A Stationary Turbine Interaction Model for Control of Wind Farms

Madjidian, Daria; Rantzer, Anders

2011

A Stationary Turbine Interaction Model for Control of Wind Farms

Daria Madjidian and Anders Rantzer

Department of Automatic Control, LTH, Lund University, Sweden
(e-mail: daria@control.lth.se, rantzer@control.lth.se)

Abstract: Turbines operating in wind farms are coupled by the wind flow. This coupling results in limited power production and increased fatigue loads on turbines operating in the wake of other turbines. To operate wind farms cost effectively, it is important to understand and address these effects. In this paper, we derive a stationary model for turbine interaction. The model has a simple intuitive structure, and the parameters have a clear interpretation. Moreover, the effect of upwind turbines on a downwind turbine can be completely determined through information from its closest neighbor. This makes the model well-suited for distributed control. In an example, we increase total power production in a farm, by coordinating the individual power production of the turbines. The example points to an interesting model property: decreasing power in an upwind turbine causes downwind turbines to pose less of an obstacle for the wind, provided that they maintain their level of power capture.

1. INTRODUCTION

While economy of scale makes it attractive to position turbines close to each other in wind farms, such a placement causes problems. When a turbine extracts power from the wind, it disturbs the wind flow behind it. This creates a coupling with turbines operating in its wake. The wind in a turbine wake is characterized by a mean wind speed deficit and an increased turbulence level. Consequently, upwind turbines limit power production and increase fatigue loads on downwind turbines.

In order to operate wind farms cost effectively in terms of power production and maintenance costs, it is important to understand and address the issue of aerodynamic coupling. Thus far, this problem has received little attention from a control perspective (see Pao et al. [2009]). The main reason for this is the lack of wake models suitable for control design.

1.1 Previous Work

Existing wake models, although abundant, are usually not developed for control purposes. Since wake modeling is a large research field, a full investigation is beyond the scope of this discussion. Therefore, only a partial overview is provided below.

A survey made by Crespo et al. [1999], classifies existing wake models into three subclasses: field models, kinematic models, and roughness element models.

Field models describe the wind speed at every point in the flow field, which makes them computationally expensive. Kinematic models provide simpler expressions than field models. They usually begin with the modeling of a single wake, by using conservation of momentum. Merged wakes are then most often described by superimposing the individual wakes on the ambient flow field. Opinions on how this should be done vary, and not all kinematic models are able to handle large wind farms (Crespo et al. [1999]). The ones that can, are sometimes based on assumptions that make them unsuitable for control purposes. For instance, Jensen [1983], and Frandsen et al [2006], both assume fixed and uniform values on thrust coefficients. However, a change in power reference to a turbine, means that the value of the thrust coefficient also changes.

Roughness element models describe the response of the ambient wind flow to a sudden change in roughness. These models are further divided into infinite cluster models, and finite cluster models.

Infinite cluster models (e.g. Frandsen [1992]) aggregate the effect of all turbines, and describe the entire wind farm as one roughness element. This results in an overall ("average") wind profile for the farm, but the individual effect of turbines is lost.

A survey by Bossayani et al [1980] compares different roughness element models, and explains how infinite cluster models are modified into finite cluster models. As opposed to modeling the farm as a single roughness element, the finite cluster models describe the wind speed at each row of turbines perpendicular to the incoming wind direction. In order to handle wind farms of any size, all models in the survey introduce a rate of replenishment, describing the influx of momentum (or power) from the free flow.

1.2 Contributions and Outline

Here, we take a new approach, and derive a stationary wind farm control model for turbines arranged in a row. The turbine interaction (wake) part captures both wind speed deficits and increased turbulence levels. The assumption is that each upwind turbine adds to deficits and turbulence levels at all turbines further downwind. The result is a turbine interaction model, that maps thrust coefficients, wind speeds, and turbulence levels at upwind turbines, to wind speeds and turbulence levels at downwind turbines. The structure of the model is intuitive, and its parameters have a clear interpretation. The model also reflects the spatially distributed structure of a wind farm, in the sense that the effect on a turbine from all other turbines can be completely determined through information from its closest upwind neighbor. This makes the model a good candidate for distributed control, which is especially important as the number of turbines in wind farms increases.

