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Coordination incentives, performance measurement, and resource allocation in public sector organizations*

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Abstract

Why are coordination problems common when public sector organizations share responsibilities, and what can be done to mitigate such problems? This paper uses a multi-task principal-agent model to examine two related reasons: the incentives to coordinate resource allocation and the difficulties of measuring performance. The analysis shows that when targets are set individually for each organization, the resulting incentives normally induce inefficient resource allocations. If the principal impose shared targets, this may improve the incentives to coordinate but the success of this instrument depends in general on the imprecision and distortion of performance measures, as well as agent motivation. Besides decreasing available resources, imprecise performance measures also affect agents' possibility to learn the function that determines value. Simulations with a least squares learning rule show that the one-shot model is a good approximation when the imprecision of performance measures is low to moderate and one parameter is initially unknown. However, substantial and lengthy deviations from equilibrium values are frequent when three parameters have to be learned.

Keywords: Public sector organizations; Coordination incentives; Performance measurement; Shared targets; Learning

JEL codes: D23, D73, D83, H11, H83

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1 Introduction

The political scientist Harold Seidman once referred to the quest for coordination in public administration as being the "twentieth-century equivalent of the medieval search for the philosopher's stone" (quote from Wilson (1989, p. 268)); a colorful illustration of the recurring theme of coordination problems in public sector organizations.[1] This paper uses a simple principal-agent model to scrutinize two closely connected reasons for why coordination problems may be more common in public sector organizations – the difficulties of accurately measuring performance, and the incentives to coordinate resource allocation when responsibilities for activities are shared – and to discuss potential remedies.

Over the last two decades, the governing of public organizations in most Western countries has moved from a reliance on rules and procedures towards management by objectives (Propper and Wilson, 2003; Andersen et al., 2008; Verbeeten, 2008). As this development entails an increased reliance on performance measures, it is important to include the effects of imperfect measures when analyzing coordination problems in public sector organizations. While accurately measuring outcomes is a problem in all organizations, it is in general more difficult in public sector organizations compared to firms. One part of the measurement problem is the lack of adequate summary measures of value in public sector organizations. There is for example no equivalent to a firm's stock value (Baker, 2002). Another issue is that performance measurement follows budget periods, whereas the relevant outcomes frequently materialize over longer periods of time. Consequently, performance is often measured with considerable imperfection in terms of both distortion (bias) and imprecision (variance) in public sector organizations (Propper and Wilson, 2003).

Imperfect measures also affect the possibilities of designing incentive systems. Seminal models of multi-task, incomplete information environments by Holmström and Milgrom (1991) and Baker (1992) show that the optimal strength of incentive schemes is relatively low-powered when task outcomes are measurable to different degrees. Incentive-pay not only allocates risks and motivates agents but also serves as an effort allocation mechanism among different tasks. With high-powered incentives, the agent's effort is excessively driven towards easy-to-measure tasks that can

[1] As an example, during a one-year period, articles about coordination problems among the following organizations appeared in the opinion pages of Sweden's largest daily newspaper ("Dagens Nyheter Debatt"): compulsory institutional and non-institutional psychiatric care (2009-08-09); schools and social services (2008-02-09); organizations treating substance abusers (2009-05-27); organizations handling land, sea and air-traffic infrastructure (2009-04-01); organizations handling fishing, sea resources and victual safety (2009-02-05); organizations supervising social services (2009-02-02); organizations involved in health and dental care for schoolchildren (2008-12-18); organizations responsible for psychiatric care of children (2008-11-19); and organizations working to stop football associated violence (2008-09-15).

form the basis of incentive pay. Indeed, as measurement problems typically require more muted incentives, such problems are a common justification for an activity to be the responsibility of a public sector organization (Acemoglu et al., 2008).

Another feature of importance for the possibilities to solve coordination problems is that most public sector organizations do not sell their services and products at market prices, or make profits. Consequently, the price mechanism and cross-unit incentive schemes based on profit sharing is not available to coordinate activities when organizations are interdependent. Public sector organizations are overall very limited in their use of monetary incentives (Burgess and Ratto, 2003; Propper and Wilson, 2003; Heinrich and Marschke, 2010), and the source of motivation is implicit and/or intrinsic rather than explicit incentive schemes (Wilson, 1989).

In the model developed here, the principal determines performance measures while two agents decide on how to allocate resources between two types of activities each – one where responsibility is shared (*joint activities*), and another for which one agent is solely responsible (*core activities*). To analyze coordination incentives in the presence of measurement problems, the model incorporates interdependent agents – as in e.g. Itoh (1991); Kretschmer and Puranam (2008); Baiman and Baldenius (2009) – in a Holmström and Milgrom (1991) type of model with imprecise and distorted performance measures. In line with the discussion above, I assume that the principal does not use monetary incentives and that agents are motivated. Recent principal-agent models include agents that are motivated by career concerns (Dewatripont et al., 1999; Acemoglu et al., 2008), identification with organizational objectives (Akerlof and Kranton, 2005), social esteem (Ellingsen and Johannesson, 2008), or are pro-socially motivated, either in the sense that the agent derives utility from producing (often called "warm-glow" altruism) or that the agent cares about the output ("output-oriented" or pure altruism) (Francois and Vlassopoulos, 2008).[2] The source of agents' motivation in my model may be interpreted as any of these examples.

Many, but not all, public sector organizations and some private organizations fit this description.[3] In the following, I use the term 'public sector organization' to denote organizations where non-market operation, motivated agents, and measurement problems are present. The use of budgets to determine resource allocation and information asymmetries between principals and agents are also of consequence

[2]Examples of models where agents are "warm-glow" altruists include Besley and Ghatak (2005); Prendergast (2007); Delfgaauw and Dur (2008) and Makris (2009), while models of agents with output-oriented altruism include Francois (2000); Glazer (2004), and Gailmard and Patty (2007). For evidence of motivated agents in public services see e.g. Perry and Wise (1990); Houston (2006); Gregg et al. (2011); Kolstad and Lindkvist (2012); Dur and Zoutenbier (2013).

[3]Departments of larger corporations, such as research and development, and administrative departments, which do not sell anything directly to customers, may be examples from the private sector.

for the model, but these are key characteristics of both private and public sector organizations.

I first use a one-shot game to analyze coordination incentives when targets are set individually for each agent and measures are undistorted and precise. The results show that when activities are interdependent among agents incentives that distort the allocation of resources away from efficient levels are normally present. This suggests one potential remedy: *sharing targets* between agents. Shared targets has been tried as a part of for example the New Labour government's efforts to create "Joined-up Government" in the United Kingdom (Politt, 2003; Bogdanor, 2005; Moseley and James, 2008), but targets used to evaluate performance are normally not shared between public sector organizations (e.g. Knapp et al., 2006).[4] I have neither found a quantitative, empirical examination, nor a formal, theoretical treatment of how sharing targets across organizational boundaries affect coordination incentives. Compared to other potential remedies such as vertical and horizontal integration, shared targets also have the advantage of being easily implemented.

Shared targets align incentives in a similar way to a profit sharing scheme – by rewarding performance ex post. An important difference to profit sharing is that the strength of the incentives created by shared targets is not controlled to the same extent by the principal, as the mechanism relies on implicit and/or intrinsic motivation. The analysis shows that imposing shared targets always improve efficiency when performance measures are undistorted and precise, and agents' motivation is aligned with the principal's interests. In general though, the effects depend on the interplay of motivation and the distortion of performance measures, as well as the relative importance of the tasks for value. For activities that are complements (and vice versa for substitutes), shared targets have their best chance of improving efficiency in situations with agents who are more motivated by core activities, and/or use performance measures that overestimate the value of such activities, as this normally exacerbates the coordination problems. However, while this result holds for a broad range of parameter values, it does not hold for all and agents who are more motivated by core activities may in some situations actually allocate higher shares of the resources to joint activities.

Turning to the other source of measurement problems, imprecision in the form of variance unambiguously decreases value (as in e.g. Feltham and Xie, 1994; Baker, 2002). Higher variance implies more risk borne by agents, which leads risk-averse agents to demand higher wages. Higher wages in turn decrease the available resources and therefore also decrease the value created. As sharing targets implies responsibility for more performance measures and thus increases total variance, this

[4] It is hardly a new idea though; Hood (2005, p. 35) mentions that already in 1650, imperial China introduced a practice of holding one officeholder responsible and punished (or rewarded) for the actions of another.

decreases the usefulness of shared targets (at least in the cases where agents' wages constitute a non-negligible share of total resources). Imprecision may also have another consequence: if agents do not know their value functions in every detail, noisy measures may make it difficult for agents to learn how to allocate resources efficiently. To examine this issue, a learning rule needs to be added to the model. A problem is that there is no consensus in the earlier literature on which learning rules players actually use in games.[5] Furthermore, game-theoretical learning rules, like for example the experience-weighted attraction rule (Camerer and Ho, 1999) and individual evolutionary learning (Arifovic and Ledyard, 2004, 2011) normally include evaluation of hypothetical strategies. The connection between the choice of strategies and outcomes is thus known to the agents, but this connection is precisely what the agents in my game do not know and have to estimate. For these reasons, I use a similar rule to the adaptive learning models in the macroeconomic literature, where the agents behave as econometricians in order to estimate unknown parameters (e.g. Sargent, 1993; Evans and Honkapohja, 2009).

