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New Aspects of Ultra Thin Absorbers

Alireza Kazemzadeh and Anders Karlsson
Abstract

Ultra thin absorbers are studied in detail to provide a comprehensive model for their absorption mechanism. It is shown that the transmission line (TML) approach is not able to model the absorber frequency response correctly. It results in large errors when the thickness to wavelength ($d/\lambda$) ratio is below a certain level. It is explained that large amplitudes of high order Floquet modes and excitation of non-transverse component of the scattered field at the absorption frequency are the reasons for the TML model inaccuracy. It is illustrated that at small $d/\lambda$ ratios, the structure becomes a localized lossy resonator which exhibits absorption even in a finite size array configuration. For a resistive squared patch periodic array, the resonance frequency can be estimated fairly accurately by the cavity model of a single perfect conductor patch antenna.

1 Introduction

Conventional absorbers like Salisbury [6], Jaumann [3, 4], and circuit analog absorbers [12] consist of dielectric layers, quarter of a wavelength thick at mid-frequency. The analysis of these absorbers is based on the dominance of a single propagating TEM mode between layers and ignorance of the possible mutual interaction of the higher order evanescent modes. In the case of homogenous resistive sheet layers (Salisbury and Jaumann absorbers), the evanescent modes are not excited and for the circuit analog absorbers their effects are negligible because of the large electrical distance ($\lambda/4$ at mid-frequency) between the FSS layers. This paper investigates the effect of these modes on the performance of the absorber when the distance between the layers is reduced such that an ultra thin absorber is formed.

Ultra-thin absorbers are characterized by a thickness of $\lambda/10$ or less at the operating frequency. Different designs of ultra thin absorbers are proposed by authors [5, 8, 10, 11]. Although the geometry of the periodic array unit cell and the design approach are different among the papers, the common idea is to construct a high impedance ground plane by aid of a meta-surface. Engheta [5] has explained that the surface impedance of the periodic arrangement (Metamaterial) interacting with the reactive impedance of the ground plane can generate a magnetic wall (open circuit) at a distance shorter than $\lambda/4$ above the ground plane. By placing a resistive sheet over this magnetic wall, power can be absorbed from the incident EM wave. The analysis of the absorber used by Engheta is based on the single mode transmission line model. Improvement in the structure thickness can be achieved by merging the resistive sheet and the periodic structure to have resistive cell elements [10].

In this paper it is shown that by decreasing the thickness of the absorber from the conventional quarter-wavelength to lower levels, the accuracy of the TML model decreases significantly and the model is not applicable below a certain $d/\lambda$ ratio. Different methods and approaches both in the time and frequency domain are applied to provide a comprehensive explanation for the absorption mechanism of ultra thin absorbers. At small $d/\lambda$ ratios a perceptible non-transverse electric field component is produced by high order evanescent Floquet modes. This dominant component is
Figure 1: The structure of the ultra thin absorber a) The side view. b) The front view. c) Circuit model for TM mode.

not taken into account in an ordinary TML model. It results in a lossy standing wave pattern underneath the resistive element. The phenomena can be illustrated by the damping oscillation behavior of the scattered field in time domain. Close agreement between the resonance frequency of a single PEC patch element and absorption frequency of the corresponding periodic array ultra thin absorber verifies that the absorption mechanism is due to a lossy resonance. It is shown that for sufficiently large space between the array elements where the cavity mode is the dominant field distribution, the absorber is able to operate even in finite size configurations.

2 Shortcomings of Transmission Line Model

It is well-known that for sufficiently large $d/\lambda$ ratios (about 1/4) the frequency response of the absorber can be simulated accurately by the TML model. In the model, the scattering (reflection, transmission) properties of the periodic array is modeled by its equivalent surface impedance. This approach is used widely in design of circuit analog absorbers [12]. Engheta [5], utilizes the same concept for formulating the reflection properties of the ultra thin absorbers. It is shown here that the approach is not applicable for small $d/\lambda$ ratios.

To proceed, a 1D ultra thin absorber formed by strip gratings is considered, see Fig. 1. For this geometry only TM polarization exhibits absorption and the equivalent surface impedance of the grating for this polarization is a series RC network [9]. Therefore, the TML model of the absorber (if applicable) is a series RC network parallel with a short circuit transmission line, see Fig. 1(c). Since ultra thin absorbers are narrowband (e.g. see Figs 7,6), accurate estimation of the absorption frequency is vital for a realistic model. One way to examine the accuracy of the TML model, is to compare the absorption frequency of the TML model to an accurate full wave simulation. The relative absorption frequency error for a typical strip grating is shown in Fig. 2 as a function of $d/\lambda$ ratio. The exponential growth of the
error at small \( d/\lambda \) ratios demonstrates clearly the disability of the TML approach for analyzing the ultra thin absorbers. In the following parts the reasons for the inaccuracy of the TML model are revealed and a comprehensive explanation for the absorption mechanism is provided.

