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de Wit, Carlos Canudas; Braun, K.; Åström, Karl Johan

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Adaptive Friction Compensation in DC-Motor Drives

C. CANUDAS, K. J. ÅSTRÖM, FELLOW, IEEE, AND K. BRAUN

Abstract—**A control scheme is proposed where the nonlinear effects of friction are compensated adaptively. When the friction is compensated, the motor drive can approximately be described by a constant coefficient linear model. Standard methods can be applied to design a regulator for such a model. This results in a control law which is a combination of a fixed linear controller and an adaptive part which compensates for nonlinear friction effects. Experiments have clearly shown that both static and dynamic friction have nonsymmetric characteristics. They depend on the direction of motion. This is considered in the design of the adaptive friction compensation. The proposes scheme has been implemented and tested on a laboratory prototype with good results. The control law is implemented on an IBM PC. The ideas, algorithm, and experimental results are described. The results are relevant for many precision drives, such as those found in industrial robots.**

I. Introduction

Adaptive control has predominantly dealt with generic models where all parameters are unknown. Such an approach has the advantage that it is general but also has the disadvantage that many parameters have to be estimated. Much of the work on adaptive control has also been confined to linear systems. In practice, many adaptive problems exist where the system can be described as partially known in the sense that part of the system dynamics is known and another part unknown. In this communication we consider a problem of this type, namely, a servo with nonlinear friction. Friction, which is always present to some degree, causes difficulties and gives rise to poor performance in precision servos in robots and other applications.

Velocity control of a servo motor with friction is considered. It is assumed that static and viscous frictions can be described as nonlinear functions of the angular velocity. The friction characteristics depend on the direction of the motion. The model can thus be split into two parts, depending on the direction of motion. The model isolates the friction torque effects and cancels them by feedback compensation.

Adaptive friction compensation has been considered before [17]. It was treated with model reference techniques in [7] and more recently in [15] and [10]. This work differs in the friction model and in the adaptive control law used.

The adaptive scheme introduced here attempts to use the *a priori* information available, i.e., the structure of the nonlinearity and the knowledge of some of the parameters. It seems natural to use adaptive schemes with explicit identification which utilizes this *a priori* information. Only those parameters which are not known *a priori* are estimated.The estimates are used to compensate for the friction-torque effects, and a linear control design is used to control the approximately linear system that is obtained when the friction effects are compensated. The final control structure can be viewed as a combination of a fixed linear controller and a feedback adaptive compensation.

The communication is organized as follows. Friction models proposed in the literature are discussed in Section II. A model where the friction torque is a piecewise-linear function of motor speed is established. This model captures static and dynamic friction effects. A strategy for friction compensation is presented in Section III. Section IV briefly describes the control laws for the linear system obtained when the friction effects are compensated. The design is a standard pole placement control. Section V proposes an adaptive version of the fixed friction compensation and proposes a possible design approach. The proposed ideas have been implemented on a laboratory prototype. The digital control laws were implemented using an IBM personal computer. The results of some experiments are shown in Section VI. Some conclusions are given in Section VII.

II. Mathematical Models

A dc motor with a permanent magnet was used in our experiments. Such motors are commonly used in robots and precision servos. The motor is provided with an electronic amplifier with current feedback. If all inertias are reflected to the motor axis, the motor can be described by the following model:

$$J\frac{d\omega}{dt}=KI(t)-T_f(t)+T_l(t). \tag{1}$$

Here J is the total moment of inertia reflected to the motor axis, K is the current constant, I is the motor current, T_f is the friction torque, and T_l is load disturbance torque. For the purpose of the investigation of the friction compensation, phenomena like compliance and torque ripple are not included in the model (1).