The results from simulations of a repeated version of the model show that if performance measures are not too imprecise, the allocated shares are close to equilibrium values and the one-shot model is a rather good approximation. However, with three initially unknown parameters and more noisy measures there can be substantial and lengthy deviations from equilibrium values. The interdependence among activities provides an added dimension of learning difficulties, as agents are affected by each other's learning. The sizes of these deviations are such that they may dwarf the problems created by coordination incentives. Therefore, the results imply that imprecise performance measures in one organization may be a concern also for other interdependent organizations, and that investing substantial resources to develop more precise measures may be worthwhile.

The next section describes the basic model. Section 3 contains benchmark results in a situation with an informed principal under conditions when the agents' motivation is aligned to principal's interest, and each individual performance measure is precise and undistorted. Section 4 examines the effects of distortion and misaligned motivation, and section 5 the effects of imprecision. Section 6 contains concluding remarks.

2 A model of resource allocation

This section presents the basic set up for the model. The model includes three players, one principal and two agents $i = 1, 2$. For instance, the principal could be

[5] See Camerer (2003) for an overview of experimental results, and Salmon (2001) and Wilcox (2006) for the difficulties of estimating learning rules in experiments.

a political committee, and the agents two managers of sub-units, where some part of the services is a shared responsibility. Examples include important public sector organizations such as schools and social services, and hospitals and primary care units (see footnote 1 for more examples). While the principal has the authority to design the structure of resource allocation and rewards, the relationship between the agents is not hierarchical.

The principal is interested in maximizing the total value of services given the amount of resources available. Total resources are denoted R and are normalized to 1. The services provided by both agents consists of two parts: activities in set A_i (*core activities*) are directed towards target groups that are solely the responsibility of agent i, whereas activities in set B_i (*joint activities*) are directed towards target groups where responsibility is shared between the agents (e.g. all children of certain ages in contact with social services are also students in some school). It is not possible, due to information asymmetries, to contract directly upon delivery of specific activities. The principal therefore allocates resources (R_i) in advance to the agents, such that $R = R_1 + R_2$. The agents receive a fixed wage, w_i, which is taken out of R_i. Agents allocate the remainder of the resources, $r_i = R_i - w_i$, between activities in A_i and B_i. Let $a_i \in [0, r_i]$ be the share of agent i's resources allocated to core activities, and $b_i \in [0, r_i]$ the share allocated to joint activities. As R is normalized to 1, R_i, w_i, r_i, a_i and b_i should be interpreted as shares of total resources. For each set of activities, let the (real-valued) functions mapping resource allocations to value be

$$V(A_i) = \theta a_i + \tau_A a_i b_i \tag{1}$$

and

$$V(B_i) = \rho b_i + \tau_B a_i b_i + \varphi b_i b_j \tag{2}$$

which yields the combined value function for each agent i

$$V_i = V(A_i) + V(B_i) = \theta a_i + \rho b_i + \tau a_i b_i + \varphi b_i b_j \tag{3}$$

where $\tau = \tau_A + \tau_B$. Total value is $V = V_1 + V_2$. Variations of this formulation are fairly common in organizational economics and in models of interdependent agents.[6] For my purposes, I believe it captures important trade-offs faced by managers of public sector organizations, and how interdependence is of vital importance for coordination problems, as interactions of activities within an organization (the term $\tau a_i b_i$) and between organizations ($\varphi b_i b_j$) are included. Following e.g. Siggelkow

[6] See for example Marschak and Radner (1972); Cremer (1990); Siggelkow (2002), and Kretschmer and Puranam (2008). All results in the paper hold qualitatively for a value function with negative, squared terms of a and b, which are often added to model decreasing returns in models that lack a budget constraint.

(2002), two arguments of a value function are said to be interdependent if the cross-partial derivative is different from zero. Furthermore, they are complements if this derivative is positive and substitutes if it is negative. The interdependence between arguments x and y is stronger than between y and z if $|\frac{\partial^2 V}{\partial x \partial y}| > |\frac{\partial^2 V}{\partial y \partial z}|$. In (3), the stronger the interdependence between activities in A_i and B_i, the higher the $|\tau|$, and the stronger the interdependence between activities in B_i and B_j, the higher the $|\varphi|$. I assume everywhere that agents are identical. To simplify notation, the indexes denoting agent i and j are subsequently omitted whenever possible.

I impose a few restrictions on the parameters: θ, ρ, τ are all > 0 and such that a strictly positive amount of resources is allocated: $a = b = 0$ is thus ruled out. φ can take on both positive and negative values, reflecting that activities b_1 and b_2 could be both complements and substitutes. I also assume $\tau \geq |\varphi|$, which rules out inefficiencies created because the basic division of labor is sub-optimal. That is, if $\tau < |\varphi|$ one could argue that it would be better to break up the organizations into three and pool activities in B_1 and B_2 into one organization.

As value cannot be directly observed, agents maximize value as measured by a number of performance measures. Following Baker (2002), performance measures have two dimensions of imperfection: *imprecision* and *distortion*.[7] A measure is imprecise if it is measured with noise, but is otherwise unbiased. A distorted measure is biased. Let $P = \{p^{A_1}, p^{B_1}, p^{A_2}, p^{B_2}\}$ be the set of available performance measures, each measure corresponding to a set of activities, and let

$$p^k = d^k V(k) + \varepsilon^k; k = \{A_1, B_1, A_2, B_2\} \tag{4}$$

where $d^k \in [0, D], D \in \mathbb{R}_+$, is a measure of distortion and ε^k is a normally distributed random term with mean zero, $\varepsilon^k \sim N(0, v^k)$. The random error terms represents influences on the performance measure that are outside an agent's control. Measures are undistorted when $d^k = 1$, whereas a measure where $d^k < 1 (d^k > 1)$ underestimates (overestimates) value.

The principal specifies a subset of performance measures for each agent, where $P^I = \{p^{A_i}, p^{B_i}\}$ is each agent's set of performance measures under *individual targets*, and $P^S = \{p^{A_i}, p^{B_i}, p^{B_j}\}$ is the corresponding set under *shared targets*. I assume that the measures are independent, in the sense that the presence of one measure does not affect the other measures. This assumption implies that the (measured) marginal value of core activities ($\partial p^k / \partial a$) is not changed by the introduction of shared targets.

Each performance measure in the chosen subset is compared to a benchmark, or a standard, denoted $\bar{p}^k$. Explicit benchmarks are common in all types of organizations. Moreover, it is difficult to conceive of performance measures that are not at least

[7]In the accounting literature distortion is often called incongruity (e.g. Feltham and Xie, 1994; Budde, 2007), while others have used the term alignment (e.g. Schnedler, 2008).

implicitly evaluated against some standard. This assumption also has the technical advantage that an agent's utility does not automatically increase with the number of performance measures.

The agents' expected utility depends on a fixed wage ($w_1 = w_2 = w,$) and the created value as measured by the performance measures in comparison to the benchmarks:

$$\mathbb{E}(u) = \mathbb{E}[-exp(-\delta(w + \boldsymbol{m}'(\boldsymbol{p} - \bar{\boldsymbol{p}})))] \tag{5}$$

where $\delta > 0$ measures the agent's risk aversion, $\boldsymbol{p}$ is the performance measures of a P^I or P^S arranged in a (column) vector with typical element p^k, $\bar{\boldsymbol{p}}$ is a (column) vector of benchmarks with typical element $\bar{p}^k$, and $\boldsymbol{m}$ is a (column) vector with typical element m^k.[8]

The vector $\boldsymbol{m}$ signifies the extent to which an agent is motivated – higher m^k implies that the agent cares more about performance measure p^k and the corresponding set of activities – and also the extent to which the agent's motivation is aligned to the principal's interest. If $m^k = m^l > 0$ for all $l, k \in \{A_i, B_i, B_j\}$, then it is only the marginal value of each allocation that guides the agent's choice of allocation, and the agent's motivation is not in conflict with the principal's interest.[9] In this case, $\boldsymbol{m}$ is just a scalar.

The levels of the benchmarks affect the allocations indirectly. To see how, first define each agent's participation constraint as

$$\mathbb{E}(u) \geq \bar{u} \tag{6}$$

where $\bar{u}$ is the (commonly known) outside option available to the agents. Furthermore, agents maximize utility subject to a budget constraint:

$$r \geq a + b. \tag{7}$$

As resources are dependent on wages, and wages are determined by equation (5) and (6), the benchmarks affect the allocation through the constraints.

Another thing to note is the effect of the benchmarks in combination with motivation. If $\bar{p}^k < \max_{a,b} \mathbb{E}(p^k)$, higher m^k implies that the principal can set a lower wage all else equal, whereas if $\bar{p}^k > \max_{a,b} \mathbb{E}(p^k)$, more motivated agents require a higher wage. If agents are motivated by career concerns, this seems reasonable. That is, if agents exceed what is expected of them, this reflects positively on their future career possibilities, and vice versa. Similarly, agents driven by desire for social esteem, identification with organizational objectives, or care about output for

[8] This formulation is a variant of the canonical model of constant absolute risk-aversion developed by Holmström and Milgrom (1987, 1991).

