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# A Numerical Study of Vortex-Induced Vibrations (VIV) in an Elastic Cantilever

**Johan Lorentzon**[†,1], **Per-Erik Austrell**[2], **Robert-Zoltan Szasz**[1] **and Johan Revstedt**[1]
[1]Division of Fluid Mechanics
[2]Division of Structural Mechanics
Faculty of Engineering, Lund University, Sweden
e–mail:pi06jl6@student.lth.se[†]

**Summary** This study treats the subject fluid-structure interaction (FSI) for incompressible flow with small vibrations. The open source packages DEAL.II and OpenFOAM have been used to create a coupling between a finite element formulation for the structure and finite volume formulation for the fluid. A staggered solution algorithm have been implemented in C++ and verified against empirical data of Vortex-Induced Vibration (VIV) frequencies.

## Introduction

A cantilever is placed in a domain of a velocity driven fluid. The traction differential acting upon the structure induce a deformation and the movement of the structure affects the fluid as well. This mutual influence referred to as fluid-structure interaction (FSI), is known to cause several interesting phenomena. Among such is vortex-induced vibration[1] (VIV), where the forced movement of a fluid around the structure gives upon point of release from the structure, an angular momentum manifested as a vortex in the fluid with an oscillating transversal force component.

## Mathematical and Numerical Description

A physical domain consisting of fluid and structure is described by velocity field ($\mathbf{U}$,$\mathbf{v}$) and pressure $p$, displacement field ($\mathbf{q}$) and pressure in a continuum model. The equations governing the motion of an incompressible Newtonian fluid and an elastic structure ($D_{ijkl}\epsilon_{kl}$) with damping ($C_{il}v_l$) then takes the following form in reduced variables ($^*$) in the fluid domain and state space formalism for the structure with small strain operator ($\epsilon(\mathbf{q})$),

$$\nabla^* \cdot \mathbf{U}^* = \mathbf{0}, \tag{1}$$

$$\frac{DU_i^*}{Dt^*} = -\partial_i p^* + \frac{1}{Re}\nabla^{*2}U_i^* + b_i^*, \tag{2}$$

$$\dot{q}_i - v_i = 0, \tag{3}$$

$$\partial_t v_i + C_{il}v_l - \partial_j D_{ijkl}\epsilon_{kl}(\mathbf{q}) = f_i. \tag{4}$$

The coupling boundary between fluid and structure is a traction term, i.e. the sum of the pressure force and the viscous force. Both PDE sets have the same character and therefore a monolithic approach is feasible [2]. However, the problem can become too large to handle or unstable, therefore a staggered algorithm is preferred where even the individually domains can further, by divide and conquer, be partitioned [4, 5]. In solving Eqn (2) and (4) it is assumed that the problem can be formulated in two steps, the solution of the physical domain in a steady state formalism, followed by semi-discretization in time.

[1]For an excellent review, C.H.K. Williamson and R. Govardhan, Vortex-induced vibrations. Ann. Rev.Fluid.Mech **36** (2004) 443-455.

Further, by assumption of fixed point solution the fluid domain is solved separately from the structure domain, using FVM respectively FEM.

```
Time Loop
 Staggered Loop
    Solve Fluid State
    Transfer Traction to Solid State Solver
    Solve Solid State
    Exit Staggered Loop if change of deformation < tolerance
    Transfer Deformation to Fluid State Solver
  End Staggered Loop
 End Time Loop
```

The norm for convergence is with respect to the displacement field. However, while combining two solvers the time must be adaptive with respect to the $CFL$ condition in order to meet the convergence criteria. The open source packages used in this study are OpenFOAM[2]. and DEAL.II[3]. The staggered algorithm allows the FSI solver to be run on separate machines/threads. The test case and the staggered algorithm originates from a study using OpenFOAM [5]. The fix-point iteration to locate the quasi-static equilibrium point between the solvers use the Aitkens relaxation method[4] to accelerate the sub cycle loop, the staggered loop.

