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A Generalization of the Predictable Degree Property to Rational Convolutional Encoding Matrices

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Abstract — The predictable degree property was introduced by Forney [1] for polynomial convolutional encoding matrices. In this paper two generalizations to rational convolutional encoding matrices are discussed.

I. INTRODUCTION

The predictable degree property, introduced by Forney [1], is a useful analytic tool when we study the structural properties of convolutional encoding matrices.

Let G(D) be a rate R = k/c binary polynomial encoding matrix with v_i as the constraint length of the i-th row. For any polynomial input g(D) the output y(D) = g(D)G(D) is also polynomial. We have

\[ \text{deg} \, y(D) = \text{deg} \, g(D) \cdot \text{deg} \, G(D) = \text{deg} \sum_{i=1}^{m} w_i(D) g_i(D) \]

\[ \leq \max_{1 \leq i \leq c} \{ \text{deg} \, w_i(D) + v_i \}, \quad (1) \]

Definition 1 A polynomial encoding matrix G(D) is said to have the predictable degree property if for all polynomial inputs g(D) we have equality in (1).

Let [G(D)]tᵢ be the (0, 1)-matrix with 1 in the position (i, j) where \( \text{deg} \, g_j(D) = v_i \) and 0, otherwise. Then we have

Theorem 1 Let G(D) be a polynomial encoding matrix. Then G(D) has the predictable degree property if and only if [G(D)]tᵢ are of full rank.

Since a basic encoding matrix is minimal-basic if and only if [G(D)]tᵢ is of full rank ([1] [4]) we have the following theorem which is due to Forney [1]:

Theorem 2 Let G(D) be a basic encoding matrix. Then G(D) has the predictable degree property if and only if it is minimal-basic.

In [6] we gave an example of a basic encoding matrix that is minimal but not minimal-basic. That minimal encoding matrix does not have the predictable degree property.

II. THE PREDICTABLE DEGREE PROPERTY FOR RATIONAL ENCODING MATRICES

Let \( g(D) = (g_1(D), \ldots, g_l(D)) \), where \( g_i(D), \ldots, g_l(D) \in \mathbb{F}_2[D] \). Denote by \( \mathcal{P}^* = \{ p(D) \in \mathbb{F}_2[D] \mid p(D) \text{ is irreducible} \} \cup \{ D^{-1} \} \).

For any \( p \in \mathcal{P}^* \) we define

\[ e_p(g(D)) = \min \{ e_p(g_1(D)), \ldots, e_p(g_l(D)) \}, \quad (3) \]

where \( e_p(g_i(D)) \) is an exponential valuation of \( g_i(D) \) [3].

For any rational input \( g(D) \) the output \( y(D) = g(D)G(D) \) is also rational. We have

\[ e_p(g(D)) = e_p \left( \sum_{i=1}^{m} w_i(D) g_i(D) \right) \geq \max_{1 \leq i \leq c} \{ e_p(w_i(D)) + e_p(g_i(D)) \}. \quad (4) \]

Definition 2 A rational encoding matrix \( G(D) \) is said to have the predictable degree property if for \( p = D^{-1} \) and all rational inputs \( g(D) \) we have equality in (4).

Let G(D) be a rational encoding matrix. As a counterpart to \([G(D)]_t\), for polynomial encoding matrices, for any \( p \in \mathcal{P}^* \) we introduce the \( b \times c \) matrix \([G(D)]_t(p)\) to be a matrix whose element in the position \((i, j)\) is equal to the coefficient of the lowest term of \( g_j(D) \), written as a Laurent series of \( p \), if \( e_p(g_j(D)) = e_p(g(D)) \) and equal to 0, otherwise.

Then we can prove

Theorem 3 Let G(D) be a rational encoding matrix. Then G(D) has the predictable degree property if and only if \([G(D)]_t(D^{-1})\) has full rank.

III. THE PREDICTABLE EXPONENTIAL VALUATION PROPERTY

Definition 3 A rational encoding matrix G(D) is said to have the predictable exponential valuation property if we have equality in (4) for all \( p \in \mathcal{P}^* \).

Theorem 4 Let G(D) be a rational encoding matrix. Then G(D) has the predictable exponential valuation property if and only if \([G(D)]_t(p) \mod p \text{ has full rank for all } p \in \mathcal{P}^* \).

A rational encoding matrix is said to be canonical if it can be realized with a minimal number of delay elements in controller canonical form [5].

Theorem 5 Let G(D) be a rational encoding matrix and assume that \( e_p(g(D)) \leq 0, 1 \leq i \leq b, \forall p \in \mathcal{P}^* \). Then G(D) has the predictable exponential valuation property if and only if G(D) is canonical.

The predictable exponential valuation property is not equivalent to being canonical.

REFERENCES


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