The strip grating absorber of the above example can be analyzed by mode matching technique. By expanding the fields in Floquet modes and investigating the behavior of the modes at the absorption frequency, valuable information about the physics of the absorption process can be obtained. Normal incidence in the direction of \( z \) in Fig. 1 is considered. Therefore, excitation of any field component in the \( z \) direction is referred as a non-transverse component. According to Fig. 1, the geometry of the problem is \( y \)-coordinate independent. Therefore, the fields for the TM mode can be expanded by Floquet modes as:

\[
E_{x,m}(x, z) = \sum_n \left\{ \alpha^+_{mn} e^{j k_{z, mn} z} + \alpha^-_{mn} e^{-j k_{z, mn} z} \right\} e^{j k_{x, n} x} \tag{2.1}
\]

\[
E_{z,m}(x, z) = \sum_n \left\{ \left( -\frac{k_{x, n}}{k_{z, mn}} \right) \alpha^+_{mn} e^{j k_{z, mn} z} + \left( \frac{k_{x, n}}{k_{z, mn}} \right) \alpha^-_{mn} e^{-j k_{z, mn} z} \right\} e^{j k_{x, n} x} \tag{2.2}
\]

\[
H_{ym}(x, z) = \sum_n \left\{ \left( \omega \epsilon_m \right) \alpha^+_{mn} e^{j k_{z, mn} z} + \left( -\omega \epsilon_m \right) \alpha^-_{mn} e^{-j k_{z, mn} z} \right\} e^{j k_{x, n} x} \tag{2.3}
\]

where

\[
k_{x, n} = \frac{2\pi n}{a}, \quad n = \ldots, -1, 0, 1, \ldots \tag{2.4}
\]

\[
k_{x, n}^2 + k_{z, mn}^2 = \omega^2 \mu_0 \epsilon_m, \quad \text{Im}(k_{z, mn}) \geq 0 \quad m = 1, 2 \tag{2.5}
\]

In the above equations index \( m \) represents the region, which can be air or the substrate, see Fig. 1. The index \( n \) refers to the Floquet mode number. By imposing the boundary conditions, the unknown coefficients are calculated. The mode index \( n = 0 \) refers to the propagating TEM mode which is used in the TML model. This mode contains no \( z \)-directed component at normal incidence. The excitation of a \( z \)-component electric field for mode indices \(| n | \geq 1\) is of great importance. For the conventional quarter-wavelength layer, the amplitude of \( E_z \) is small in comparison to the transverse component of the E-field and takes a negligible share in the reactive part of the grating impedance. But this is not the case for small \( d/\lambda \) ratios, where the correct modeling of the absorption mechanism requires full consideration of this dominant component. The following example illustrates the significance of the non-transverse component.

For a fixed value of strip grating width \((w)\) and periodicity \((a)\), the equivalent surface impedance of the grating is calculated. The calculated capacitance is referred as \( C_{\text{grating}} \). Then the transmission line model is formed using the calculated impedance of the grating and the exact physical thickness of the absorber \( l = d \), see Fig. 1(c). Obviously the TML model frequency response differs from the full wave simulation for small \( d/\lambda \) ratios. Next the parameters of the transmission line model,
the capacitance \((C)\) and transmission line length \((l)\), are adjusted such that it results in the same frequency response as the full wave analysis. Henceforth, this new circuit model is referred to as the adjusted TML model and its capacitance as \(C_{\text{adj}}\). In the adjusted TML model the increase of its capacitance \((C_{\text{adj}})\) value from the grating capacitance \((C_{\text{grating}})\) level may represent the growth of the \(E_z\) component. The relative capacitance increase as a function of \(d/\lambda\) ratio for a typical grating is shown in Fig. 3. The adjusted capacitance can be considered as a superposition of the grating capacitance which is formed between adjacent array elements and an extra capacitance formed by the \(E_z\) component between the ground plane and the resistive element. The \(E_z\) component and consequently the extra capacitance are thickness dependent. This effect is not considered in an ordinary TML model which is formed by just considering the grating capacitance as the reactive part of the equivalent impedance.

