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# Madness in the Method: A Paradox of Inquiry Learning

Emmanuel J. Genot<sup>a</sup> and Agneta Gulz<sup>b</sup>

**Abstract**—Hintikka’s *Interrogative Model of Inquiry* (IMI) rationalizes the process of discovery (as opposed to justification), and has been proposed as an epistemological basis for inquiry learning. We show that some key steps of inquiry learning still cannot be rationalized within the IMI, and suggest possible developments of the IMI that could offer a suitable logical and epistemological basis for inquiry learning.

## I. INTRODUCTION

Epistemological models distinguish *contexts of discovery* from *contexts of justification*, and usually assume that inferences carried in the former cannot be rationalized. Some formal models of inquiry explicitly tackle discovery of new facts driven by problem-solving. One of the most general is Hintikka’s *Interrogative Model of Inquiry* (IMI) which describes inquiry as a ‘game’ where Inquirer asks ‘small’ instrumental to Nature, in order to answer a ‘big’ question. The IMI vindicates Sherlock Holmes’ method, where deduction guides interrogation. Hakkarainen and Sintonen (hereafter H&S) argue that “representing inquiry as a step-by-step procedure, captures the dynamics of theory building—and hence learning” [6, p.39], and that the IMI offers an epistemological basis for inquiry learning. They back their claim with empirical results, but eschew the question whether the formal results of the IMI also support it.

This paper answers that question, and uncovers an unexpected consequence. Like Polonius who sees the method in Hamlet’s madness, the IMI rationalizes discovery by grounding it in deductions.<sup>1</sup> But it also entails that in some contexts of discovery *critical deductive steps cannot be rationalized*, and these contexts include those studied by H&S. However, we resist the anti-methodology conclusion that there is a deeper ‘madness in the method’. We suggest that the IMI captures the effect of *interaction* in inquiry learning, and conclude on possible ways to extend the model to a full account of its role in theory formation (and learning).

Sec. II presents the IMI and illustrates it with an example from Sherlock Holmes. Sec. III uses this example to introduce key concepts and results of the IMI. Sec. IV presents a reconstruction of the Sherlock Holmes case that highlights the role of guesses that the IMI cannot rationalize, and shows them to be critical in H&S’s study. We conclude on how the IMI should be developed to actually support education practices.

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<sup>1</sup>“Though this be madness, yet there is method in it.” William Shakespeare, *The Tragedy of Hamlet, Prince of Denmark*, Act 2, scene 2.

## II. THE INTERROGATIVE MODEL OF INQUIRY

### A. The Game of Inquiry

Early formulations of the IMI goes back to the 1980s but we will consider it (for this exposition) as a generalization of algorithmic learning-theoretic models that appeared in the 1990s, esp. the ‘first-order paradigm’ of [14], in which a *problem* may be characterized by a pair  $\langle T, Q \rangle$ , where  $T$  is a background theory expressed some (first-order) language  $\mathcal{L}$ ;<sup>2</sup> and  $Q$  a (principal) *question*—usually a binary question, that partitions possible states of Nature compatible with  $T$ , denoted hereafter  $\mathbf{S}(T)$ . Nature chooses a state  $s \in \mathbf{S}(T)$  and a *data stream* (an infinite sequence of basic sentences of  $\mathcal{L}$ ) that in the limit fully characterize the features of  $s$  expressible in  $\mathcal{L}$ ; then Nature reveals one datum at a time. A learning strategy is a function taking as argument finite segments of the data stream, and returning either an answer in  $Q$  or ‘?’ (suspension of judgment).

The model of [12] generalizes the above one by dropping some idealizations. Nature, instead of a complete data stream, chooses a set  $A_s$  of *available answers* in  $s$ , that can be expressed by sentences in  $\mathcal{L}$  of arbitrary complexity (and may then be analyzed by ‘analytical’ moves).  $A_s$  determines which properties and entities are resp. observable and identifiable. The data stream is built by Inquirer, using instrumental questions to supplement the information  $T$  gives her about  $s$ , and may therefore remain incomplete.<sup>3</sup> An *interrogative learning strategy* takes as argument a finite sequence of data, and outputs a (possibly empty) subset of ‘small’ questions (aimed at generating the extension of the data sequence) along with the current conjectured answer to  $Q$  (or suspension of judgment). Finally,  $Q$  may be a *why-* or *how-*question (with  $Q = \{q_i\}$ ), in which case Inquirer *assumes* that  $q_i$  holds, and aims at finding conditions which, together with  $T$ , entail  $q_i$ . Answers to a *why-* or *how-*question ‘compact’ a whole line of inquiry (cf. [11], [10, ch. 7] and § V).

