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ABSTRACT: The natural material wood is commonly graded with regard to the parallel to the grain strength and stiffness properties and taking into account different growth characteristics such as knots and grain deviations. In this paper the impact of knots and grain deviations on the fracture perpendicular to the grain of timber is analysed by means of numerical models. The results are used for the calibration of an analytical model. With this model it is possible to evaluate the impact of growth characteristics on the perpendicular to the grain fracture and compare the results with test data from literature. The evaluation shows that certain growth characteristics increase the strength perpendicular to the grain. This is in contrast with current grading procedures, where such growth characteristics are considered as being strength decreasing. The results are compared with a model for the description of the effects of growth characteristics on the distribution characteristics of the strength perpendicular to the grain. This strongest link model can be used to describe phenomena with a parallel system of failure events.

1. INTRODUCTION

Although wood is a natural material exhibiting various inhomogeneities, fracture of wood is most often studied on small clear specimens and modeled neglecting these inhomogeneities. However, for the prediction of the structural behavior of full scale glulam members the influence of growth characteristics, like e.g. knots, grain deviations and cracks has to be accounted for.

The failure of boards loaded in tension parallel to grain is often initiated by growth characteristics. For a safe and reliable use of timber in load-carrying structures it is necessary to grade timber according to certain criteria. These criteria are commonly chosen with respect to the bending (or tensile) strength of timber. For other strength properties like tension perpendicular to the grain or shear no adequate grading criteria have been specified. However, it is well known that for tension perpendicular to grain the stressed volume has an important impact on the effective strength and that wood checks and other characteristics initiate failure. This effect is often described by means of the Weibull weakest link theory (Weibull, 1939).

Cracks in wood normally propagate parallel to the grain due to the low strength and fracture en-

Table 1: Shear stresses at failure in the reduced cross-section of beams with variable notch height in 3-point bending, separated into samples with and without knots (Larsen and Riberholt, 1972)

α [−]	$\tau_{u,mean}$ (CoV) [N/mm^2] (%)	$\tau_{u,0.05}$ [N/mm^2]	$\tau_{u,0.10}$ [N/mm^2]
	all		
1	4.38 (3.1)	4.02	4.12
0.75	2.93 (29.6)	1.70	1.90
0.5	2.19 (26.5)	1.36	1.51
0.25	2.00 (38.0)	1.02	1.17
	with knots		
1	-	-	-
0.75	3.63 (30.6)	1.78	2.04
0.5	2.41 (20.9)	1.55	1.71
0.25	2.77 (31.6)	1.47	1.70
	without knots		
1	-	-	-
0.75	2.72 (24.1)	1.75	1.92
0.5	2.12 (27.9)	1.32	1.45
0.25	1.78 (31.8)	1.02	1.15

ergy of wood perpendicular to the grain. In the vicinity of knots the wood fibers deviate from the global grain direction. This leads to an increase in effective resistance against crack growth. Therefore knots and grain deviations along the crack path can lead to an increase of the load-carrying capacity of e.g. end-notched beams (Jockwer, 2014).

Early experimental research on the impact of knots on the fracture behaviour of timber is reported in (Larsen and Riberholt, 1972). 200 end-notched solid timber beams of the quality grade "ungraded" with varying height of the reduced cross-section $\alpha \cdot h$ were tested in 3-point bending. Fracture of the notch corner was the failure mode in the tests and the results are summarized in Tab. 1. The failure load of the notched beams with knots was found to be higher both on mean and 5^{th}– and 10^{th}–percentile levels.

Riberholt et al. (1991) tested a large series of end-notched beams in order to study the influence of various geometrical parameters on the load-carrying capacity. A crack retarding effect of knots was observed and specimens with knots showed higher load-carrying capacities. In addition to full-size specimens also tests on small clear specimens were carried out. One specimen contained a knot along the crack surface and showed a considerable higher fracture energy.

Similar impacts of knots on the load-carrying behavior of end-notched beams were detected in the tests reported in (Möhler and Mistler, 1978). A reduction of load-carrying capacity was observed for beams with checks along the crack path.