A wind farm model consists of three parts: a model for the ambient wind entering the farm, turbine models, and a model describing the aerodynamic interaction between the turbines. The interaction model constitutes the main part of this paper and is presented in Section 2. Section 3 begins by describing the ambient wind and the turbines. It then links these models to the interaction model to form a complete model of the farm. In Section 4, we present an example where we increase total power production by coordinating the power production of the turbines. Finally, we point to an interesting result: by decreasing power production in an upwind turbine, all downwind turbines can reduce their thrust while maintaining the same power capture.

2. WIND TURBINE INTERACTION

The basic mechanisms behind turbine interaction are explained in Burton et al. [2008]. Each turbine extracting power produces a wake, characterized by a mean wind speed deficit and an increased turbulence level. Since the thrust coefficient determines the momentum extracted from the flow, it is directly linked to the deficit. The wind speed gradient between the wake and the free stream results in additional shear generated turbulence. Therefore, the thrust coefficient is also directly related to the added wake turbulence. The shear generated turbulence transfers momentum from the free flow to the wake and causes the wake to expand. Therefore, as the wake travels downstream, it gradually becomes wider but shallower until the flow has fully recovered far downstream. It should also be noted that in addition to shear generated turbulence, the turbine itself generates turbulence directly. This extra component is caused by the vortices shed by the blades, and from having placed an object (the turbine) in the wind field. However, this type of turbulence decays quickly, and (due the the long distances between turbines in farms) does not add significantly to turbulence levels at downwind turbines.

Our goal is to develop a model of turbine interaction, with the additional property that the wind speed at a turbine can be determined from information available at its closest neighbors. We tailor our model to a farm consisting of a row of N equidistant turbines, where the mean values of the thrust coefficients are between 0 and 1. We do not consider wake meandering, and therefore assume that the wind direction is parallel to the row of turbines at all times. The setup is shown in Figure 1.

To capture the structure of aerodynamic coupling between turbines, we need a model where upwind turbines add to the wind speed deficits and turbulence levels of downwind turbines. Based on the previous discussion, the deficit and turbulence that a turbine adds at another turbine

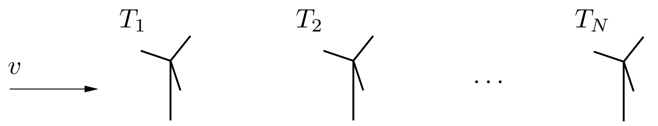

Fig. 1. Row of turbines

depend on the thrust coefficient of the upwind turbine and the distance between the two turbines. A large thrust coefficient implies that more momentum is extracted from the flow. Therefore, both deficit and added turbulence should increase when the thrust coefficient increases. Since ambient wind flow strives to reduce wake effects, the deficit and added turbulence should decrease with the distance between turbines.

We also impose additional standard requirements on the model (see Frandsen et al [2006] and Frandsen [2007]).

- i Mean wind speeds at all turbines must be positive, and not higher than ambient mean wind speed. In addition, in an infinitely long row of turbines, where all turbines extract maximum amount of power, there is an asymptotic wind speed.
- ii Turbulence levels at all turbines must be bounded from below by ambient turbulence intensity, and bounded from above.

We now present the interaction model. Let v be the ambient mean wind speed, and denote the thrust coefficient of turbine n as C_{T_n}. A model for the deficit experienced by turbine 2 in the row is given by:

$$v_2 = (1 - k_1 C_{T_1})v$$

where $k_1 > 0$ is a distance parameter. The larger the distance between the turbines, the smaller k_1 will be.

Turbine 2 and 3 will be in the wake of the turbine 1. Suppose for a moment, that turbine 2 is switched off, which implies $C_{T_2} = 0$. The deficit at the third turbine can then be modeled as:

$$v_3 = (1 - k_2 C_{T_1})v$$

where $0 < k_2 < k_1$. This implies $v_3 > v_2$, and the first turbine therefore shapes the wind field around the second and third turbine non-uniformly. To take this into account when merging the wakes of turbine 1 and turbine 2, we add their effects:

$$v_3 = (1 - k_1 C_{T_2} - k_2 C_{T_1})v$$

Similarly, the deficit at turbine number $n + 1$ can be expressed as:

$$v_{n+1} = (1 - k_1 C_{T_n} - \ldots - k_n C_{T_1})v \tag{1}$$

where $0 < k_n < \ldots < k_1$. The model implies that upwind turbines, $1, \ldots, n$, subtract wind from a downwind turbine, $n + 1$, according to their thrust coefficients and their distance to turbine $n + 1$.

