A priori bounds on the onset frequency of wideband antennas

Sohl, Christian; Gustafsson, Mats

2008

A PRIORI BOUNDS ON THE ONSET FREQUENCY OF WIDEBAND ANTENNAS

Christian Sohl and Mats Gustafsson

Department of Electrical and Information Technology, Lund University, Sweden
Email: {Christian.Sohl, Mats.Gustafsson}@eit.lth.se

ABSTRACT

This paper presents new bounds on the onset frequency and partial realized gain of wideband antennas. The result is a sum rule quantifying the antenna performance in terms of its low-frequency properties via certain static boundary-value problems. The theoretical findings are compared with numerical simulations using the method of moments.

1. INTRODUCTION

This conference paper is based on a recent approach on physical bounds on antennas set forth in Refs. 1 and 2. For this purpose, consider an antenna of arbitrary shape modeled by linear and time-translational invariant constitutive relations in terms of the electric and magnetic susceptibilities $\chi_\mathrm{e} = \chi_\mathrm{e}(\boldsymbol{x})$ and $\chi_\mathrm{m} = \chi_\mathrm{m}(\boldsymbol{x})$, respectively.[1] Based on the holomorphic properties of the forward scattering dyadic, a sum rule for the partial realized gain g (with respect to the spatial $\hat{\boldsymbol{k}}$-direction and electric $\hat{\boldsymbol{e}}$-polarization) is derived in Refs. 1 and 2, *viz.*,

$$\int_0^\infty \frac{g(k;\hat{\boldsymbol{k}},\hat{\boldsymbol{e}})}{k^4}\,\mathrm{d}k = \frac{\eta}{2}\left(\hat{\boldsymbol{e}}^*\cdot\boldsymbol{\gamma}(\chi_\mathrm{e})\cdot\hat{\boldsymbol{e}} + (\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}}^*)\cdot\boldsymbol{\gamma}(\chi_\mathrm{m})\cdot(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{e}})\right), \quad (1)$$

where $\eta = \eta(-\hat{\boldsymbol{k}},\hat{\boldsymbol{e}}^*)$ is a real-valued number in the unit interval $[0,1)$. Here, the static polarizability dyadic $\boldsymbol{\gamma}$ is defined by (ℓ takes any of the values e and m depending on whether the problem is of electric or magnetic nature)

$$\boldsymbol{\gamma}(\chi_\ell) = \sum_{i,j=1}^{3}(\hat{\boldsymbol{a}}_i\cdot\boldsymbol{\gamma}_{ij})\hat{\boldsymbol{a}}_i\hat{\boldsymbol{a}}_j,$$

where $\hat{\boldsymbol{a}}_1$, $\hat{\boldsymbol{a}}_2$ and $\hat{\boldsymbol{a}}_3$ form an arbitrary set of linearly independent unit vectors, and

$$\boldsymbol{\gamma}_{ij} = \int_{\mathbb{R}^3}\chi_\ell(\boldsymbol{x})(\hat{\boldsymbol{a}}_j - \nabla\psi_j(\boldsymbol{x}))\,\mathrm{dV}_{\boldsymbol{x}}.$$

The scalar potential ψ_j is the unique solution of the static boundary-value problem

$$\begin{cases}\nabla\cdot((\chi_\ell(\boldsymbol{x})+1)\nabla\psi_j(\boldsymbol{x})) = \hat{\boldsymbol{a}}_j\cdot\nabla\chi_\ell(\boldsymbol{x}) \\ \psi_j(\boldsymbol{x}) = \mathcal{O}(x^{-2}) \text{ as } x\to\infty\end{cases} \qquad \boldsymbol{x}\in\mathbb{R}^3,$$

where $x = |\boldsymbol{x}|$. It is surprising to see that the integral on the left-hand side of (1) is related to the static or low-frequency behavior of the antenna.

