Some distance properties of tailbiting codes

Handlery, Marc; Höst, Stefan; Johannesson, Rolf; Zyablov, Viktor V.

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Some Distance Properties of Tailbiting Codes

Marc Handley, Stefan Höst, Rolf Johannesson
Department of Information Technology
Lund University, Box 118
S-221 00 Lund, Sweden
e-mail: {marc, stefanh, rolf}@dit.lth.se

Víctor V. Zyablov
Inst. for Problems of Information Transm.
Russian Academy of Science
Moscow, Russia
e-mail: zyablov@iitp.ru

Abstract — The active tailbiting segment distance for convolutional codes is introduced. Together with the earlier defined active burst distance, it describes the error correcting capability of a tailbiting code encoded by a convolutional encoder. Lower bounds on the new active distance as well as an upper bound on the ratio between tailbiting length and memory of encoder such that its minimum distance $d_{\text{min}}$ equals the free distance $d_{\text{free}}$ of the corresponding convolutional code are presented.

I. INTRODUCTION

Tailbiting codes can be obtained by terminating rate $R = b/c$ convolutional codes into block codes of length $l$ $c$-tuples [1]. For simplicity, we consider only binary codes. The error correcting capability of a block code is estimated via the minimum distance argument, we define the active tailbiting segment distance $d_{\text{act}}^a$. We also give an upper bound on the length of a tailbiting code so that $d_{\text{min}}$ equals $d_{\text{free}}$ of the corresponding convolutional code. This is useful when analyzing concatenated coding schemes containing tailbiting encoders.

II. THE ACTIVE TAILBITING SEGMENT DISTANCE

Consider the convolutional code $C$ encoded by a rate $R = b/c$ encoder. The binary matrix $\sigma_s$ denotes the encoder state at time $t$. Let $S_{\sigma_t}^{\sigma_t'}$, $0 \leq t < t'$, be the set of state sequences $\sigma_t', \sigma_{t+1}', \ldots, \sigma_{t'}$ that start in state $\sigma_t$ and terminate in state $\sigma_{t'}$, and do not have two consecutive zero states with zero input in between. Let $j_b$ be the smallest positive integer such that $S_{\sigma_t}^{\sigma_t'} \neq \emptyset$. The $j_b$th order active burst distance $[2]$ is $d_{\text{act}}^a = \min_{\sigma_t, \sigma_{t+j_b}} \{ w_H(u_0, u_{j_b}) \}$, $j_b \geq j_b$. For any code $C$, $d_{\text{act}}^a$ is invariant over the set of its canonical encoders [2]. It is lower-bounded by a linearly increasing function $d_{\text{act}}^a \geq a_j \cdot b^j$, $j \geq j_b$, where $a_j$ is the asymptotic slope, and $b$ is chosen as large as possible. Let $j_b$ be the smallest number of steps that, starting in the all-zero state, take us to any reachable state.

Definition 1 The $j$th order active tailbiting segment distance is $d_{\text{act}}^a \triangleq \min_{\sigma_t, \sigma_{t+j_b}} \{ w_H(u_0, u_{j_b}) \}$, where $\sigma$ denotes any possible encoder state.

Theorem 1 The active tailbiting segment distance is lower-bounded by $d_{\text{act}}^a \geq a_j b^j$, for all $j \geq 0$.

III. PROPERTIES OF TAILBITING CODES VIA THE ACTIVE DISTANCES

We define an incorrect path at the receiver to be any trellis path differing from the transmitted path. For any $k_1, k_2 < l$, let $e[k_1, k_2]$ denote the Hamming weight of the error pattern $e[k_1, k_2] = e_{k_1} e_{k_1+1} \ldots e_{k_2}$, where $e_i$, $0 \leq i < l$, are $c$-tuples and all indices are evaluated modulo $l$. Then, we have

Theorem 2 Consider a tailbiting code $C_{tb}^c$ of length $l$ $c$-tuples encoded by a convolutional encoder. Then, for $j_b < l$, a maximum likelihood (ML) decoder corrects all error patterns $e[k_1, k_2]$ that satisfy $e[k_1, k_2] \mod l < \min \{ d_{\text{act}}^a, d_{\text{act}}^a / 2 \}$ for $0 \leq l < l$, $j_b \leq j_b < l$. For $j_b \geq l$, all error patterns that satisfy $e[k_1, k_2] \mod l < d_{\text{act}}^a / 2$ are corrected.

Example 1 A tailbiting code of length $l = 18$ $c$-tuples encoded by $G(D) = (1 + D + D^2) + D^3$ with $d_{\text{min}} = 5$ is used on a binary symmetric channel. From Theorem 2 follows that $d_{\text{act}}^a \geq 10$.

Consider a tailbiting code of length $l$ $c$-tuples with minimum distance $d_{\text{min}}$. The free tailbiting length $l_{\text{free}}$ is the shortest length $l$ for which $d_{\text{min}}$ will remain equal to $d_{\text{free}}$ of the corresponding convolutional code for all tailbiting lengths greater than or equal to $l_{\text{free}}$. The free tailbiting length is upper-bounded by $l_{\text{free}} \leq \lfloor d_{\text{free}} / c \rfloor$.

IV. ENSEMBLE PROPERTIES OF THE ACTIVE TAILBITING SEGMENT DISTANCE

The concept of the active distances can be generalized to time-varying convolutional encoders.

Theorem 3 There exists a rate $R = b/c$ convolutional code $C$ encoded by a time-varying encoder of memory $m$ such that $a_j \cdot b^j > p(j + 1)c + O(\log m)$, for $j = O(m) \geq j_0$, $m \to \infty$ where $p = h^{-1}(1 - R)$ is the Gilbert-Varshamov parameter, $h()$ is the binary entropy function, and $j_0$ is the smallest integer satisfying $(1 - R)(j + 1)c \geq 4 \log m$.

Using the Heller asymptotic bound we obtain

Theorem 4 There exists a tailbiting code $C_{tb}^c$ encoded by a time-varying encoder of memory $m$, such that $\lim_{m \to \infty} l_{\text{free}} / m \leq \frac{1}{b^j}$.

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