<sup>2</sup>A first-order language  $\mathcal{L}$  can express statements about individuals, their properties and relations; combinations of such statements (with Boolean operators *not*, *and*, *or*, and *if...then...*); and their existential and universal generalizations (with quantifiers *there exists...* and *for all...* respectively). A *basic* sentence of  $\mathcal{L}$  contains only individual names and relations symbols, i.e. no Boolean operator other than (possibly) an initial negation, and no quantifier. In what follows, we implicitly restrict the meaning of ‘deduction’ to ‘first-order deduction’—i.e. relations between premises and conclusions couched in some first-order language.

<sup>3</sup>Introducing  $A_s$  weakens the assumptions that: (a) data streams are all ways complete in the limit; (b) all predicates (names) of  $\mathcal{L}$  denote observable qualities (identifiable objects); and: (c) a datum needs no analysis. The IMI also drops the idealization that: (d) Nature always chooses  $s$  in  $\mathbf{S}(T)$ , and: (e) all answers in  $A_s$  are true in  $s$ . Cases where (d-e) hold define the special case of *Pure Discovery* (cf. III-A).

## B. Information-seeking

The success of Inquirer’s strategy depends in part on the set of questions she is ready to ask at a given point (which evolve throughout inquiry), and in part on  $A_s$ . Hintikka calls *range of attention* the set of *yes-no* questions Inquirer considers possible[7], but keeps its role implicit in the results of [12]. We will make it explicit, since it is critical to understand how the IMI bears upon learning practices of empirical agents. We also generalize the range of attention to include questions other than *yes-no* (cf. sec. III).

Together with  $T$ , answers to instrumental questions induce an *information bi-partition* over  $\mathbf{S}(T)$ : the first cell comprises scenarios compatible with the answers, and the other, those which are not. At the outset, the first cell is identical with  $\mathbf{S}(T)$ : all possible states compatible with  $T$  are *indiscernible* from each other, and  $s$  is assumed to be one of them. The partition is refined when new answers are accepted. Answers gradually ‘hack off’ scenarios incompatible with them. The assumptions that  $T$  and  $A_s$  are truthful may be revised (cf. infra) reopening possibilities. Instrumental questions may also trigger ‘sub-inquiries’ (e.g. *why-* and *how-*questions, or questions with statistical answers requiring parameters estimation) about some problem  $\langle T, Q' \rangle$  (where answers already obtained from  $A_s$  may also be assumed) possibly halting investigations of  $\langle T, Q \rangle$ .

An inquiry about  $\langle T, Q \rangle$  terminates when Inquirer is able to tell whether the first cell of the partition (compatible with the answers and  $T$ ) is identical with some  $q_i \in Q$ , i.e. suffices to identify  $s$  ‘enough’ to answer  $Q$ . This may sometimes be impossible (e.g. for inductive problems) but one can then strengthen  $T$  with additional assumptions (including e.g. extrapolations for unobserved values). It is also sometimes possible to devise methods that rather than waiting for an answer to  $Q$ , emit an initial conjecture and adopt a policy for changing it later in face of new data.<sup>4</sup> The model handles retraction of answers by ‘bracketing’ and excluding them from further information processing (sometimes re-opening  $Q$  by preventing identification of  $s$ ); bracketing can also be extended to handle revisions of  $T$  [4]. Reasoning probabilistically from answer known to be uncertain is discussed in [8].