As an example of how (1) can be used in modern antenna design, consider a planar antenna Λ enclosed by a circular disk $\Lambda_+ = \{\boldsymbol{x}\in\mathbb{R}^3 : x\leq a\}$ of radius a.[2] Let $\hat{\boldsymbol{n}}$ denote the outward-directed unit normal vector of the disk, and choose $\hat{\boldsymbol{k}} = \hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{e}} = \hat{\boldsymbol{\rho}}$, corresponding to a direction of observation and an electric polarization which are perpendicular and parallel to the disk, respectively. Introduce the frequency band $f\in[3.1, 10.6]\,\mathrm{GHz}$, or equivalently $k\in[0.65, 2.22]\,\mathrm{cm}^{-1}$, as the appropriate frequency band for ultra-wideband (UWB) communication in North America. Assume that Λ is specified to have a partial realized gain

$$g(k;\hat{\boldsymbol{n}},\hat{\boldsymbol{\rho}}) \geq \begin{cases} g_\mathrm{p}(\hat{\boldsymbol{n}},\hat{\boldsymbol{\rho}})k^4/k_1^4 & k\in[0,k_1] \\ g_\mathrm{p}(\hat{\boldsymbol{n}},\hat{\boldsymbol{\rho}}) & k\in[k_1,k_2]\,, \\ 0 & \text{otherwise}\end{cases} \qquad (2)$$

where $k_1 = 0.65\,\mathrm{cm}^{-1}$ and $k_2 = 2.22\,\mathrm{cm}^{-1}$. Then, for a given threshold $g_\mathrm{p}(\hat{\boldsymbol{n}},\hat{\boldsymbol{\rho}})$, it is desirable to determine the smallest radius a such that it is feasible for Λ to have a partial realized gain which satisfies (2).

Based on (2), a straightforward calculation of (1) yields

$$\int_0^\infty \frac{g(k;\hat{\boldsymbol{n}},\hat{\boldsymbol{\rho}})}{k^4}\,\mathrm{d}k \geq g_\mathrm{p}(\hat{\boldsymbol{n}},\hat{\boldsymbol{\rho}})\left(\frac{1}{k_1^3} + \int_{k_1}^{k_2}\frac{\mathrm{d}k}{k^4}\right) = \frac{g_\mathrm{p}(\hat{\boldsymbol{n}},\hat{\boldsymbol{\rho}})}{3}\frac{4k_2^3 - k_1^3}{k_1^3k_2^3}.$$

From the analysis in Ref. 1, it follows that the polarizability dyadics for the perfectly electric conducting circular disk are $\boldsymbol{\gamma}(\chi_\mathrm{e}) = 16a^3\mathbf{I}_\perp/3$ and $\boldsymbol{\gamma}(\chi_\mathrm{m}) = \mathbf{0}$,

[1] The results in this paper are formulated for isotropic susceptibilities, but they can easily be extended to include anisotropic or bi-anisotropic material models.

[2] Here, the support Λ is defined by $\Lambda = \Lambda_\mathrm{e}\cup\Lambda_\mathrm{m}$, where $\Lambda_\ell = \{\boldsymbol{x}\in\mathbb{R}^3 : \chi_\ell(\boldsymbol{x})\neq 0\}$ and ℓ takes any of the values e and m.

respectively, where $\mathbf{I}_\perp = \mathbf{I} - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}$ denotes the projection dyadic in $\mathbb{R}^3$. Hence, by inserting (**??**) into (1), one obtains

$$\frac{g_\mathrm{p}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{\rho}})}{a^3} \leq 0.55\eta(-\hat{\boldsymbol{n}}, \hat{\boldsymbol{\rho}}),$$

where a now measures the radius of the disk in units of centimeters. For example, by invoking the upper bound $\eta(-\hat{\boldsymbol{k}}, \hat{\boldsymbol{e}}^*) < 1$, it is concluded that the minimum radius of the disk is 1.8 cm for $g_\mathrm{p}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{\rho}}) = 3$, and 1.9 cm for $g_\mathrm{p}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{\rho}}) = 4$. For many antennas, η is close to $1/2$ and a more realistic bound is therefore 2.2 cm and 2.4 cm for $g_\mathrm{p}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{\rho}}) = 3$ and $g_\mathrm{p}(\hat{\boldsymbol{n}}, \hat{\boldsymbol{\rho}}) = 4$, respectively.

The conference presentation will focus on the use of this sum in antenna design, and how static considerations can offer fundamental insights into the behavior of wideband antennas, *e.g.*, by establishing estimates on the onset antenna frequency. The theoretical findings will be compared with several numerical simulations using the method of moments.

2. REFERENCES

[1] M. Gustafsson, C. Sohl, and G. Kristensson. Physical limitations on antennas of arbitrary shape. *Proc. R. Soc. A*, 463:2589–2607, 2007.

[2] C. Sohl and M. Gustafsson. A priori estimates on the partial realized gain of ultra-wideband (UWB) antennas. *Quart. J. Mech. Appl. Math.*, 2008.