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Abstract

The effective refractive-index as a function of vacuum wavelength is approximated by Lagrange interpolation polynomials. The root-mean-square value of the chromatic dispersion is then calculated analytically. It is demonstrated that use of fourth degree polynomials is far more efficient than use of second degree polynomials. The rms-value of the chromatic dispersion over the wavelength range [1.25 μm, 1.60 μm] is calculated and minimized for step-index fibers, triangular-index fibers, and α-power fibers. The full vector solution of Maxwell's equations is used. It is demonstrated that the approximate model of the refractive-index, used in this paper and in other papers, induces an error in the rms-value which is not negligible when designing dispersion-flattened fibers.

1 Introduction

The predominant transmission medium in long-distance, high-capacity telecommunication is the single-mode optical fiber. If wavelength multiplexing is to be used in order to further increase transmission capacity, then the chromatic dispersion should be kept low over a range of vacuum wavelengths [1]. The problem of how to create such dispersion-flattened fibers is approached in this paper. The vacuum wavelength range is chosen as [1.25 μm, 1.60 μm] which is a range of low attenuation in pure silica glass.

The chromatic dispersion is defined in Section 2 and the relation to pulse-broadening is explained. The approximate refractive-index model to be used is defined and an efficient method of calculating the root-mean-square value of the chromatic dispersion over a broad wavelength range is presented. The error induced by the approximate refractive-index model is investigated in the special case of a step-index fiber. This error has recently been analyzed by Safaai-Jazi and Lu [2].

The rms-value of the chromatic dispersion is minimized in Section 3. The analysis is limited to step-index fibers, triangular-index fibers, and α-power fibers. The refractive-index is, in the cases of triangular-index fibers and α-power fibers, assumed to be continuous at the core-cladding boundary. These fibers are all described by two variables: the core radius a and the relative refractive-index increase N_1 in the core center. The concept of "relative refractive-index increase" is defined in Section 2.3. The condition that the first higher-order mode should appear at 1.25 μm yields a curved line in the N_1-a-plane. The rms-value of the chromatic dispersion is calculated at different points on this line. The point of minimum chromatic dispersion is then easily located.

2 Chromatic dispersion

2.1 Pulse broadening

The field components of a guided mode of an optical fiber include the factor

$$e^{j(\omega t-\beta z)}$$

where β is the propagation constant.

The group velocity v_g and the group delay τ_g are defined as

$$v_g = \frac{d\omega}{d\beta} \tag{2.1}$$

$$\tau_g = \frac{1}{v_g} \tag{2.2}$$

The group velocity, and thus the group delay, varies with the vacuum wavelength. The chromatic dispersion is defined as

$$C = \frac{d\tau_g}{d\lambda_0} \tag{2.3}$$

where λ_0 is the vacuum wavelength.

The chromatic dispersion is of importance when calculating the broadening an optical pulse undergoes when travelling along an optical fiber [1]. The basic formula in this context is [3].

$$\sigma_{\mathrm{out}}^2 = \sigma_{\mathrm{in}}^2 + (\Delta\lambda_0 LC)^2$$

where

σ	temporal width of light pulse
$\Delta\lambda_0$	spectral width of light pulse
L	length of fiber
C	chromatic dispersion

The effective refractive-index n_e is defined as

$$n_e = \frac{\beta}{k_0} \tag{2.4}$$

where k_0 is the vacuum propagation constant. A useful formula for the chromatic dispersion in a single-mode fiber is

$$C = -\frac{\lambda_0}{c}\frac{d^2 n_e}{d\lambda_0^2} \tag{2.5}$$

where c is the speed of light in a vacuum and n_e is the effective refractive-index of the fundamental mode. The formula (2.5) is derived from (2.1) – (2.4).

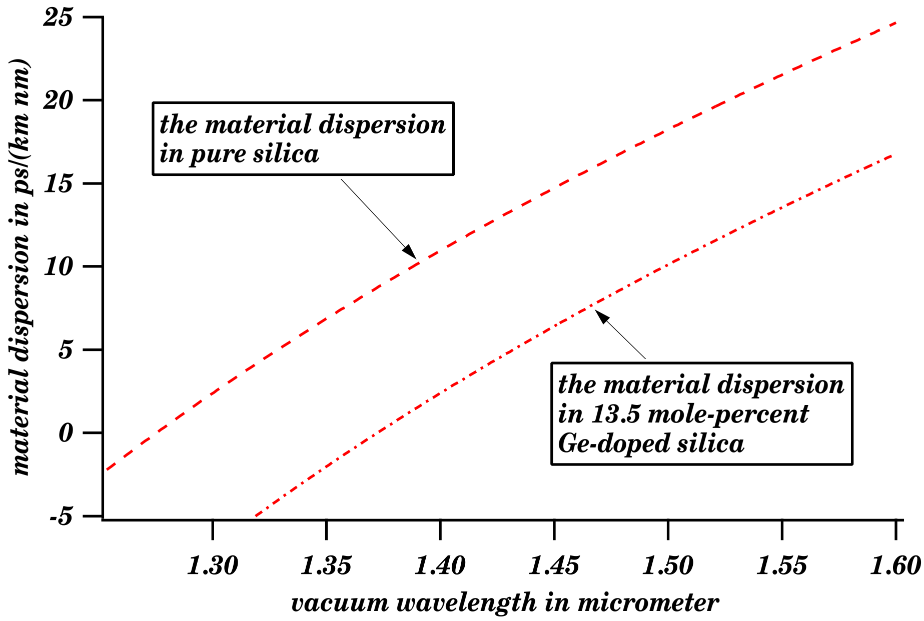

Figure 1: The material dispersion as a function of the vacuum wavelength.