Jockwer (2014) analyzed impacts on the variation of load-carrying capacity of end-notched beams. In this study it was shown that the large variation of load-carrying capacities can not be explained only by the variation of the elastic material properties and fracture energy in mode 1 (opening mode). In addition a model uncertainty with a considerably high coefficient of variation $CoV \approx 23\%$ has to be included in order to be able to meet the test results. The variation of this model uncertainty can be justified by the presence of knots along the crack path. In (Jockwer, 2014) it is described how the crack retarding effect of knots was studied in tests by means of optical measurement systems.

This paper aims at investigating and quantifying the impact of different growth characteristics on the fracture behaviour of timber. In numerical models the impact of the shape of the crack path is analysed. Analytical models are used to study the impact of varying fracture energy along the crack path. The results of these studies are used to evaluate the distribution characteristics of the load-carrying capacity of specimens loaded (locally) in tension perpendicular to the grain.

2. GRADING OF TIMBER AND MATERIAL PROPERTIES

2.1. *Growth characteristics*

The mechanical properties of solid timber mainly depend on the physical and structural characteristics of wood (Forest Products Laboratory, 2010). Key physical parameters are the wood density and the moisture content (*MC*). In sound, non-decayed wood, the structural characteristics with the highest impact on the fracture behavior and the mechanical properties are:

- amount and size of knots and knot clusters
- cross grain

Table 2: Distribution parameters of material properties being of relevance for the fracture of glulam equivalent to strength class GL24h at MC = 12% according to EN 338 (2009), JCSS (2001) and Jockwer (2014)

Parameter	Unit	Symbol	Mean	5^{th} *perc.*	*CoV*	PDF
MOE ∥ to the grain	$[N/mm^2]$	$E_{0,mean}$	11'500	9'600	13 %	lognormal
MOE ⊥ to the grain	$[N/mm^2]$	$E_{90,mean}$	300	250	13 %	lognormal
Shear modulus	$[N/mm^2]$	$G_{v,mean}$	650	540	13 %	lognormal
Fracture energy Mode 1 (clear wood)	[N/mm]	$G_{f,I,mean}$	0.3	0.218	20 %	lognormal
Fracture energy Mode 2 (clear wood)	[N/mm]	$G_{f,II,mean}$	1.15	0.695	30 %	lognormal

- orientation and width of the annual rings (the latter as a visible indicator for the density)
- presence / absence / distance from pith (as a result of the cutting pattern).

2.2. *Strength grading*

In order to reach the intended mechanical properties of glued-laminated products the raw material has to be strength graded. The procedure is to grade the timber applying either visual or machine grading techniques. Visual grading is based on an assessment of boards with regard to their outer appearance (e.g. DIN 4047-1 (2012)). The timber species, the knot area ratio, slope of grain, growth ring spacing and defects such as splits and bark pockets are identified. Machine grading involves measuring different parameters like dynamic or static MOE, density, sizes and number of knots and knot clusters, moisture content, etc. known to correlate with the desired mechanical properties (bending strength and bending MOE) and offering the possibility of determining them based on statistical relationships (Glos, 1983).

2.3. *Material characteristics and mechanical properties of glulam*

Selected mechanical properties of glulam of the common strength class GL24h can be represented by parameters as summarized in Tab. 2 (EN 14080 (2013); JCSS (2001)). The variations in the properties have been modeled with different distribution functions. Strength properties are commonly represented by Weibull or lognormal distributions (Sørensen and Hoffmeyer, 2001) whereas bending stiffness and density can be represented by normal distributions (Steiger and Arnold, 2009). The lognormal distribution can, for sake of simplicity, be chosen for the representations of all material properties as suggested by (JCSS, 2001; COST Action E 24, 2005). The coefficient of variation (*CoV*) for solid timber can be calculated with that distribution function from the mean and 5^{th}–percentile values of the stiffness properties to $CoV = 23\%$ (EN 338 (2009)). The respective value for glulam is $CoV = 11\%$ (EN 14080 (2013)). An equal $CoV = 13\%$ is suggested for the stiffness of both solid timber and glulam in (JCSS, 2001).