Friction Models

Friction models have been extensively discussed in the literature [5], [15], [7]. In spite of this, there is considerable disagreement on the proper model structure. It is well established that the friction torque is a function of the angular velocity. There is, however, disagreement concerning the character of the function. In the classical

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C. Canudas is with the Laboratoire d'Automatique de Grenoble, B.P. 46, 38402, Saint-Martin-d'Hères, France.
K. J. Åström is with the Department of Automatic Control, Lund Institute of Technology, Box 118, S-22100, Lund, Sweden.
K. Braun was with the Fachgruppe fur Automatik, ETH-Zentrum, CH-8092 Zurich, Switzerland.
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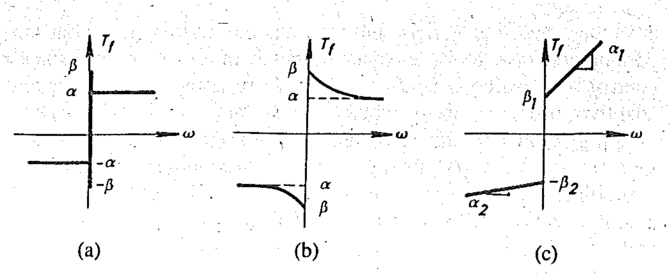


Fig. 1. Different friction models.

Coulomb friction model there is a constant friction torque opposing the motion when $\omega = 0$. For zero velocity the striction will oppose all motions as long as the torques are smaller in magnitude than the striction torque. This model is represented in Fig. 1(a). The model has been well established in connection with slow speeds in numerically controlled machines.

The model shown in Fig. 1(b) was proposed in [7]. A very different model was proposed in [15]. There, the model was based on experimental studies of a stabilized platform with ball bearings on the gimbals. In [15] the model

$$\frac{dT_f}{dt} + aT_f = T_c \operatorname{sgn}(\omega)$$

is proposed, where the parameter a depends on ω. Notice that this model introduces additional dynamics but that it does not include any static friction characteristics. The friction model used in our studies is shown in Fig. 1(c). This model includes Coulomb friction and viscous friction. The friction curve is, however, not symmetric.

The following model is used:

$$T_f(\omega) = \begin{cases} \alpha_1\omega + \beta_1, & \omega > 0 \\ \alpha_2\omega + \beta_2, & \omega < 0 \end{cases}. \tag{2}$$

Neglecting the load disturbances torque and the resonances modes of the motor couplings, the motor can thus be described by (1) where the friction torque is given by (2).

III. Friction Compensation

The nonlinear friction limits the performance of the closed-loop system. The influences of the nonlinearities can to some extent be reduced by high-gain linear feedback. This is suggested in [16]. This approach has, however, some severe limitations because the nonlinearities will dominate any linear compensation for small errors. The effects of the friction can also be alleviated by mounting force sensors, which measure the friction levels, and using them in a linearizing feedback loop around the torque motor, as suggested in [8]. The selection of the adequate techniques to compensate for the friction torques depends on the choice of the friction model. For the dynamic model proposed in [15], it is possible to predict the friction behavior and compensate it by feedforward. An alternative approach is to reduce the effects of the friction terms by a nonlinear compensation. It is easy to see how this can be done. Neglecting the load disturbance torques T_l, (1) and (2) can be written as

$$J\frac{d\omega}{dt} = KI(t) - T_f(\omega). \tag{3}$$

Introduce

$$I(t) = u(t) + \frac{\hat{T}_f(\omega)}{\hat{K}} \tag{4}$$

where $u(t)$ is a new control variable, $\hat{T}_f$ is an estimate of the function T_f, and $\hat{K}$ is an estimate of the current constant K. Then

$$J\frac{d\omega}{dt} = Ku(t) + \left\{\frac{K}{\hat{K}}\hat{T}_f(\omega) - T_f(\omega)\right\}. \tag{5}$$

If the estimates are good, the terms within the curly brackets vanish and the system obtained with the nonlinear feedback looks like a frictionless motor. It is, therefore, natural to call the feedback (4) a friction compensation. The success of the compensation clearly depends on the accuracy of the estimates of K and T_f. The parameter K is the torque constant of the motor whose value can be found from catalogues. It can also be measured. A complication is that K is not a constant. For many motors, K will also depend on the relative oscillation of the rotor and stator at high frequencies (ripple torque). The friction torque T_f is a function of the angular velocity. To obtain T_f, it is important to know the shape of the function and to have a good estimate of the angular velocity ω. In our investigation we have used functions of the form (2). This simple model makes it possible to deal with variations and asymmetries of the friction torque which are not included in other models.