## C. The Sherlock Holmes sense of “deduction”

An example from Sherlock Holmes inquiry in *The Case of Silver Blaze* ideally illustrates the type of reasoning the IMI captures. In this short story, Holmes assists Inspector Gregory in the investigation of the theft of Silver Blaze (a race horse) and the murder of his trainer. The principal question is: *who stole Silver Blaze and killed his trainer?* During the night of the theft, a stable-boy was drugged and Silver

Blaze’s trainer was killed. Gregory holds a suspect, Fitzroy Simpson, and has already settled the following (instrumental) questions: (1) *Does Simpson have motive?* (he is indebted from betting on horses); (2) *Did he have an opportunity?* (he was near the stable the evening before the theft, stopped the maid carrying the food, and was eventually driven out by a watchdog and the stable boy); (3) *Does he own a weapon?* (he owns a weighted walking stick); and: (4) *Can he be placed at the crime scene?* (his scarf was found near the trainer’s body).

Gregory’s by-the-books strategy uses questions that must be specified for each investigation, and are then applicable to almost every potential suspect. This strategy keeps questioning simple (there are no strategic dependencies between questions) and gives a basis for probabilistic inference: a high ‘yes’ count increases suspicion (culprits usually have one), and a high ‘no’ decreases it (innocents usually have one). Although the former count may result from a coincidence, the probability remains low as long as answers are statistically independent. Acceptability of a conclusion based on it depends on ruling out cases where they are not, and in which method is known to be unreliable, i.e. when either the high ‘yes’ or ‘no’ have a hidden common cause (e.g. when an innocent is framed, or a culprit has carefully planned and executed his plot). Simpson’s guilt is the natural hypothesis (which Holmes concedes at the outset), and is strengthened by Gregory’s reasoning.

Holmes however describes the case as one where “[t]he difficulty is to detach the framework of fact—of absolute undeniable fact—from the embellishments of theorists and reporters” [3, p.522]. Holmes’ own expectations are instrumental in his decision to investigate,<sup>5</sup> but does not favor any hypothesis (even only for testing it, e.g. with Simpson’s guilt). Instead, he proceeds trying to identify the thief, narrowing down the range of suspects without explicitly listing them, attempting instead to find discriminating properties, using *yes-no* questions. One of them is whether the dog kept in the stable had barked at the thief,<sup>6</sup> and Holmes sums up later his conclusions he drew learning that the dog had not:

The Simpson incident had shown me that a dog was kept in the stables, and yet, though someone had been in and had fetched out a horse, he had not barked enough to arouse the two lads in the loft. Obviously the midnight visitor was someone whom the dog knew well. [3, p.540]

Holmes’ instrumental question may seem irrelevant to those who do not anticipate his reasoning, and Holmes’ reputation plays a role in their judgment: the horse’s owner does not consider the incident significant, but Gregory and

<sup>4</sup>An example is the *halting problem*, in which one must determine whether the current run of a program  $p$  that runs either finitely or infinitely many steps, will actually be finite or infinite. An ‘impatient’ method that conjectures that  $p$  is currently at the beginning of an infinite run, and repeats this conjecture indefinitely unless  $p$  stops (in which case it states it, and halt) solves the problem on the current run, but also on every possible run. The relation between the halting problem and empirical inductive problems is discussed in [13].

<sup>5</sup>Holmes confesses that “[he] could not believe it possible that the most remarkable horse in England could long remain concealed [and] expected to hear that he had been found, and that his abductor was the murderer” [3, p.522].

<sup>6</sup>Holmes does not ask the question explicitly, but obtains an answer from Gregory in the following dialogue: “Is there any point to which you would wish to draw my attention?” “To the curious incident of the dog in the night-time.” “The dog did nothing in the night-time.” “That was the curious incident,” remarked Sherlock Holmes.” [3, p.540]

Watson do, knowing that Holmes seldom attend to insignificant facts. Holmes trusts his assumptions and reports about the facts, and conservatively so (the ‘yes’ count vs. Simpson could make one doubt that the dog is a good watchdog). His conclusion reduces the set of potential suspects (ruling out Simpson) without explicitly tracking probabilities.

### III. THE ROLE OF DEDUCTION IN INQUIRY

#### A. *Pure Discovery*

The inquiry game described in § II-A and II-B is with asymmetric information, since Inquirer does not know whether  $T$  is true in  $s$ , nor which answers are in  $A_s$ , and whether they are reliable. Nonetheless, Sherlock Holmes’ method, as illustrated in *The Case of Silver Blaze*, takes evidence at face value, then follows a line of deductions, sometimes taking educated guesses (but keeping track of them to go back if needed many cases) in order to avoid considering too many cases in parallel. Holmes usually reconsiders his grounds for accepting answers or relying on background assumptions only in the face of contradictory evidence (and even then, does not always reason probabilistically).

