Fast & Loose: Resource-bounded Reasoning in Wason's Selection Task

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Introduction

- In Wason's **Selection Task** (**ST**) [Was68] subjects **do not perform as classical-logical reasoners**.
- If **ST** is treated as a **"loose probabilistic task"** [OC94], observed inferential behavior is **optimal for Bayesian reasoners**.
- [OC94] acknowledge that **Bayesian computations are too complex** for bounded-rational reasoners, but no model so far.
- We obtain **convergence to the Bayesian solution** under assumptions generalizing Bayesian Rationality w/ severely bounded cognitive resources.

Instructions for the Selection Task

"Here are four cards, all of which have a number on one side and a letter on the other, followed by a rule that applies only to them:

A K 4 7

(Rule) If there is a vowel on the one side (P), then there is an even number on the other side (Q)

Which card (if any) must be turned in order to decide if the rule is true—without turning unnecessary cards?"

Components of the Model (Overview)

1. Interrogative Model of Learning : generalizes Bayesian (probabilistic) learning, captures interrogative strategies. [Hin07, GG14].
2. Non-classical semantics: subsumes truth-functional semantics [Was68] and probabilistic semantics proved equivalent to the Bayesian model [Duc09]
3. Cooperative Principle [Gri89]: constraints the expectations based on the instructions (generalize constraints on Bayesian priors).

Model (1): Interrogative Learning

- Sequential two-player game w/asymmetric info.: Inquirer (I) vs Nature (N).
- Problem=$\langle T, ? \rangle$: T = theory; $?$ = partition of models of T $\mathrm{Mod}(T)$.
- N chooses a state $s \in \mathrm{Mod}(T)$ and a set of available answers Ans.
- I must identify s 'up to' inclusion in some cell of Q:
  - ▷ I uses 'instrumental' questions, N answers **only if in Ans**.
  - ▷ I stops asking questions at information fixpoints.

Model (2): Generalized Adams Hypothesis

- **Adams' Hypothesis** captures the Bayesian model of **ST** [Duc09]

$$\mathrm{Prob}(\text{If P then Q}) = \mathrm{Prob}(Q|P) \text{ if } P > \mathbf{0,1} \text{ otherwise} \qquad \text{(AH)}$$

- **Generalized Adams Hypothesis** (**GAH**): X expects **If P, then Q** to hold to the same extent that X expects **Q** to hold, knowing **P**.

Model (3): Cooperative Principle

Quality Rule is **true**

Quantity+Relation **P** and **Q** are mentioned, **not-Q** is not:

(Rule*) If there is a vowel on the one side, then there is an even number on the other side more likely than a non-even one.

- Rule*=true in state s iff **P/Q**-occurrences in s (denoted m_s) outnumber **P/not-Q** occurrences in s (denoted n_s).

Model (4): Putting it Together

- Modeler's representation:

$$\mathrm{Mod}(T) = \{(A, y_1), (K, y_2), (x_3, 4), (x_4, 7) : x_i \in \{A, \ldots, Z\}, y_i \in \mathbb{R}\} \quad (1)$$

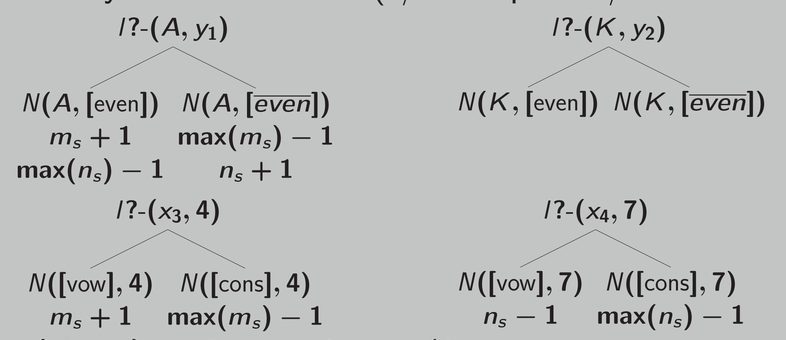
$$? = \{\text{Yes}, \text{No}\} \text{ where } = \begin{cases} s \in \text{Yes} & \text{if } m_s > n_s \\ s \in \text{No} & \text{otherwise} \end{cases} \quad (2)$$

$$\forall s \in \mathrm{Mod}(T), \mathrm{Ans}(s) = \{a_1, a_2, a_3, a_4\} \,\&\, a_1 = y_1, a_2 = y_2, a_3 = x_3, a_4 = x_4 \quad (3)$$

- Inquirer's representation: linguistic competence & categorizing letters as vowels or consonants (vow/cons) & numbers as even or not (even/$\overline{even}$).