2.2 The refractive-index of pure silica

A formula for the refractive-index of pure silica glass is [4, 5]

$$n_s = C_0 + C_1\lambda_0^2 + C_2\lambda_0^4 + \frac{C_3}{(\lambda_0^2 - 0.035)} + \frac{C_4}{(\lambda_0^2 - 0.035)^2} + \frac{C_5}{(\lambda_0^2 - 0.035)^3} \tag{2.6}$$

where λ_0 is in micrometer and the coefficients are given in Table 1. Examples which can be used as check-values are

$$n_s(\lambda_0 = 1.25 \text{ micrometer}) = 1.447825545$$
$$n_s(\lambda_0 = 1.60 \text{ micrometer}) = 1.443787805$$

The material dispersion is defined as

$$C_m = -\frac{\lambda_0}{c}\frac{d^2 n_s}{d\lambda_0^2} \tag{2.7}$$

A formula for the refractive-index of 13.5 mole-percent Ge-doped silica is also given in [4, 5]. Material dispersion curves are given in Figure 1.

2.3 An approximate refractive-index model

The actual refractive-index profile $n(r, \lambda_0)$ of an optical fiber is a function of the radial coordinate r and of the vacuum wavelength λ_0. The actual refractive-index profile $n(r, \lambda_0)$ can be written

$$n(r, \lambda_0) = N(r, \lambda_0)\, n_s(\lambda_0)$$

$$\begin{aligned} C_0 &= +\ 1.4508554 \\ C_1 &= -\ 0.0031268 \\ C_2 &= -\ 0.0000381 \\ C_3 &= +\ 0.0030270 \\ C_4 &= -\ 0.0000779 \\ C_5 &= +\ 0.0000018 \end{aligned}$$

Table 1: The coefficients in the approximation formula for the refractive-index of pure silica glass [4, 5].

where $n_s(\lambda_0)$ is the refractive-index of pure silica and $N(r, \lambda_0)$ is "the normalized refractive-index" or "the relative refractive-index increase".

An approximation, applied by e.g. Yip and Jiang [6], is to assume that the normalized refractive-index N is a function of the radial coordinate only, i.e.

$$n(r, \lambda_0) = N(r)\, n_s(\lambda_0) \tag{2.8}$$

The necessity of resorting to an approximate refractive-index model is stated and discussed in Ref. [2]. A more accurate refractive-index model must probably be based on some interpolation technique. A linear interpolation technique is used by Garth [7].

2.4 The rms-value f of the chromatic dispersion

A computer program has been implemented which computes the root-mean-square value f of the chromatic dispersion of the fundamental mode over a wavelength range.

$$f = \left(\frac{1}{\lambda_2 - \lambda_1} \int_{\lambda_1}^{\lambda_2} C^2(\lambda_0)\, d\lambda_0 \right)^{1/2} \tag{2.9}$$

This rms-value is the function f which is to be minimized.

The computer program applies the power-series expansion method developed in Ref. [8]. This method yields the full vector solution of Maxwell's equations, see Appendix A.

The computer program calculates the effective refractive-index for a number of equidistant vacuum wavelengths. This is done by solving the characteristic equation by a secant root-searching method. The material dispersion is included through (2.8). The effective refractive-index as a function of the vacuum wavelength is represented by Lagrange interpolation polynomials [9]. The rms-value f is then calculated analytically using (2.5) and (2.9), see Appendix B.

The rms-value f of the material dispersion between $\lambda_1 = 1.25$ μm and $\lambda_2 = 1.60$ μm was calculated to 14.6 ps/(km nm) using the formula for the refractive-index of pure silica (2.6) and the described interpolation technique. The calculation

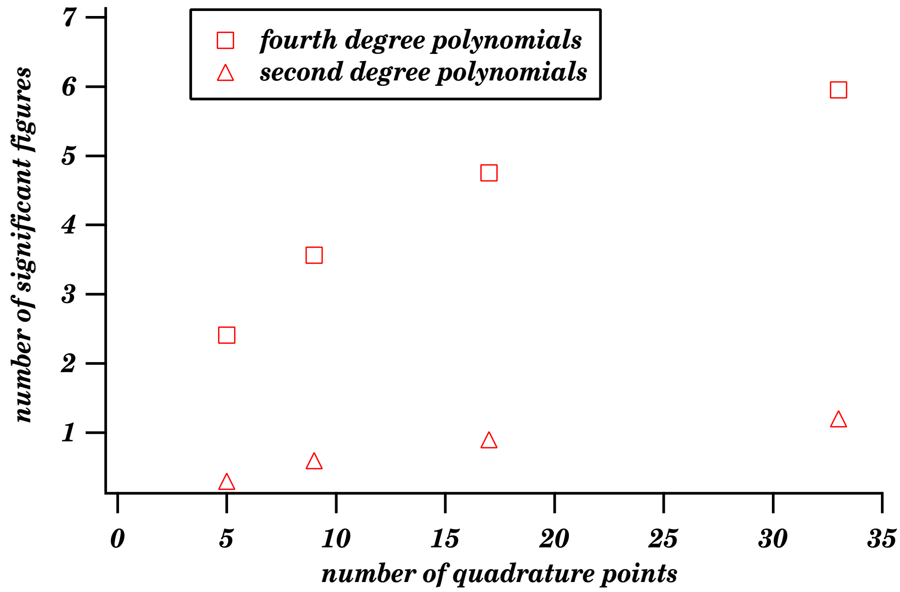

Figure 2: The number of significant figures in the rms-value f of the material dispersion as a function of the number of quadrature points, i.e. as a function of the number of refractive-index evaluations.