An inquirer can, as Sherlock Holmes, undertake inquiry ‘as if’ it were what Hintikka calls a *Pure Discovery* (PD) problem, i.e. “a type of inquiry in which all answers [...] can be treated as being true [and] [one] do[es] not have to worry about justifying what we find.” [10, p. 98]. A context can turn out not to be a PD-context in a variety of ways, internal to inquiry (contradiction between answers to ‘control’ questions and expectations based on  $T$ , or between answers from multiple sources) or external (failure of action undertaken based on the outcome of a given inquiry). The IMI handles such contexts through *defeasible* reasoning (‘bracketing’ unsafe premises in  $T$  or uncertain answers in order to circumscription of a ‘safe’ PD subcontext), and maintaining PD behavior; or by suspending PD-like behavior altogether when no such subcontext can be isolated given one’s current evidence (and reasoning probabilistically). Subsequently, the IMI addresses first issues arising in PD-contexts, and then extends the conclusions (when possible) to others.

Mismatch between Inquirer’s the range of attention and  $A_s$  is the prime issue of interrogative inquiry, and occurs when either Inquirer asks a question to which the answer is not in  $A_s$ , or fails to ask a relevant question whose answer is in  $A_s$  (as with Gregory, failing to ‘ask’ about the dog). A related issue is the strategic problem of choosing the next best ‘small’ question given one’s current information ( $T$  and past answers). Both arise in PD and non-PD contexts alike, encompassing e.g. in the latter the opportunity to use ‘control’ questions for new sources, etc. How Inquirer addresses these problems depends on how she manages her range of attention, in the extended sense of § II-A.

#### B. *Building blocks of interrogative strategies*

The IMI captures the dynamics of *discovery of new facts* through ‘small’ questions, as a goal-directed process, possibly conjecturing some answer  $q_i$  and testing it (cf. n. 4).

These strategies supervene on one’s current information ( $T$  and the answers accepted so far), which is ‘mined out’ for open questions, before they are selected and ask sequentially. This process is inferential, in the following sense: even if Inquirer’s information partition excludes that “neither  $A$  nor  $B$ ” holds in  $s$ , the question whether  $A$  or  $B$  holds (possibly together, if compatible) will enter her the range of attention if she establishes that  $T$  entails that “ $A$  or  $B$ ” holds.<sup>7</sup> Once Inquirer performs the inference, she may choose to raise the question “Which of  $A$  or  $B$  holds?”—or a sequence of *yes-no* questions about  $A$  and  $B$ —and use it to refine her information partition. If no answer is obtained, she may need to *reason by cases*, or mine  $T$  (and past answers) to find equivalences between  $A$  and  $B$  on the one hand, and some  $A'$  and  $B'$  on the other, so as to reformulate her questions.

The same holds *mutatis mutandis* for statement like “There is an  $x$  s.t.  $\phi(x)$ ” (where  $\phi(\cdot)$  is a description) that open *wh*-questions about the object (or person, location, etc.) satisfying the description. Without an answer, one must reason to introduce an arbitrary name  $\alpha$  standing for the (so far unknown) object satisfying the description, avoiding any other assumption about  $\alpha$  other than  $\phi(\alpha)$ , until (possibly)  $\alpha$  is identified with a known entity. Again, it may be possible to mine  $T$  to obtain a description  $\psi(\cdot)$  such that  $T$  (possibly together with past answers) entails that “If  $x$  is s.t.  $\phi(x)$ , then it is s.t.  $\psi(x)$ ” and ask the question about  $\psi(\cdot)$  instead.

