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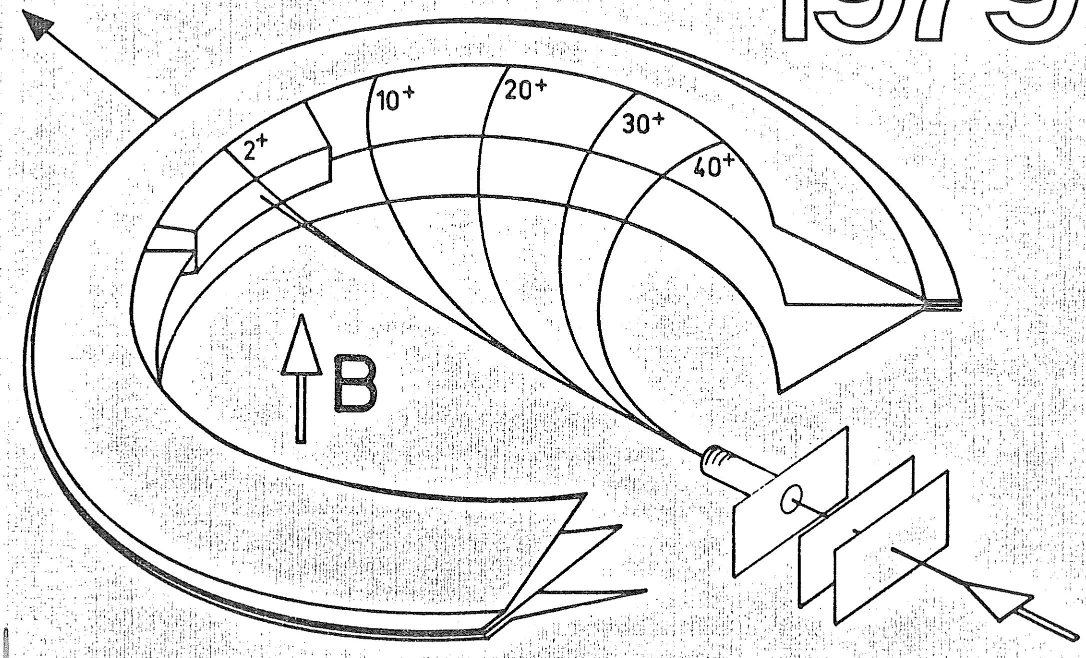

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DYNAMICAL CALCULATIONS OF SPONTANEOUS-FISSION HALF-LIVES

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Abstract

DYNAMICAL CALCULATIONS OF SPONTANEOUS-FISSION HALF-LIVES.

Spontaneous-fission half-lives are calculated for the heaviest even-even nuclei ($Z = 96-110$). The penetration of the fission barrier is treated as a one-dimensional problem, in the WKB approximation. The potential energy is calculated as a sum of the Myers-Swiatecki droplet model energy and the shell correction obtained by the Strutinsky method. Quadrupole axial and non-axial deformations and axial deformations of multipolarities three, five and six are considered. The mass tensor (only six components) is calculated in the cranking approximation. All microscopic calculations are based on the modified oscillator single-particle potential. The action integral is minimized by two different variational methods. No free parameters are used. – The calculated half-lives reproduce the experimental values with an accuracy slighly better than two orders (around a factor of 50), on the average.

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1. INTRODUCTION

There is a continuous interest in the theoretical reproduction of the spontaneous-fission half-lives T_{sf} for known nuclei and in corresponding predictions for unknown nuclides. This is connected with a continuous progress in the synthesis of new elements and also of new isotopes of already known elements. An important reason for this interest is also the fact that the present description of T_{sf} is still rather far from satisfactory.

This paper is a continuation of our previous research [1, 2], in which a theoretical reproduction on the halv-lives T_{sf} without making use of any free parameter has been tried. In the earlier, both dynamic (e.g. Ref.[3]) and static (e.g. Ref.[4]), calculations, a free parameter has been used. Henceforth, we shall refer to Ref.[1] as I.

We shall concentrate on even-even nuclei with atomic numbers Z = 96–110. The half-lives T_{sf} are calculated dynamically. The main differences with respect to the paper I are: accounting for the ϵ_6 degree of freedom (describing the deformation of multipolarity 6) in the calculation of the fission barrier, application of the Ritz method in the variational calculation of the barrier penetrability – as a test for the variational method elaborated and applied earlier (see I) – consideration of a larger number of isotopes.

