



LUND UNIVERSITY

Crack Tip Singularity in a Buckling Thin Sheet

Ståhle, Per; Li, Chong; Bjerken, Christina

Published in:
Nordic Seminar on Computational Mechanics

2006

[Link to publication](#)

Citation for published version (APA):

Ståhle, P., Li, C., & Bjerken, C. (2006). Crack Tip Singularity in a Buckling Thin Sheet. In G. Sandberg (Ed.), *Nordic Seminar on Computational Mechanics* (pp. 43-46)
http://www.byggmek.lth.se/fileadmin/byggnadsmekanik/publications/web_NSCM19.pdf

Total number of authors:

3

General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

Crack Tip Singularity in a Buckling Thin Sheet

Per Ståhle* and Chong Li

Division of Solid Mechanics
Lund University, Lund, Sweden
e-mail: per.stahle@solid.lth.se

Christina Bjerken

Division of Materials Science
Malm University, Malm, Sweden

Summary The effect of buckling on stress distribution in the crack tip vicinity is explored. The computed stress state reveal that the buckling leads to a weaker crack tip singularity than the one of linear elastic fracture mechanics. The change of the singularity has been studied in this work by post-buckling analysis using FE method. The weaker singularity has been taken into account in a modified fracture mechanical theory. The implications for fracture mechanical predictions are discussed.

Introduction

When thin cracked plates are subjected to tensile loading in a direction perpendicular to the crack surface buckling is observed when the load exceed a critical limit. Experimental data by Dixon and Strannigan [1], shows that the maximum stress at a crack tip was about 30 % for a buckled sample in comparison to a sample where buckling was artificially held back. Li *et al.* [2] suggests that the asymptotic stresses are singular but the singularity is weaker than the square root stress singularity of linear fracture mechanics. The change of the singularity implies that the stress intensity factor become undefined after buckling.

The buckling of thin plates containing cracks has been studied in previous works although most investigators address prediction of the critical buckling load for various crack orientations (cf. Markström and Storåkers [3] and Sih and Lee [4]). Recently Brighenti [5] also conducted an investigation of postbuckled cracked plates using nonlinear FE analyses although the work mainly focus on the critical buckling load.

Problem formulation

Consider a thin and rectangular shaped plate containing a crack. The plate consists of a homogeneous, isotropic elastic material described by the modulus of elasticity E and Poissons ratio ν . The bending stiffness of the plate is $D = Et^3/[12(1 - \nu^2)]$. One of two studied geometries, an edge crack is given in Fig. 1. The other geometry is a centred cracked panel. The right half of that geometry is defined also by Fig. 1.

The in plane stresses are divided into a membrane part given as a force per unit of thickness N_{xx} , N_{yy} and N_{xy} and bending moment per unit of thickness M_{xx} , M_{yy} and M_{xy} .

For the centered crack, the applied displacement on the constrained opposed edges $v = v_\infty$, $u = w = 0$ and $dw/dy = 0$ on $0 \leq x \leq b$ ($|x| \leq b$ for the centre cracked plate) and $|y| = h$. All other boundaries are traction free.

Because of the symmetric geometry and load across $x = 0$ and across $y = 0$ only one quarter of the plate is studied. Plausible buckling modes are symmetry and antisymmetry for the out of plane displacement w . The lowest buckling mode is obtained for $w(x, y) = w(x, -y) = w(-x, y)$

meaning that the plate moves either strictly toward $z > 0$ or to $z < 0$ (cf. Markström and Storåkers [3]).

For low load, in the vicinity of the crack tip, stresses are assumed to be singular as expected for linear fracture mechanics

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} f_{yy}(\theta), \quad (1)$$

where the polar coordinates $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$ are used. The angular function $f_{yy}(\theta)$ is known. Straight ahead of the crack tip $f_{yy}(0)$ equals unity.

At post buckling loads it is assumed that the crack tip stresses may be written as

$$\sigma_{yy} = K(2\pi r)^s g_{yy}(\theta), \quad (2)$$

where $g_{yy}(\theta)$ is chosen to be unity straight ahead of the crack tip.

