived in [20]. The question is whether a similar analytical solution can be found for the general decentralized setting of $\mathbf{OP}_2$.
3) *Structure of infinite dimensional elements.* Typically, wind propagation delays between adjacent turbine are much larger compared to the time constants of turbine dynamics. Thus, discretizing the time axes and absorbing delays into the dynamics will substantially increase the order. In the case of a large-scale farm, this will result in numerically unfeasible solutions. The question is whether there exists a solution to $\mathbf{OP}_2$ similar to that of $\mathbf{OP}_1$ in [11]. Namely, a solution whose computational burden does not depend on the length of the delays.

The issues listed above are subject to ongoing research. Meanwhile, in order to assess the potential of the proposed distributed feedforward control scheme, an approximate solution of $\mathbf{OP}_2$ will be presented in the following subsection.

B. Decentralized model matching and Hadamard product

Unstable poles in $\mathbf{OP}_2$ stand for dynamics of external signals and modes and are located on the imaginary axes. Thus, the stabilization part of the problem can be circumvented by an arbitrary small shift of the unstable poles into the open left half plane. Motivated by this we will assume that

$\boldsymbol{\mathcal{A}_1}$: $\bar{G}_1$, $\bar{G}_2$ and $\bar{G}_3$ are stable.

We will assume also that

$\boldsymbol{\mathcal{A}_2}$: $\bar{G}_1(\infty) = 0$,

$\boldsymbol{\mathcal{A}_3}$: $\bar{G}_{2/3}(\infty)$ have full row/column rank respectively,

$\boldsymbol{\mathcal{A}_4}$: $\bar{G}_{2,3}$ have no purely imaginary transmission zeros.

Assumption $\boldsymbol{\mathcal{A}_2}$ is technical and imposes no loss of the generality, [11]. The assumptions $\boldsymbol{\mathcal{A}_3}$ and $\boldsymbol{\mathcal{A}_4}$ are standard and rule out problem redundancy and singularity. Following the same arguments as in [11, § II], the domain of the optimization parameter $\bar{K}$ can be replaced by

$$H_D^2 := \{\text{diag}\{K_1, \ldots, K_N\} \mid K_1, \ldots, K_N \in H^2\}.$$

and the problem can be effectively rewritten as

$$\bar{K}^* = \underset{\bar{K} \in H_D^2}{\text{argmin}} ||\bar{G}_1 - \bar{G}_3 \bar{K} \bar{G}_2||_2. \tag{4}$$

Frequency domain solution to this problem can be derived following the idea from [13]. Applying vec operator to (4) and using the properties of Khatri-Rao product, we can reshape it as

$$\bar{K}^* = \underset{\bar{K} \in H_D^2}{\text{argmin}} ||\text{vec}(\bar{G}_1) - (\bar{G}_2' \odot \bar{G}_3)\text{dvec}(\bar{K})||_2.$$

This way the problem is reduced to a standard one-side model matching optimization with no structural constraint on the parameter. Denote $\tilde{G}_1 := \text{vec}(\bar{G}_1)$, $\tilde{G}_3 := (\bar{G}_2' \odot \bar{G}_3)$ and introduce spectral factorization

$$U_3^{\sim} U_3 = \tilde{G}_3^{\sim} \tilde{G}_3, \quad U_3, U_3^{-1} \in H^{\infty}.$$

Note that due to the properties of Khatri-Rao product, [21],

$$\tilde{G}_3^{\sim} \tilde{G}_3 = (\bar{G}_2' \odot \bar{G}_3)^{\sim} (\bar{G}_2' \odot \bar{G}_3) = (G_2 G_2^{\sim})' \circ (G_3^{\sim} G_3).$$

At this point, standard Hilbert space arguments can be applied as in [11] to derive the optimal solution $\text{dvec}(K^*) = U_3^{-1}\{U_3^{-\sim} \tilde{G}_3^{\sim} \tilde{G}_1\}_-$ and the corresponding minimal achievable norm $||\{U_3^{-\sim} \tilde{G}_3^{\sim} \tilde{G}_1\}_+||_2$. The following result can now be formulated.

Theorem 1: Let the assumptions $\boldsymbol{\mathcal{A}_{2\text{-}4}}$ hold. Then, $\mathbf{OP}_2$ is solvable and the optimal $\bar{K}$ with the corresponding minimal $||\bar{H}||_2$ are given by

$$dvec(\bar{K}^*) = U_3^{-1}\{U_3^{-\sim}(\bar{G}_2' \odot \bar{G}_3)^{\sim} vec(\bar{G}_1)\}_-,$$
$$min||\bar{H}^*||_2 = ||\{U_3^{-\sim}(\bar{G}_2' \odot \bar{G}_3)^{\sim} vec(\bar{G}_1)\}_+||_2,$$

where U_3 is a spectral factor satisfying

$$U_3^{\sim} U_3 = (G_2 G_2^{\sim})' \circ (G_3^{\sim} G_3), \quad U_3, U_3^{-1} \in H^{\infty}.$$

Note that despite the use of Kronecker product in derivations, the dimension of the resulting spectral factorization is compatible with the dimensions of the original problem data. The order of the factorization, however, is infinite due to delays involved in the problem. Thus, in order to calculate the solution the time axis has to be discretized.

Remark 2: Even for finite order problems (without delays), the result of Theorem 1 does not reveal the order of the optimal solution and its structure. It seems that the main deficiency at this point is in efficient state-space formulae for the solution.