[9] I treat motivation as exogenously given throughout. See Rob and Zemsky (2002) for a model where the utility of cooperation and the corporate culture is endogenously determined by the incentive structure and the history of cooperation in the organization.

other reasons could also be expected to demand compensation for not being able to achieve what is expected of them.

Benchmarks could be thought of as being determined exogenously to the organizations, or as being determined by the principal. I will take the first view here, and assume that each benchmark is some fixed, positive number. The reason is that for the type of organizations under consideration – e.g. schools and hospitals – benchmarks are commonly set up as comparisons to other units, or are determined by government regulations, and/or by professional organizations. A principal is thus not likely to be able to choose any benchmark.[10]

Throughout, I assume that the principal knows that the agents are identical and splits the initial allocation in half. The timing of the model is:

1. The principal learns total resources, R.
2. The principal specifies performance measures and offers a fixed wage.
3. If each agent's participation constraint is met, the principal allocates resources to the agents. Otherwise, return to step 2 and let the principal offer a new wage level.
4. Agents decide how to allocate the given resources between a and b, which determines total value.

3 Complete information, perfect measures, and aligned motivation

To show the effects of interdependence in a simple way, this section compares the resource allocations of agents with that of an informed principal under conditions when the agents' motivation is aligned to principal's interest, and each individual performance measure is precise and undistorted. Therefore, assume $v^k = 0, d^k = 1$, and $m^k = m^l > 0$ for all $k, l \in \{A_i, B_i, B_j\}$. Assume also that the details of the model as laid out above, including the effect of their own and the other agent's allocation on the performance measures, are common knowledge among the two agents. Given the procedure stipulated in the previous section and that the agents' utility functions are strictly concave, their allocations constitute a unique sub-game perfect Nash equilibrium. An informed principal chooses an allocation to directly maximize

$$V^* = p^{A_1} + p^{B_1} + p^{A_2} + p^{B_2} \tag{8}$$

[10]A benchmark such as "the average test scores of schools should be among the top ten percent in the country" could perhaps be thought of as something in between. How such benchmarks should be chosen optimally is an interesting question for further research.

subject to

$$2r^* \geq 2a^* + 2b^*. \tag{9}$$

Proposition 1 compares V^* to V^I, the value created by two identical agents with individual targets. To make the comparison interesting, I assume that the available resources are the same for agents with individual targets and the informed principal, so $r = r^*$.[11] All calculations are found in the Appendix.

Proposition 1: Suppose agents are identical and $\varphi \neq 0$. If

(*i*) $\theta \geq \rho + r\tau$, then all resources are allocated to activities in A_1 and A_2, $(a, b) = (r, 0)$ and $V^* = V^I$;

(*ii*) $\theta \leq \rho - r(\tau - \varphi)$, then all resources are allocated to activities in B_1 and B_2, $(a, b) = (0, r)$ and $V^* = V^I$;

(*iii*) $\rho - r(\tau - \varphi) < \theta < \rho + r\tau$, then $a, b > 0$ and $a + b = r, V^* > V^I$. Moreover, the difference in value is increasing in $|\varphi|$.

The reason for (*i*) and (*ii*) is of course that the value of a dominates the value of b and vice versa, so interdependence need not be taken into account.[12] From here on I analyze only the case where strictly positive shares of resources are allocated to both tasks.

As shown by (*iii*), with individual targets – whenever there is interdependence between the two agents and a and b are positive – there exist incentives to allocate resources in a sub-optimal way. Thus, even when favorable (indeed, implausible) assumptions of agent motivation and performance measures are made, some mechanism needs to be in place to manage interdependencies. This result is in line with results from models of coordination incentives in the literature on private firms (e.g. Rantakari, 2008; Kretschmer and Puranam, 2008; Baiman and Baldenius, 2009), but the result does not depend on agents having different preferences to the principal. The allocations in this case are

$$(a^I, b^I) = \left(r - \frac{\rho + r\tau - \theta}{2\tau - \varphi}, \frac{\rho + r\tau - \theta}{2\tau - \varphi} \right) \tag{10}$$

$$(a^*, b^*) = \left(r - \frac{\rho + r\tau - \theta}{2(\tau - \varphi)}, \frac{\rho + r\tau - \theta}{2(\tau - \varphi)} \right) \tag{11}$$

[11] In principle, with an informed principal there is no need for agents in the model, as their only task is to allocate resources. The principal could therefore choose not to hire any agents and save the wages. This comparison is not very informative though.

[12] Note that the parameter values in the proposition hold for the individual targets case, but the parameter condition for $(a, b) = (0, r)$ is different when the principal is fully informed: $\rho - r(\tau - 2\varphi) < \theta < \rho + r\tau$. That is, the principal allocates positive shares to both a and b for a narrower range of parameters.

which implies that $b^I < b^*$ when joint activities are complements ($\varphi > 0$) and $b^I > b^*$ when they are substitutes ($\varphi < 0$).

Corollary 1 describes how the agents can be made to internalize the externality created by interdependence with the help of shared targets. Then, $P^S = \{p^{A_i}, p^{B_i}, p^{B_j}\}$ and the resulting value is denoted V^S.

Corollary 1: Suppose the agents are subject to shared targets, then $V^* = V^S$.

Thus, first-best can be achieved by letting agents share targets when performance measures are precise and undistorted, and agents' motivation is in line with the principal's interest. The next sections relax some of the assumptions made in this section and examine if and when shared targets can improve upon individual targets.

4 Distortion and misaligned motivation

Performance measures are of course seldom, if ever, "perfect" and agents may often have diverging motivations compared to their principal. This section examines the effect of distorted performance measures and misaligned motivation, while keeping the assumptions of common knowledge and precise performance measures.

As discussed in section 2, the wage level w, and in turn available resources r, depend on the difference between $\max_{a,b} p^k$ and $\bar{p}^k$. This difference also influences how changes in m^k and d^k affect w, and consequently the resources as $r = R_i - w$.[13] In order to focus on the "pure" effects of motivation and distortion on the choice of allocations, I abstract from the resource effects here and assume that r is fixed in this section. Section 5 returns to this issue.

How does distortion affect the allocations? It is *not* necessarily true in the model that a distorted performance measure decrease value, even if r is fixed. Recall that individual targets with undistorted and precise measures yield an inefficient allocation, b being too low in the case of complements and too high in the case of substitutes. Thus, a distorted measure that either overestimates the marginal value of b, or underestimates the marginal value by the "right" amount, could induce a first-best allocation. Solving a similar maximization problem under individual targets as in section 3 but with $d^k > 0 \, \forall \, k \in \{A_i, B_i\}$, i.e. maximizing

$$u = -exp\left[-\delta\left(w + m \sum_{k \in \{A_i, B_i\}} d^k V(k) - \bar{p}^k\right)\right] \qquad (12)$$

[13] If agents are motivated enough, or measures overestimate value enough, wages may be driven to zero. While this does not seem to be a very common state of affairs in the public sector or for managers in general, it is not an unthinkable concept for other types of agents. For instance, internships with zero or very low compensation are common in many industries.

yields the following allocation

$$b^I = \frac{d^{B_i}(\rho + \tau_B r) + d^{A_i}(r\tau_A - \theta)}{d^{A_i}2\tau_A + d^{B_i}(2\tau_B - \varphi)} \tag{13}$$

If we compare this expression to (10), it can be shown that if

$$\frac{d^{B_i}}{d^{A_i}} = \frac{\theta + b^* 2\tau_A - r\tau_A}{\rho + \tau_B r - b^*(2\tau_B - \varphi)} \tag{14}$$

then $b^I = b^*$ and there is no loss of value even with individual targets. The point is that it is the ratio of measured marginal values that matters for the resource allocation, and therefore it is the *combination* of performance measures that is important, rather than the individual measures. This implies that distortion works differently here compared to e.g. Feltham and Xie (1994) and Baker (2002), where distortion always imply a loss of value. Kaarbøe and Olsen (2008), Schnedler (2008) and Thiele (2010) also show that distortion may increase value. In their models, this is driven by distortion of non-verifiable measures, by different effort costs, and by different ability over tasks, respectively. That is, distorted performance measures may be efficient if they correct for something else that causes deviations from efficient allocations. In this model, distortion may correct for the externality created by interdependence.