2. DESCRIPTION OF THE CALCULATIONS

In the calculation of the half-life, $T_{sf} = \ln 2/(nP)$, where n is the number of assaults of a given nucleus on the fission barrier per unit time, a basic factor is the probability P of the penetration of the nucleus through the fission barrier for a given assault. Similarly, as in previous investigations [1, 3, 5, 6], the barrier is treated as one-dimensional. A two-dimensional treatment has been studied in Ref.[7] for the case of a simple barrier. In the one-dimensional case, the penetrability calculated in the WKB approximation is [5]

$$P = [1 + \exp S(L_{min})]^{-1} \tag{1}$$

where

$$S(L) = 2 \int_{s_1}^{s_2} \sqrt{\frac{2}{\hbar^2} [V(s) - E]\, B(s)}\, ds \tag{2}$$

with

$$B(s) = \sum_{i,j} B_{\alpha_i \alpha_j}(s) \frac{d\alpha_i}{ds} \frac{d\alpha_j}{ds} \tag{3}$$

is the action integral calculated along a trajectory L given in the deformation space. V(s) is the potential energy along L, E is the energy of the fissioning nucleus, α_i and α_j (i, j = 1, . . . , n) are the deformation parameters, and s is the parameter describing the position on the trajectory L. B(s) is the effective-mass (inertia) parameter of the nucleus 'moving' along L and constructed from the components B of the mass tensor.

By the dynamical calculation of T_{sf}, we mean that, having both the potential energy $V(\alpha_1, \ldots, \alpha_n)$ and the mass tensor $B_{\alpha_i \alpha_j}(\alpha_1, \ldots, \alpha_n)$ dependent on deformation, we look (by variational methods) for the trajectory L_{min} for which the action S(L) becomes minimum.

The deformation of the nucleus is described by the following parameters: (quadrupole) ϵ, (hexadecapole) ϵ_4, (multipolarity 6) ϵ_6 (see Ref.[8]) and (reflection-asymmetric) ϵ_{35}, being a combination of the parameters ϵ_3 and ϵ_5 corresponding to the deformations of multipolarities 3 and 5, respectively [9]. Besides these axially symmetric deformations, the non-axial quadrupole deformation γ [10] is also taken into account. The full dynamical calculation is performed in the (ϵ, ϵ_4) space. Other degrees of freedom are accounted for either in an approximate dynamical way (γ) (see I) or only statically (ϵ_6, ϵ_{35}), i.e. only through the potential energy.

The potential energy is calculated as a sum of the smooth part, described by the Myers-Swiatecki droplet model [11, 12] and the shell correction evaluated by the Strutinsky method [13].

The mass tensor is obtained by the cranking model (e.g. Refs [5, 14]). All microscopic calculations (shell correction and mass tensor) are based on the modified oscillator single-particle potential with the 'A = 242 parameters' (see Ref. [8]).

Minimization of the action integral (2) is performed by the variational method. The potential energy and the mass tensor are calculated at 198 grid points in the (ϵ, ϵ_4)-plane, the same as in I, covering a wide region in this plane. A procedure based on dynamical programming methods [15] and elaborated in Ref.[16] allows for a numerically fast choice of the minimal path L_{min} among all possible zigzag-type trajectories connecting two fixed end points and passing through the grid points situated in the barrier region (i.e. in the region where the potential energy $V(\alpha_1, \ldots, \alpha_n)$ is larger than the energy E of a fissioning