As stated earlier, we aim for a model where the wind speed at a turbine can be determined from information available at its closest upwind neighbor. This can be achieved if the deficit decays exponentially with distance. For the time being, we will assume that this is the case, and relax the assumption later on. Let $k = k_1$, and $k_i = k^i$, $i = 2, \dots, N-1$, and define

$$z_n = \sum_{i=1}^{n} k^i C_{T_{(n+1-i)}}$$

Then, (1) can be written recursively:

$$\begin{aligned} v_{n+1} &= (1 - z_n)v = (1 - kC_{T_n} - kz_{n-1})v \\ &= (1 - kC_{T_n})v - kv + k(1 - z_{n-1})v \\ &= (1 - kC_{T_n})v - k(v - v_n) \end{aligned} \tag{2}$$

The first term on the right hand side describes the deficit caused by the closest neighbor n, whereas the second term describes the total deficit caused by other upwind turbines.

By defining the relative mean deficit at turbine n as:

$$\delta_n = \frac{v - v_n}{v} \tag{3}$$

we can express (2) as:

$$\delta_{n+1} = k\delta_n + kC_{T_n}$$

This gives a new interpretation. The wind speed deficit at a turbine depends on the deficit at the previous turbine, and the effect of the previous turbine.

We can also rewrite (2) as

$$v_{n+1} = v_n + \underbrace{(1-k)(v - v_n)}_{\text{recovery}} \underbrace{-kvC_{T_n}}_{\text{effect of neighbor}} \tag{4}$$

which states that the wind speed at a turbine depends on the wind speed at the previous turbine, a deficit dependent recovery term, and the effect of the previous turbine.

The wind speed at turbine n, will be described as a mean wind speed with turbulent fluctuations, $w_n(t)$, superimposed:

$$v_n + w_n(t)$$

where w_n is a zero mean stationary process, with variance σ_n^2. Again, assuming an exponential decay in added wake turbulence, with the same distance parameter k, we can model the dependence of σ_{n+1} on upwind turbines as:

$$\begin{aligned} \sigma_{n+1} &= (1 + kC_{T_n} + \dots + k^n C_{T_1})\sigma \\ &= \sigma + k\sigma C_{T_n} + \frac{k\sigma}{v}(v - v_n) \end{aligned} \tag{5}$$

where σ^2 is the variance of the incoming ambient wind speed. Equation (5) states that the added turbulence at a turbine depends on the deficit at the previous turbine, and the effect of the previous turbine.

Even though the assumption on exponential decay is arbitrary, (4) and (5) still give an idea of how to obtain a parametrized stationary turbine interaction model. To allow more flexibility, we separate the terms by introducing one distance parameter for each:

$$v_{n+1} = v_n + k'(v - v_n) - kvC_{T_n} \tag{6}$$

$$\sigma_{n+1} = \sigma + \frac{c'\sigma}{v}(v - v_n) + c\sigma C_{T_n} \tag{7}$$

where k', k, c', and c are all positive.

Remark 1. By using (3), (6) can be expressed as a first order system:

$$\delta_{n+1} = (1 - k')\delta_n + kC_{T_n} \tag{8}$$

The speed of recovery corresponds to the pole of the system: $1 - k'$.

Expressions (6) and (7) give an intuitive map from thrust coefficients, distance, ambient mean wind speed, and ambient wind speed variation to mean wind speeds and wind speed variations at the turbines. Deficit and added turbulence both increase with thrust coefficients and decrease with distance. The speed of recovery can be tuned through k' and c', and the effect of the nearest upwind neighbor can be tuned through k, and c.

By using requirements i) and ii) stated earlier in this section, we can provide some bounds on k and k'.