To see how misaligned motivation affects the results, let the elements of the motivation vector be $m^k \geq 0 \; \forall \; k \; \in \{A_i, B_i, B_j\}$ and not necessarily equal. The agent then maximizes

$$u = -exp\left[-\delta\left(w + \sum_{k \in \{A_i, B_i, B_j\}} m^k\left(p^k - \bar{p}^k\right)\right)\right] \tag{15}$$

subject to the same restrictions as before. Compare this expression to (12) to see that motivation affects the allocation in a similar way to distorted performance measures. In a general formulation, with distortion included, the allocation to joint activities with shared targets becomes

$$b^S = \frac{m^{B_i}d^{B_i}(\rho + r\tau_B) + m^{A_i}d^{A_i}(r\tau_A - \theta)}{m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}(2\tau_B - \varphi) - m^{B_j}d^{B_j}\varphi} \tag{16}$$

As long as $m^{B_j}, d^{B_j} > 0$, expression (16) shows that shared targets always imply a higher b when φ is positive, and a lower b when φ is negative, compared to individual targets (when either m^{B_j}, d^{B_j} or φ is zero, the allocation is equal to the one with individual targets). Shared targets can therefore only be an improvement when individual targets result in $b^I < b^*$ for $\varphi > 0$ (complements), and $b^I > b^*$ for $\varphi < 0$ (substitutes).[14]

[14] I still assume that $r = a + b$ and $a, b > 0$, so the changes to the allocation from misaligned motivation and distortion do not warrant a corner solution.

For complements (and reversed for substitutes) it may seem as if increased motivation for core activities, or distorted measures that overestimate the value of such activities, should imply a higher a and increase the possibility that shared targets improve the allocation. Similarly, increased motivation for, or overestimation of, the value of joint activities should have the opposite effect. However, proposition 2 shows that while this intuition holds for a broad range of parameter values, it does not hold for all:

Proposition 2: Let b be given by

$$b^I = \frac{m^{B_i} d^{B_i} (\rho + r\tau_B) + m^{A_i} d^{A_i} (r\tau_A - \theta)}{m^{A_i} d^{A_i} 2\tau_A + m^{B_i} d^{B_i} (2\tau_B - \varphi)},$$

then i):

$$\frac{\partial b}{\partial m^{A_i}} < 0, \frac{\partial b}{\partial d^{A_i}} < 0 \tag{17}$$

when

$$\theta (2\tau_B - \varphi r) + \tau_A (2\rho + \varphi r) > 0; \tag{18}$$

and ii):

$$\frac{\partial b}{\partial m^{B_i}} < 0, \frac{\partial b}{\partial d^{B_i}} < 0 \tag{19}$$

when

$$\theta (2\tau_B - \varphi) > \tau_A (2\rho + r (4\tau_B - \varphi)). \tag{20}$$

Regarding i), increased motivation for core activities, m^{A_i} (or increased distortion, d^{A_i}), normally decreases b and increases the possibility for shared targets to work. But as there are no parameter restrictions set on τ_B and τ_A individually (only on $\tau = \tau_A + \tau_B > 0$), the inequality in (18) can be reversed when τ_B is small enough relative to φr. This would require that allocations to joint activities have a relatively large effect on core activities, but not the other way around (τ_A is large relative to τ_B).

About ii), increased motivation for joint activities m^{B_i} (or increased d^{B_i}) may increase b as there are many parameter values for which the inequality in (20) is reversed. The inequality holds when a affects the value of b strongly (τ_B is high), but b does not have a positive effect on a (τ_A is relatively low, zero, or negative). Then, increased motivation for joint activities may decrease the share of resources allocated to these activities.

In sum, Proposition 1 and Corollary 1 do not hold generally. There are instances when distorted measures and agent motivation may neutralize the inefficiency found with individual targets. However, for a broad range of parameter values, shared targets are more likely to improve coordination incentives for complements when agents are highly motivated by core activities, or performance measures overestimate the value of core activities (vice versa for substitutes).

5 Imprecision

To see the first effect of imprecise measures on resource allocation clearly, let $\boldsymbol{p}$ be composed of the undistorted performance measures under shared targets and $\boldsymbol{m}$ be a scalar, so that the loss of value would be zero absent noise. The size of the imprecision of a performance measure depends on the variance, v^k. When $v^k > 0$ and the error term is normally distributed, the agents' expected utility functions can be shown to be

$$\begin{aligned}\mathbb{E}(u) &= \mathbb{E}\left[-exp\left(-\delta\left(w + m \sum_{k\in\{A_i,B_i,B_j\}} V(k) + \varepsilon^k - \bar{p}^k\right)\right)\right] \\ &= -exp\left[-\delta\left(w + m \sum_{k\in\{A_i,B_i,B_j\}} V(k) - \bar{p}^k - \frac{\delta m v^k}{2}\right)\right]. \qquad (21)\end{aligned}$$

The agent's utility is still increasing in the fixed wage and in measured value, but is always decreasing in the variance of the performance measures because the (risk-averse) agents are forced to bear more risk. A negative influence on agents' utility must increase the wages paid. As wages have to be taken out of available resources, this decreases the amount that can be allocated to produce value. It is also evident that all else equal, more motivated agents will require more compensation for bearing risk, which seems reasonable if motivation derives from for example career concerns, or social esteem. This also shows that the relationship between motivation and wages (and in turn resources) is again not straightforward. It is not simply the case that highly motivated agents demand lower wages.

The discussion in this and the previous sections points to differences between shared targets and individual targets because 1) shared targets take the interdependence, motivation, and distortion differently into account, and 2) because shared targets may change the amount of resources available for allocation. Such resource effects can in turn be the result of *i*) adding a measure, which increases wages whenever there is some imprecision of the added measure; *ii*) $\mathbb{E}(p^k) \neq \bar{p}^k$ for some k, which may increase or decrease wages; and *iii*) any change of the amount of resources available may also change the relative allocation between a and b, as r is a part of the expressions for a and b.[15] This last point implies that noise affects the results in a different way to models of private firms because of the budget constraint, which is typically absent in such models (e.g. Feltham and Xie, 1994; Baker, 2002).

Assume for the sake of simplicity that motivation is aligned and there is no distortion, so that the only difference in value with shared targets absent any resource

[15] In fact, a change in r is only neutral if $\rho = \theta$. To see this, differentiate the ratio of a/b with respect to r, which yields $\partial(a/b)/\partial r = (2\tau - \varphi)(\rho - \theta)/(\rho - \theta + r\tau)^2$. As the denominator and $2\tau - \varphi$ must be greater than zero, the expression is only zero when $\rho = \theta$.

effects is due to the externality induced by interdependence. Using the participation constraint in (6), the wage level can be expressed as $w = -\frac{ln(-\bar{u})}{\delta} - m\sum_k V(k) - \bar{p}^k - \frac{\delta m v^k}{2}$. In turn, the difference in resources under shared targets compared to individual targets is

$$\begin{aligned}\Delta^r \equiv & r^S - r^I = \frac{1}{2} - w^S - \frac{1}{2} + w^I = w^I - w^S \\ = & m\left(V^S(A) - V^I(A) + V^S(B_i) - V^I(B_i) + V^S(B_j) - \bar{p}^{B_j} - \frac{\delta m v^{B_j}}{2}\right)\end{aligned} \tag{22}$$

In Appendix A.4, I show that the expression for $V^S - V^I \equiv \Delta^V$ can be written as:

$$\Delta^V = \frac{(\rho + r^I\tau - \theta)^2\,(-\varphi)^2}{4(2\tau-\varphi)^2(\tau-\varphi)} + \frac{\Delta^r\left(\tau^2\Delta^r + 2\tau(\rho + r^I\tau - \theta) + 4\theta(\tau-\varphi)\right)}{4(\tau-\varphi)} \tag{23}$$

The first term of the resulting expression represent the gain in value from having shared targets correct the interdependence externality. As discussed in section (4), this difference can be made larger or smaller by misaligned motivation and distortion. The second term represent the resource effects 1*b*)-3*b*). It is also clear from the expression that if $\Delta^r > 0 (< 0)$, implying $r^S > r^I$ ($r^S < r^I$), then the second term is always positive (negative). Together, (22) and (23) imply that for example the variance from an added measure always affects value negatively, while the total effect is ambiguous. The $V(k)$ terms is in turn again affected by the resources, and while it is possible obtain an expression only in terms of parameters, it becomes rather opaque and does not add much intuition, so I refrain from showing it. In any case, as long as wages are a small share of total resources, so that the difference between wages under shared targets and individual targets is also likely to be small, resource effects will not be a major problem. The next section examines a potentially more problematic consequence of imprecise measures.

5.1 Learning with imprecise performance measures

The one-shot game relies on assumptions that agents know how resource allocations determine value, both for themselves and for the other agent. To examine how imprecise performance measures affect the agents' possibilities of learning their value function, this section simulates a repeated version of the model. The simulations also shed more light on when the one-shot model is a reasonable approximation, as the rather strict assumption of common knowledge is relaxed.

I assume that the agents still have some knowledge of how their own allocations affect the performance measures (as the agents would not be needed otherwise). In particular, I assume that they know the functional form of the mapping from shares of resources to measured value, but must learn some of the parameters. It seems reasonable, and is supported by empirical evidence, that the values of interdependent activities are more difficult to assess (e.g. Sherman and Keller, 2011), so I let first φ, and then all of φ, τ_A and τ_B, be unknown. The resource allocation of the other agent is also unknown beforehand, but revealed after each period. When agents choose their best replies, they use the other agents choice in the previous period, b_{jt-1}; i.e. they assume that the other agent's choice of b is stationary. Given this uncertainty, I also assume that agents choose myopic best replies, i.e. they are not forward looking in terms of resource allocation. Myopia can be motivated by the fact that agents may be replaced. If agents know that they are learning over time, it may similarly be regarded as rational to only take the current period into account. From a different point of view, it may instead reflect an aspect of bounded rationality.[16] Both stationarity and myopic best responses are common assumptions in game-theoretic learning models (e.g. Fudenberg and Levine, 2009).