Inference from  $T$  and past answers, opening questions or making implicit definitions explicit, are primary means to increase one’s range of attention through reasoning. This IMI models by counting *inferential* moves on a par with *interrogative* ones. Hintikka calls *presupposition* of a question the statements that opens it, and the fundamental ‘rule’ of the game of inquiry is that a question can be asked as soon as its presupposition has been inferred (making it available for an interrogative move). With our extended notion of range of attention, the rule can be rephrased as: *a question enters the Inquirer’s range of attention when its presupposition is obtained by an inferential move.*

In *Silver Blaze*’s case, Gregory’s strategy derives *deductively* from his background knowledge a (testable) reformulation of the question ‘Is Simpson guilty?’; but the support the answers he obtains give to his hypothesis is probabilistic (cf. § IV-A). Holmes also reformulates a question (‘Who is the culprit?’) and the way he arrives at the instrumental question that specifies it, and the conclusion(s) he draws from the answer, are deductive. But Holmes ‘small’ question has the form “Is it the case that  $A$  or not?”, where  $A$  is: “the dog barked at the thief”, and the possibility to ask it depends on the language he use alone (irrespective of the current information state). More generally, for some language  $\mathcal{L}$ , any grammatically correct statement  $A$  or description  $\phi(a)$  built with the vocabulary of  $\mathcal{L}$  (where  $a$  is a proper name or an indexical like ‘this’ or ‘that’) can in principle be built into a

<sup>7</sup>In this case, “Is it the case that neither  $A$ , nor  $B$ ?” is a control questions w.r.t.  $T$ . Obtaining an answer that contradicts  $T$  (and some past answers) may lead to revise it, or reject the answer (and possibly the source). Again, these strategies are only implemented when Inquirer has already ceased to assume that the context is one of PD.

yes-no question without the need of further inference from one's current information.<sup>8</sup>

### C. The Deduction, Yes-No and Strategy Theorem

We conclude this section with informal summaries of the main formal results of the IMI pertaining to interrogative strategies, before we apply them to our example, and ultimately, to H&S' conclusions, in the next section. Neglecting the distinction between statements, and propositions they express, the main strategic problem of interrogative inquiry is “[g]iven the list of the propositions one has reached in a line of inquiry, which question should one ask next?” [10, p.98]. Given the role of presupposition, this is equivalently expressed as: “[w]hich proposition should one use as the presupposition of the next question?” [10, p.98].

Three results proved in [12] can be combined to answer this question: the *Deduction Theorem*, the *Yes-No Theorem*, and the *Strategy Theorem*. Together, they fully vindicate Sherlock Holmes' (and Conan Doyle's) view that the deduction guides inquiry (at least in the PD case). The *Deduction Theorem* simply states that if an answer  $q_i \in Q$  can be established *interrogatively* in  $s$  assuming  $T$ , then  $q_i$  can be established *deductively* (without using questions) from  $T$  and a finite subset  $A'_s$  of  $A_s$ . Equivalently: answers act as additional premises, and interrogative reasoning reduces to *deduction from  $T$  strengthened by a finite set of answers*.<sup>9</sup>

The *Yes-No Theorem* is perhaps more surprising, but no less straightforward, and states that:  $q_i$  can be established interrogatively in  $s$  assuming  $T$  iff  $q_i$  can be established interrogatively in  $s$  assuming  $T$  using only yes-no questions. The yes-no theorem is best understood as stating that every interrogative argument can be *reconstructed* as an argument proceeding with yes-no questions alone.<sup>10</sup>

The *Strategy Theorem* rests on an observation about *deductive* proofs. Obtaining the shortest proof for a conclusion  $c$  from a set of premises  $P$  (when  $c$  actually follows from  $P$ ) requires to: (a) examine the least number of cases; and: (b) introduce the smallest number of (arbitrary) names. Proof rules that open cases and introduce names in deductive reasoning, are *the same as* inferential rules that open questions in interrogative reasoning. Hence, taking  $P = T$  and  $c = q_i$  for some  $q_i \in Q$ , answers in  $A_s$  eliminate cases, and dispense from introducing arbitrary individuals. Given the Deduction Theorem, this means that, when  $q_i$  can be interrogatively obtained in  $s$  (given  $T$ ) the shortest *interrogative* derivation is identical with the shortest *deductive* derivation of  $q_i$  from

<sup>8</sup>If  $A$  or  $\phi(a)$  include vague terms (or imprecise categories), disambiguation is needed to obtain an answer, but *sequence of yes-no questions* (further specifying a ‘prototype’ in the current context) will suffice.

<sup>9</sup>Because of the possibility of mismatch, the converse of the Deduction Theorem only holds on the condition that elements of  $A_s$  needed to obtain (interrogatively)  $q_i$  from  $T$  are answers to questions in Inquirer's range of attention.

