



LUND UNIVERSITY

A priori bounds on the onset frequency of wideband antennas

Sohl, Christian; Gustafsson, Mats

2008

[Link to publication](#)

Citation for published version (APA):

Sohl, C., & Gustafsson, M. (2008). *A priori bounds on the onset frequency of wideband antennas*. 104-105. Paper presented at Nordic Radio Science and Communication Conference, 2008, Växjö, Sweden.

Total number of authors:

2

General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

A PRIORI BOUNDS ON THE ONSET FREQUENCY OF WIDEBAND ANTENNAS

Christian Sohl and Mats Gustafsson

Department of Electrical and Information Technology, Lund University, Sweden
 Email: {Christian.Sohl, Mats.Gustafsson}@eit.lth.se

ABSTRACT

This paper presents new bounds on the onset frequency and partial realized gain of wideband antennas. The result is a sum rule quantifying the antenna performance in terms of its low-frequency properties via certain static boundary-value problems. The theoretical findings are compared with numerical simulations using the method of moments.

1. INTRODUCTION

This conference paper is based on a recent approach on physical bounds on antennas set forth in Refs. 1 and 2. For this purpose, consider an antenna of arbitrary shape modeled by linear and time-translational invariant constitutive relations in terms of the electric and magnetic susceptibilities $\chi_e = \chi_e(\mathbf{x})$ and $\chi_m = \chi_m(\mathbf{x})$, respectively.¹ Based on the holomorphic properties of the forward scattering dyadic, a sum rule for the partial realized gain g (with respect to the spatial $\hat{\mathbf{k}}$ -direction and electric $\hat{\mathbf{e}}$ -polarization) is derived in Refs. 1 and 2, *viz.*,

$$\int_0^\infty \frac{g(k; \hat{\mathbf{k}}, \hat{\mathbf{e}})}{k^4} dk = \frac{\eta}{2} \left(\hat{\mathbf{e}}^* \cdot \boldsymbol{\gamma}(\chi_e) \cdot \hat{\mathbf{e}} + (\hat{\mathbf{k}} \times \hat{\mathbf{e}}^*) \cdot \boldsymbol{\gamma}(\chi_m) \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{e}}) \right), \quad (1)$$

where $\eta = \eta(-\hat{\mathbf{k}}, \hat{\mathbf{e}}^*)$ is a real-valued number in the unit interval $[0, 1]$. Here, the static polarizability dyadic $\boldsymbol{\gamma}$ is defined by (ℓ takes any of the values e and m depending on whether the problem is of electric or magnetic nature)

$$\boldsymbol{\gamma}(\chi_\ell) = \sum_{i,j=1}^3 (\hat{\mathbf{a}}_i \cdot \boldsymbol{\gamma}_{ij}) \hat{\mathbf{a}}_i \hat{\mathbf{a}}_j,$$

where $\hat{\mathbf{a}}_1$, $\hat{\mathbf{a}}_2$ and $\hat{\mathbf{a}}_3$ form an arbitrary set of linearly independent unit vectors, and

$$\boldsymbol{\gamma}_{ij} = \int_{\mathbb{R}^3} \chi_\ell(\mathbf{x}) (\hat{\mathbf{a}}_j - \nabla \psi_j(\mathbf{x})) dV_{\mathbf{x}}.$$

¹The results in this paper are formulated for isotropic susceptibilities, but they can easily be extended to include anisotropic or bi-anisotropic material models.

The scalar potential ψ_j is the unique solution of the static boundary-value problem

$$\begin{cases} \nabla \cdot ((\chi_\ell(\mathbf{x}) + 1) \nabla \psi_j(\mathbf{x})) = \hat{\mathbf{a}}_j \cdot \nabla \chi_\ell(\mathbf{x}) & \mathbf{x} \in \mathbb{R}^3, \\ \psi_j(\mathbf{x}) = \mathcal{O}(x^{-2}) \text{ as } x \rightarrow \infty \end{cases}$$

where $x = |\mathbf{x}|$. It is surprising to see that the integral on the left-hand side of (1) is related to the static or low-frequency behavior of the antenna.