of (2.9) was performed for different numbers of quadrature points, i.e. for different numbers of vacuum wavelengths, and the use of repeated second and fourth degree polynomials was compared, see Figure 2. The result clearly demonstrates that use of fourth degree polynomials is far more efficient than use of second degree polynomials. The cost in terms of computer time to evaluate the rms-value f of the chromatic dispersion for a given refractive-index profile is proportional to the number of effective refractive-index evaluations. Hence, the old effective refractive-index evaluations are re-used when the number of quadrature points is increased and the sequence of numbers on the abscissa in Figure 2 is

$$5, 9, \ldots, n, n+n-1, \ldots$$

2.5 Error induced by approximate refractive-index model

In order to investigate the magnitude of the error induced by the approximation (2.8), the chromatic dispersion of a step-index fiber is calculated. The core of this step-index fiber is assumed to be 13.5 mole-percent Ge-doped silica and the cladding is assumed to be pure silica. The core radius of the step-index fiber was determined by the condition that the cut-off wavelength should be equal to 1.25 μm. The relative refractive-index increase N_1 in the core varies from 1.0144 at 1.25 μm to 1.0147 at 1.60 μm. When computing the waveguide dispersion, the refractive-indices are held constant at their 1.25 μm-values. When applying the approximation (2.8) the relative refractive-index increase N_1 in the core is held constant at the 1.25 μm

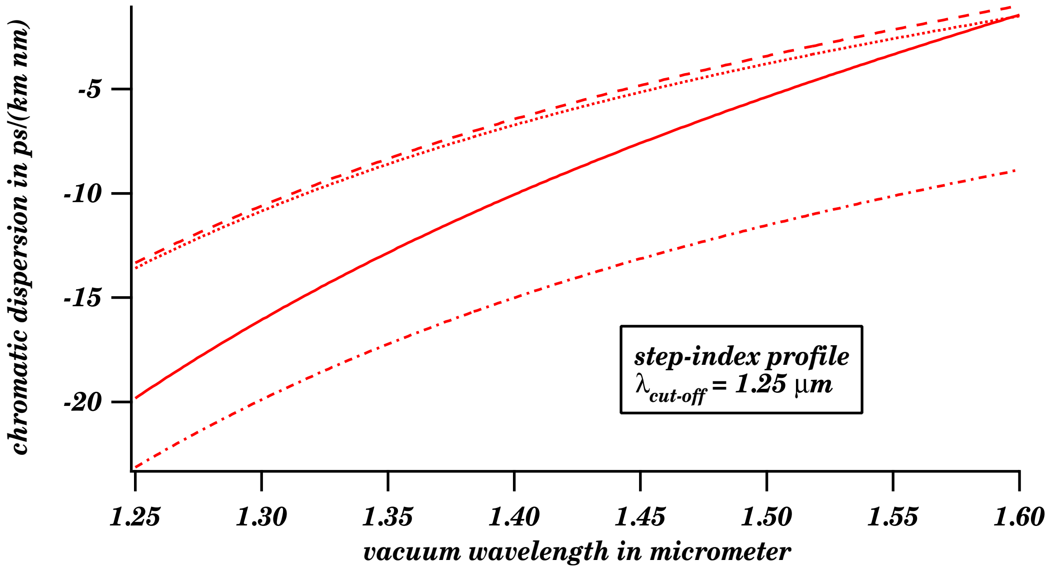

Figure 3: Four different calculations of the chromatic dispersion in a step-index fiber in which the core is 13.5 mole-percent Ge-doped silica and the cladding is pure silica. The solid line is the exact chromatic dispersion. The dotted line is obtained when approximation (2.8) is used. The dashed line is the sum of waveguide dispersion and pure silica material dispersion. The dashed-dotted line is the sum of waveguide dispersion and doped silica material dispersion.

value, i.e. $N_1 = 1.0144$. It is seen in Figure 3 that applying (2.8) is only slightly better than simply adding the waveguide and the material dispersion. However, in the absence of a better alternative, the approximation (2.8) will be used in this paper.

The rms-value f of the exact chromatic dispersion between $\lambda_1 = 1.25$ μm and $\lambda_2 = 1.60$ μm is calculated to 10.7 ps/(km nm). When N_1 is held constant, the rms-value f is calculated to 7.3 ps/(km nm). Thus, the error induced by the approximate refractive-index model (2.8) is, in this special case, 3.4 ps/(km nm).

3 Minimization

3.1 Step-index profiles

A normalized step-index profile is

$$N(r) = \begin{cases} N_1 & 0 \le r < a \\ 1 & r > a \end{cases}$$

where a is the core radius and the corresponding actual refractive-index profile is

$$n(r, \lambda_0) = \begin{cases} N_1 n_s(\lambda_0) & 0 \le r < a \\ n_s(\lambda_0) & r > a \end{cases}$$

