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*Published in:*
[Host publication title missing]

*DOI:*
10.1109/ISIT.1997.613022

1997

# Active Distances and Cascaded Convolutional Codes[1]

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*Abstract* — **A family of active distances for convolutional codes is introduced. Lower bounds are derived for the ensemble of periodically time-varying convolutional codes.**

## I. Introduction

The "extended distances" were introduced by Thommesen and Justesen [1] for unit memory (UM) convolutional codes. We present (non-trivial) extensions to encoder memories $m \geq 1$ and call them *active distances* since they stay "active" in the sense that we consider only those codewords which do not pass two consecutive zero states [2].

## II. Active Distances

Consider the ensemble of binary, rate $R = b/c$, periodically time-varying convolutional codes encoded by a polynomial generator matrix of memory $m$ and period $T$,

$$G = \begin{pmatrix} G_0(t) & \cdots & G_m(t+m) & & \\ & G_0(t+1) & & \cdots & G_m(t+m+1) \\ & & \ddots & & & \ddots \end{pmatrix} \quad (1)$$

in which each digit in each of the matrices $G_i(t+T)$ for $0 \leq i \leq m$ and $0 \leq t \leq T-1$, is chosen independently and equally likely to be 0 and 1.

Let $\mathcal{U}^r_{[t_1-m,t_2+m]}$ be the set of information sequences $\boldsymbol{u}_{t_1-m} \cdots \boldsymbol{u}_{t_2+m}$ such that the first $m$ and the last $m$ subblocks are zero and they do not contain $m+1$ consecutive zero subblocks.

Let $\mathcal{U}^c_{[t_1-m,t_2]}$ be the set of information sequences $\boldsymbol{u}_{t_1-m} \ldots \boldsymbol{u}_{t_2}$ such that the first $m$ subblocks are zero and they do not contain $m+1$ consecutive zero subblocks.

Let $\mathcal{U}^s_{[t_1-m,t_2]}$ be the set of information sequences $\boldsymbol{u}_{t_1-m} \ldots \boldsymbol{u}_{t_2}$ such that at least one subblock is nonzero and they do not contain $m+1$ consecutive zero subblocks.

Next we introduce the truncated time-varying generator matrix

$$G_{[t,t+j]} = \begin{pmatrix} G_m(t) & & \\ \vdots & \ddots & \\ G_0(t) & & G_m(t+j) \\ & \ddots & \vdots \\ & & G_o(t+j) \end{pmatrix}. \quad (2)$$

**Definition 1** *Let $C$ be a time-varying convolutional code encoded by a time-varying, polynomial generator matrix. Then the* $j$th order active row distance *is*

$$a^r_j \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}^r_{[t-m,t+j+m]}} w_H(\boldsymbol{u}_{[t-m,t+j+m]} G_{[t,t+j+m]}), \quad (3)$$

*the* $j$th order active column distance *is*

$$a^c_j \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}^c_{[t-m,t+j]}} w_H(\boldsymbol{u}_{[t-m,t+j]} G_{[t,t+j]}), \quad (4)$$

*and the* $j$th order active segment distance *is*

$$a^s_j \stackrel{\text{def}}{=} \min_t \min_{\mathcal{U}^s_{[t-m,t+j]}} w_H(\boldsymbol{u}_{[t-m,t+j]} G_{[t,t+j]}). \quad (5)$$

For a convolutional code encoded by a time-varying, noncatastrophic, polynomial generator matrix we define its free distance as $d_{\text{free}} \stackrel{\text{def}}{=} \min_j a^r_j$.

## III. Cascaded Codes

Consider a scheme with two convolutional codes in cascade.

**Theorem 1** *There exist cascaded convolutional codes in the ensemble of periodically time-varying cascaded convolutional codes whose active distance satisfies*

$$\delta^r_l \stackrel{\text{def}}{=} \frac{a^r_j}{mc} \geq (l+1)h^{-1}(1 - \frac{l}{l+1}R) - O(\frac{\log_2 m}{m}) \quad (6)$$

*for* $l \geq l^r_0 = O(\frac{1}{m})$,

$$\delta^c_l \stackrel{\text{def}}{=} \frac{a^c_j}{mc} \geq lh^{-1}(1-R) - O(\frac{\log_2 m}{m}) \quad (7)$$

*for* $l \geq l^c_0 = O(\frac{\log_2 m}{m})$, *and*

$$\delta^s_l \stackrel{\text{def}}{=} \frac{a^s_j}{mc} \geq lh^{-1}(1 - \frac{l+1}{l}R) - O(\frac{\log_2 m}{m}) \quad (8)$$

*for* $l \geq l^s_0 = \frac{R}{1-R} + O(\frac{\log_2 m}{m})$.

By minimizing the lower bound on the active row distance we obtain nothing but the main term in Costello's lower bound on the free distance, viz., $\frac{R}{-\log_2(2^{1-R}-1)}$.

## References

[1] C. Thommesen and J. Justesen, *Bounds on distances and error exponents of unit-memory codes*, IEEE Trans. Inform. Theory, vol. IT-29, pp. 637–649, July 1983.

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[1]This research was supported in part by the Royal Swedish Academy of Sciences in cooperation with the Russian Academy of Sciences and in part by the Swedish Research Council for Engineering Sciences under Grant 94-83.