5.2 The learning rule

For simplicity, I use a regime of individual targets and exemplify the rule below with the situation where φ, τ_A and τ_B are unknown. This implies that agents use p^{A_i}, p^{B_i} and what they know about the parameters and allocations in their own value function to "back out" the values of the unknown parameters. In period 1, agents use initial beliefs of the unknown parameters to make their choice. For τ_A in periods $t > 1$ agents use p^{A_i} and the known terms θ, a_i, b_i to get an estimate:

$$\begin{aligned}(\hat{\tau}_A)_t &= \frac{1}{t-1}\sum_{s=1}^{t-1}\frac{1}{a_{is}b_{is}}\left(p_s^{A_i}-\theta a_{is}\right)\\ &= \frac{1}{t-1}\sum_{s=1}^{t-1}\frac{1}{a_{is}b_{is}}\left(\theta a_{is}+\tau_A a_{is}b_{is}+\varepsilon_s^{A_i}-\theta a_{is}\right)\\ &= \frac{1}{t-1}\sum_{s=1}^{t-1}\left(\tau_A+\frac{\varepsilon_s^{A_i}}{a_{is}b_{is}}\right). \end{aligned} \tag{24}$$

That is, agents take the average of the backed out values of τ_A and the error term of the performance measure over the past periods. Effectively, agents regard the error terms as having mean zero. The error term is scaled up by the term a_ib_i, which implies that the lower the values of a_i and b_i, the more the error term influences the estimation. This is so since τ_A is not observed separately from a_ib_i. If $a_{it} = 0$ or $b_{it} = 0$, the performance measure contains no information about the value of τ_A and I assume that $(\hat{\tau}_A)_t = (\hat{\tau}_A)_{t-1}$. When φ is the only unknown, agents use a similar rule to estimate that parameter but instead use p^{B_i} and the terms ρ, a_i, b_i, b_j and τ_B.

[16]However, this type of bounded rationality is less compatible with agents motivated by career concerns, which requires intertemporal reasoning.

For τ_B and φ things are a bit more complicated. As

$$p_t^{B_i} = \rho b_{it} + \tau_B a_{it} b_{it} + \varphi b_{it} b_{jt} + \varepsilon_t^{B_i} \tag{25}$$

contains two unknown parameters to be estimated, agents need to estimate these parameters jointly over several periods. Therefore, in periods 1 and 2 I assume that agents do not update their beliefs about φ, but use their initial beliefs $\hat{\varphi}_0$ to estimate τ_B in the same way as τ_A. That is, agents focus on the within organization interaction between a_i and b_i first. In the first period, choices are made based on initial beliefs. In the second, there is one observation to estimate τ_B from, which yields an estimate for $t = 2$ equal to

$$\begin{aligned}(\hat{\tau}_B)_2 &= p_1^{B_i} - \rho b_{i1} - \tau_B a_{i1} b_{i1} - \hat{\varphi}_0 b_{i1} b_{j1} \\ &= \tau_B + \frac{1}{a_{i1} b_{i1}} \left((\varphi - \hat{\varphi}_0) \, b_{i1} b_{j1} + \varepsilon_1^{B_i} \right)\end{aligned} \tag{26}$$

For periods $t > 2$, I assume that the agents in every period estimate the parameters by an ordinary least squares (OLS) estimation. In each period t, combine the performance measure and the known terms ρ and b_i into

$$y_{it} = p_{t-1}^{B_i} - \rho b_{it-1}. \tag{27}$$

Then, define the matrix $\mathbf{X}_{it}$ and the vector $\mathbf{y}_{it}$ as

$$\mathbf{X}_{it} = \begin{bmatrix} (a_{i1} b_{i1}) & (b_{i1} b_{j1}) \\ \vdots & \vdots \\ (a_{it-1} b_{it-1}) & (b_{it-1} b_{jt-1}) \end{bmatrix}, \mathbf{y}_{it} = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{it-1} \end{bmatrix} \tag{28}$$

and let agent i's point estimate of the parameters at time t be written as the OLS estimator:

$$\hat{\boldsymbol{\beta}}_{it} = \begin{bmatrix} \hat{\tau_{Bit}} \\ \hat{\varphi}_{it} \end{bmatrix} = \left(\mathbf{X}_{it}' \mathbf{X}_{it} \right)^{-1} \mathbf{X}_{it}' \mathbf{y}_{it}. \tag{29}$$

Note that the above learning rules imply that the initial beliefs of the unknown parameters are discarded after the first observation (with the exception of $\hat{\varphi}_0$ when all three parameters are unknown).

5.3 Simulation set-up and results

For simplicity, I assume that total resources and wages, as well as distortion and motivation – factors that co-determined the equilibrium of the one-shot game – are time invariant. As in the previous sections, I study the case when $a + b = r$. The true parameter values in all versions of the simulation model are exogenous and time invariant, and such that both a and b are greater than zero in equilibrium.

Furthermore, the level of distortion is not important for the analysis in this section, so I exemplify only with undistorted performance measures.

The stage game is repeated for T periods. In each period $t = 1, 2, ..., T$, the two agents choose a myopic best reply allocation, using the estimations of the unknown parameters. In $t = 1$, there is no history, so I assume that the players maximize, taking just their initial beliefs and the constraints into account.

In each repetition, a value for all unknown parameters in the initial period is selected by a uniform randomization. The range of permissible initial beliefs about φ is $\hat{\varphi}_0 \in [0, \tau]$. That is, the agents are assumed to believe that the interdependence within their organization is at least not less "important" than the interdependence among the organizations. I only consider complements in the simulation, therefore the lower bound is 0 and agents are not initially allowed to incorrectly perceive inputs into joint activities as substitutes. When unknown, $(\hat{\tau_A})_0$ and $(\hat{\tau_B})_0$ are also in $[0, \tau]$.

In periods $t > 1$, all players observe the outcome of their performance measures in the previous period. Using the learning rule, agents update their assessments of the unknown parameters. Agents then decide how to allocate the given resources between a and b by choosing a myopic best reply conditional on their beliefs. Using the last period's play by agent j and solving a similar program as in the one-shot model yields the following best reply function for b:

$$b_{it} = \frac{\rho + \bar{r}\left((\hat{\tau_A})_{it} + (\hat{\tau_B})_{it}\right) - \theta + \hat{\varphi}_{it} b_{jt-1}}{2\left((\hat{\tau_A})_{it} + (\hat{\tau_B})_{it}\right)}. \tag{30}$$

whereas a_{it} is determined as $a_{it} = \bar{r} - b_{it}$. The simulations run for $T = 30$ periods and each variation is repeated 10,000 times. I use the following values of the true parameters: $\theta = 1.1, \rho = 1.004, \tau_A = 0.4, \tau_B = 0.4, \varphi = 0.4$ and $\bar{r} = 0.48$, which yields $b^I = 0.24$ in equilibrium.

There are no restrictions on the parameters after the initial round, in order to let the learning rule run its course. However, there may be situations where it is unreasonable to assume that agents would always let the parameter estimate fully determine resource allocation. This may be the case when, for example, allocating no resources to an area of activities is not an option, or if they realize that measures are noisy. To model this, I let the play of b_{it} be confined to three intervals: 1) $b_{it} \in [0, 0.48]$; 2) $b_{it} \in [0.01, 0.47]$; and 3) $b_{it} \in [0.1, 0.38]$. That is, if the parameter estimates imply a choice of b (and as a consequence a) outside the specified range, the upper or lower bound is chosen instead. The first scenario implies essentially no restrictions except that agents cannot spend more than available resources, while the second imposes mild restrictions that rule out situations where no learning occurs (recall that the learning rule provides no information about parameters when $b_{it} = 0$

Table 1: Average total absolute differences (%)

