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Some Distance Properties of Tailbiting Codes
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Abstract — The active tailbiting segment distance for convolutional codes is introduced. Together with
the earlier defined active burst distance, it describes the error correcting capability of a tailbiting code
encoded by a convolutional encoder. Lower bounds on the new active distance as well as an upper bound on
the ratio between tailbiting length and memory of encoder such that its minimum distance \( d_{\text{min}} \) equals the
free distance \( d_{\text{free}} \) of the corresponding convolutional code are presented.

I. INTRODUCTION
Tailbiting codes can be obtained by terminating rate \( R = b/c \) convolutional codes into block codes of length \( l \) \( c \)-tuples [1].
For simplicity, we consider only binary codes. The error correcting capability of a block code is estimated via \( d_{\text{free}} \). There
is no description which error patterns with more than \( \lfloor d_{\text{free}} - 1 \rfloor \) errors can be corrected. In order to describe the error correcting capability of a tailbiting code beyond the minimum distance argument, we define the active tailbiting segment distance \( d_{\text{aj}}^{lb} \). We also give an upper bound on the length of a tailbiting code so that \( d_{\text{min}} \) equals \( d_{\text{free}} \) of the corresponding convolutional code. This is useful when analyzing concatenated coding schemes containing tailbiting encoders.

II. THE ACTIVE TAILBITING SEGMENT DISTANCE
Consider the convolutional code \( C \) encoded by a rate \( R = b/c \) encoder. The binary matrix \( \sigma_j \) denotes the encoder state at time \( t \). Let \( S_{[0,t]}^{(j)} \neq \emptyset, 0 \leq t_1 < t_2 \), be the set of state sequences \( \sigma_{[1,t]} = \sigma_{[1,t_1]} \ldots \sigma_{[1,t_2]} \) that start in state \( \sigma_1 \) and terminate in state \( \sigma_t \), and do not have two consecutive zero states with zero input in between. Let \( j \) be the smallest positive integer such that \( S_{[0,t]}^{(j)} \neq \emptyset \). The \( j \)-th order active burst distance \( d_{\text{aj}}^{lb} \) is the\(^{[2]} \) \( \min_{1 \leq j \leq t} \{ w_h(\nu_{[0,j]}) \} \), \( j \geq j_0 \). For any code \( C \), \( a_j^{lb} \) is invariant over the set of its canonical encoders \( [2] \). It is
lower-bounded by a linearly increasing function \( a_j^{lb} \geq a_j^{lb} + \beta_j \), \( j \geq j_0 \), where \( \alpha \) is the asymptotic slope, and \( \beta \) is chosen as large as possible. Let \( j \) be the smallest number of steps that,
starting in the allzero state, take us to any reachable state.

Definition 1 The \( j \)-th order active tailbiting segment distance is \( a_j^{lb} = \min_{\sigma} \{ \nu_{[0,j]}(\sigma_{[0,j]}) \} \), where \( \sigma \) denotes any possible encoder state.

Theorem 1 The active tailbiting segment distance is lower-bounded by \( a_j^{lb} \geq \alpha(j + 1), \) for all \( j \geq 0 \).

III. PROPERTIES OF TAILBITING CODES VIA THE ACTIVE DISTANCES
We define an incorrect path at the receiver to be any trellis path differing from the transmitted path. For any \( k_1, k_2 < l \), let \( e_{[k_1,k_2]} \) denote the Hamming weight of the error pattern \( e_{[k_1,k_2]} = e_{k_1} e_{k_1+1} \ldots e_{k_2} \), where \( e_i, 0 \leq i < l \), are \( c \)-tuples and all indices are evaluated modulo \( l \). Then, we have

Theorem 2 Consider a tailbiting code \( C^{lb} \) of length \( l \) \( c \)-tuples encoded by a convolutional encoder. Then, for \( j_0 < l \), a maximum likelihood (ML) decoder corrects all error patterns \( e_{[0,l]} \) that satisfy \( e_{[0,l]} \mod l \leq \min \{ a_j^{lb} / 2, a_{l-1}^{lb} / 2 \} \) \( \forall 0 \leq l < l, j_0 \leq j < l \). For \( j_0 \geq l \), all error patterns that satisfy \( e_{[0,l]} \mod l \leq a_{l-1}^{lb} / 2 \) are corrected.

Example 1 A tailbiting code of length \( l = 18 \) \( 2 \)-tuples encoded by \( G(D) = (1 + D + D^2 + 1 + D^3) \) with \( d_{\text{min}} = 5 \) is used on a binary symmetric channel. From Theorem 2 follows that \( e_{[0,17]} \mod l \leq \min \{ a_j^{lb} / 2, a_{17}^{lb} / 2 \} \) is corrected by an ML decoder although the error pattern contains four channel errors which exceeds \( d_{\text{free}} \).

Consider a tailbiting code of length \( l \) \( c \)-tuples with minimum distance \( d_{\text{min}} \). The free tailbiting length \( l_{\text{free}} \) is the shortest length \( l \) for which \( d_{\text{min}} \) will remain equal to \( d_{\text{free}} \) of the corresponding convolutional code for all tailbiting lengths greater than or equal to \( l_{\text{free}} \). The free tailbiting length is upper-bounded by \( l_{\text{free}} \leq \lfloor d_{\text{free}} / \alpha \rfloor \).

IV. ENSEMBLE PROPERTIES OF THE ACTIVE TAILBITING SEGMENT DISTANCE
The concept of the active distances can be generalized to time-varying convolutional encoders.

Theorem 3 There exists a rate \( R = b/c \) convolutional code \( C \) encoded by a time-varying encoder of memory \( m \) such that \( a_j^{lb} > \beta(j + 1) + O(\log m) \), for \( j = O(m) \geq j_0, m \rightarrow \infty \) where \( \beta = h^{-1}(1-R) \) is the Gilbert-Varshamov parameter, \( h(1-R) \) is the binary entropy function, and \( j_0 \) is the smallest integer satisfying \( (1-R)(j+1) \geq 4 \log m \).

Using the Heller asymptotic bound we obtain

Theorem 4 There exists a tailbiting code \( C^{lb} \) encoded by a time-varying encoder of memory \( m \), such that \( \lim_{m \rightarrow \infty} l_{\text{free}} / m \leq \frac{1}{2} \).

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