<sup>10</sup>This understanding eschews the issue of mismatch. In the left-to-right direction, every *whether*-question about  $A$  or  $B$ , or *wh*-question about  $\phi(\cdot)$ , that receives (say) answer  $A$  or  $\phi(a)$  suffices for the yes-no questions about  $A$  or  $\phi(a)$  to enter Inquirer's range of attention for the purpose of reconstructing an argument. The antecedent of the right-to-left direction holds when the yes-no questions are already in the range of attention and satisfies the consequent by definition.

$T$  and a finite subset  $A'_s$  of  $A_s$ , where  $A \in A'_s$  or  $\phi(a) \in A'_s$  are introduced resp. when “ $A$  or . . .” or “There is an  $x$  that  $\phi(x)$ ” are obtained from  $T$  and past answers.

Deduction (for the first-order case) is only *semi-decidable*: when some conclusion  $c$  follows from a set of premises  $P$ , there is always a finite proof. However, there may not be a finite proof that  $c$  does *not* follow from  $P$ , if it does not. Subsequently, the Strategy Theorem entails that there cannot be any general mechanical (algorithmic) method for solving interrogative problems by: (1) trying first to deduce some  $q_i$  from  $T$ ; (2) ask questions if step (1) is not successful; and: (3) if step (2) is also unsuccessful, reiterate (1) with other potential answer to  $Q$ . However, it *does* entail that having some idea about *which cases would have to be ruled out* to deduce some  $q_i \in Q$  from  $T$ , gives a good idea of *which question one should ask* to establish interrogatively  $q_i$  from  $T$  (assuming that the answers would be obtained).

## IV. DEDUCTION ABDUCTED

### A. Abduction

Hintikka has suggested that the Strategy Theorem offers important insights about *abduction* [9], [10, ch. 2], esp. in contrast with inference to the best explanation (IBE). The latter occurs when: (a) Inquirer further partitions the states compatible with  $T$  and the answers she has received, and: (b) accepts (defeasibly) one of the answers. This reasoning can be rationalized, assuming a probability distribution over the refined partition; and an acceptance rule that fires if probabilities are raised (conditional on past answers) over a fixed threshold. Gregory's strategy is naturally reconstructed as a case of IBE, where the acceptance rule ‘fires’ because the answers are independent, and the probability of a coincidence is low. If the probabilistic constraints are precise enough, the outcome of IBE can be uniquely determined, but involves (probabilistic) justification, and is definitely non-PD.

By contrast *abduction* (in Hintikka's sense) routinely occurs in PD contexts (or contexts that Inquirer still assumes to be PD), and when Inquirer anticipates a (possible) course of the interrogative derivation, and attempts to steer the course of the investigation towards it. It depends on the ‘deductive insight’ that some instrumental questions are such that their answer can strengthen  $T$  enough to reduce the admissible states to those making some  $q_i \in Q$  true. Holmes' question about the dog does not single out one suspect (although it excludes Simpson), but nonetheless ‘partially’ answers the principal question (narrowing down the range of suspects).

Hintikka's reconstruction rationalizes abduction as a *strategic* inference (an insight from deduction) although it is not in general mechanizable (because of semi-decidability); while IBE is a purely mechanical procedure, under probabilistic constraints. However, abduction involving yes-no questions cannot always be fully rationalized: yes-no questions that do not ‘break down’ questions whose presupposition are inferred from  $T$  and previous answers, involve intuitive leaps. The difficulty also affects IBE: the relevant partition of cases may be inferred from  $T$ , but on occasion must be imposed by ‘abductive’ yes-no questions [5]. Asking

about the dog would be as ‘abductive’ for reasoning with IBE, as it is for reasoning deductively.

### B. Serendipity

Hintikka reconstructs Holmes’ reasoning in [10, ch.7, §2] as an explanatory reasoning (answer to a *why*-question).<sup>11</sup> The information that (a) no dog barked at the thief; and (b) there was a watchdog, provide *ad explanandum* conditions, alongside the general truth that watchdogs do not bark at their masters. Once the stable-boys ruled out (one was drugged, the other two asleep in the loft) the only individual fitting the description ‘master to any watchdog kept in the stables’ is Silver Blaze’s trainer. Once Holmes has reached this conclusion, the principal question also changes to a pair (a *why*-question about the trainer’s motives, and a *how*-question about the circumstances of his death). Learning about the dog incident makes Holmes ‘bracket’ his own expectations that the thief is an assassin (cf. n. 5).