2.4. *Knot clusters and clear wood sections*

The natural structure of the spruce wood makes a distinction between clear wood and knot sections possible. Fink et al. (2013) used a constant length of the knot sections of 150 mm for the description of the structure of the boards although the length of the knot sections in reality is varying. It is suggested by (Fink et al., 2013) to model the clear wood section as being Gamma-distributed with an expected length of $d_{mean} = 530$ mm and a standard deviation of $\sigma = 250$ mm.

2.5. *Cross grain*

In a previous investigation reported in (Oscarsson et al., 2014) 450 glulam laminations were scanned for surface fibre directions. Here, the same data set was used to quantify the cross grain. The median of the nominal grain direction on the edges of the laminations (for all laminations)was used as a measure in order not to have the data corrupted by the local grain deviation close to knots. A median deviation from perfectly aligned grain in the range of up to 2 degrees was found to be quite common and the extreme 10% fractile values of the median included deviations of approximately 4-6 degrees.

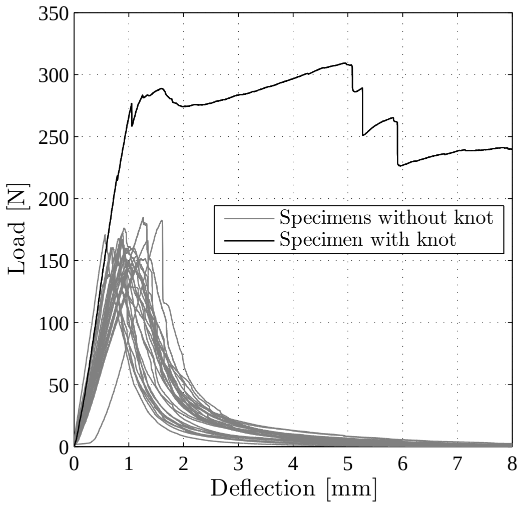

Figure 1: Force-displacment behaviour of a SENB specimen without and with knots (Jockwer, 2014).

2.6. *Fracture related material parameters*

The fracture energy of mode 1 is commonly determined by testing single edge notched beam (SENB) specimens as specified in a Draft Standard of CIB-W18 by Larsen and Gustafsson (1990), also known as the Nordtest method (Nordtest, 1993). Parameters for PDFs fitted to the fracture energy values of individual data are summarised in Tab. 2. The correlation between density and $G_{\mathrm{f,I}}$ is low for the observed range of densities being of relevance for structural applications. No general trend of the fracture energy with regard to the spatial distribution in grain direction can be found. However, there is a considerable impact of the presence of knots in the fracture plane as can be seen in Fig. 1. This impact can be explained by the resulting larger fracture surface due to grain deviations and the doweling effect of the knot interfering the separation of the specimen and allowing further load transfer along the fracture plane. Such extended studies do not exist for the mode 2 fracture energy $G_{\mathrm{f,II}}$. However, a similar impact of growth characteristics can be assumed.

3. IMPACT OF GROWTH CHARACTERISTICS ON THE FRACTURE OF WOOD

The impact of knots and grain deviations and of cross grain on the fracture energy of a SENB and the crack propagation load of a notched beam, respectively, is analyzed by means of numerical models.