Another way to illustrate the existence and significance of the \(E_z\) component in the absorption process is to investigate the amplitudes of the higher order Floquet modes that contribute to the expansion of the \(E_z\) component inside and outside of the absorption band, see Eq. 2.2. For this purpose a strip grating ultra thin absorber \((w/a = 0.6, d/\lambda \simeq 1/54)\) is considered with absorption frequency 5.53 GHz. The normalized amplitudes (to the incident wave) of high order Floquet modes (excluding mode number zero corresponding to the propagating TEM mode) at/out of the absorption frequency band are shown in Fig. 4. The level of mode amplitudes is 15 dB higher in average at the absorption frequency, implying the major role of high order modes in the absorption process. The high level of mode amplitudes at the absorption frequency results in a strong \(E_z\) component which has an \(x\)-directed (Fig. 1) complex valued propagation wavenumber, producing damped standing wave pattern (damped oscillation in time domain) under the resistive strip(s). In other words, the absorption is due to transfer of power from the incident TEM mode to a lossy cavity (arrays of cavities) supporting TM mode at the resonance frequency. This idea is demonstrated and verified in the following sections.

2.1 Time Domain Analysis

As the \(d/\lambda\) ratio decreases the non-transverse component of the electric field \((E_z)\) strengthens and results in a standing wave pattern under the resistive element at absorption frequency. This phenomena can be seen clearly in the time domain. A strip grating ultra thin absorber is considered with the frequency response of Fig. 6 (the solid curve). The absorber is illuminated by a wideband Gaussian signal, see the dashed curve of Fig. 5. In the time domain solver a test point is selected close to the edge of the resistive element inside the substrate. The time domain response of the \(E_z\) component of the scattered field at the test point is shown in Fig. 5. It should be noted that the electric field of the input signal is oriented in the \(x\) direction but for the ultra thin absorber of our example \((d/\lambda \simeq 1/45)\) the scattered field at the test point is dominated by the \(z\)-component. The damped oscillatory behavior of the output signal \((E_z\) at test point) implies a lossy resonance which can be modeled by a complex pole in the \(\omega\)-plane. This can be verified by observing the normalized
Figure 2: The relative absorption frequency error of the TML model compared to full wave simulation, as a function of thickness to wavelength ratio. \((a = 25 \text{ mm}, w = 15 \text{ mm}, \epsilon_r = 1)\)

Figure 3: The relative capacitance increase \((\frac{C_{\text{adj}}}{C_{\text{grating}}})\) as a function of thickness to wavelength ratio. \((a = 40 \text{ mm}, w = 27 \text{ mm}, \epsilon_r = 1.2)\)
Figure 4: Comparison of the amplitude of higher order modes (Mode Number $\geq 1$) at the absorption frequency ($f = 5.53$ GHz) and out of absorption band ($f = 3.5$ GHz)

spectrum of the output signal in Fig. 6 (the circled curve). An interesting feature of Fig. 6 is the simultaneous occurrence of absorption (reflection coefficient deep) and peak of the test point signal spectrum. This emphasizes the fact that the absorption is due to a transition of the real valued $z$-directed wavenumber of the incident wave to a complex $x$-directed wavenumber that dissipates the power.

The complex pole of the time domain oscillation and the complex wavenumber of the standing wave are estimated accurately by eigenmode analysis of the cavity models of the next section.

3 TM Mode Cavity Model

It was shown that in an ultra thin absorber a resonance occurs in the substrate. This implies that a relation must exist between the resistive element dimensions and the resonance wavelength. An example is provided to verify this relationship. Consider an ultra thin absorber with a periodic array of resistive square patches [9]. For fixed values of width, substrate thickness and periodicity ($w=15$ mm, $d=1$ mm, $a=30$ mm) the absorber frequency response is calculated for different substrate permittivities. The narrowband frequency response of the absorber can be modeled by a complex pole in the frequency plane. For two different substrate permittivity the complex poles are given in the Table.1 ($f_{\text{absorption}}$).

Next a single perfect conductor, square patch antenna is considered with the same width, substrate thickness and permittivity as above. There are many cavity models for estimating the resonance frequency of the patch antenna [1, 2, 7]. Among the models, the cavity model with the perfect magnetic conducting (PMC) walls is
**Figure 5:** Time domain response of the $E_z$ component at the test point.