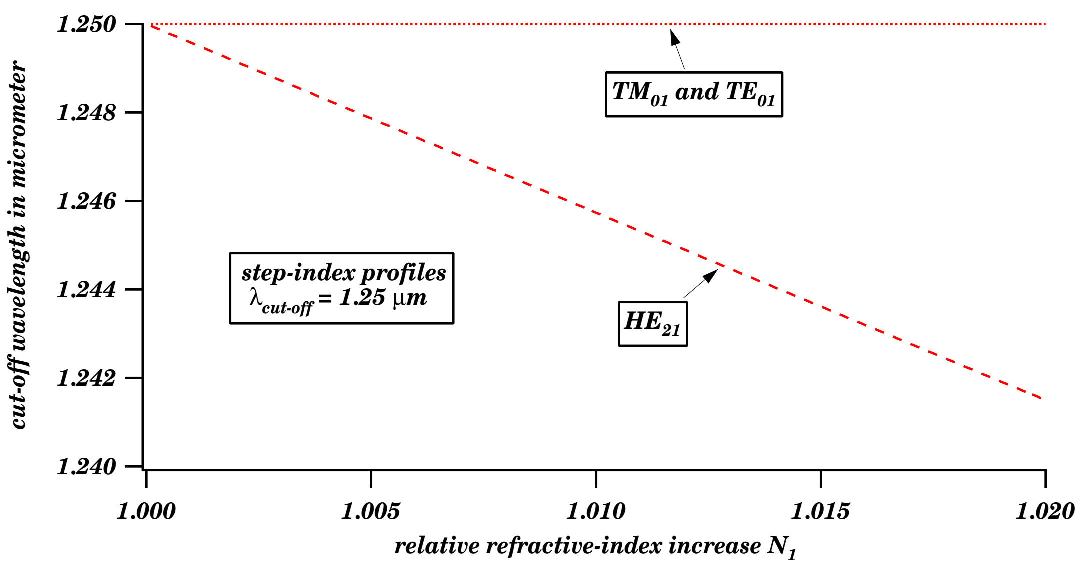

Figure 4: The cut-off vacuum wavelengths for the first higher order modes in a step-index fiber ($\lambda_{\text{cut-off}} = 1.25\ \mu$m) as a function of the relative refractive-index increase N_1 in the core.

The minimization problem can now be stated. Seek the parameters N_1 and a of the step-index profile which minimize the rms-value f of the chromatic dispersion over the vacuum wavelength range $[\lambda_1, \lambda_2]$ where $\lambda_1 = 1.25\ 10^{-6}$ m and $\lambda_2 = 1.60\ 10^{-6}$ m subject to the constraint

$$\lambda_c \quad = \quad 1.25\ 10^{-6}\ \text{m} \tag{3.1}$$

where λ_c is the vacuum cut-off wavelength for the first higher-order mode.

The fundamental mode in a step-index fiber is HE_{11} and the first higher-order modes to appear are TM_{01} and TE_{01}. The mode HE_{21} appears at a slightly shorter wavelength, see Figure 4.

The exact cut-off condition for the TM_{01} and TE_{01} modes is

$$V = j_{01} = 2.405 \tag{3.2}$$

where j_{01} is the first zero of the Bessel function J_0 and the normalized frequency V is

$$V = \frac{2\pi}{\lambda_0}\ a\ \sqrt{N_1^2 - 1}\ \ n_s \tag{3.3}$$

Substitute (3.1), (2.6), and (3.2) into (3.3) and the cut-off condition can be written as

$$a\ \sqrt{N_1^2 - 1} = \frac{j_{01}\lambda_c}{2\pi n_s(\lambda_c)} = 3.30\ 10^{-7}\ \text{m}$$

This constraint turns the rms-value f into a function of only one parameter, i.e.

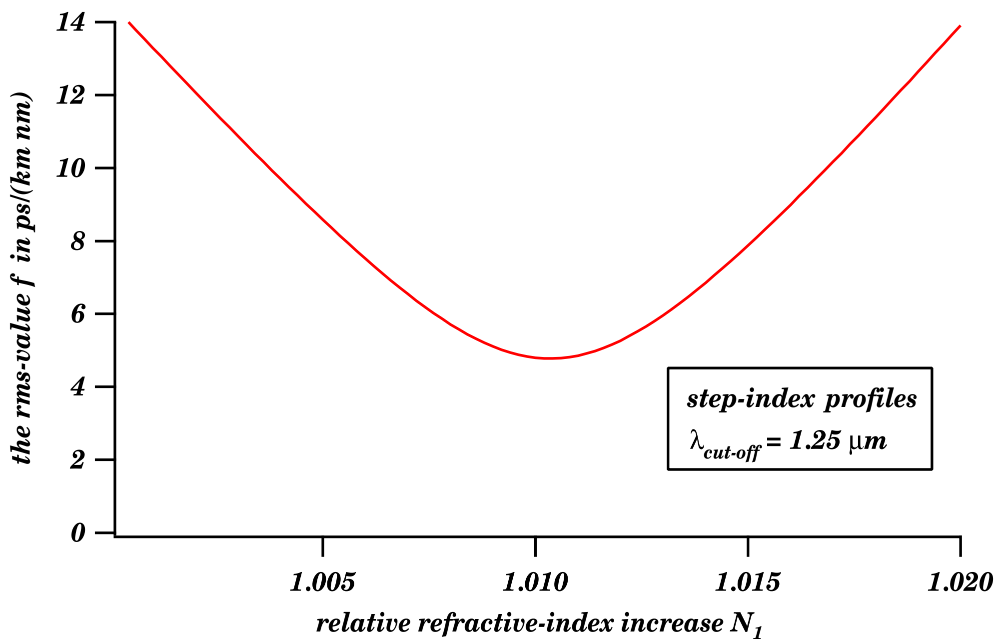

Figure 5: The root-mean-square value f of the chromatic dispersion over the vacuum wavelength range [1.25 μm, 1.60 μm] for a step-index fiber as a function of the relative refractive-index increase N_1 in the core.

$f = f(N_1)$. The graph of $f(N_1)$ is given in Figure 5. The minimum is

$$\left\{ \begin{array}{lcl} f_{\min} & = & 4.79 \text{ ps/(km nm)} \\ N_1 & = & 1.0103 \\ a & = & 2.29\ 10^{-6} \text{ m} \end{array} \right.$$

where a is the core radius.