	Panel 1: $b_{it} \in [0, \bar{r}]$					
	Unknown parameter: φ			Unknown parameters: τ_A, τ_B, φ		
Periods	(1)	(2)	(3)	(4)	(5)	(6)
All	1.73 (0.79)	3.12(0.97)	10.0(3.55)	7.84(17.8)	21.5(27.1)	41.9(31.0)
1-5	9.62 (4.75)	11.4(4.60)	22.9(8.23)	23.3(18.2)	40.8(22.8)	62.3(23.4)
6-10	0.22 (0.11)	2.16(1.15)	11.0(6.00)	4.96(18.6)	19.2(30.1)	46.2(38.5)
11-20	0.15 (0.08)	1.48(0.77)	7.51(3.99)	4.72(18.6)	17.5(29.8)	37.2(36.8)
21-30	0.11 (0.06)	1.12(0.61)	5.66(3.12)	4.66(18.6)	17.0(29.7)	34.3(35.7)
	Panel 2: $b_{it} \in [0.01, \bar{r} - 0.01]$					
	Unknown parameter: φ			Unknown parameters: τ_A, τ_B, φ		
Periods	(1)	(2)	(3)	(4)	(5)	(6)
All	1.73(0.79)	3.12(0.97)	10.0(3.55)	5.14(1.97)	14.8(7.33)	33.4(16.2)
1-5	9.62(4.75)	11.4(4.60)	22.9(8.23)	21.1(11.2)	38.5(18.3)	58.0(18.5)
6-10	0.22(0.11)	2.16(1.15)	11.0(6.00)	2.14(2.07)	12.2(12.1)	37.8(25.9)
11-20	0.15(0.08)	1.48(0.77)	7.51(3.99)	1.92(1.91)	9.78(8.48)	27.8(20.8)
21-30	0.11(0.06)	1.12(0.61)	5.66(3.12)	1.86(1.86)	9.31(7.93)	24.4(18.5)
	Panel 3: $b_{it} \in [0.1, \bar{r} - 0.1]$					
	Unknown parameter: φ			Unknown parameters: τ_A, τ_B, φ		
Periods	(1)	(2)	(3)	(4)	(5)	(6)
All	1.72(0.78)	3.12(0.96)	10.0(3.55)	4.87(3.02)	14.3(6.72)	32.4(13.6)
1-5	9.58(4.66)	11.3(4.52)	22.9(8.19)	18.9(8.07)	31.8(9.99)	49.7(12.6)
6-10	0.22(0.11)	2.16(1.15)	11.0(6.00)	2.25(3.29)	11.9(8.50)	34.8(18.8)
11-20	0.15(0.08)	1.48(0.77)	7.51(3.99)	2.04(3.19)	10.7(7.86)	28.3(16.5)
21-30	0.11(0.06)	1.12(0.61)	5.66(3.12)	1.97(3.16)	10.3(7.70)	26.5(15.8)

Standard errors in parentheses.
In column (1) and (4), $\sigma^k = 0.001 \times V(k) = 0.000287$.
In column (2) and (5), $\sigma^k = 0.01 \times V(k) = 0.00287$.
In column (3) and (6), $\sigma^k = 0.05 \times V(k) = 0.01435$.

or $b_{it} = \bar{r}$). The third scenario imposes more substantial restrictions.

The results reported in table 1 are the total absolute differences in percent between each agent's choice of b_{it} and the equilibrium value as given by the parameters (b^I),[17] averaged over the 10,000 repetitions. That is, the value for a period $t \in T$ is

$$\frac{1}{10000} \sum_{rep=1}^{10000} \left(\frac{|b_{1t}^{rep} - b^I| + |b_{2t}^{rep} - b^I|}{b^I} \right) \times 100. \tag{31}$$

Columns (1)-(3) of table 1 show the results of simulations where only φ is unknown, while τ_A, τ_B and φ are unknown in columns (4)-(6). Panels 1-3 correspond to the three ranges for b_{it} discussed above. The error terms are normally distributed with mean zero and a standard deviation, $\sigma^k, k \in \{A_i, B_i\}$, of 0.1 (columns (1) and

[17]Note that this is the equilibrium value under individual targets, not the efficient share b^*.

(4)), 1 (columns (2) and (5)), and 5 percent (columns (3) and (6)) of the equilibrium value of the performance measures, given by $p^{A_i} = V(A_i) = 0.287$ and $p^{B_i} = V(B_i) = 0.287$.

The results show that when there is only one unknown parameter, the agents' assessments converge fast to the true value of b. There are more deviations from equilibrium values of b when the standard deviation is quite high, i.e $\sigma^k = 0.05 \times V(k)$, but the absolute difference added over both agents is still less than 8 percent in periods 11-20, and less than 6 percent in periods 21-30. There are also practically no situations where the parameter estimates result in values of $b_{it} = 0$ or $b_{it} = \bar{r}$ (which precludes learning in the next period according to the learning rule), which is shown by the fact that the differences are not affected by the change of permissible range for b_{it}. The values are almost identical over Panels 1-3.

Things look worse when there are three unknown parameters. When $\sigma^k = 0.001 \times V(k)$, agents manage to learn rather fast, and it is only in period 1-5 where difference to equilibrium values is really substantial in all panels. But with more imprecision the differences become quite large, especially for $\sigma^k = 0.05 \times V(k)$ when the differences in periods 21-30 are between 24-34 percent. One explanation is that the noisy performance measures cause agents to choose $b = 0$ or $b = \bar{r}$, as can be seen by the difference between Panel 1 on the one hand, and Panel 2 and 3 on the other. Such situations may be interpreted as coordination breakdowns, or the discontinuing of a project, but in many instances it may not be plausible that all of the resources go to one area even if agents were to believe that the other type of activity is not worth doing. However, this source of deviation is ruled out in Panels 2 and 3, and there are still substantial deviations from equilibrium values left after 30 periods. Moreover, the differences between these two panels are small for most periods, which indicates that restrictions on b_{it} do not further learning over extended periods of time.

A concern may be that the deviations are due to too wide ranges for the initial beliefs. This does not seem to be the case though; while the deviations decrease in all periods, the average total absolute difference in periods 21-30 is still over 25 percent when I use $\hat{\tau_{A0}}, \hat{\tau_{B0}}, \hat{\varphi}_0 \in [0.3, 0.5]$ and $b_{j0} \in [0.2, 0.28]$ in a specification otherwise similar to column (6) (results available on request).

Another potential problem is that the learning rule implies that agents use all earlier periods when they re-estimate the parameters in each period. Each new period thus receives less weight than the previous, and early outcomes based on potentially bad estimates affect the learning throughout the 30 periods. A simple way to test if this is the case is to vary the maximum number of periods used for estimation, so that information from the early periods stop being used after a while. However, shortening the maximum number of periods used for estimation below 30 in specifications similar to column (6) yields consistently higher average total absolute differences for all maxima in $\{3, 5, 10, 20, 25\}$, and all panels (results available on request).

It is of course difficult to say in general what a reasonable amount of noise is, since it depends on the activity and the measure in question. But the results provide another potential explanation of coordination problems in public sector organizations: if agents have to use noisy performance measures to estimate the value of resource allocations, it may take a long time to learn the equilibrium allocations even if the agents use a very efficient learning procedure such as OLS. If a period is taken to be one year (a very common budget period), 30 periods is a substantial amount of time. If the equilibrium allocation corresponds to the efficient allocation, then this also implies substantial inefficiency. The interdependence among activities provides an added dimension of difficulty as agents are affected by each other's learning. Therefore, imprecise performance measures in one organization may be a concern for other, interdependent organizations.

6 Concluding remarks

This paper suggests that coordination problems ought to be more common in public sector organizations than in private sector organizations; not because organizational coordination problems differ in kind, but because performance measurement problems are more severe and the instruments available to create coordination incentives are more limited and blunt.

First, unless the interdependencies of agents are managed somehow, resource allocation is likely to be inefficient. The model shows that interdependencies may lead to inefficient resource allocations when measures are assigned individually to organizations, even if agents' motivation is aligned with the principal, and performance measures are undistorted and precise. Sharing targets solve the coordination problem with perfect measures and aligned motivation. It may also improve incentives to coordinate when measures are distorted and motivation misaligned, but the success depend on the interplay of distortion, motivation, and the relative importance of core and joint activities for value. For complements (and vice versa for substitutes), shared targets are most likely to improve coordination incentives when agents are highly motivated by core activities, and/or performance measures overestimate the value of core activities.

An interesting question for the usefulness of shared targets is therefore whether motivation can be expected go in any particular direction? I would argue that we should expect agents to normally give higher priority to core activities. This could be for reasons of career concerns or because of identification with organizational objectives, or both. If performance measures indicate the ability of a manager to potential employers and core activities are the manager's own responsibility whereas responsibilities for joint activities are shared, then measures of core activities reasonably constitute a more informative indication of the manager's ability. The manager would thus have incentives to give core activities higher priority. It is also reasonable to expect managers to be more likely to choose professions where they identify with the core activities of their organization. For example, people who are interested in teaching are more likely to become teachers and subsequently headmasters, than to self-select into the social services. If this reasoning is correct, complements are likely to present more severe coordination problems than substitutes.

Imprecision in the form of variance of the performance measures has two distinct effects, both adverse. First, noisy measures increase the risk borne by agents, for which risk-averse agents demand compensation. Compensation, in the form of wages, is taken out of available resources and there is consequently less resources to allocate to productive activities. As adding measures increases total variance, this channel affects the choice between individual and shared targets as well. Second, if the agents have to learn at least some of the parameters of their value function, noisy measures may result in a very long learning period. In the simulations presented here, agents use a least squares learning rule to estimate the parameters. Although this rule is likely to be an idealized way of learning, allocations with noisy measures are frequently far from equilibrium values after 30 periods when three parameters have to be learned. None of these effects of imprecision are of course particular to public sector organizations but may be aggravated in such organizations, due to the relative difficulty of measuring outcomes. In such situations, it thus seems worthwhile to invest substantial resources to develop more precise performance measures.