The velocity estimate has been generated by a tachometer or by a Kalman filter. In our first experiments we simply attempted to introduce a friction compensation based on (2), where the parameters were adjusted manually. The experiments performed were simply to adjust the parameters α_1, β_1, α_2, and β_2 so that the motor behaved like a frictionless motor. These experiments clearly indicated the benefits of friction compensation but also the necessity of having different parameters for different direction rotation. The experiments also showed that it was possible to achieve friction compensation using a friction model like (2) except for very slow tracking rates. It was also found that the coefficients in the friction model (2) varied with temperature and time. They may also vary with changes of the operation conditions. This motivates making the friction compensation adaptive.

IV. Control Design

Although the main thrust of this work is to discuss friction compensation, it is necessary to also add a conventional feedback loop to evaluate the final results. A natural approach is to design the feedback loop under the assumptions of perfect friction compensation. The system is then described by

$$J\frac{d\omega}{dt} = Ku(t), \tag{6}$$

and it is easily verified that the control law

$$u(t) = K_r\left[-\omega(t) + \frac{1}{T_i}\int_0^t (\omega_r(\tau) - \omega(\tau))\, d\tau\right] \tag{7}$$

with

$$K_r = \frac{2\xi\omega_0 J}{K} \qquad T_i = \frac{K}{J\omega_0^2} \tag{8}$$

gives a closed-loop system with the transfer function

$$G(s) = \frac{\omega_0^2}{s^2 + 2\xi\omega_0 + \omega_0^2}. \tag{9}$$

An equivalent discrete-time control law of the form

$$u(t) = u(t-h) + s_0[\omega_r(t) - \omega(t-h)] + s_1[\omega_r(t-h) - \omega(t-h)] \tag{10}$$

was actually used in the experiments. The parameters of the regulator were determined by pole-placement design [1]. ω_r is the desired reference signal.

V. Adaptive Friction Compensation

In a typical servo application the moment of inertia J and the current constant K may be regarded as known. To obtain the friction compensation it is necessary to obtain estimates of the friction torque functions T_f. With the representation (2) this reduces to estimation of the parameters α_1, α_2, β_1, and β_2.

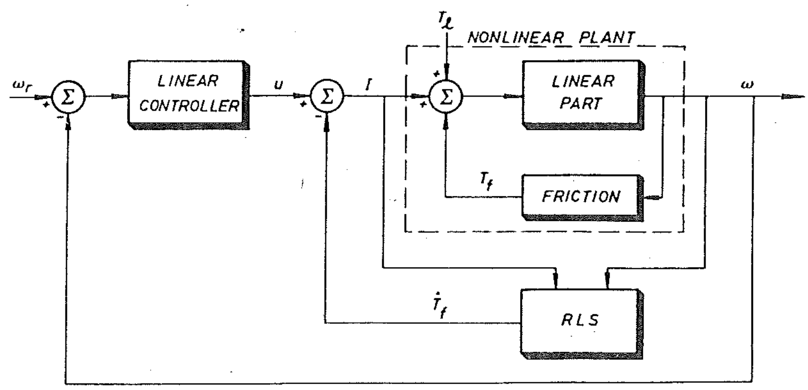


Fig. 2. Block diagram of motor controller with adaptive friction compensation.

Parameter Estimation Methods

Since the parameters α_1, α_2, β_1, and β_2 appear in the continuous-time formulation, it is natural to estimate the parameters in this form. Standard linear parameters estimation methods may be applied to the equation

$$J\frac{d\tilde{\omega}}{dt}=K\tilde{I}(t)-\alpha_i\tilde{\omega}-\beta_i \tag{11}$$

where $\tilde{\omega}$ denotes a filtered version of ω. The signals were sampled and filtered. Standard recursive estimation methods are used to generate the estimates. The filter can be optimized based on knowledge of the noise and the known parameters. More details of these techniques can be found in [4], [6], [9], [13], and [14].