The normalized refractive-index profile and the chromatic dispersion of this "optimal" step-index profile are given in Figure 6 and Figure 7, respectively. A relative refractive-index increase as high as $N_1 = 1.0103$ would give high attenuation. If, somewhat arbitrarily, it is assumed that the maximum relative refractive index-increase allowed is $N_1 = 1.005$, then it is evident from Figure 5 that the minimum rms-value of the chromatic dispersion is $f(1.005)$ which is equal to 8.59 ps/(km nm).

3.2 Triangular-index profiles

A normalized triangular-index profile is

$$N(r) = \left\{ \begin{array}{ll} N_1 + (1 - N_1)\,\frac{r}{a} & 0 \le r \le a \\ 1 & r \ge a \end{array} \right.$$

The minimization problem is to determine the parameters N_1 and a of the triangular-index profile which minimize the rms-value f of the chromatic dispersion. The

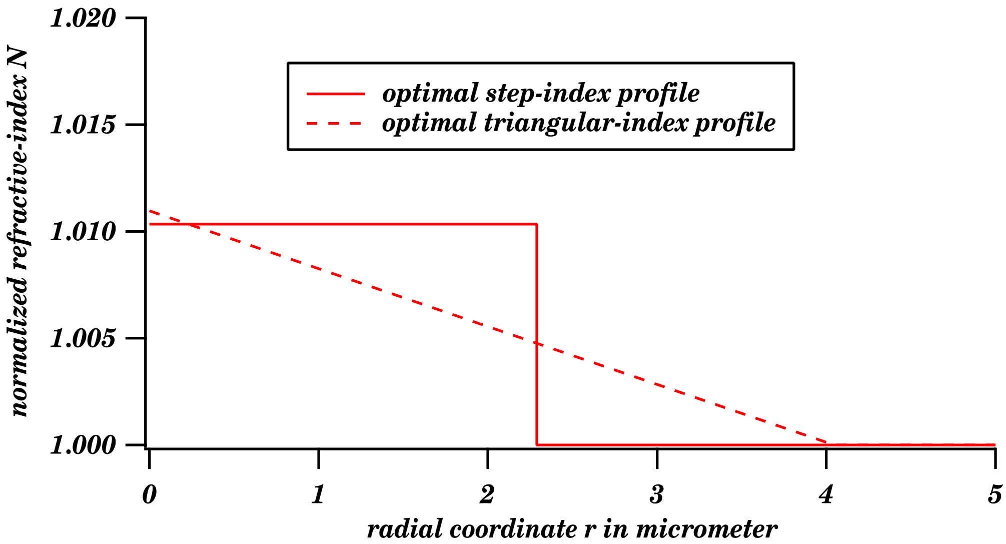

Figure 6: The step- and triangular-index profiles which minimize the rms-value f of the chromatic dispersion over the vacuum wavelength range [1.25 μm, 1.60 μm] subject to the constraint $\lambda_{\text{cut-off}} = 1.25$ μm.

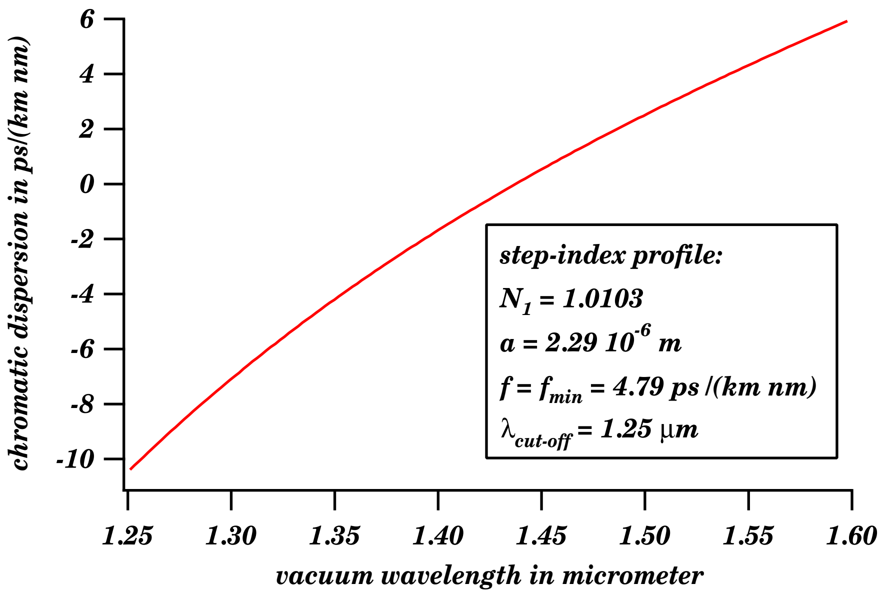

Figure 7: The chromatic dispersion of the step-index profile which minimizes the rms-value f of the chromatic dispersion over the vacuum wavelength range [1.25 μm, 1.60 μm].