Requirement i) states that $v_n \leq v$, for $C_{T_n} \in [0, 1]$, $n = 1, \dots, N$, and any $N \in \mathbb{N}$. Since $v_n = (1 - \delta_n)v$, the requirement translates to $\delta_n \geq 0$. From (8) we note that this is satisfied *if and only if* $k' \leq 1$. Let $C_{T_n} = 1$, $n = 1, \dots, N$. Then $v_n > 0$ is satisfied if and only if $\delta_n < 1$. If $k' \in (0, 1]$, from (8) we see that $\exists \bar{\delta} > 0$, such that $\delta_n < \bar{\delta}$, $n = 1, \dots$, and $\lim_{n\to\infty} \delta_n = \bar{\delta}$. $\bar{\delta}$ satisfies

$$\bar{\delta} = (1 - k')\bar{\delta} + k \Rightarrow \bar{\delta} = \frac{k}{k'}$$

The asymptotic wind speed is given by $\bar{v} = (1 - \bar{\delta})v$. We have that, $\bar{v} > 0$, *if and only if*

$$0 < k < k' \leq 1 \tag{9}$$

Since c' and c are positive, (9) also implies $\sigma_n \geq \sigma$, and that there is $\bar{\sigma} > 0$ such that $\sigma_n < \bar{\sigma}$ for $n = 1, \dots, N$, and any $N \in \mathbb{N}$. This shows that requirement i) and ii) are satisfied if and only if (9) holds.

Remark 2. The model can be made more general. First, the equidistance assumption is not necessary. Different distances can be handled, by introducing $k_{i,j}$ to model the coupling between turbines i and j, and assuming that the deficit decay satisfies

$$k_{i,j} = \prod_{l=0}^{j-1} k_{(i+l, i+l+1)}, \quad \text{for } j > i$$

Also, the term C_{T_i} in (1), can be replaced by a more general expression $f_i(C_{T_i})$, where f_i is monotonically increasing and

$$f_i : \quad [0, 1] \to [0, 1]$$

Similarly, when modeling turbulence levels, we can introduce $g_i(C_{T_i})$, where g_i is monotonically increasing and

$$g_i : \quad [0, 1] \to \mathbb{R}_+$$

Proceeding the same way as above results in:

$$v_{n+1} = v_n + k'_{n,n+1}(v - v_n) - k_{n,n+1} v f_n(C_{T_n})$$

$$\sigma_{n+1} = \sigma + \frac{c'_{n,n+1}\sigma}{v}(v - v_n) + c_{n,n+1}\sigma g_n(C_{T_n})$$

where $k'_{n,n+1}$, $k_{n,n+1}$, $c'_{n,n+1}$, and $c_{n,n+1}$ are all positive.

3. WIND FARM MODEL

It is common practice to model the wind speed at a turbine as a mean wind speed, v, with fluctuations, w, superimposed. The fluctuations have zero mean when averaged over a period of about 10 minutes and are roughly Gaussian (Burton et al. [2008]). This makes it natural to model incoming wind speed at a turbine as, $v+w(t)$, where v denotes the mean wind speed over a 10 minute interval, and w is a stationary process with zero mean, and variance

σ^2. The standard deviation σ is usually defined implicitly through the *turbulence intensity*, $T_i = \frac{\sigma}{v}$.

In order for a turbine to extract power, it needs to interact with the wind. The total wind power passing through the area swept by the turbine rotor is given by:

$$P_{\text{wind}} = \frac{1}{2}\rho\pi R^2 v^3$$

where ρ is the density of the air, and R is the rotor radius. The power, P, extracted by the turbine, depends on the pitch angle β, and tip speed ratio λ:

$$P = C_P(\lambda, \beta) P_{\text{wind}}$$

C_P is the *power coefficient* of the turbine, and determines the portion of total wind power that is extracted. The tip speed ratio is defined as $\lambda = \frac{R\omega_r}{v}$, where ω_r is the angular velocity of the rotor.

We define the *available power*, P_a at the turbine as the maximum amount of power that the turbine can extract:

$$P_a = \min(C_{P,\max} P_{\text{wind}}, P_{\max}) \tag{10}$$

where $C_{P,\max}$ is the peak of the C_P curve, and $P_{\max}$ is the rated extracted power for the turbine.

There will also be a thrust force F_T on the rotor:

$$F_T = \frac{1}{2}\rho\pi R^2 C_T(\lambda, \beta) v^2$$

where the thrust coefficient, C_T, depends on tip speed ratio and pitch angle. As discussed in Section 2, the thrust coefficient is directly linked to the wind speed deficit and added turbulence that the turbine induces downwind.