A 4-card Poll (1): Relevance of Answers

Relevance 1 Answers affect estimates for m_s & n_s; questions can be considered independently of one-another. (I/N: Inquirer's/Nature'smove)



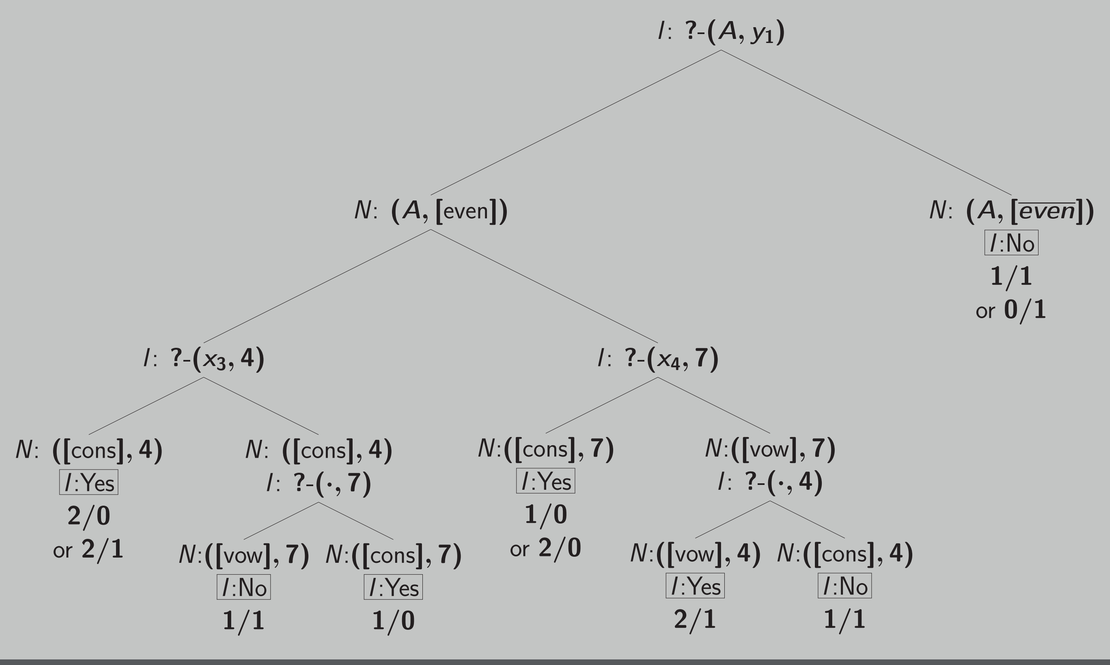
Relevance 2 (A, y_1) is always relevant (for estimating m_s or n_s); $(x_3, 4)$ and $(x_4, 7)$ are possibly relevant (for m_s and n_s, resp.) or irrelevant; (y_2) is always irrelevant.

Relevance 3 Questions can be ordered by relevance:

$$?\text{-}(A, y_1) \succ ?\text{-}(x_3, 4) \sim ?\text{-}(x_4, 7) \succ ?\text{-}(K, y_2)$$

A 4-card Poll (2): General Learning Strategy

1. Relevant cards only, most relevant first.
2. I stops at fixpoint when she can solve $\langle T, ? \rangle$ based on the ratio m_s/n_s, i.e. output Yes or No.



A 4-card Poll (3): Tie-Breakers

1. **Under indifference** vis-à-vis Rule, no single best sequence:

$$?\text{-}(A, \cdot)^\frown ?\text{-}(\cdot, 4)^\frown ?\text{-}(\cdot, 7) \sim ?\text{-}(A, \cdot)^\frown ?\text{-}(\cdot, 7)^\frown ?\text{-}(\cdot, 4)$$

2. **Under assumptions of [OC94]**:
   (i) Initially estimates Rule=false: **not-Q** is at least as likely as **Q**, given **P**.
   (ii) Questions aim at most surprising first (potential **P/Q** cases).

$$?\text{-}(A, \cdot)^\frown ?\text{-}(\cdot, 4)^\frown ?\text{-}(\cdot, 7) \succ ?\text{-}(A, \cdot)^\frown ?\text{-}(\cdot, 7)^\frown ?\text{-}(\cdot, 4)$$

Conclusion: Low-Cost Rational Analysis

- With **CP** and pragmatic inferences, **ST** is solvable without computation of probabilities/utilities or elimination of dominated strategies which are major contributors to computational costs.
- Local reasoning is cognitively less taxing, but converges towards optimal data selection without approximation or heuristics.
- Variations in initial attitude towards Rule/Rule* and criterion for picking questions can account for relevance-based selection reversals from [GKSH01] (future work!).

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