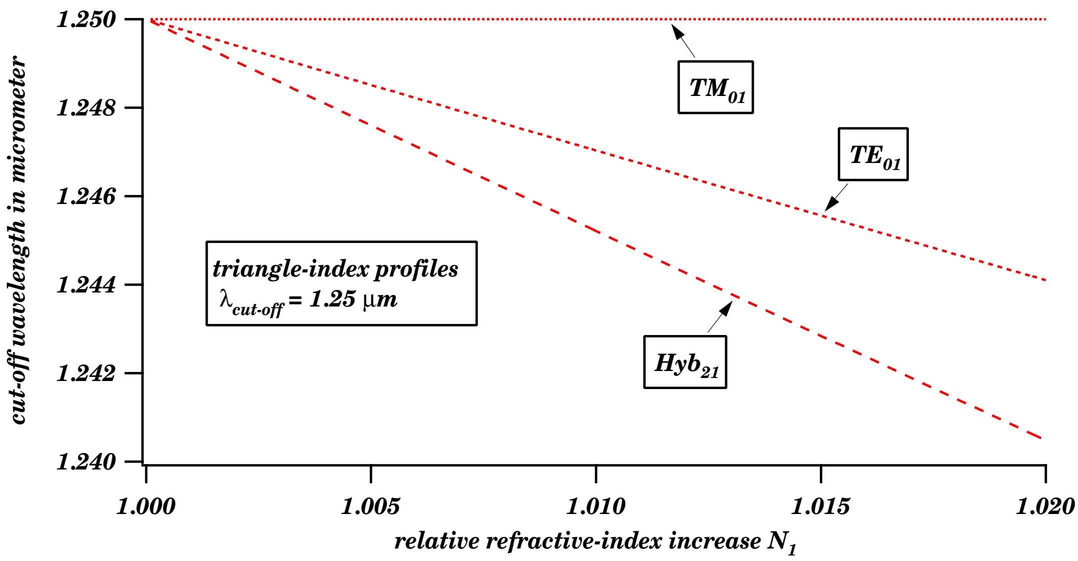

Figure 8: The cut-off vacuum wavelengths for the first higher order modes in a triangular-index fiber ($\lambda_{\text{cut-off}} = 1.25\ \mu$m) as a function of the relative refractive-index increase N_1 in the core center.

fundamental mode in a triangular-index fiber is Hyb_{11} and the first higher-order modes are, in order of appearance as the wavelength is decreased, TM_{01} TE_{01} and Hyb_{21} see Figure 8. The hybrid mode notation is after Morishita [10]. The rms-value f, as a function of N_1 , when the TM_{01} cut-off is held at 1.25 μm, is given in Figure 9. The minimum is

$$\begin{cases} f_{\text{min}} & = & 6.71\ \text{ps/(km nm)} \\ N_1 & = & 1.0110 \\ a & = & 4.05\ 10^{-6}\ \text{m} \end{cases}$$

Notice that this "optimal" triangular-index profile gives a higher rms-value f than the corresponding "optimal" step-index profile in Section 3.1.

3.3 α-power profiles

A normalized α-power profile is, cf. [11], and see Figure 10

$$N(r) = \begin{cases} \sqrt{N_1^2 - (N_1^2 - 1)\ (\frac{r}{a})^\alpha} & 0 \le r \le a \\ 1 & r \ge a \end{cases}$$

Notice that $\alpha = 1$ almost, but not exactly, corresponds to a triangular-index profile and that $\alpha = \infty$ exactly corresponds to a step-index profile. Some exact normalized frequencies for the TE_{01} cut-off in α-power profiles are given by Oyamada and Okoshi [12]. The tabulated data in Ref. [12] are calculated by scalar analysis, but in the special case of TE-modes scalar and exact analyzes coincide. The fundamental

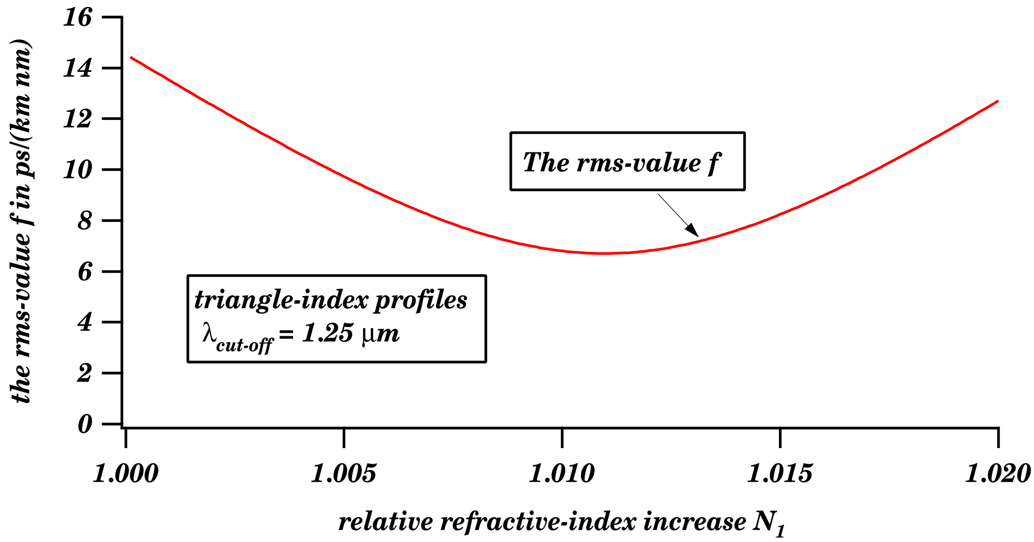

Figure 9: The root-mean-square value f of the chromatic dispersion over the vacuum wavelength range [1.25 μm, 1.60 μm] for a triangle-index fiber as a function of the relative refractive-index increase N_1 in the core center.

mode in an α-power fiber is Hyb_{11} and the first higher-order mode to appear is TM_{01}. The condition that the TM_{01} cut-off is 1.25 μm yields a curved line in the N_1-a-plane, see Figure 11. The TE_{01} cut-off is given in Figure 12. The step-index fiber provides the lowest root-mean-square chromatic dispersion, see Figure 13.

4 Conclusion

The rms-value of the chromatic dispersion can be efficiently evaluated by Lagrange interpolation followed by analytical differentiation and integration. The error in this rms-value, induced by the approximate model (2.8) of the refractive-index, is calculated to as much as 3.4 ps/(km nm) for a strongly doped, step-index fiber. It has been demonstrated that with this approximate model of the refractive-index and within the class of α-power profiles, the step-index fiber provides the lowest rms-value of the chromatic dispersion over the wavelength range [1.25 μm, 1.60 μm], namely 4.79 ps/(km nm).