C. Preliminary simulation results

To illustrate application of the proposed control scheme, we consider a farm consisting of $N = 5$ turbines with an incoming mean wind speed of $V_{\text{nom}} = 15$ m/s. We choose the power set point for the farm as $P_{\text{farm}} = 17$ MW and distribute the powers among the turbines as in [6]: $P_{\text{nom}_{1,2}} = 4$ MW, $P_{\text{nom}_{3,4,5}} = 3$ MW. The nominal wind speeds experienced by each of the turbines are computed according to the static model in [22], $V_{\text{nom}_{1\ldots5}} = \{15, 14.7, 14.5, 14.42, 14.37\}$ m/s. We choose the weights for input and output signals of each turbine to be the same as in Section III, up to the shift of imaginary axis poles by $\epsilon = 0.01$. The weight for the overall farm power production is chosen to be static $W_\Sigma = 0.1$. We assume that the wind propagation delay is $h = 2$ sec. This value is not typical for real wind farms, where the delays are considerably longer. We choose h to be small in order to avoid numerical problems related to discretization of the time axis. Hence, the results presented below serve illustration purposes only. Our aim is to explain what kind of cooperation can be achieved under the proposed control scheme and what the benefits of such a cooperation are.

Denote the "cooperative" controllers obtained by solving $\mathbf{OP}_2$ and the "non-cooperative" controllers obtained by solving $\mathbf{OP}_1$ for each individual turbine by $\mathcal{K}_i$ and $\tilde{\mathcal{K}}_i$, respectively, for $i = 1 \ldots 5$. To get the first insight into the behavior of distributed feedforward control scheme, let us compare the impulse responses of $\mathcal{K}_1$ and $\tilde{\mathcal{K}}_1$, depicted in Figure 5. Note that the impulse response of $\mathcal{K}_1$ differs from that of $\tilde{\mathcal{K}}_1$ by sharp peaks occurring at the time multiples of h. These peaks reflect the fact that $\mathcal{K}_1$ "expects" downwind turbines to experience similar wind fluctuations as the first turbine, but with delays of h sec, $2h$ sec and so on. To take this into account, $\mathcal{K}_1$ adjusts the power production of the first turbine at the right times in order to compensate for anticipated power changes of its downwind neighbors. The same behavior is present in impulse responses of the other turbines in the row, which have not been presented due to space limitations. The number of peaks in their impulse responses relates to the number of downwind turbines.

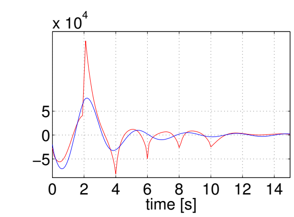

Fig. 5. Impulse responses of $\mathcal{K}_1$ (red) and $\tilde{\mathcal{K}}_1$ (blue)

As in § III-B, we study the system's response to an incoming wind gust. We assume that the wind gust starts at time zero, lasts 1.5 sec and has an amplitude of 0.5 m/s.

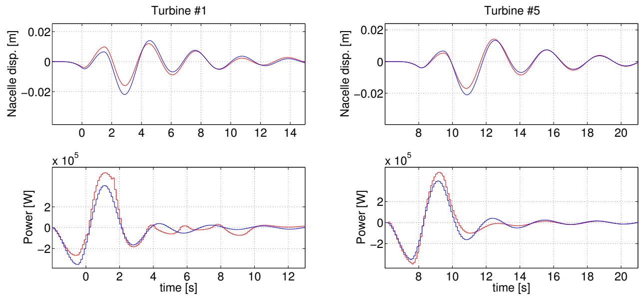

Fig. 6. Simulation results: Nacelle displacements and power deviations of the first turbine (left) and the last turbine (right). (blue — feedforward with $\tilde{\mathcal{K}}_i$; red — feedforward with $\mathcal{K}_i$)

Figure 6 shows nacelle displacement and power production of the first and the last turbine. We see that cooperation between controllers accommodates more aggressive control actions. As a result cooperative controllers slightly reduce tower oscillations compared to noncooperative ones. This comes at the expense of individual turbine power variations. However, the main benefit of considering the whole farm ($\mathcal{K}_i$ via $\mathbf{OP}_2$) rather then individual turbines ($\tilde{\mathcal{K}}_i$ via $\mathbf{OP}_1$) becomes evident once the overall farm power production is examined, see Figure 7. We observe that cooperation between turbines significantly reduces fluctuations in the overall farm power production without causing deterioration in terms of load reduction.

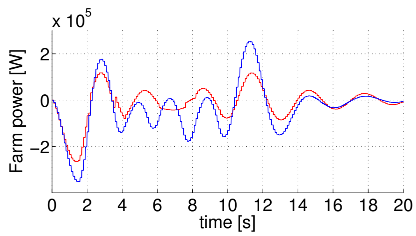

Fig. 7. Variation in overall farm power (blue — $\tilde{\mathcal{K}}_i$; red — $\mathcal{K}_i$)

V. SUMMARY

A problem of wind farm control for load mitigation was addressed in a simple setup of collective pitch controlled turbines arranged in a row. A distributed feedforward control scheme was considered, in which the wind speed measurements from upwind turbines are transmitted to their closest downwind neighbors. In the first part, the problem of using previewed wind speed measurements was formulated as H^2 model matching optimization and solved using the result from [11]. Simulation results were presented illustrating the potential of preview for the reduction of thrust force and tower oscillations. In the second part the possibility of cooperation between turbines was considered. The problem was formulated as H^2 decentralized model matching optimization and a number of open theoretical challenges associated with it were discussed. An approximate frequency domain solution of the problem was presented and the benefits of cooperation were illustrated by simulations.

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