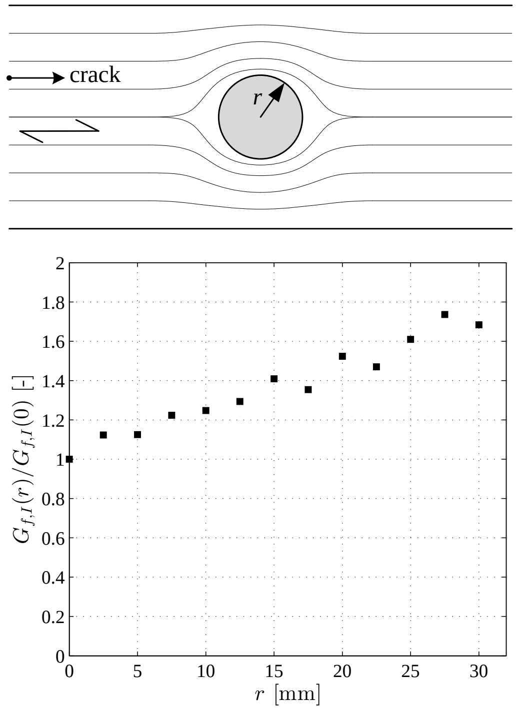

Figure 2: Grain deviation in the region of a knot and impact of the amplitude of grain deviation on the fracture energy $G_{f,I}$.

3.1. *Impact of grain deviations*

There are various impacts of knots on the fracture behaviour of timber. One of them is the deviation of grain direction in the vicinity of knots which leads to an increase of the fracture surface. In Fig. 2(a) an example of the deviation of the crack is illustrated as it might occur in the vicinity of a knot. The impact of the deviation on the load-deflection behaviour can be analysed using SENB specimens. The numerical study was performed by means of the software ABAQUS applying enriched element (xFEM) as described in (Qiu et al., 2014). The results of the study are summarized in Fig. 2(b): The increase in fracture energy depends on the size and slope of the grain deviation.

An additional impact of knots on the fracture energy is caused by the their reinforcing effect on the crack. The reinforcing effect can be assumed to depend on the knot size within one knot cluster. In tests on SENB which included knots in the crack

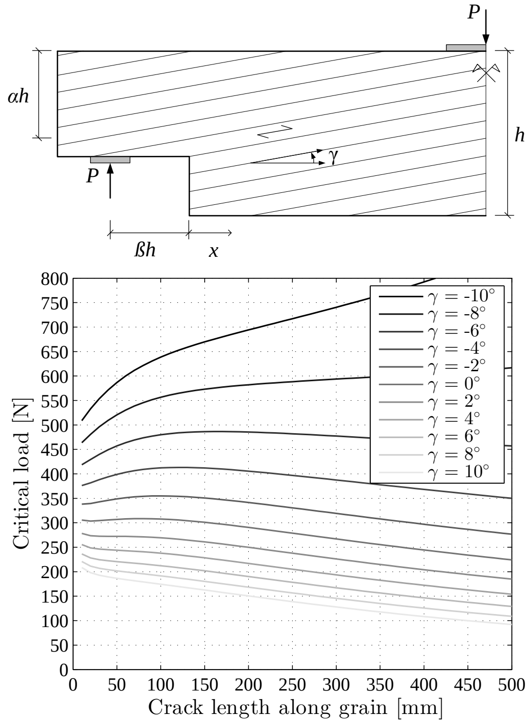

Figure 3: Model of a beam with cross grain and development of the critical load causing crack propagation.

plane a relative increase of up to factor 10 can be observed (Fig. 1).

3.1.1. *Impact of cross grain*

The global grain direction of glulam beams is commonly oriented parallel to the beam axis. Hence, a crack will develop along the beam axis, leading to a separation of the notched beam in an upper and a lower part.

In case of an inclination of the grain direction, not only the directions of the orthotropic material properties are different but also the remaining cross-section of the lower and the upper beam change during crack growth.

In Fig. 3(a) an example of a global inclination of the grain direction of a notched beam by $\gamma = 10^\circ$ is illustrated. The numerical study was performed by means of a MATLAB based FE model using the compliance method for the calculation of energy release rate and crack propagation load. The ultimate load and also the failure behavior changes considerably already for small inclinations in the order of $\pm 10^\circ$ as shown in Fig. 3(b).

4. Load-carrying capacity of a notched beam

4.1. *Notched beam model*

Gustafsson (1988) proposed an analytical model for the estimation of the load-carrying capacity of end-notched beams. The strength equation is set up by balancing energies during crack growth initiated in an end-notched beam. The energy release rate is calculated by derivation of beam deflection as a function of crack length. The analysis delivers the load-carrying capacity at a given notch length βh.