Appendix A A power-series expansion method

The method in Ref. [8] is developed for a cylindrical dielectric waveguide with a piece-wise polynomial permittivity profile in the radial direction. The relative permittivity is equal to the square of the refractive-index. A system of four ordinary differential equations is derived from Maxwell's equations. This system has two bounded solutions in the core. These solutions are constructed by a sequence of power-series expansions. The two bounded solutions in the cladding are expressed in

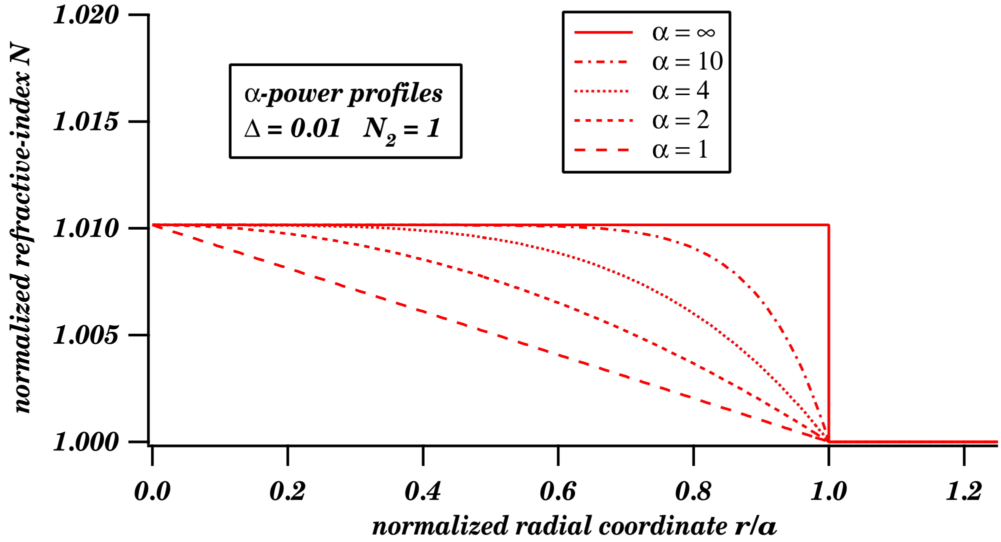

Figure 10: The normalized refractive-index profile for different α-values.

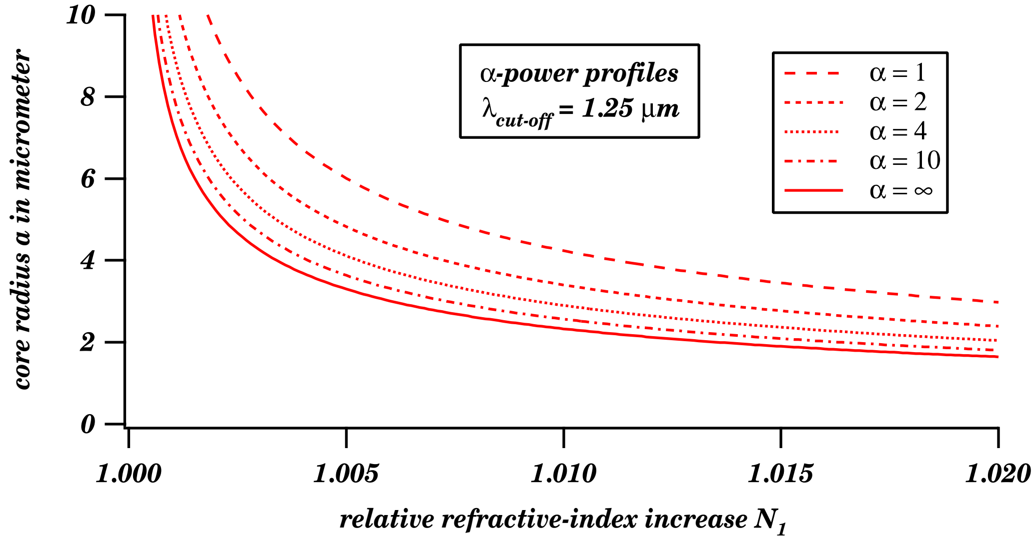

Figure 11: The condition $\lambda_{\text{cut-off}} = 1.25\ \mu$m yields a curved line in the N_1-a-plane.

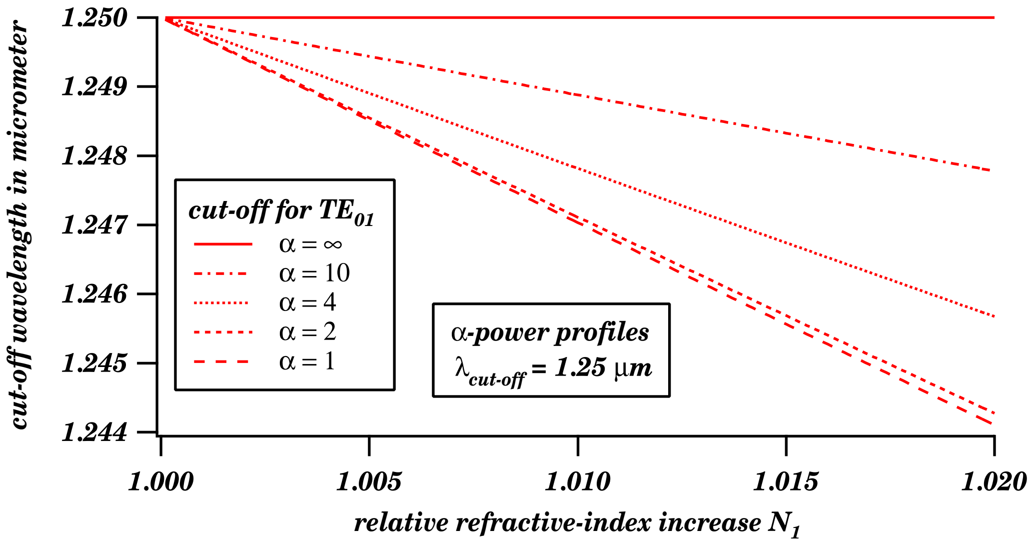

Figure 12: The cut-off vacuum wavelength for the TE_{01} mode as a function of the relative refractive-index increase N_1 in the core center for different α-power profiles. Cut-off for TM_{01} is 1.25 μm.