Each turbine has a control variable, u, which can be generator torque and/or pitch angle for an uncontrolled turbine, or e.g. power reference for a power controlled turbine (see Section 4). By manipulating u, tip speed ratio and pitch angle can be controlled. Assuming that the mapping $(u, v) \to (\lambda, \beta)$ is well defined, we can define

$$\begin{aligned} c_P(u, v) &= C_P(\lambda(u, v), \beta(u, v)) \\ c_T(u, v) &= C_T(\lambda(u, v), \beta(u, v)) \end{aligned}$$

Given ambient mean wind speed v, and ambient turbulence intensity T_i, the model for turbine $n+1$ along the row is given by:

$$\begin{aligned} P_{n+1} &= \frac{1}{2}\rho\pi R^2 c_P(u_{n+1}, v_{n+1}) v_{n+1}^3 \\ y_{n+1} &= f(u_{n+1}, v_{n+1}, \sigma_{n+1}) \\ v_{n+1} &= v_n + k'(v - v_n) - kv c_T(u_n, v_n), \quad v_1 = v \\ \sigma_{n+1} &= \sigma + \frac{c'\sigma}{v}(v - v_n) + c\sigma c_T(u_n, v_n), \quad \sigma_1 = T_i v \end{aligned}$$

where y_n represents outputs of interest for turbine n (e.g. a measure of fatigue loading), and c_P and c_T are the power and thrust coefficients as a function of control action u_n, and mean wind speed v_n.

4. EXAMPLES

4.1 NREL turbines

The wind farm in the examples below will consist of NREL 5 MW turbine models. The turbines are variable speed and (collective) pitch controlled, and described in detail in Jonkman et al [2009], and Grunnet et al [2010]. The generator has an efficiency of $\mu = 0.944$ which implies $P_{\max} = 5/\mu = 5.30$ MW. Power and thrust coefficients are shown in Figure 2.

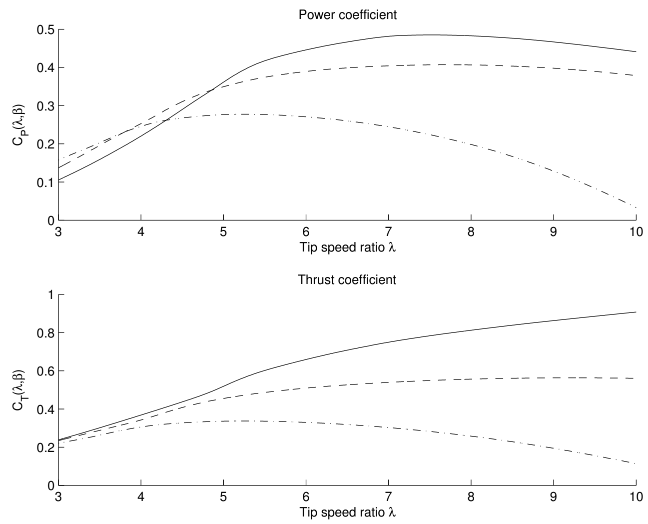

Fig. 2. Power and thrust coefficient for the NREL 5 MW turbine. Solid: $\beta = 0^o$, dashed: $\beta = 2^o$, dash-dotted: $\beta = 4^o$. $\lambda^* = 7.6$, $\beta^* = 0^o$

We assume that each turbine is equipped with a standard controller that manipulates generator torque and pitch angle. The controller has three regions of operation, illustrated by Figure 3. In region 1, the wind speed is too low to produce power. In region 2, the controller tries to extract maximum power. This is done by fixing the pitch angle to the optimal angle for power capture, β^*. β^* is the value that results in the highest value of the power coefficient (see Figure 2). The controller then varies generator torque to track the optimal tip speed ratio, λ^*. In region 3, the controller strives to maintain a power reference, u. This is achieved by keeping the rotational speed close to $\omega_r = \min(\frac{\lambda^* v}{R}, \omega_{r,\text{rated}})$ by varying the pitch angle, where $\omega_{r,\text{rated}}$ is the rated rotor speed. The generator torque is used to produce the desired power.

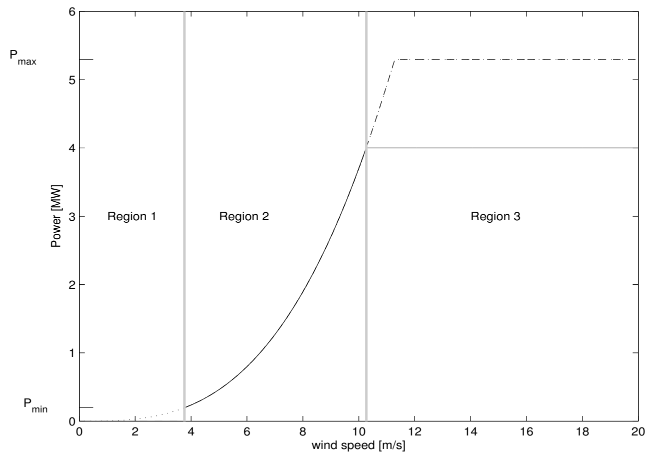

Fig. 3. Power capture curve for the NREL 5 MW turbine. $u = 4$ MW (solid), maximum power capture (dashed). The gray dotted lines show the operating regions for the controller.

