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The Effect of the Tailbiting Restriction on Feedback Encoders

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Abstract — **It is shown that for short and moderate relative tailbiting lengths and high signal-to-noise ratios systematic feedback encoders have better bit error performance than nonsystematic feedforward encoders. Conditions for when tailbiting will fail are given and it is described how the encoder starting state can be obtained for feedback encoders in both controller and observer canonical form.**

I. Systematic versus Nonsystematic Tailbiting Encoders

Comparing the bit error performance between tailbiting codes encoded by systematic and nonsystematic encoders [1] shows that for a bad channel systematic encoders, feedforward or feedback, give the best performance. Simulations also show that the best encoders to use when the channel quality is unknown are the systematic feedback ones. In a good channel we show that the type of encoder having the best bit error performance depends on the relative tailbiting length, i.e., the tailbiting length/memory. For a good channel, ML-decoding, and a rate $R = b/c$ tailbiting code of length L an upper bound on the bit error probability can be expressed as $P_b \leq \frac{1}{L}\sum_{d=d_{min}}^{cL} b_d P_d$, where b_d is the sum of all bit errors for all codewords of weight d and P_d is the probability that a word of weight d is chosen instead of the allzero word. For a given length L and memory m the encoder giving the lowest bit error probability in a good channel is the one with as large minimum distance as possible and the smallest $b_{d_{min}}$ as possible. For rate $R = 1/2$ a search has been made for these encoders at various lengths and encoder memories. We can identify three regions where different encoder types give the best performance. For very short relative tailbiting lengths the best feedforward encoders are systematic and give the same bit error probability as the best systematic feedback encoders. For short and medium relative tailbiting lengths, systematic feedback encoders are typically a factor of 1.5-2 better than the feedforward ones. For long relative tailbiting lengths feedforward encoders give typically a factor of 2 better performance than the systematic feedback encoders. The explanation for this lies in the type of codeword which leads to the minimum distance. We show that this in turn depends on the relative tailbiting length.

II. Tailbiting failure

A rate $R = b/c$ feedback convolutional encoder of memory m can be viewed as consisting of b linear feedback shift registers (LFSRs), where the longest shift register has length m. For a given LFSR we define the *cycle characteristic* of the LFSR as the set of all possible cycles of its output. Consider first a rate $R = 1/c$ encoder. Assume that the LFSR has a cycle of length p. Then if we are in one of the states that belongs to this cycle and feed the encoder with only zeros at the input, corresponding to an allzero information sequence, the encoder returns to the same state after p steps. If the tailbiting length (number of trellis sections) L is a multiple of p, then we have more than one codeword corresponding to an all-zero input since the allzero codeword corresponds also to the allzero input. This means that for this L, we have no one-to-one mapping between the blocks of information bits and the codewords, and the tailbiting technique cannot work. Every polynomial has at least one cycle of length 1, the zero cycle corresponding to the allzero codeword, which is not a trouble maker, but for any multiple of any other cycle, the tailbiting technique fails. If we have a general rate $R = b/c$ encoder the tailbiting technique does not work for any multiple of the cycles in the cycle characteristic of any of the b LFSRs. See also [2][3][4].

III. Finding the Encoder Starting State

For polynomial convolutional encoders realized in controller canonical form the initial state of the encoder is simply given by the reciprocal of the last m input b-tuples, but for systematic feedback encoders the starting state depends on all of the information bits to be encoded. Several methods are presently known for finding the starting state in the controller canonical form. Certain algebraic equations may be set up and solved to obtain the starting state [2][4]. In some cases the number of delay elements can be reduced by realizing the encoder in observer canonical form. For example, the minimal realization of rate $R = 2/3$ and $R = 3/4$ systematic feedback encoders is the observer canonical form [5]. We give a method for finding the starting state for this form.

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