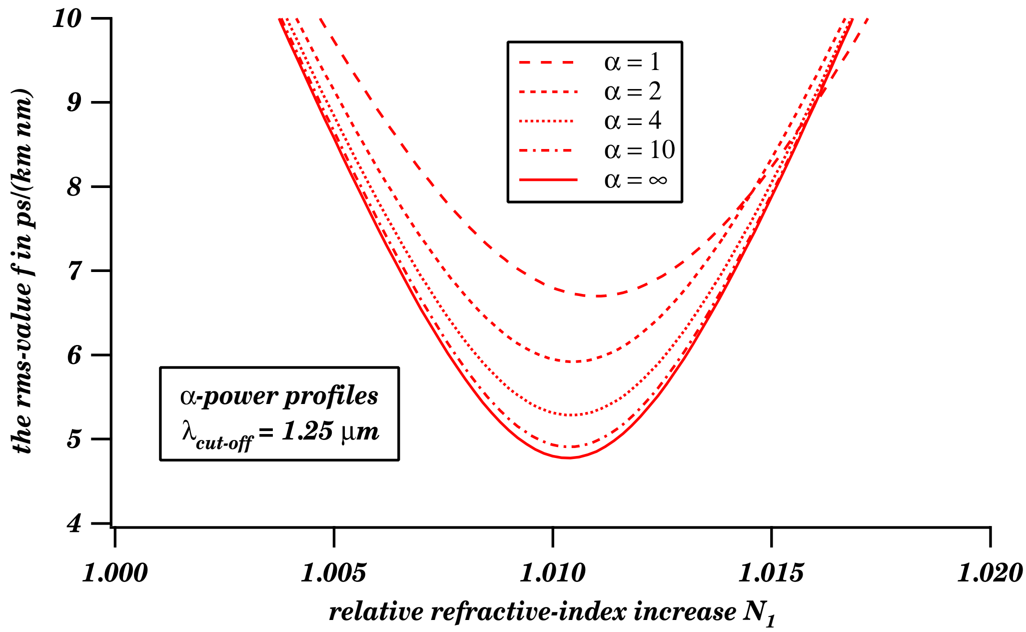

Figure 13: The rms-value f of the chromatic dispersion over the vacuum wavelength range [1.25 μm, 1.60 μm] for different α-power fibers as a function of the relative refractive-index increase N_1 in the core center.

modified Bessel functions. The characteristic equation is obtained from the boundary conditions at the core-cladding interface. The propagation constant, and thus the effective refractive-index n_e, is obtained as a root of the characteristic equation. One detail, not explicitly mentioned in Ref. [8], is that when the azimuthal mode number is equal to zero, the characteristic equation splits into a TM-case and a TE-case. These cases should, of course, be treated separately, otherwise double-roots and closely spaced roots appear when solving the characteristic equation.

Appendix B Calculation of chromatic dispersion

The relevant mathematical problem in the evaluation of the rms-value of the chromatic dispersion (2.9) is to calculate the integral

$$\int_a^b x^2 \left(\frac{d^2 f}{dx^2}\right)^2 dx$$

in which x stands for wavelength and f stands for effective refractive-index. The function f is approximated by the fourth-degree polynomial passing through the following five points [9]

$$x_p = \frac{a+b}{2} + p\frac{b-a}{4} \qquad p = -2, -1, 0, 1, 2$$

This polynomial is differentiated twice.

$$\frac{d^2 f}{dx^2} = \left(\frac{4}{b-a}\right)^2 \frac{d^2 f}{dp^2}$$

where

$$\begin{aligned}
\frac{d^2 f}{dp^2} \quad = \quad & \frac{-f_{-2} + 16 f_{-1} - 30 f_0 + 16 f_1 - f_2}{12} + \\
& + \frac{-f_{-2} + 2 f_{-1} - 2 f_1 + f_2}{2} p + \\
& + \frac{f_{-2} - 4 f_{-1} + 6 f_0 - 4 f_1 + 6 f_2}{2} p^2
\end{aligned}$$

The three coefficients in this polynomial in p should be calculated numerically.

$$\begin{aligned}
\int_a^b x^2 \left(\frac{d^2 f}{dx^2}\right)^2 dx \quad = \quad & \left(\frac{a+b}{2}\right)^2 \left(\frac{4}{b-a}\right)^3 \int_{-2}^{2} \left(\frac{d^2 f}{dp^2}\right)^2 dp + \\
& + \left(\frac{a+b}{2}\right) \left(\frac{4}{b-a}\right)^2 \int_{-2}^{2} 2p \left(\frac{d^2 f}{dp^2}\right)^2 dp + \\
& + \left(\frac{4}{b-a}\right) \int_{-2}^{2} p^2 \left(\frac{d^2 f}{dp^2}\right)^2 dp
\end{aligned}$$

Integration gives the final expression which is used as a quadrature rule.

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