On farm level, the control input to a turbine is the power reference u. A turbine will only respond to farm control if

$$P_{\min} \leq u \leq P_a$$

where $P_{\min}$ is the lowest level of power production a turbine can sustain, and the available power, P_a, is given by (10). This corresponds to operation in region 3.

Figures 4 and 5 show the power and thrust coefficients as function of power reference and wind speed. The high valued flat part of the curves correspond to operation in region 2. The slanting part of the curves correspond to operation in region 3.

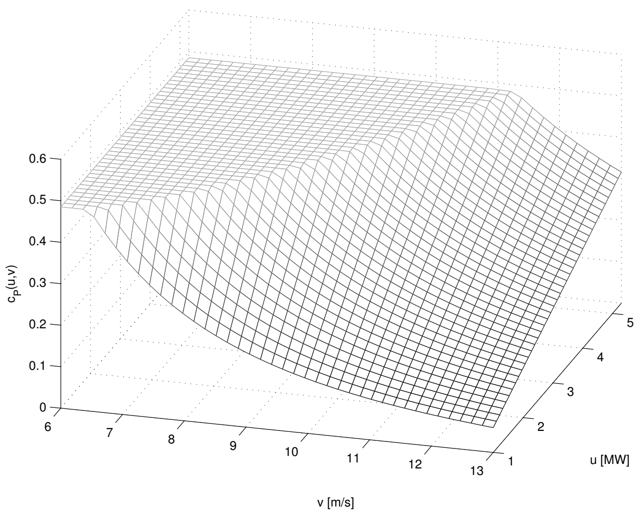

Fig. 4. Power coefficient for the NREL 5 MW turbine as function of wind speed and power reference.

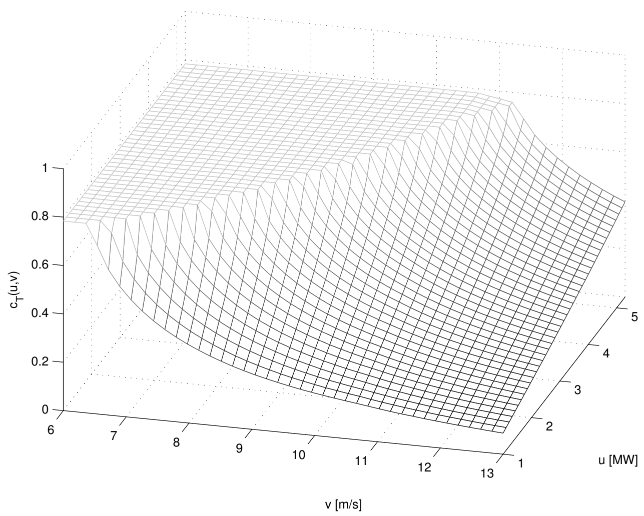

Fig. 5. Thrust coefficient for the NREL 5 MW turbine as function of wind speed and power reference.

4.2 Wind field parameters

Let incoming mean wind speed be 11 m/s, and ambient turbulence intensity 0.1. The coupling parameters are set to $k' = c' = 0.35$, $k = 0.1$, and $c = 0.92$. This means that if all turbines would operate in region 2, the second turbine would experience a mean wind speed deficit of 8%. The asymptotic mean wind speed deficit far downwind would be 22%. If we define the turbulence intensity at turbine n, as $T_{\mathrm{i,n}} = \frac{\sigma_n}{v}$, the second turbine would experience a turbulence intensity of 0.17. The asymptotic turbulence intensity far downwind would be 0.3.