As an example of how (1) can be used in modern antenna design, consider a planar antenna Λ enclosed by a circular disk $\Lambda_+ = \{\mathbf{x} \in \mathbb{R}^3 : x \leq a\}$ of radius a .² Let $\hat{\mathbf{n}}$ denote the outward-directed unit normal vector of the disk, and choose $\hat{\mathbf{k}} = \hat{\mathbf{n}}$ and $\hat{\mathbf{e}} = \hat{\boldsymbol{\rho}}$, corresponding to a direction of observation and an electric polarization which are perpendicular and parallel to the disk, respectively. Introduce the frequency band $f \in [3.1, 10.6]$ GHz, or equivalently $k \in [0.65, 2.22]$ cm⁻¹, as the appropriate frequency band for ultra-wideband (UWB) communication in North America. Assume that Λ is specified to have a partial realized gain

$$g(k; \hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) \geq \begin{cases} g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) k^4 / k_1^4 & k \in [0, k_1] \\ g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) & k \in [k_1, k_2], \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

where $k_1 = 0.65$ cm⁻¹ and $k_2 = 2.22$ cm⁻¹. Then, for a given threshold $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})$, it is desirable to determine the smallest radius a such that it is feasible for Λ to have a partial realized gain which satisfies (2).

Based on (2), a straightforward calculation of (1) yields

$$\int_0^\infty \frac{g(k; \hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})}{k^4} dk \geq g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) \left(\frac{1}{k_1^3} + \int_{k_1}^{k_2} \frac{dk}{k^4} \right) = \frac{g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})}{3} \frac{4k_2^3 - k_1^3}{k_1^3 k_2^3}.$$

From the analysis in Ref. 1, it follows that the polarizability dyadics for the perfectly electric conducting circular disk are $\boldsymbol{\gamma}(\chi_e) = 16a^3 \mathbf{I}_\perp / 3$ and $\boldsymbol{\gamma}(\chi_m) = \mathbf{0}$,

²Here, the support Λ is defined by $\Lambda = \Lambda_e \cup \Lambda_m$, where $\Lambda_\ell = \{\mathbf{x} \in \mathbb{R}^3 : \chi_\ell(\mathbf{x}) \neq 0\}$ and ℓ takes any of the values e and m.

respectively, where $\mathbf{I}_\perp = \mathbf{I} - \hat{\mathbf{n}}\hat{\mathbf{n}}$ denotes the projection dyadic in \mathbb{R}^3 . Hence, by inserting (??) into (1), one obtains

$$\frac{g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}})}{a^3} \leq 0.55\eta(-\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}),$$

where a now measures the radius of the disk in units of centimeters. For example, by invoking the upper bound $\eta(-\hat{\mathbf{k}}, \hat{\mathbf{e}}^*) < 1$, it is concluded that the minimum radius of the disk is 1.8 cm for $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 3$, and 1.9 cm for $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 4$. For many antennas, η is close to 1/2 and a more realistic bound is therefore 2.2 cm and 2.4 cm for $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 3$ and $g_p(\hat{\mathbf{n}}, \hat{\boldsymbol{\rho}}) = 4$, respectively.

The conference presentation will focus on the use of this sum in antenna design, and how static considerations can offer fundamental insights into the behavior of wideband antennas, *e.g.*, by establishing estimates on the onset antenna frequency. The theoretical findings will be compared with several numerical simulations using the method of moments.

2. REFERENCES

- [1] M. Gustafsson, C. Sohl, and G. Kristensson. Physical limitations on antennas of arbitrary shape. *Proc. R. Soc. A*, 463:2589–2607, 2007.
- [2] C. Sohl and M. Gustafsson. A priori estimates on the partial realized gain of ultra-wideband (UWB) antennas. *Quart. J. Mech. Appl. Math.*, 2008.