Numerical method

To perform the buckling and the post-buckling analysis, the finite element method is utilized through the commercial software ABAQUS [6]. With regard to the symmetry, the calculations are performed using one quarter of the central cracked plate and one half of the edge cracked plate. The following dimensions are used to describe the model: $h/w = 1.5$ and $t/w = 0.0001$. Two different crack lengths are modeled for the centre cracked plate, i.e. $a/w = 0.1$ and 0.2 , respectively, and for the edge cracked plate a crack with $a/w = 0.1$ is investigated. The size of the plate is assumed sufficient to represent very large plates. The relation between bending stiffness and tensile stiffness enters the equations only through the ratio t/a , which is very small.

The element net uses 6-node triangular thin shell elements with five degrees of freedom per node. Quadratic shape functions are used. The mesh region near crack tip is made denser as the distance to the tip decreases. A denser element net is also used, where the number of nodes is doubled on the edges and the distribution is kept the same as for the coarser case. The coarser case consists of approximately 10 000 elements and the denser case up to approximately 37 000 elements. The length of shortest side of the elements for the entire model is $1.310^{-6}a$ for the coarser case and $6.510^{-7}a$ for the denser case.

To obtain the post buckling responses a modified Riks method was used. This method appears to be most successful among some suggested methods solving unstable problems Ramm [7] and Crisfield [8]. It is able to obtain static equilibrium solutions for unstable problem where the load magnitude is determined by one single scalar parameter. To be able to analyze the post-buckling response the problem needs to be converted into a continuous response instead of bifurcation. A geometric imperfection is introduced as a superimposed displacement field of a buckling mode from the eigenvalue estimation and a scale factor is applied to the field to scale the applied imperfection. The basic of the modified Riks method is the Newton-Raphson method (cf. Riks [9], [10]). The load magnitude taken into consideration as an additional unknown thus the method solves both the load and the displacement at the same time on the equilibrium path. The implemented Riks method by ABAQUS [6] uses an arc length to designate the progress of the calculation, Riks [11]. The choice of the size of the arc length is governed by the convergence rate-dependent, automatic

incrementation algorithm for static cases within the Abaqus standard solver. The finite element implementation of the modification of Riks method is known to suffer from an inability to deal with wrongly chosen direction of integration path along the response curve, thus the solution occasionally get stuck in a loop (Silver et al. [12]; ABAQUS [6]).

Results

The analysis for the central cracked plate could be carried out to an extent of where the applied load reached 100 times for the corresponding buckling load while for the edge cracked plate only about 2 times the corresponding buckling load was reached.

The critical load is expected to increase with increasing thickness in proportion to t^2 . For dimensional reasons the critical buckling stress may therefore be written: $\sigma_k = \lambda E(t/a)^2$, where λ is a geometry and load case dependent dimensionless parameter. For the central crack with $a/w = 0.1, 0.2$ and for the edge crack with $a/w = 0.1$ the result is 2.2883, 2.674 and 28.1835 respectively.

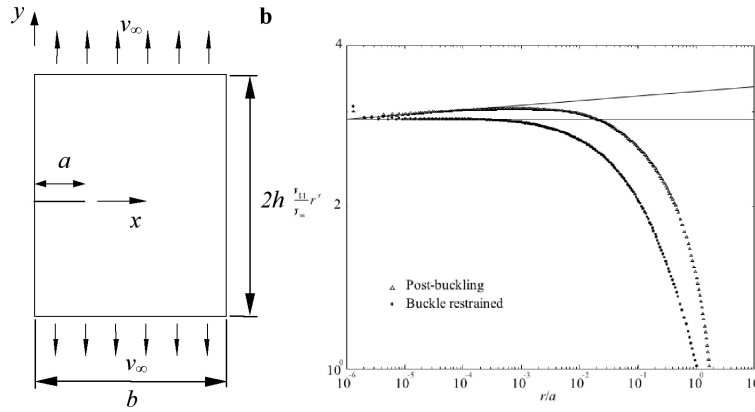


Figure 1: Comparison of stresses normalized by the applied stresses and scaled by the $r|s|$ -dependence of the unbuckled case straight ahead of the crack tip between pre- and post-buckling for the edge crack with $a/w = 0.1$