**Figure 6:** Frequency response of the absorber (solid curve) and the normalized spectrum of the $E_z$ component at test point (circled curve).
Table 1: Comparison of the resonant frequency of cavity model (GHz) for a single square patch and absorption frequency of the corresponding ultra thin absorber (GHz).

<table>
<thead>
<tr>
<th>$\epsilon_r$</th>
<th>$f_{\text{resonance}}$ (Enhanced Cavity Model)</th>
<th>$f_{\text{absorption}}$ (Full wave simulation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6.73 + j0.18</td>
<td>6.59 + j0.23</td>
</tr>
<tr>
<td>3.2</td>
<td>5.35 + j0.09</td>
<td>5.32 + j0.15</td>
</tr>
</tbody>
</table>

the simplest one, which results in real value wave-numbers [1]. An enhanced cavity model is proposed by [7], which is based on Carver and Coffey’s [2] design equation using modal-expansion technique. The method considers an admittance condition at the walls (taking the radiation resistance into account), resulting in a more accurate complex wave-number. This model is applied to the square patch of our example ($w=15\,\text{mm}, d=1\,\text{mm}$). The values obtained for the resonance frequencies are tabulated in Table 1. Comparison of the absorption frequency of the ultra thin absorber to the resonance frequency of the cavity model for the corresponding single element patch antenna, reveals many facts. First of all, it verifies the idea of occurrence a TM mode resonance (excitation of a dominant $E_z$ component) at small $d/\lambda$ ratios in ultra thin absorbers. Secondly, it provides a reliable estimation of the absorption frequency (less than 3% error), which is significantly better than the TML model, see Fig. 2. This small error is expected since in the cavity model the resistivity of the patch element is ignored. Finally, for $w/a$ ratios not close to 1, where the capacitance formed at the gap of neighboring elements is smaller than the capacitance formed by the dominant $E_z$ component of the scattered field and the ground plane, the absorption frequency of the periodic array can be estimated accurately from the resonance of a single element patch. This suggests that ultra thin absorbers should be able to operate in finite size array configurations, independent of neighboring elements. This is examined in the next section.

4 Finite Size Ultra Thin Absorber

It was shown that for sufficiently large gaps between neighboring elements ($w/a < 0.8$), the cavity model of a single patch can estimate the absorption frequency of the periodic array ultra thin absorber accurately. It suggests that the mutual coupling between neighboring elements are imperceptible and the dominant field distribution is formed by the eigenmode of the cavities. Since the TM mode resonance has a localized field distribution close to the vicinity of the resistive element, such an ultra thin absorber should be able to absorb power even in finite size array configurations. This feature of ultra thin absorbers is important for practical applications.

To illustrate the above statements a square patch periodic absorber is considered [9]. The frequency response of the absorber is plotted in Fig. 7. The resonance frequency of the absorber is $f_r = 6.06\,\text{GHz}$. Next a finite size array formed by the unit cell of the periodic structure is considered. The unit cell is composed of a resistive square patch with width $w$ over a finite size square shape ground plane.
of width $a$ (periodicity of the ultra thin absorber). The RCS of a $3 \times 3$ elements array and also a single element patch absorber are calculated at the resonance frequency and out of the absorption band of the periodic array. The polar plots of these RCS values are shown in Figs. 8 and 9. As seen from the figures the finite size arrays exhibit significant RCS reduction in the backscattering direction at the resonance frequency. The single element absorber clearly verifies the existence of a local lossy resonance for which the frequency and spatial field distribution are almost independent of neighboring elements.

5 Conclusion

Ultra thin absorbers are studied in detail and some new features of their operation mechanism are introduced. It is shown that the conventional TML model approach is not applicable for this type of absorbers. Therefore, full wave analysis both in the time and frequency domains are applied to provide a correct explanation of the absorption physics. It is shown that large amplitudes of higher order Floquet modes and consequently excitation of a perceptible non-transverse electric field at the absorption frequency are the main reasons of the TML model inaccuracy. Time domain simulation of the absorber demonstrates the occurrence of a resonance phenomena in the vicinity of the resistive element. The cavity model of a single element patch antenna can estimate the absorption frequency accurately and verify the idea of a TM mode resonance. For sufficiently large gaps between array elements it is shown that the absorber can operate even in finite size extensions.
Figure 8: Comparison of RCS for single element patch absorber at and out of resonance frequency band. ($f_r = 6.06$ GHz.)

Figure 9: Comparison of RCS for 3 by 3 elements patch array absorber at and out of resonance frequency band. ($f_r = 6.06$ GHz.)
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