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A Calculations

A.1 Proposition 1

Four scenarios may be possible depending on the parameter values in the value functions (the scenario where $a_i = b_i = 0$ is ruled out by assumption):

1. $a_i > 0, b_i = 0$
2. $a_i = 0, b_i > 0$
3. $a_i > 0, b_i > 0, a_i + b_i = r$
4. $a_i > 0, b_i > 0, a_i + b_i < r$

The agents' and the principal's maximization problems are described first below; and then the value created is compared in each of the four cases.

The agents' problem

As the agents are assumed to be identical, it is enough to show the solutions for one agent. As $v^k = 0$ and there is no uncertainty, the expectations operator is dropped and as all elements of $\boldsymbol{m}$ are equal, this vector is reduced to the scalar m. Under individual targets an agent maximize:

$$\max_{a_i,b_i} u_i = -exp\left[-\delta\left(w + m\left(\sum_{p^k \in P_i^I} p^k - \bar{p}^k\right)\right)\right] \tag{32}$$

This expression is maximized subject to

$$r_i \geq a_i + b_i \tag{33}$$

$$a_i, b_i \geq 0 \tag{34}$$

which yields the following Lagranian:

$$\begin{aligned} L = -exp\left[-\delta\left(w + m(\theta a_i + \rho b_i + \tau a_i b_i + \varphi b_i b_j - \bar{p}^k)\right)\right] \\ +\lambda\left(r_i - a_i - b_i\right) - \mu_a\left(-a_i\right) - \mu_b\left(-b_i\right) \end{aligned} \tag{35}$$

The corresponding Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial a_i} = m\left(\theta + \tau b_i\right) exp(\cdot) - \lambda + \mu_a = 0 \tag{36}$$

$$\frac{\partial L}{\partial b_i} = m\left(\rho + \tau a_i + \varphi b_j\right) exp(\cdot) - \lambda + \mu_b = 0 \tag{37}$$

$$\lambda \geq 0, \lambda = 0 \text{ if } a_i + b_i < r_i \tag{38}$$

$$\mu_a \geq 0, \mu_a = 0 \text{ if } a_i > 0 \tag{39}$$

$$\mu_b \geq 0, \mu_b = 0 \text{ if } b_i > 0 \tag{40}$$

(36) and (39) imply

$$m\left(\theta+\tau b_i\right) exp\left(\cdot\right)-\lambda \leq 0 \tag{41}$$

where (41) is equal to 0 if $a_i > 0$. (37) and (40) imply

$$m\left(\rho+\tau a_i+\varphi b_j\right) exp\left(\cdot\right)-\lambda \leq 0 \tag{42}$$

where (42) is equal to 0 if $b_i > 0$.

The principal's problem

An perfectly informed and risk neutral principal would maximize value directly according to

$$V^* = V_1 + V_2 = \theta(a_1+a_2)+\rho(b_1+b_2)+\tau(a_1b_1+a_2b_2)+\varphi(b_1b_2+b_1b_2) \tag{43}$$

subject to

$$1 \geq r_1 + r_2 + w_1 + w_2 \qquad\qquad a_i, b_i \geq 0 \tag{44}$$

Agents are identical so resources allocated to each agent are $r_1 = r_2 = r \geq a_i + b_i$. This yields the following Lagranian

$$L = \theta(a_1+a_2)+\rho(b_1+b_2)+\tau(a_1b_1+a_2b_2)+\varphi(b_1b_2+b_1b_2) \tag{45}$$

$$+\lambda(2r-a_1-b_1-a_2-b_2)-\mu_1(-a_1)-\mu_2(-a_2)-\mu_3(-b_1)-\mu_4(-b_2) \tag{46}$$

and the following Kuhn-Tucker conditions

$$\frac{\partial L}{\partial a_1} = \theta+\tau b_1-\lambda+\mu_1 = 0 \tag{47}$$

$$\frac{\partial L}{\partial b_1} = \rho+\tau a_1+2\varphi b_2-\lambda+\mu_2 = 0 \tag{48}$$

$$\frac{\partial L}{\partial a_2} = \theta+\tau b_2-\lambda+\mu_3 = 0 \tag{49}$$

$$\frac{\partial L}{\partial b_2} = \rho+\tau a_2+2\varphi b_1-\lambda+\mu_4 = 0 \tag{50}$$

$$\lambda \geq 0, \lambda = 0 \text{ if } a_i + b_i < r_i \tag{51}$$

$$\mu_1 \geq 0, \mu_1 = 0 \text{ if } a_1 > 0 \tag{52}$$

$$\mu_2 \geq 0, \mu_2 = 0 \text{ if } b_1 > 0 \tag{53}$$

$$\mu_3 \geq 0, \mu_3 = 0 \text{ if } a_2 > 0 \tag{54}$$

$$\mu_4 \geq 0, \mu_4 = 0 \text{ if } b_2 > 0 \tag{55}$$

In turn, these equations implies that the following conditions hold

$$\theta + \tau b_1 - \lambda \leq 0 \; (= 0 \text{ if } a_1 > 0) \tag{56}$$
$$\rho + \tau a_1 + 2\varphi b_2 - \lambda \leq 0 \; (= 0 \text{ if } b_1 > 0) \tag{57}$$
$$\theta + \tau b_2 - \lambda \leq 0 \; (= 0 \text{ if } a_2 > 0) \tag{58}$$
$$\rho + \tau a_2 + 2\varphi b_1 - \lambda \leq 0 \; (= 0 \text{ if } b_2 > 0) \tag{59}$$

Value in scenario 1-4

Below, the derived conditions for the principal and the agents are compared in the four scenarios:
1. *Agents*: As $\rho, \tau > 0$, (42) implies that $\lambda > 0$, i.e. there is a positive marginal value of allocating additional resources to agent i. Thus, $(a_i, b_i) = (r_i, 0)$ is the candidate for a maximum point in this scenario. For each agent, $V^I(r_i, 0) = \theta r_i$.

Principal: Use (57) and (59) to see that as $\rho, \tau > 0, \lambda > 0$. Thus, for each agent $(a_i^*, b_i^*) = (r_i, 0)$ is the candidate for a maximum point in this scenario. Each agent produce a value of $V^*(r_i, 0) = \theta r_i$.

2. *Agents*: As $\theta, \tau > 0$, (41) implies that $\lambda > 0$. Thus, in max $(a_i, b_i) = (0, r_i)$, which yields $V^I(0, r_i) = \rho r_i + r_i^2 \varphi$ for each agent.

Principal: $\theta, \tau > 0$, so $\lambda > 0$ according to (56) and (58). Therefore, $(a_i^*, b_i^*) = (0, r_i)$ is the candidate for the maximum point. Value per agent is $V^*(0, r_i) = \rho r_i + r_i^2 \varphi$.

As $V^I = V^*$ in both scenarios, this concludes (i) and (ii). See scenario 3 for the parameter values that imply that max is in (i) and (ii).

3. *Agents*: Here (41) and (42) holds with equality, which makes the first-order conditions for agent $i = 1, 2$ equal to

$$m(\theta + \tau b_i) exp(\cdot) - \lambda = 0 \tag{60}$$
$$m(\rho + \tau a_i + \varphi b_j) exp(\cdot) - \lambda = 0 \tag{61}$$
$$r_i - a_i - b_i = 0 \tag{62}$$

Using that $b_i = b_j$ and the three conditions to solve for a_i, b_i:

$$b_i = \frac{\rho + r_i \tau - \theta}{2\tau - \varphi} \tag{63}$$
$$a_i = r_i - \frac{\rho + r_i \tau - \theta}{2\tau - \varphi} \tag{64}$$

To get these allocations the following must hold $2\tau > \varphi$, $\theta + \tau b_i \geq 0$, $\rho + r_i \tau > \theta$ and $(\rho + r_i \tau - \theta)/(2\tau - \varphi) < r_i \Leftrightarrow \rho - r_i(\tau - \varphi) < \theta$. The first two hold by definition, whereas the second two are the conditions stated in the proposition, which we thus assume hold in this case.