Stresses in the x direction straight ahead of the crack tip normalized with the applied stresses and the r -dependence of the unbuckled case are plotted against the distance from the crack tip normalized with the crack length in a log-log diagram in Fig. 2. The comparison is made between the last steps of the post-buckling analyses versus a buckle restrained case. The stress responses of the buckle restrained case should maintain the characteristics of a case where linear fracture mechanism is valid; therefore the slope of the curve should correspond to the square-root singularity in a region close to the crack tip. By selecting the region between $10^{-5} < r/a < 10^{-4}$ an exponent $|s|$ in the $r^{|s|}$ depending stresses, is less than 1% below the theoretical value, $-1/2$. A difference of inclination between the buckled case and the buckle restrained case indicates that a different crack tip singularity is at play after the occurrence of buckling. To clearly identify the differences of the singularity the normalized stress response is rescaled with r^s where $s = -0.505$, is plotted against r/a in Fig. 5, where s is the singularity calculated from the buckle restrained cases. The result shows that the singularity is weaker in a post-buckled state. This result is in accordance with the preliminary calculations by Li *et al.* [2].

Conclusions

A thin plate containing a central crack or an edge crack under tensile load in the direction perpendicular to the crack surface was studied. FEM calculations of the post-buckling behaviour using non-linear geometries have been conducted. The post-buckling calculation was performed by the modified Riks method where the plate is considered as imperfect prior buckling.

The post-buckling analyses have shown that within a region of $0.0001a$ ahead of the crack tip the stress field is found to be of the form $r^{-s} f(\theta)$

The field possesses a weaker singularity than a square root singularity that is found for in-plane deformation. Already when the buckling load is exceeded by 100 times the singularity decreases from -0.5 to -0.49, this applied load that causes a 2% drop of the singularity may seem quite small. However, maximum stress in a sheet of paper containing a crack of the length of 20% of the width at the moment of fracture is several thousand times larger than its buckling load and the drop of singularity could be of considerably larger as load of such magnitude is reached.

Different a/w relations for the central cracks appears to have little influence on the weakening of singularity. As a result by the presented data it can be concluded that if the apply load remain below 100 times the buckling load of the specimen then fracture criteria by linear fracture mechanics should retain validity for a central crack. References

References

- [1] J. R. Dixon and J. S. Strannigan. Stress distribution and buckling in thin sheets with central slits. *2nd International Conference on Fracture*, Brighton, 105-118, (1969)
- [2] C. Li, R. Espinosa and P. Ståhle. Fracture mechanics for membranes. *The 15th European Conference of Fracture*, Stocholm, (2004)
- [3] K. Markström and B. Storåkers. Buckling of cracked members under tension. *International Journals of Solid Mechanics*, **16**, 217-229 (1980)
- [4] G. C. Sih and Y. D. Lee. Tensile and compressive buckling of plates weakened by cracks. *Theoretical and Applied Fracture Mechanics*, **6**, 129-138, (1986)
- [5] R. Brighenti. Buckling of cracked thin-plates under tension or compression. *Thin-walled Structures*, 1-16, (2004)
- [6] ABAQUS. Analysis Users Manual ver. 6.4. Hibbit, Karlsson & Sorensen Inc., (2003)
- [7] E. Ramm. Strategies for Tracing the Nonlinear Response Near Limit Points. *Nonlinear Finite Element Analysis in Structural Mechanics*, Edited by E. Wunderlich, E. Stein, and K. J. Bathe, Springer-Verlag, Berlin, (1981)
- [8] M. A. Crisfield. A fast incremental/iterative solution procedure that handles snap-through. *Computers & Structures*, **13**, 55-62, (1981)
- [9] E. Riks. The application of Newtons method to the problem of elastic stability. *Journal of Applied Mechanics*, **39**, 1060-1066, (1972)
- [10] E. Riks. An incremental approach to the solution of snapping and buckling problems. *International Journal of Solids and Structures*, **15**, 529-551, (1979)
- [11] E. Riks, C. C. Rankin and F. A. Brogan. An incremental approach to the solution of snapping and buckling problems. *International Journal of Solids and Structures*, **15**, 529-551, (1979)
- [12] M. J. Silver, J. D. Hinkle and L. D. Peterson. The buckling behavior of a central crack in a plate under tension. *Engineering Fracture Mechanics*, **43**, no. 4, 529-548, (1992)