An alternative is to derive a zero-order hold model for the motor representation (1), (2). This gives

$$\omega(t)+a_{1i}\omega(t-h)=b_{0i}I(t-h)+b_{1i}. \tag{12}$$

With this approach it is necessary to estimate six parameters instead of four.

Another possibility is to sample with such a short rate that the derivative can be approximated by forward differences, i.e.,

$$\omega(t+h)=\omega(t)+\frac{h}{J}\left[KI(t)-\alpha_i\omega(t)-\beta_i\right]. \tag{13}$$

This approximation retains the number of parameters of the physical model. Many other alternatives exist, such as the Tusting approximation, etc.

Estimation Algorithms

All estimation algorithms can be characterized by the error model

$$\epsilon(t_i)=f(t_i)-\phi^T(t_i)\theta \tag{14}$$

where the function f and the regression vector ϕ are functions of the data and θ is vector of the unknown parameters. A recursive least-squares algorithm is then given by the normal equations. The closed-loop scheme with the adaptive friction compensation is shown in Fig. 2.

A Possible Design Path

The previous sections covered different friction models and alternative methods to construct adaptive predictors. The necessity to add a conventional feedback loop to evaluate the effectiveness of the friction compensation was also mentioned. To illustrate one possible path to implement the previous ideas, choose a discrete-time predictor and a pole-placement control design policy. We can then proceed as follows. The discrete time models (12) or (13) can be reformulated as the following general model:

$$\omega(t+h)=\begin{cases}(A_1+\tilde{A}_1)\omega(t)+(B_1+\tilde{B}_1)I(t)+\delta_1, & \text{if } \omega(t)>0\\ (A_2+\tilde{A}_2)\omega(t)+(B_2+\tilde{B}_2)I(t)+\delta_2, & \text{if } \omega(t)<0\end{cases} \tag{15}$$

where A and B are polynomials in the delay operator q^{-1} of the appropriate order. The polynomials A_1, B_1, A_2, B_2 are the known model part of the plant. $\tilde{A}_1$, $\tilde{B}_1$, $\tilde{A}_2$, $\tilde{B}_2$ are the unknown model part of the plant which is provided by the model uncertainty and by the nonlinear feedback of the process. The operator q^{-1} indicates a delay operation of one period h.

We can always let $A_1 = A_2 = A$ and $B_1 = B_2 = B$. The difference in each case will be absorbed by the corresponding polynomial uncertainty $\tilde{A}_i$ and $\tilde{B}_i$. Then (15) can be reduced to a more compact form:

$$\omega(t+h)=A\omega(t)+BI(t)+g(t) \tag{16}$$

where

$$\begin{aligned}g(t)&=g_1(t)m(t)+g_2(t)(1-m(t))\\ g_1(t)&=\tilde{A}_1\omega(t)+\tilde{B}_1I(t)+\delta_1\\ g_2(t)&=\tilde{A}_2\omega(t)+\tilde{B}_2I(t)+\delta_2\end{aligned} \tag{17}$$

and

$$m(t)=\begin{cases}1, & \text{if } \omega(t)>0\\ 0, & \text{if } \omega(t)<0\end{cases}. \tag{18}$$

For the model (13) the polynomials just defined are $A = 1$, $B = hK/J$, and $\tilde{A}_i = -h\alpha_i/J$, $\tilde{B}_i = 0$, $\delta_i = -h\beta_i/J$ for $i = 1, 2$. The functions $g(t)$ contain the friction effects to be canceled. By the arguments discussed in Section III, the nonlinear model (16) can be linearized if the following control law is applied:

$$I(t)=u(t)+\bar{g}(t) \tag{19}$$

where $\bar{g}(t)$ is equal to $g(t)/B$. Replacing the foregoing control law in the process model (16) gives

$$\omega(t+h)=A\omega(t)+Bu(t) \quad \text{or } \bar{A}\omega(t)=q^{-1}Bu(t),$$
$$\bar{A}=1-q^{-1}A. \tag{20}$$