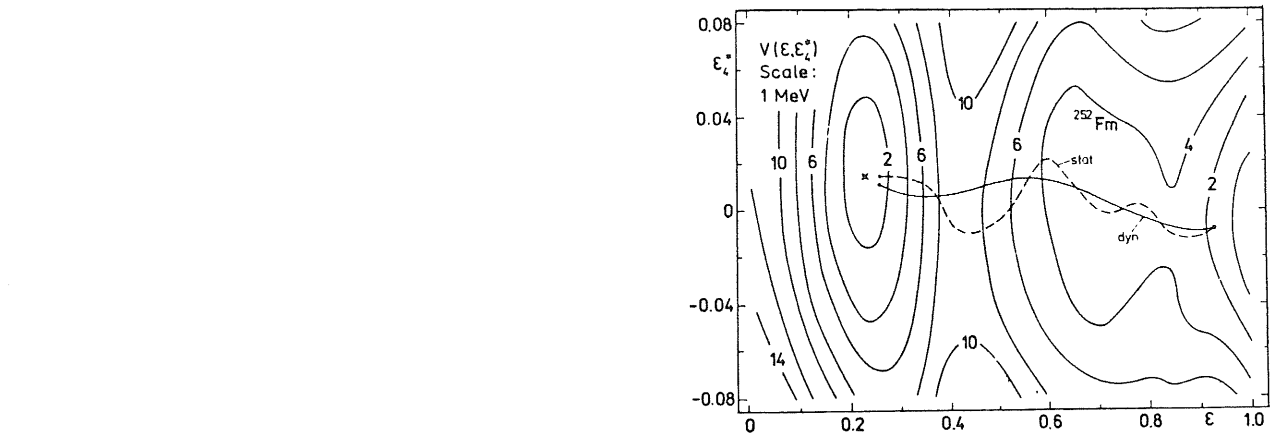


FIG.1. Potential energy of ^{252}Fm calculated as a function of the deformation parameters ϵ and ϵ_4^*. Static and dynamic fission trajectories are indicated.

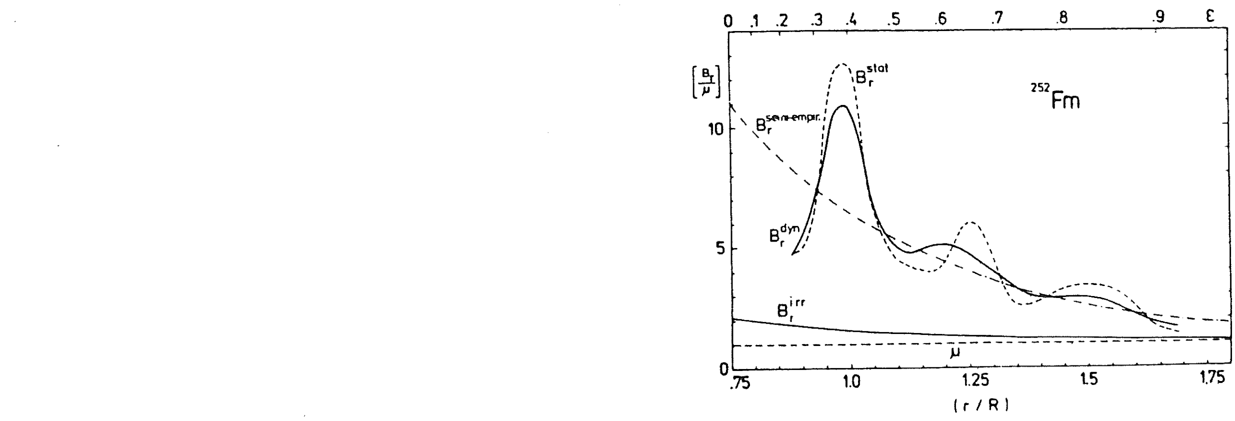


FIG.2. Effective mass parameter calculated along static and dynamic trajectories for ^{252}Fm.

nucleus. As the two end points we take the point of minimal potential energy (first minimum), situated before the entrance into the barrier, and a point corresponding to the same potential energy but situated behind the exit point out of the barrier, on the static fission trajectory. The sensitivity to changes in the end points is checked.

As the dynamical fission trajectories are found to be smooth and not to deviate very much from the straight line connecting the end points (Fig.1), we found it reasonable to use also the following Ritz method for finding L_{min} [2]: The deviation of a fission trajectory from the straight line connecting the end points is expanded in a Fourier series (sine functions), and the coefficients of the expansion are treated as the variational parameters. Minimization of the action S(L) shows that the series is rapidly convergent. The main contribution is obtained from the first two terms; the contribution of the third term is already small and that of the fourth is only of the order of two pro-mille (2×10^{-3}). This method, which uses smooth trial trajectories L, may be considered to be a test of the above method using L lines of the zigzag-type. The half-lives T_{sf} obtained by both methods appear to be practically the same.