Table 1. Power extracted in each turbine (MW)

i	P_i	$P_{\mathrm{a,i}}$	$P_i/P_{\mathrm{a,i}}$	$P^0_{\mathrm{a,i}}$	$P_i - P^0_{\mathrm{a,i}}$	σ_i/σ^0_i
1	4.76	4.92	0.97	4.92	-0.16	1.00
2	3.72	3.97	0.94	3.87	-0.15	0.96
3	3.21	3.47	0.93	3.26	-0.048	0.93
4	2.95	3.18	0.93	2.90	0.047	0.92
5	2.78	3.00	0.93	2.68	0.10	0.92
6	2.68	2.89	0.93	2.54	0.13	0.92
7	2.67	2.82	0.95	2.46	0.21	0.92
8	2.66	2.75	0.97	2.40	0.26	0.93
9	2.64	2.67	0.99	2.37	0.27	0.95
10	2.58	2.58	1.00	2.34	0.23	0.97
$\sum$	**30.64**	**32.11**	–	**29.74**	**0.90**	–

4.3 Examples

Consider a wind farm with N turbines, where each turbine tries to capture as much power as possible. We will now try to increase the total power capture in the farm by limiting power capture at some of the turbines. The idea is that by decreasing power capture at upwind turbines, their thrust coefficients also decrease. This in turn, increases available power at turbines further downwind.

We proceed as follows: let each turbine capture as much power as possible, i.e. $u_i = P_{\max},\ i = 1, \ldots, N$, and set $n = 1$.

(1) Grid over u_n to find

$$u_n^* = \arg\max_{u_n} \sum_{i=1}^{N} P_i$$

(2) Set $u_n = u_n^*$, and $n = n + 1$. Go to step 1.

Table 1 shows the result for a farm with $N = 10$ turbines. P_i is the power captured by turbine i; $P_{a,i}$ is the available power at turbine i; and σ_i is the wind speed standard deviation at turbine i. $P^0_{a,i}$ and σ^0_i denote the captured power and wind speed standard deviation at turbine i when each turbine extracts maximum power. Limiting some of the upwind turbines, resulted in a total power capture increase of 3.0%. The turbulence levels at all turbines except turbine 1 were also decreased.

Figure 6 shows the power increase when running the same algorithm for different values of N. Although, the power distribution might not be optimal, the result indicates that for larger farms, the benefit in coordinating turbines could be larger. A reason for this is illustrated by the following example.

Consider a farm with $N = 10$ turbines, where all turbines individually extract as much power as possible. Fix the power references of turbines 2 through $N - 1$ to the current available power. Then decrease the power capture of turbine 1 by 0.5 MW. Figure 7 shows the *change* in wind speed, Δv_i, thrust coefficient, ΔC_{T_i}, and available power, $\Delta P_{a,i}$, at each turbine. The increase in available power becomes larger further downwind. The reason is that the change at turbine 1 increases available power at turbines

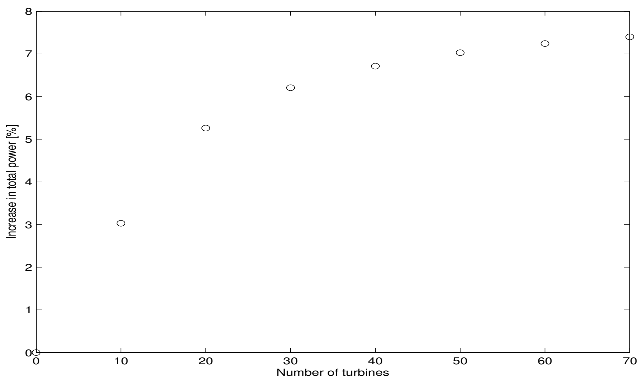

Fig. 6. Relative gain in total captured power for different numbers of turbines.

2-10. Turbine 2 already has a power setpoint it needs to maintain. To do this, the turbine needs to decrease its power capture. This leads to a simultaneous decrease in thrust coefficient, which further increases available power at turbines 3-10. Similarly, turbine 3 is also limited and needs to decrease its power capture, and so on. The change at turbine 1, therefore, results in a larger and larger increase in available power downwind. Having many turbines in a farm, can thus increase the impact of decreasing power in a single turbine.

Based on the results in Figure 7, we observe that the model suggests that a power decrease at an upwind turbine can have two effects. The first is that it causes all turbines further downwind, that maintain their level of power production, to pose less of an obstacle for the wind. The second is that the increase in available power becomes larger further downwind. While the first effect is intuitive, the latter is due to the choice of of k, and k'. It occurs for large enough ratios, $\frac{k}{k'}$ (i.e small enough distances between the turbines). Whether this effect can be found in actual wind farms has to be concluded by experiments.