The most general linear controller is given by the following equation:

$$Ru(t)=T\omega_r(t)-S\omega(t);\ \omega_r(t)=A_m(1)\omega_r(t) \tag{21}$$

where $\omega_r(t)$ is the reference signal, $\omega(t)$ is the reference signal, $\omega(t)$ the process output, and $u(t)$ the input applied to the linearized system (20). A_m is the polynomial which describes the desired closed-loop

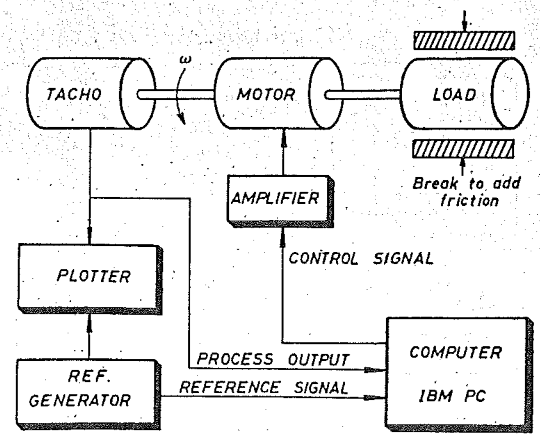


Fig. 3. Experimental setup.

characteristics, and the R, S, T polynomials will be found by solving the Diophantine equations:

$$R\bar{A}+q^{-1}BS=TBA_m \qquad T=A_nA_0 \qquad R=DR' \tag{22}$$

where A_0 is the observer polynomial, A_n is a notch filter, and D is the internal model. (These polynomials can be included or not; the simplest case is $A_0 = A_n = D = 1$). The adaptive nonlinear compensation algorithm based on the same previous linear philosophy can be described by the next sets of equations.

Adaptive Predictor: Let

$$\hat{\omega}(t+h \mid t)=A\omega(t)+Bu(t)-\hat{g}(t) \tag{23}$$

where

$$\hat{g}(t)=\phi(t)^T\hat{\theta}(t)=[\phi_1(t)^Tm(t),\ \phi_2(t)(1-m(t))]\cdot\begin{bmatrix}\hat{\theta}_1(t)\\ \hat{\theta}_2(t)\end{bmatrix}$$

$$\phi_1(t)^T=[\omega(t),\ \cdots,\ \omega(t-hn\tilde{a}1),\ u(t),\ \cdots,\ u(t-hn\tilde{b}1),\ 1] \tag{24}$$

$$\phi_2(t)^T=[\omega(t),\ \cdots,\ \omega(t-hn\tilde{a}2),\ u(t),\ \cdots,\ u(t-hn\tilde{b}2),\ 1]$$

$$\hat{\theta}_1(t)^T=[\tilde{a}_1^1(t),\ \cdots,\ \tilde{a}_{n\tilde{a}1}^1(t),\ \tilde{b}_0^1(t),\ \cdots,\ \tilde{b}_{n\tilde{b}1}^1(t),\ \delta_1(t)] \tag{25}$$

$$\hat{\theta}_2(t)^T=[\tilde{a}_1^2(t),\ \cdots,\ \tilde{a}_{n\tilde{a}2}^2(t),\ \tilde{b}_0^2(t),\ \cdots,\ \tilde{b}_{n\tilde{b}2}^2(t),\ \delta_2(t)].$$

Predictor Error: We have

$$e(t)=\omega(t)-\hat{\omega}(t \mid t-h)=g(t-h)-\hat{g}(t-h). \tag{26}$$

Parameter Estimation Algorithm: Use a recursive least squares (RLS) algorithm.

Adaptive Control Law: The following holds:

$$Ru(t)=T\omega_r-S\omega(t)+R\hat{\bar{g}}(t), \qquad \hat{\bar{g}}(t)=\hat{g}(t)/B \tag{27}$$

where $n_{\tilde{a}i}$ and $n_{\tilde{b}i}$ are the degrees of the polynomials $\tilde{A}_i$ and $\tilde{B}_i$, respectively.