Holmes eventually recounts a purely deductive reasoning, that reconstructs the process of his investigation (as an application of the Deduction Theorem would). The crux of Holmes’ *interrogative* reasoning is how he picks premises (a) and (b). Since (a) is vacuously true (and uninformative) if no dog is indeed kept in the stables, one needs (b) to draw a useful conclusion. Holmes explains his reasoning as going from (a) to (b), and the former is in turn suggested by the dog barking at Simpson in the evening, but not at the thief in the night-time—from which Holmes then goes to extend his background assumptions to include the general truth (a) and the *ad explanandum* premise (b).

While the Strategy Theorem captures perfectly Holmes’ line of reasoning, it cannot fully rationalize it, because it depends on a *yes-no* question that enters Holmes’ range of attention (but not in Gregory’s) without being inferred from his background information. Actually *Holmes’ picking premise (a) and anticipating its effect also depends on anticipating the answer to that question*. While (a) is part of the common ground that Holmes, Gregory and Watson share, its usefulness (as constraint on the information partition) is only revealed after (b) is learned. The same goes for the ‘general truth’ that watchdogs abstain from barking at their masters alone.

The trigger for Holmes’ line of reasoning is *serendipity*, or “observing an unanticipated, anomalous, and strategic datum which becomes the occasion for developing a new or extending an existing theory” [1, p.260]. Gregory is not aware of the datum (b) in the same way as Holmes is, but there is no ‘reason’ one way or another. Still, Holmes’ reasoning strategy can be vindicated on purely deductive grounds, but his ‘abducted’ deduction sheds a very different light on H&S’s conclusions.

<sup>11</sup>The statement that “a dog was kept in the stables, and yet [...] had not barked enough to arouse the two lads in the loft” can be extracted as an interpolation formula (a formula that follows from the premises, and entails the conclusion, using only their common vocabulary) form the proof, and answers the *why*-question about the conclusion that “the midnight visitor was someone whom the dog knew well”. The reconstruction uses an extremely parsimonious first-order language, with two properties, one relation, and two names.

### C. Abduction in collaborative learning

The study reported by H&S in [6] shows that, in a computer-supported collaborative learning environment, children engage in higher-level inquiry processes, and H&S appeal to the IMI conceptual framework to interpret the results. Subjects were elementary school pupils, and completed four science projects, by answering broad questions collaboratively, or smaller questions individually,<sup>12</sup> using in both cases resources shared with the whole group through the CSILE software environment, that lets users register (public) notes in an initially empty database, with either an informative or interrogative content (labeled “Problem” or “I Need To Understand” cf. [6, 32]).

The experiment lends naturally to the IMI interpretation. Informative notes, when they were accepted, constrained the information partition of all members of the group (H&S do not use the technical description, though). Interrogative notes were classified as ‘principal’ (for the main problems, or sub-inquiries) or subordinate to others. And a qualitative evaluation determined how close children had come to pre-determined answers—whether they had moved from “initial intuitive theories” to a “new conceptual understanding” [6, p. 38] mirroring the scientific theories describing the phenomena they studied. Individual reasoning strategies were not explicitly studied, but how children monitored each others’ questions was. The general conclusion was that:

The epistemic value of CSILE students’ knowledge-seeking inquiry seems partially to be based on a process in which social communication pushed [...] inquiry further than [they] might originally have been able to go.

[CSILE] appeared to foster engagement in higher-level practices of inquiry [and] epistemological awareness concerning the process of inquiry.[6, p.38–39]

The IMI captures more precisely the “epistemological awareness” than H&S do realize. A question asked publicly enters the range of attention of all members of the group. Mining the database for an answer reveals whether it can be answered on the basis of the information in it alone, or not. This in turn yields *introspective knowledge* (knowing that one knows or knowing that one does not know). While *yes-no* questions (whose presuppositions are trivial) do not increase knowledge in that sense, they can trigger *strategic* reasoning, anticipations, and ‘deductive’ insights.<sup>13</sup> Explicitly attempting to capture such phenomena could have made the appeal to the IMI in H&S’s study more fruitful (and precise). However, the CSILE environment, which does not track the inferential steps, does not suffice for that purpose.