Growth characteristics along the crack path can be accounted for in the model by assigning variations of the values of fracture energy to the respective position along the crack path $\beta h + x$.

Additional lamellas can be inserted in the model in order to simulate the interaction between these lamellas and in order to model weakest link effects during failure of a glulam beam.

4.1.1. *Monte Carlo simulations*

In a Monte Carlo based set of simulations the material properties according to Tab. 2 were used. Within the clear wood sections the fracture energy was modeled as lognormally distributed with $G_{\mathrm{f,mean}} = 0.3$ N/mm and $CoV = 20\%$. The variations in stiffness and fracture energy in the clear wood sections only lead to a minor variation of the load-carrying capacities (dash-dotted line *"Reference"* in Fig. 4). Hence, it has to be accounted for additional variations by inserting knot sections along the crack path as explained in Section 2.4. The notched beam shows a brittle failure behavior due to the strong decrease of the crack propagation load with increasing crack length. This leads to a diminishing impact of the knot section on the load-carrying capacity with increasing crack length. The impact of the growth characteristics on the load-carrying capacity is illustrated in Fig. 4 and can be described as follows:

- **knots**: the distance between the knot sections has a strong impact on the distribution of load-carrying capacities. For smaller distances the mean value of the load-carrying capacity increases. The variation of the distances between knot sections has only a minor impact on the distribution of load-carrying capacities.

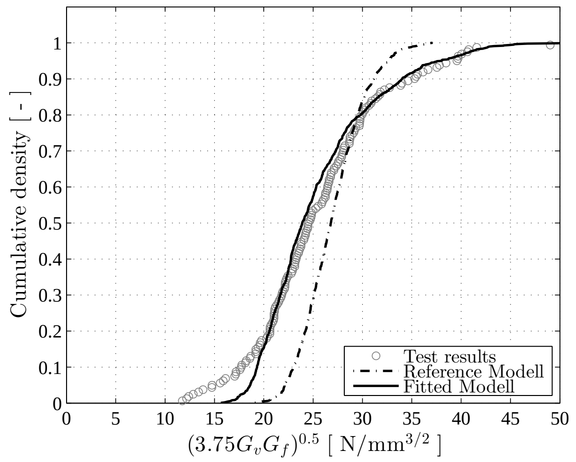

Figure 4: Comparison of the cumulative density distribution of test results with the models accounting for different growth characteristics.

- **grain deviations**: Within the knot section the effective fracture energy is higher compared to the clear wood section due to grain deviations and the reinforcing effect of the knots. A knot factor F_{knot} is introduced to describe the relative increase of fracture energy in the knot section. This factor has a strong impact on the load-carrying capacity. The best fit between the analytical model and test results from literature (Jockwer, 2014) is achieved for $F_{\text{knot,mean}} = 2.0$ with $CoV = 40\%$.

The fitted analytical model as illustrated in Fig. 4 gives best agreement with the test results when reducing the load-carrying capacity to 80% of the reference values. This reduction is in line with the studies by e.g. Franke (2008). The general trend is, that the presence of knot sections leads to an increase of the load-carrying capacity of notched beams. This effect is in contrast to the current procedures where only the weakening effect of the knot sections on the bending strength is accounted for.

5. STRONGEST LINK MODEL

5.1. *Background*

As discussed in Section 2 it is difficult to specify grading criteria to guarantee for strength properties other than bending and tension parallel to the grain, like e.g. tensile strength perpendicular to grain and shear strength. For tension perpendicular to grain it

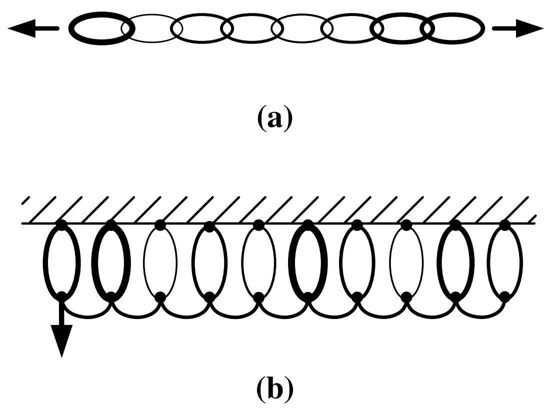

Figure 5: Illustration of the weakest link model (a) and of the strongest link model (b).