Typical assumptions of the pole-placement design are needed: coprimeness between the polynomials A, B, and the stability of $(B+\tilde{B}_i)^{-1}$ for $i = 1, 2$. The closed-loop properties of the foregoing set of equations are analyzed in [4]. The previous algorithm has been implemented in an experimental set for the model (13). The results are described in the following section.

VI. Experiments

The ideas have been tested experimentally on a simple servo. The experimental setup is shown in Fig. 3. It consists of a servo composed of a dc motor with gear and load. The motor speed is measured using a tachometer. There is friction in the motor bearings and in the gear train. The friction can also be increased by a simple mechanical arrangement. The first experiments were performed using dedicated analog hardware which was built using operational amplifiers. In this experiment it was attempted to reduce the friction by a fixed nonlinear compensator as discussed in Section III. A nonlinear friction compensation of the form (4) was introduced, and the parameters were adjusted manually. It was found that friction compensation is indeed possible but that the parameters of the friction compensation depend on the operating condition. The adjustment of the parameters of the friction compensation is also quite critical. Fig. 4 shows that degradation of the closed-loop responses may occur if the friction parameters are not chosen properly.

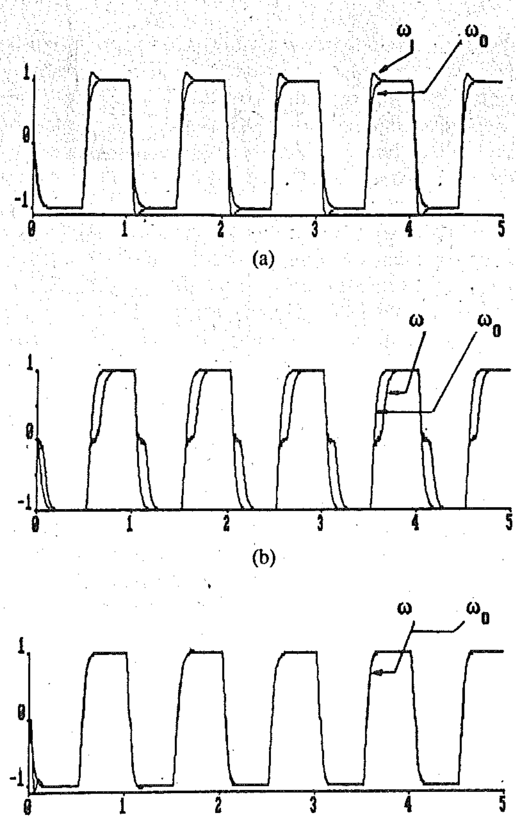


Fig. 4. Simulation illustrating parameters sensitivity of fixed-gain friction compensator. Zero-friction behavior is ω_0; process output is ω. (a) Overestimation of friction levels. (b) Underestimation of friction levels. (c) Adaptive friction compensation.

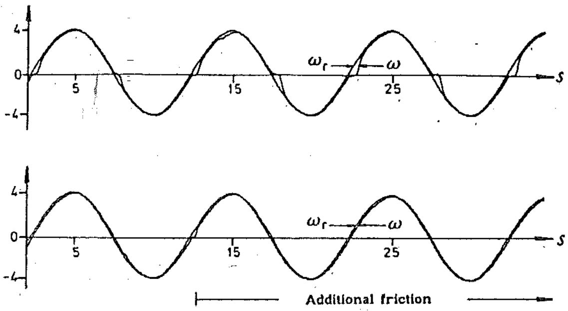
The results in Fig. 4 were obtained by simulation. Similar phenomena would also be found experimentally.

The experiments with adaptive friction compensation were performed under computer control. An IBM PC-XT with the 8087 floating point chip and Data Translation AD and DA converters were used. The major part of the software was written in Microsoft Pascal. The Meta WINDOW package was used for the graphics. Concurrency was obtained by a simple scheduler written in Assembler. This allowed the control program to run in the foreground and graphics and man–machine communication in the background. The minimal sampling rate is 55 ms. For more details of the implementation aspects see [2].