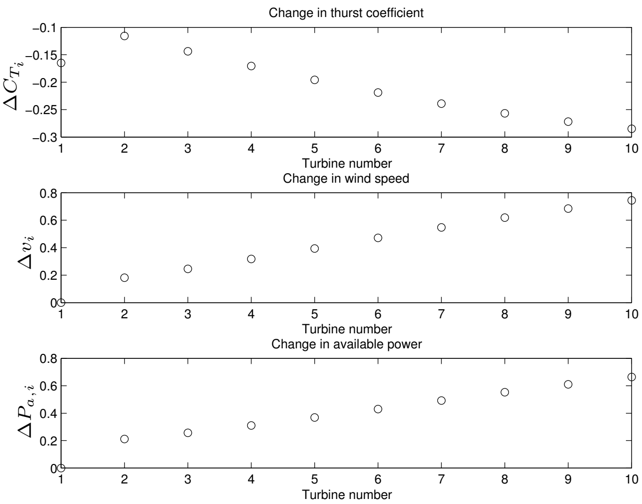

Fig. 7. Result of decreasing power in turbine 1 by 0.5 MW, while maintaining the power capture in turbines 2-9. The values in the plots show the *change* relative to the case before the power decrease at turbine 1. Note that while $\Delta P_{a,i}$, and Δv_i increase downwind, the same is not true for the *absolute* values $P_{a,i}$, and v_i.

5. FUTURE WORK

Due to high confidentiality, and that real wind farms are mostly commercial, it takes time to obtain data for validation. However, efforts are being made on this front, and model validation will be carried out in the near future.

Another natural step is to examine to what extent the simplicity and distributed structure of the model can be exploited for distributed control purposes.

We would also like to extend the model to fit more complicated wind farm topologies, such as grids.

ACKNOWLEDGEMENTS

This work was supported by the European Community's Seventh Framework Programme under grant agreement number 224548, acronym AEOLUS, and the Swedish Research Council through the Linnaeus Center LCCC.

REFERENCES

Tony Burton, David Sharpe, Nick Jenkins, and Ervin Bossayani. *Wind Energy Handbook.* John Wiley and Sons, 2nd edition, 2008

Sten Frandsen. On the Wind Speed Reduction in the Centre of Large Clusters of Wind Turbines. *Journal of Wind Engineering and Industrial Aerodynamics*, volume 39, pages 251-265, Elsevier Science Publishers, B.V., Amsterdam, 1992

E.A. Bossayani, C. Maclean, G.E. Whittle, P.D. Dunn, N.H, Lipman, and P.J. Musgrove. The Efficiency of Wind Turbine Clusters. In *Proc. Third International Symposium on Wind Energy Systems*, August 1980

A. Crespo, J. Hernández, and S. Frandsen. Survey of Modeling Methods for Wind Turbine Wakes and Wind Farms. *Wind Energy*, volume 2, pages 1-24, 1999

N.O. Jensen. A Note on Wind Generator Interaction. *Technical Report Risø-M-2411*, Risø National Laboratory, Roskilde, 1983

Sten Frandsen, Rebecca Barhelmie, Sara Pryor, Ole Rathmann, Søren Larsen, and Jørgen Højstrup. Analytical Modelling of Wind Speed Deficit in Large Offshore Wind Farms. *Wind Energy*, volume 9, pages 39-53, 2006.

J. Jonkman, S. Butterfield, W. Musial, and G. Scott. Definition of a 5-MW Reference Wind Turbine for Offshore System Development. *Technical Report NREL/TP-500-38060*, National Renewable Energy Laboratory, Golden, Colorado, February 2010

Jacob Deleuran Grunnet, Mohsen Soltani, Torben Knudsen, Martin Kragelund, and Thomas Bak. Aeolus Toolbox for Dynamic Wind Farm Model, Simulation and Control. In *Proc. European Wind Energy Conference*, April 2010

Lucy Y. Pao, and Kathryn E. Johnson. A Tutorial on the Dynamics and Control of Wind Turbines and Wind Farms. In *Proc. American Control Conference*, June 2009.

Sten Troæs Frandsen *Turbulence and Turbulence Generated Structural Loading in Wind Turbine Clusters.* Ph.D. Thesis, Risø National Laboratory, Risø-R-1188(EN), 2007