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Active Distances and Cascaded Convolutional Codes

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Abstract — A family of active distances for convolutional codes is introduced. Lower bounds are derived for the ensemble of periodically time-varying convolutional codes.

I. INTRODUCTION

The "extended distances" were introduced by Thommesen and Justesen [1] for unit memory (UM) convolutional codes. We present (non-trivial) extensions to encoder memories \( m \geq 1 \) and call them active distances since they stay "active" in the sense that we consider only those codewords which do not pass two consecutive zero states [2].

II. ACTIVE DISTANCES

Consider the ensemble of binary, rate \( R = b/c \), periodically time-varying convolutional codes encoded by a polynomial generator matrix of memory \( m \) and period \( T \),

\[
G = \begin{pmatrix}
G_0(t) & \cdots & G_m(t + m) \\
G_0(t + 1) & \cdots & G_m(t + m + 1) \\
\vdots & \ddots & \vdots \\
\end{pmatrix}
\]

in which each digit in each of the matrices \( G_i(t + T) \) for \( 0 \leq t \leq m \) and \( 0 \leq t < T - 1 \), is chosen independently and equally likely to be 0 and 1.

Let \( U_{t-m, t+j+m} \) be the set of information sequences \( u_{t-m} \ldots u_{t+j+m} \) such that the first \( m \) and the last \( m \) subblocks are zero and they do not contain \( m+1 \) consecutive zero subblocks.

Let \( U_{t, t+j} \) be the set of information sequences \( u_{t-m} \ldots u_{t+j} \) such that at least one subblock is nonzero and they do not contain \( m+1 \) consecutive zero subblocks.

Next we introduce the truncated time-varying generator matrix

\[
G_{[t, t+j]} = \begin{pmatrix}
G_m(t) \\
G_0(t) & \cdots & G_m(t + j) \\
\vdots & \ddots & \vdots \\
G_0(t + j) \\
\end{pmatrix}
\]

Definition 1 Let \( C \) be a time-varying convolutional code encoded by a time-varying, polynomial generator matrix. Then the \( j \)th order active row distance is

\[
a_j^r = \min_t \min_{U_{[t-m, t+j+m]}} w_H(u_{[t-m, t+j+m]} G_{[t, t+j+m]}),
\]

the \( j \)th order active column distance is

\[
a_j^c = \min_t \min_{U_{[t-m, t+j]}} w_H(u_{[t-m, t+j]} G_{[t+t+j]}),
\]

and the \( j \)th order active segment distance is

\[
a_j^s = \min_t \min_{U_{[t-t+j]}} w_H(u_{[t-t+j]} G_{[t+t+j]}).
\]

For a convolutional code encoded by a time-varying, non-catastrophic, polynomial generator matrix we define its free distance as \( d_{\text{free}} \).

III. CASCADED CODES

Consider a scheme with two convolutional codes in cascade. Theorem 1 There exist cascaded convolutional codes in the ensemble of periodically time-varying cascaded convolutional codes whose active distance satisfies

\[
\delta_i^* \equiv a_j^* \geq (l + 1) h^{-1} (1 - \frac{l}{l + 1}) R - O(\log_2 m)
\]

for \( l \geq l_0 = O(\frac{K}{R}) \).

REFERENCES