3. RESULTS AND DISCUSSION

An example of the potential energy calculated as a function of ϵ and ϵ_4 (the parameter $\epsilon_4^* = \epsilon_4 - 0.2\,\epsilon + 0.06$, equivalent to ϵ_4, is introduced to make an average fission trajectory for heavy elements especially simple: $\epsilon_4^* \approx 0$) is given in Fig.1. We see that the dynamical trajectory (dyn) is shorter and smoother than the static one (stat), i.e. the trajectory obtained when only the potential energy is minimized or, equivalently, when S(L) is minimized with the mass tensor independent of deformation. This is typical for all nuclei.

Although the dynamical trajectory deviates considerably from the static one, the corresponding potential barriers (along both trajectories) are close to one another. This is because, when deviating from the static trajectory, we are close to the minimum of the potential energy and the changes of this energy are of second order. The difference in the effective-mass parameter is larger. The parameter calculated along the dynamical path (B^{dyn}) is smaller in the region of the largest values of the barrier than that obtained along the static path (B^{stat}). This can be seen in Fig.2, where B^{dyn} is considerably smaller than B^{stat} in the region of the saddle point which is around $\epsilon = 0.4$ (as seen in Fig.1). This difference results in the smaller life-times T_{sf} when calculated dynamically, than those obtained statically; so, it makes dynamics significant. It can also be seen that B^{dyn} is a smoother function of the deformation than B^{stat}, and this makes B^{dyn} somewhat less sensitive to details of the shell structure of the nucleus and, thus, to details of its description, than B^{stat}. In the figure, we also show a one-parameter semi-empirical effective mass $B^{semi\text{-}empir}$ as introduced in Ref.[17] The parameter is fitted to the experimental T_{sf} [17] (see also Ref.[4]). We see that $B^{semi\text{-}empir}$ is rather close to our effective B, when in the latter the shell effects are smoothed out.

The effect of the deformation ϵ_6 (investigated only statically in this paper) on the fission barrier is considerable. It is illustrated in Fig.3 for ^{252}Fm. We can see that minimization of the potential energy with respect to this additional degree of freedom decreases the energy mainly in the region of the first minimum.

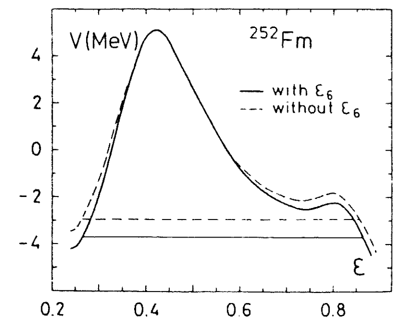


FIG.3. Effect of the inclusion of the deformation ϵ_6 on the static fission barrier for ^{252}Fm.

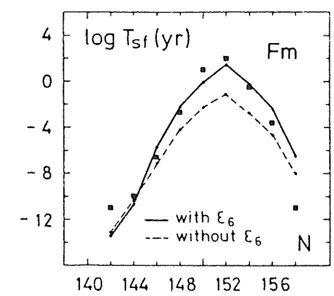


FIG.4. Effect of accounting for ϵ_6 on the half-lives T_{sf} for the Fm isotopes.

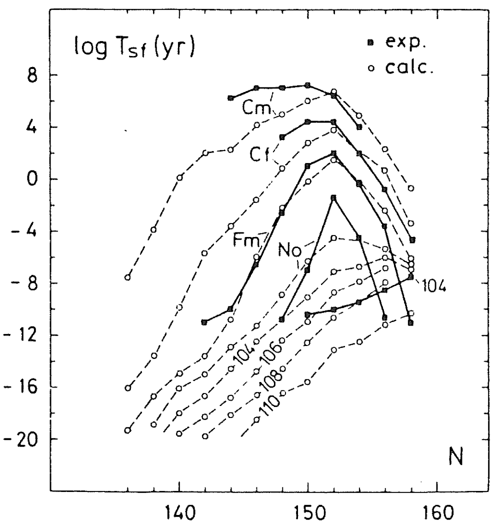
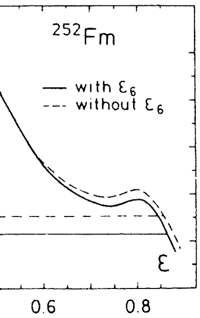
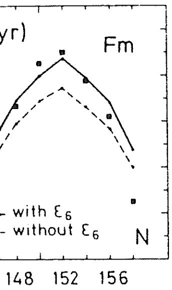


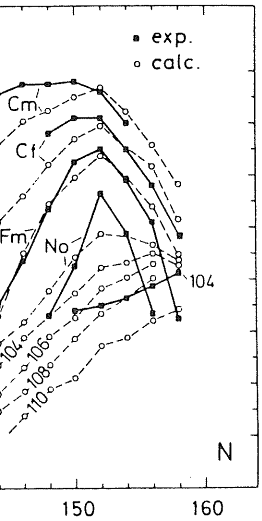
FIG.5. Final results for the calculated half-lives T_{sf} (in years). The experimental values are also shown.



mation ϵ_6 on the static fission barrier for ^{252}Fm.