Principal: The candidate point can be solved from the fact that (56)-(59) holds with equality and that $r_i - a_i - b_i = 0$. The resulting allocations are:

$$b_i^* = \frac{\rho + r_i\tau - \theta}{2(\tau - \varphi)} \tag{65}$$

$$a_i^* = r_i - \frac{\rho + r_i\tau - \theta}{2(\tau - \varphi)} \tag{66}$$

For these to hold, $\varphi < \tau$ and $\theta + \tau b^* \geq 0$, which holds by assumption, and $\theta < \rho + r_i\tau$ and $(\rho + r_i\tau) - \theta)/2(\tau - \varphi) < r_i$ which corresponds to the conditions of the proposition in the principal's case. To compare the principal's allocations to the agents, I use the fact that $a_i = r_i - b_i$ and compare allocations for one agent as follows (and drop the indexes as there should not be any risk of confusion):

$$\begin{aligned} V^I - V^* &= \theta(a - a^*) + \rho(b - b^*) + \tau(ab - a^*b^*) + \varphi(b^2 - b^{*2}) \\ &= \theta(r - b - r^* + b^*) + \rho(b - b^*) + \tau((r - b)b - (r^* - b^*)b^*) + \varphi(b^2 - b^{*2}) \\ &= (b - b^*)(\rho - \theta) - (b^2 - b^{*2})(\tau - \varphi) + (rb - r^*b^*)\tau + (r - r^*)\theta \\ &= (b - b^*)(\rho + r\tau - \theta) - (b^2 - b^{*2})(\tau - \varphi) \end{aligned} \tag{67}$$

Where the last equality is the result of $r = r^*$, which holds according to the stated assumptions. $V^* > V^I$ when $l < 0$, which is the case if

$$(b - b^*)(\rho + r\tau - \theta) < (b^2 - b^{*2})(\tau - \varphi) \Rightarrow (b - b^*)\frac{\rho + r\tau - \theta}{\tau - \varphi} < b^2 - b^{*2}$$

Let $(\Delta b) = b - b^*$. Then, as

$$b^* = \frac{\rho + r\tau - \theta}{2(\tau - \varphi)} \Rightarrow \frac{\rho + r\tau - \theta}{\tau - \varphi} = 2b^*$$

write

$$\begin{aligned} &(b - b^*)\frac{\rho + r\tau - \theta}{\tau - \varphi} < (b^2 - b^{*2}) \Rightarrow 2b^*(\Delta b) < (b^* + (\Delta b))^2 - b^{*2} \Rightarrow \\ &2b^*(\Delta b) < 2b^*(\Delta b) + (\Delta b)^2 \Rightarrow 0 < (\Delta b)^2 \end{aligned}$$

which holds for all $(\Delta b)^2 \neq 0$. As

$$(\Delta b) = -\frac{(\rho + r\tau - \theta)\varphi}{(2\tau - \varphi)2(\tau - \varphi)} = -\frac{b^*\varphi}{2\tau - \varphi}$$

(Δb) is only zero when $\varphi = 0$ and/or $b^* = 0$, which would be a contradiction to the stated assumptions. Moreover, as derivative of the expression with respect to φ is strictly negative if $\varphi > 0$ (complements) and strictly positive if $\varphi < 0$ (substitutes) the difference in value to the optimal allocation is increasing in all permissible absolute values of φ.

4. *Agents*: This scenario implies that $\lambda = 0$ and $r_i - a_i - b_i > 0$. The first-order conditions are

$$\theta + \tau b_i = 0 \tag{68}$$

$$\rho + \tau a_i + \varphi b_j = 0 \tag{69}$$

As θ, τ and b_i are all positive by assumption, (68) cannot hold, and this scenario cannot occur.■

A.2 Corollary 1

Using P^S, agent i maximizes:

$$\max_{a_i,b_i} u_i = -exp\left(-\delta\left(w + m\sum_{p^k\in P_i^S} p^k - \bar{p}^k\right)\right) \tag{70}$$

subject to $r_i \geq a_i + b_i$ and $a_i, b_i \geq 0$. This yields the following Lagranian:

$$L = -exp\left[-\delta\left(w + m(\theta a_i + \rho(b_i + b_j) + \tau a_i b_i + \tau_B a_j b_j + 2\varphi b_i b_j - \bar{p}^k)\right)\right]$$
$$+ \lambda(r_i - a_i - b_i) - \mu_a(-a_i) - \mu_b(-b_i)$$

Solving this problem in the same way as the agents' problem in scenario 3 above results in

$$b_i = \frac{\rho + r_i\tau - \theta}{2(\tau - \varphi)}$$

which is equal to the share allocated to $b_i^*, i = 1, 2$ in proposition 1. Therefore, $V^* = V^S$.■

A.3 Proposition 2

The share allocated to joint activities under individual targets is

$$b^I = \frac{m^{B_i}d^{B_i}(\rho + r\tau_B) + m^{A_i}d^{A_i}(r\tau_A - \theta)}{m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}(2\tau_B - \varphi)}.$$

As m^{A_i} and d^{A_i}, and m^{B_i} and d^{B_i} have similar derivatives, I exemplify with m^{A_i} and m^{B_i}. Differentiating b^I with respect to m^{A_i}, yields

$$\frac{\partial b^I}{\partial m^{A_i}} = \frac{d^{A_i}(r\tau_A - \theta)\left(m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}(2\tau_B - \varphi)\right)}{\left(m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}(2\tau_B - \varphi)\right)^2} - \frac{d^{A_i}2\tau_A\left(m^{B_i}d^{B_i}(\rho + r\tau_B) + m^{A_i}d^{A_i}(r\tau_A - \theta)\right)}{\left(m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}(2\tau_B - \varphi)\right)^2}$$

As the denominator, as well as d^{A_i} and $m^{B_i}d^{B_i}$ are strictly positive, and the term

$$(r\tau_A - \theta)\, m^{A_i} d^{A_i} 2\tau_A$$

is present on both side of the minus sign and thus cancel out, $\frac{\partial b^I}{\partial m^{A_i}}$ is negative when

$$2\tau_A\,(\rho + r\tau_B) > (r\tau_A - \theta)\,(2\tau_B - \varphi) \Rightarrow \theta\,(2\tau_B - \varphi r) + \tau_A\,(2\rho + \varphi r) > 0.$$

Differentiating with respect to m^{B_i} yield

$$\frac{\partial b^I}{\partial m^{B_i}} = \frac{d^{B_i}\,(\rho + r\tau_B)\left(m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}\,(2\tau_B - \varphi)\right)}{\left(m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}\,(2\tau_B - \varphi)\right)^2} - \frac{\left(m^{B_i}d^{B_i}\,(\rho + r\tau_B) + m^{A_i}d^{A_i}\,(r\tau_A - \theta)\right)d^{B_i}\,(2\tau_B - \varphi)}{\left(m^{A_i}d^{A_i}2\tau_A + m^{B_i}d^{B_i}\,(2\tau_B - \varphi)\right)^2}$$

As the denominator, d^{B_i} and $m^{A_i}d^{A_i}$ are strictly positive, and the term

$$(\rho + r\tau_B)\, m^{B_i} d^{B_i}\,(2\tau_B - \varphi)$$

appears on both sides of the minus sign, the derivative is negative when

$$\begin{aligned}
&2\tau_A\,(\rho + r\tau_B) + (r\tau_A - \theta)\,(2\tau_B - \varphi) < 0 \Rightarrow \\
&\theta\,(2\tau_B - \varphi) > \tau_A\,(2\rho + r\,(4\tau_B - \varphi))\,. \blacksquare
\end{aligned}$$

A.4 Comparison between shared and individual targets

Use the expression for differences in value derived in (67), that $r^I + \Delta^r = r^S$, and set $\rho + r^I\tau - \theta \equiv X, 2(\tau - \varphi) \equiv Y$, and $2\tau - \varphi \equiv Z$. Then $\Delta^V \equiv V^S - V^I$ can be written as

$$\begin{aligned}
\Delta^V &= (b^S - b^I)(\rho - \theta) - ((b^S)^2 - (b^I)^2)(\tau - \varphi) + (r^S b^S - r^I b^I)\tau + (r^S - r^I)\theta = \\
&= \left(\frac{X}{Y} + \frac{\tau\Delta^r}{Y} - \frac{X}{Z}\right)(X - r^I\tau) - \left(\left(\frac{X}{Y} + \frac{\tau\Delta^r}{Y}\right)^2 - \left(\frac{X}{Z}\right)^2\right)\left(\frac{Y}{2}\right) + \\
&\tau\left(\left[r^I + \Delta^r\right]\left(\frac{X}{Y} + \frac{\tau\Delta^r}{Y}\right) - r^I\left(\frac{X}{Z}\right)\right) + \theta\Delta^r.
\end{aligned}$$

In turn, this expression can be rewritten as

$$\left(\frac{1}{2Y^2Z^2}\right)\left[X^2Y(Z-Y)^2 + \Delta^r Y Z^2\left(\tau^2\Delta^r + 2X\tau + 2Y\theta\right)\right].$$

Rearranging and inserting the shortened expressions again, this expression becomes

$$\begin{aligned}
&\left(\frac{1}{2YZ^2}\right)\left(X^2(Z-Y)^2\right) + \left(\frac{\Delta^r}{2Y}\right)\left(\tau^2\Delta^r + 2X\tau + 2Y\theta\right) = \\
&\frac{(\rho + r^I\tau - \theta)^2\,(-\varphi)^2}{4(2\tau - \varphi)^2(\tau - \varphi)} + \frac{\Delta^r\left(\tau^2\Delta^r + 2\tau(\rho + r^I\tau - \theta) + 4\theta(\tau - \varphi)\right)}{4(\tau - \varphi)}
\end{aligned}$$

In a similar way, it is possible to obtain an expression under the assumption of no resource effects, i.e. letting $r^s = r^I$. This expression is then exactly equal to the term $\frac{(\rho+r^I\tau-\theta)^2(-\varphi)^2}{4(2\tau-\varphi)^2(\tau-\varphi)}$ above. Thus, the second term in the expression is equal to the resource effects.