### The Case Study

A cantilever of thickness $D = 0.2$ m and height of $10D$ is placed $5D$ from the inlet, $2.5D$ from the walls and $20D$ from the outlet. The wire frame of the rectangular domain is thus $(26 \times 6 \times 12.5) \cdot D$. The flow is velocity driven with uniform Dirichlet condition at the inlet (mag($\mathbf{U}$)) and Neumann conditions at the outlet. For the pressure a Neumann condition is used at the inlet and a Dirichlet condition at the outlet. At the walls, no-slip conditions are used. The unstructured grid in the fluid domain is created using scaled tetrahedral elements with a structured boundary mesh with size 0.02 m , growth rate 1.1 and 0.1 m as upper limit on cell size, while the structured grid for the structure domain $8 \times 8 \times 64$ cell partition.

### Application to VIV

The following empirical expression for the Strouhal number (St) can be used to estimate the frequency of the VIV for a cantilever in an infinite domain,

$$St = \frac{fl}{U} = 0.198(1 - \frac{19.7}{Re}). \qquad (5)$$

The result in table 1 presents the frequency of probes placed in respective domain, showing the synchronization between the frequency of the fluid motion ($f$) and the structure ($f_s$).

[2] *http://www.opencfd.co.uk/openfoam/*
[3] *http://www.dealii.org.*

It scales within the margin of error with Eqn (5). However, wall effects should also be accounted for. Table 2 gives the observed VIV in the nodamped cases with no fluid probes, note that for $U = 1$ two frequencies appear, where the higher is the first harmonic of the lower and it appears due to discretization error of a sinusoidal function. The VIV is masked

| U ($ms^{-1}$) | $f_s(Hz)$ | $f(Hz)$ | $f_i(Hz)$ |
|---|---|---|---|
| 1 | 0.7 | 1-1.3 | 1.7 |
| 10 | 8 | 7-14 | 11 |
| 25 | 15 | 17-25 | 11 |

Table 1: VIV frequency with damping from section 6.3 in [1].

| U ($ms^{-1}$) | $f_s(Hz)$ |
|---|---|
| 1 | 0.79, 0.74, 1.47, 1.53, 1.53, 1.53 |
| 10 | 6.3, 6.6, 5.2 |
| 25 | 17.07, 17.2 |

Table 2: VIV frequency without damping from table 6.1 in [1].

by the in-line frequency due to release of cantilever and for this reason a Rayleight damping was added with 0.1%. The in-line frequency $f_i$ well match reported elsewhere [3]. Figure 1-2 is the FFT spectra for U=1 in table 1.

**Conclusion**

This study presents a method to resolve the fluid-structure interaction (FSI) using a fixed-point iterative scheme with a partitioned Gauss-Seidel technique accelerated with Aitkens relaxation method. The validation of the solver involves among others,

- reproduced frequency shift in in-line movement.
- matched frequency in VIV with probes of fluid and structure.
- reproduced VIV frequency with regard to Eqn (5).

The study implicates the need for damping in this model where frequency is obtained in a real time numerical experiment.

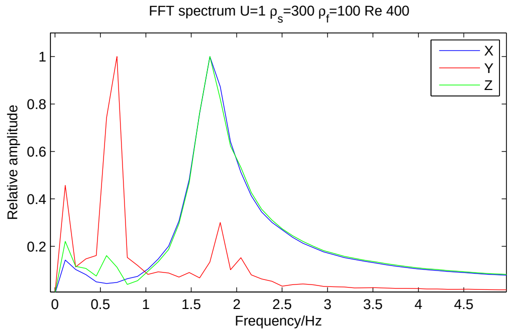


Figure 1: The FFT on marker point at U=1.

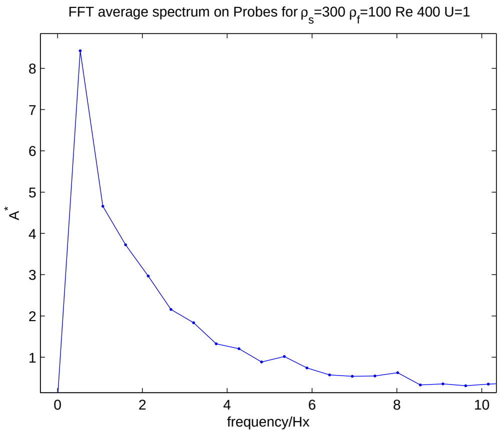


Figure 2: The averaged FFT spectrum for the fluid probes for U=1.

## References


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