; on the half-lives T_{sf} for the Fm isotopes.



alf-lives T_{sf} (in years). The experimental values

This increases the barrier for about 0.7–0.8 MeV and results in an increase of the half-life T_{sf} by about two orders. The changes in T_{sf}, due to the inclusion of ϵ_6, are explicitly given in Fig.4 for all Fm isotopes for which the experimental T_{sf} are known. We can see that T_{sf} is increased for almost all isotopes.

The final results for T_{sf} are given in Fig.5. Some more recent experimental points in the figure are taken from Refs [18–21]. We see that the microscopic calculations (with no free parameters!) are able to reproduce the experimental life-times with an accuracy better than two orders (around a factor of 50), on the average. The change in the half-life systematics, seen for Z = 102–104, is less abrupt in the calculations than in the experiment.

The calculations are performed with the droplet-model parameters of Ref.[11]. A use of the new parameters [12] fitted to the experimental masses and barriers results, as a rule, in life-times T_{sf} that are one or two orders shorter.

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DISCUSSION

H.A.O. NIFENECKER: Moretto and Babinet have pointed out that the pairing gap should be treated dynamically and that the action integral should also be minimized with respect to the pairing gap parameter. Have you include these factors in your calculations or would you consider them irrelevant?

A. SOBICZEWSKI: We did not minimize the action integral with respect to the pairing gap. Pairing is treated in a static (BCS) manner, with the strength of this interaction being fitted to the odd-even mass difference.

F. DICKMANN: I would just like to comment, in connection with wheth the pairing gap is used in the calculation of the effective mass parameter, that the gap cannot be chosen freely in the quasi-static cranking model and has to be the same as for the potential energy calculation. If we determine the gap by treating it as a variable free parameter to be determined by minimizing the action integral, then the effective mass is no longer the cranking mass. I think that for the finite deformation velocities, not treated in your paper, anti-pairin effects would reduce the gap parameter below its static value.

A.F. MICHAUDON: I should also like to stress the importance of pairing in calculating the mass inertia parameter and, therefore, the dynamic approach to scission. Usually, simple assumptions are made for the variation in the pairi force with deformation, taken either as a constant or as proportional to the surface of the deformed nucleus. But the truth of the matter seems more complicated.

For example, when pairing is not introduced as a free parameter, but is obtained from the calculation using an adequate effective force, Hartree-Fock calculations actually show that the pairing strength can exhibit large variation as a function of deformation. I refer those who are interested in this point to the paper that Dr. Berger will be presenting at this Symposium (see SM-241/C in these Proceedings).

K.W. GOEKE: Dr. Sobiczewski, as far as I understand, you have used the Inglis cranking model for evaluation of the collective mass tensor. This model disregards, however, the rather important residual interaction. Since the mass parameter enters directly into the expression for the life-time, I would expect your results to change considerably if you applied theories such as self-consistent cranking or the adiabatic time-dependent Hartree-Fock (ATDHF) theory.

A. SOBICZEWSKI: Yes, I agree that the residual interactions which are not included in the cranking model and which could be considered, for example, in the ATDHF treatment, may alter the inertia significantly. It would certainly be a good thing to obtain inertia by using ATDHF. However, before applying the ATDHF to the fission mode it would be worthwhile testing it (with consistently calculated potential energy) by an analysis of the vibrational and rotational modes, which seem to be better suited to testing inertia than fission. For cranking inertia this test has shown that the figure obtained is too small. I refer you to Ref.[14] of my paper.

J.P. THEOBALD: Did you also find evidence for highly asymmetric fission of the No isotopes when one of the fragments is a Ni or Ca isotope, such as has been predicted by Greiner and Sandulescu?

A. SOBICZEWSKI: No, we have not dealt with the fragmentation problem.