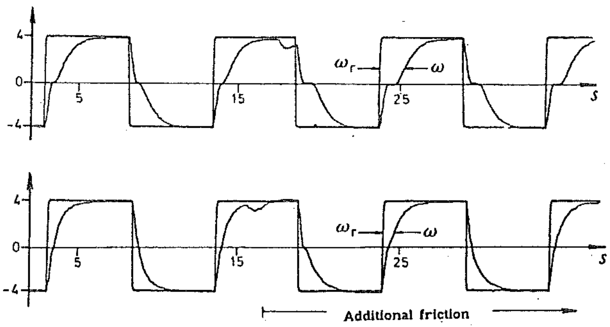
Tracking experiments were carried out with a constant-gain regulator without friction compensation and a controller with adaptive friction compensation. Some results are shown in Fig. 5. The upper traces in the figure shows the tracking performance with a linear constant gain regulator. Notice the deterioration in performance as the friction is increased. The lower traces show the corresponding curves for a regulator with adaptive friction compensation. The improvements are quite noticeable. The time history of the estimates corresponding to the lower traces are shown in Fig. 6.

VII. Conclusion

Although high-quality servos of the type found in robots and systems for tracking and pointing are largely described by linear



(a)



(b)

Fig. 5. Results of tracking using linear fixed-gain regulator and regulator with adaptive friction compensation for two different types of reference signals. Experimental results (process output ω, reference signal ω_r). (a) Upper trace: noncompensation. Lower trace: adaptive compensation. (b) Upper trace: noncompensation. Lower trace: adaptive compensation.

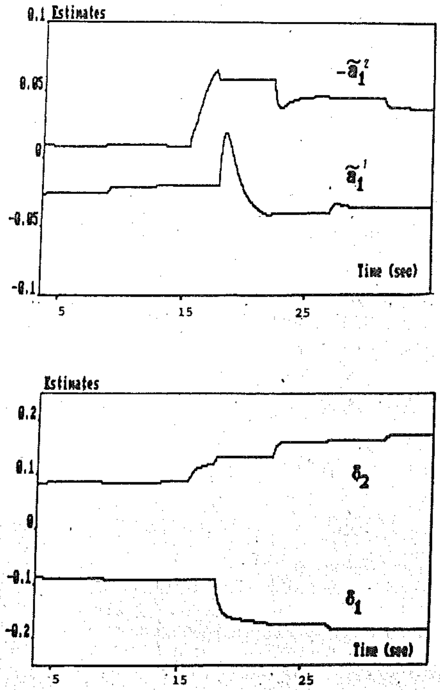


Fig. 6. Time histories of estimates $\tilde{a}_1^1$, $\tilde{a}_2^2$, δ_1, and δ_2 under friction variations.

models, their performance is often limited by nonlinear phenomena such as friction and backlash. We have discussed the possibilities of improving the performance of a servo by nonlinear compensation of friction. Models for friction have been reviewed. Several different models have been proposed. A particular form was chosen based on experiments on a servo. It was found to be essential to have a model which is asymmetric in the angular velocity. Several methods to compensate for friction have been discussed. Different ways to estimate the coefficients of the friction model have also been investigated. The adaptive techniques have been found superior because the friction depends on the operating conditions. Adaptive friction compensation has been applied to an experimental system. Its benefit has been clearly demonstrated in experiments on a servo, where the control law was implemented on an IBM PC. With regards to future work, it seems appropriate to investigate the structure of the friction models in more detail since this seems to be an issue where considerable disagreement exists in the literature. The availability of a friction model with appropriate structure is also crucial for the performance of the adaptive friction compensation.

The technique presented here can also be extended to deal with variations in inertia and other types of loads. To do this, it is necessary to augment the mathematical model and to estimate additional parameters. Work in this direction is in progress. Also notice that the friction compensation acts as a linearization loop and not as a mechanism for canceling the torque loads.

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