Transmission line theory with application to distribution cables

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1 Introduction

The motivation for this report is the following. It has been noted that for distribution cables, the loss angle for the zero sequence impedance increase with length. The zero sequence resistance is important for the earth fault protection in Petersén earthed system. A key issue is to explain the physics behind this effect. In simulations it has been noted that the resistive losses increase. Can this be explained by analysis, or is it simply caused by limitations in the simulation models?

2 Transmission line theory

Suitable references for transmission line theory are [1] and [2].

Assume that the line, or cable, has constant parameters with notations, *c* is capacitance in *F/km*, l is inductance in *H/km* and *r* is resistance in *ohm/km*. Constant parameters means that we ignore frequency dependence in resistance and inductance. Skinn-effect cause the resistance to change with frequency. The return path of the zero sequence current is influenced by frequency, so that zeros sequence resistance and inductance are influenced. At higher frequencies the zero sequence return current goes closer to ground surface that significantly increase resistance but slightly decrease zero sequence inductance.

If needed, it is straightforward to include frequency dependent *r* and *l* in the analysis.

Another aspect that might be needed fro higher frequencies, say over 100 kHz, is to include the damping effect of the cables semi-conducting layers that is located between conductor and insulation. Publication [3] shows that these layers significantly influence the damping of higher frequencies.

2.1 Wave equations

Consider a general model of a transmission line, were the physical component could be either an over-head line, or a cable with the following parameters for series impedance and shunt admittance. The parameters can be for either positive sequence, or zero sequence.

$$z = r + j\omega l$$

$$y = j\omega c$$

Two equations describe voltage, $U(x)$, and current $I(x)$ at a given position *x* along the line

$$\frac{dU(x)}{dx} = z\,I(x)$$

$$\frac{dI(x)}{dx} = yU(x)$$

2.1.1 Wave propagation constant

Introduce the following definitions. The wave propagation constant is

$$\gamma = \sqrt{z\,y}$$

2.1.2 Characteristic impedance

The characteristic impedance is

$$Z_C = \sqrt{\frac{z}{y}}$$

2.1.3 Wave velocity

The wave velocity is

$$v = \frac{1}{\sqrt{l\,c}}$$

2.1.4 Wave length

The wave length for a specific frequency f is

$$\lambda = T\,v = \frac{v}{f}$$

2.2 ABCD parameters of distributed line

The relation between voltage and current at the sending end, index *s*, and the receiving end, index *r*, is

$$\begin{pmatrix} U_S \\ I_S \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} U_R \\ I_R \end{pmatrix}$$

with

$$A = D = \cosh(\gamma\,d) \qquad \text{per unit}$$

$$B = Z_C \sinh(\gamma\,d) \qquad \text{ohm}$$

$$C = \frac{1}{Z_C}\sinh(\gamma\,d) \qquad \frac{1}{\text{ohm}}$$

where d is the distance between sending and receiving end.

2.3 Line with load at receiving end

Consider the case with a load impedance Z_R connected at the receiving end. Using the ABCD-parameters

$$\begin{pmatrix} U_S \\ I_S \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} U_R \\ I_R \end{pmatrix}$$

and that $U_R = Z_R I_R$ gives

$$U_S = (A Z_R + B)\,I_R$$
$$I_S = (C\,Z_R + D)\,I_R$$

We are interested of the impedance seen from the sending end, thus

$$Z_S = \frac{U_S}{I_S} = \frac{(A Z_R + B)}{(C Z_R + D)}$$

Open line end means that $Z_R \to \infty$, then $Z_S^{\text{open}} \to \dfrac{A}{C}$.

Short circuited line end means that $Z_R = 0$, then $Z_S^{\text{ksl}} = \dfrac{B}{D}$.

2.4 Minimal impedance magnitude for line with receiving end open

For a line with open remote end,

$$Z_S = Z_C \frac{\cosh(\gamma d)}{\sinh(\gamma d)} = Z_C \frac{\exp(\gamma d) + \exp(-\gamma d)}{\exp(\gamma d) - \exp(-\gamma d)}$$

with

$$Z_C = \sqrt{\frac{z}{y}}$$

The task is to find the distance *d* that minimize $|Z_S|$.

Since Z_C is independent of the length *d*, it is equivalent to study the impedance

$$Z_1 = \frac{\exp(\gamma d) + \exp(-\gamma d)}{\exp(\gamma d) - \exp(-\gamma d)}$$

The wave propagator $\gamma = \sqrt{z\,y}$ is rewritten as a complex number as $\gamma = a + jb$, then

$$Z_1 = \frac{\cos(bd)\left[\exp(a\,d) + \exp(-a\,d)\right] + j\sin(bd)\left[\exp(a\,d) - \exp(-a\,d)\right]}{\cos(bd)\left[\exp(a\,d) - \exp(-a\,d)\right] + j\sin(bd)\left[\exp(a\,d) + \exp(-a\,d)\right]}$$

We rewrite in hyperbolic functions

$$Z_1 = \frac{\cos(bd)\cosh(ad) + j\sin(bd)\sinh(ad)}{\cos(bd)\sinh(ad) + j\sin(bd)\cosh(ad)}$$