Also, whether the IMI vindicates collaborative learning-based education, is less clear than H&S’s optimistic conclu-

<sup>12</sup>The questions in the former case were as broad as: “how to explain gravity?”, “how did the universe begin, and how will it evolve?” and “how do cell and the circulatory system in the human body work?”. The author do not specify the questions (about electricity) in the latter.

<sup>13</sup>The effect of *yes-no* questions in interrogative games w.r.t. to introspective knowledge and strategic reasoning is discussed more technically in [5], in contexts where *preventing* strategic reasoning is critical.

sions imply. It is certainly useful to analyze “how sound questions arise” (p. 40) in their study or how “theories [...] characteristically serve[d] to chop the unmanageable why-questions into yes-no -questions” (*ibid.*). However, students had to fill an *empty database* in the first place, and their initial problem was a generalized version of Holmes’ problem in *Silver Blaze*. In this case, the students’ ‘deductive’ conclusions must be reconstructed from *interrogative reasoning from an empty theory*. The IMI can give no more insight as to how (*yes-no*) questions used for such a purely abductive task are selected, no more that it can rationalize Holmes’ question about the dog.

## V. CONCLUSION

The CSILE study shows that ‘something’ occurs in the process of inquiry-driven collaborative learning, and the IMI is able to reconstruct *post hoc* that ‘something’ as increased epistemic and strategic awareness. But the difficulty to *rationalize* critical abductive steps of inquiry, and their occurrence in the CSILE study, leaves open the issue of what can actually facilitate the student’s sophisticated reasoning. As Hintikka himself showed, Socrates teaches Meno’s slave all the geometry the illiterate slave needs to demonstrate that the diagonal of the square is incommensurable to its side, and uses only *yes-no* questions to convey the required knowledge in geometry [10, ch.4, §8]. Socrates chooses his own questions, forcing the slave to probe consequences of (provoked) some possible answers first, then (in face of contradiction) to retract his guesses, and probe the consequences of the other (correct) answers. Each time the slave is probing the consequences of a false presupposition, he could well be said to be progressing in the demonstration, but this progress can only converge if monitored by Socrates. Similarly, in the CSILE study, some questions were “based on wrong presuppositions” [6, p.38], and whether the students would had correct each other in the long run without guidance is unclear.

The role of educators is not only to guide their students to a better understanding of the *current* theories, but also to improve their ability to contribute to their *future* evolution. In this respect, the IMI does not offer a sufficient conceptual apparatus for drawing more substantial conclusions than other epistemological models, nor to offer (yet) foundations for inquiry learning. More specifically, H&S observe that there is the dialogue with Nature, [and] there is the dialogue with fellow inquirer learners, *carried out in a common language, and guided by ordinary norms of social interaction.*” [6, p.41] (Our emphasis.) So far, the IMI has only addressed the first explicitly. It also captures how “dialogue with fellow inquirers” increases “epistemological awareness”, when understood as introspective knowledge, and opportunities for strategic inferences. But the IMI does not offer a *dynamic* model of the interplay of linguistic and communicative abilities (both the “common language” and the “norms of interaction”) and conceptual abilities (“conceptual understanding”). Pending such an account, H&S’ conclusions rest on unstable ground.

J. Barrett’s recent proposal (in [2]) is highly relevant in this context. Barrett describes evolutionary inquiry games where *teams* of learners attempt to “satisfy their descriptive and predictive aims by revising their linguistic dispositions, their theoretical dispositions, or both.” [2, p.1]. The evolutionary approach characterizes “what it might mean for descriptions of the world to be faithful and hence for empirical inquiry to be successful” (*ibid.*) without presupposing a body of knowledge towards which the learners would have to converge in order to characterize success (unlike H&S’s study). Yet it maintains intelligibility of convergent knowledge, because it can model the convergence towards a *given description* (currently deemed to be faithful). This proposal is thus relevant to both epidemiologists, cognitive scientists and educators, interested in extending the IMI and finding an epistemological basis for collaborative learning.

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