Use the notation *M* for the square magnitude, that is $M = |Z_1|^2$ gives

$$M = \frac{1 + \cosh(2ad) - 2\sin^2(bd)}{1 + \cosh(2ad) - 2\cos^2(bd)}$$

The task is to find the distance *d* that gives the minimal value of $|Z_1|$ is equivalent to find the *d* that gives the minimal value of M. Rewrite

$$M = \frac{\cosh(2ad) + 1 - 2\sin^2(bd)}{\cosh(2ad) - 1 + 2\sin^2(bd)}$$

Calculating the derivative with respect to the distance *d* gives

$$\frac{dM}{dd} = \frac{d}{dd}\left(\frac{f}{g}\right) = \frac{f'g - g'f}{g^2}$$

$$f = \cosh(2ad) + 1 - 2\sin^2(bd)$$

$$f' = 2a\sinh(2ad) - 4b\sin(bd)\cos(bd)$$

$$g = \cosh(2ad) - 1 + 2\sin^2(bd)$$

$$g' = 2a\sinh(2ad) + 4b\sin(bd)\cos(bd)$$

We want $\frac{dM}{dd} = 0$ for extreme points, thus we focus on $f'g - g'f = 0$

$$f'g - g'f =$$
$$[2a\sinh(2ad) - 4b\sin(bd)\cos(bd)][\cosh(2ad) - 1 + 2\sin^2(bd)]$$
$$-$$
$$[2a\sinh(2ad) + 4b\sin(bd)\cos(bd)][\cosh(2ad) + 1 - 2\sin^2(bd)]$$

Simplifying

$$f'g - g'f = 4a\sinh(2ad)[-1 + 2\sin^2(bd)] - 8b\sin(bd)\cos(bd)\cosh(2ad)$$

$$\frac{d|Z_3|}{dd} = 0 \Rightarrow$$

Direct calculation leads to

$$a\sinh(2ad)\cos(2bd) + b\cosh(2ad)\sin(2bd) = 0$$

Rewrite as

$$A\sin(2bd + \theta) = 0$$

with

$$\theta = \arctan\frac{a\sinh(2ad)}{b\cosh(2ad)} = \arctan\left(\frac{a}{b}\cdot\tanh(2ad)\right)$$

To find the solution we need to solve $2bd + \theta = \pi$, that is, solve the equation

$$\arctan\left(\frac{a}{b}\cdot\tanh(2ad)\right) = \pi - 2bd$$

At this moment, it is an open question how to solve this equation analytically. A practical approach is to use a simple numerical iteration schemes. One iteration scheme to find the distance *d* that gives minimal $|Z_S|$ is:

$$d_1 = \frac{\pi}{2b}$$

$$\theta_k = \arctan\left(\frac{a}{b}\cdot\tanh(2ad)\right)$$

$$d_{k+1} = \frac{\pi - \theta_k}{2b}$$

Example

$$a = \mathrm{Re}(\gamma) = 0.0063$$

$$b = \mathrm{Im}(\gamma) = 0.0124\,,$$

$$d_1 = \frac{\pi}{2b} = 126.5\,\mathrm{km}$$

$$\theta_1 = \arctan\left(\frac{a}{b}\cdot\tanh(2ad_1)\right) = 0.4365\,\mathrm{rad}$$

$$d_2 = \frac{\pi - \theta_1}{2b} = 108.9\,\mathrm{km}$$

$$\theta_2 = \arctan\left(\frac{a}{b}\cdot\tanh(2ad_2)\right) = 0.4191\,\mathrm{rad}$$

$$d_3 = \frac{\pi - \theta_2}{2b} = 109.6\,\mathrm{km}$$

The iteration scheme converges quickly. A plot confirms that the distance 109.6 km gives minimal $|Z_S|$.

2.5 Minimal impedance for lossless line

To check that the result above is reasonable, we assume a lossless line, that is $a = 0$, then

$$\frac{d|Z_3|}{dd} = 0 \Leftrightarrow \sin(2bd) = 0$$

We exclude the trivial case when d=0, then

$$2bd = n\pi \quad \text{for} \quad n = 1,2,3...$$

So

$$d = n\frac{\pi}{2b} \quad \text{for} \quad n = 1,2,3...$$

The constant b is the imaginary part of the wave propagator, without losses

$$b = \mathrm{Im}(\lambda) = \mathrm{Im}(\sqrt{z\,y}) = \omega\sqrt{lc}$$

This gives

$$d = n\frac{\pi}{2\omega\sqrt{lc}} = n\frac{1}{4f_0\sqrt{lc}} = n\frac{v}{4f_0}$$

with

$$v = \frac{1}{\sqrt{lc}}$$

For our line we have

$$c_0 = 0.3317 \cdot 10^{-6} \ \text{F/km}$$

$$l_0 = 3.5 \cdot 10^{-3} \ \text{H/km}$$

This gives

$$v \approx 29.35 \cdot 10^3 \ \text{km/s}$$

We calculate *d* for $f_0 = 50\,\text{Hz}$ and n=1, thus

$$d_1 = \frac{29.35 \cdot 10^3}{4 \cdot 50} \approx 147\,\text{km}$$

$$d_k \approx k \cdot 147\,\text{km} \quad \text{for k} = 1, 2, 3...$$

References

[1] Hallén, E., *Electricitetslära*, Erik Hallén, Stockholm, 1953.

[2] Glover, J. D. et al, *Power Systems Analysis and Design*, Fourth Edition, Thomson., 2008.

[3] Steinbrich, K., “Influence of semiconducting layers on the attenuation behaviour of single-core power cables”, IEE Proceeding on Generation, Transmission and Distribution, Vol. 152, No. 2, March 2005.