is, however, well known that the size of the stressed volume is important and also that failure often is initiated at checks and growth characteristics. The more or less random distribution and the number and size of these weak spots causes the volume effect. The probability of failure in tension perpendicular to grain of wood is often described using the 2-parameter Weibull distribution (Weibull, 1939):

$$p_f(\sigma) = P(R < \sigma) = 1 - e^{-\left(\frac{\sigma}{f_c}\right)^m} \tag{1}$$

where σ is the stress in a unit volume of the material and f_c and m are material parameters which define magnitude and scatter in strength. This distribution together with the Weibull weakest link theory gives the strength distribution as a serial system of failure events like e.g. a linear chain containing n unit volumes (Fig. 5a) as

$$p_f(\sigma) = 1 - e^{-n\left(\frac{\sigma}{f_c}\right)^m} \tag{2}$$

A very convenient feature of this extreme value distribution is that its CoV equals the one of the unit volume strength distribution. A simple generalization of Eq. 2 makes it applicable to arbitrary volumes of material in which the stress may be nonhomogenous, but still required to be finite.

The Weibull weakest link theory is, however, not applicable to structural elements with notches, cracks or other shaping that reveals a stress singularity since the theory for such situations in general predicts either zero strength or no crack propagation, no matter the magnitude of load (Gustafsson and Enquist, 1988). For end-notched beams the Weibull weakest link theory is contradicted by test results (Larsen and Riberholt, 1972) as discussed in Section 1. The higher strength observed may instead be described by a strongest link concept.

5.2. *Strongest link model*

A strongest link model with a sequential system of failure events is proposed in (Gustafsson, 2014) and can be illustrated e.g. by the resistance that a zipper gives towards being opened as illustrated in Fig. 5(b): the zipper link providing the highest resistance is decisive. Such a model can be applied in cases where global failure is governed by crack propagation along a crack path of certain length, and more generally where failure of two or more structural elements, or points, precedes global structural failure. If, e.g., the strength distribution of a single link can be described by Eq. 1, then the strongest link strength distribution of a zipper with n links is

$$p_f(\sigma) = \left(1 - e^{-\left(\frac{\sigma}{f_c}\right)^m}\right)^n \tag{3}$$

The strongest link model results for more links in a higher strength and in a smaller CoV. Moreover, for heterogeneous materials with a given mean link or material strength, an increased structural strength is predicted.

In crack propagation analysis the link strength f_c can be interpreted as the fracture toughness K_c of the material determined experimentally for constant stress intensity, K, for a crack propagation of length Δx. The parameter m is then a measure of the scatter in K_c. σ in the ratio σ/f_c represents the stress intensity K. From conventional fracture mechanics analysis the function $K = K(P,x)$ can be determined, P being the external load and x the length of the crack. If K is constant along the crack path then:

$$p_f(P) = \left(1 - e^{-\left(\frac{PK_1}{K_c}\right)^m}\right)^n \tag{4}$$

K_1 represents the value of K when $P = 1$. The application of Eq. 4 to strength analysis of end-notched beams can be done by representing the fracture toughness and the stress intensity in terms of the critical energy release rate G_c and the energy release rate G, respectively, i.e. $K_c = (G_v G_c)^{0.5}$ and $PK_1 = (G_v G)^{0.5}$. If G varies along the crack propagation length L, then

$$p_f(P) = \prod_{i=1}^{n} \left(1 - e^{-\left(\sqrt{\frac{G(P,x_i)}{G_c}}\right)^m}\right) \tag{5}$$

where $n = L/\Delta x$ and $x_i = (i - 0.5)\Delta x$, with Δx being the reference length for the material parameters G_c and m.

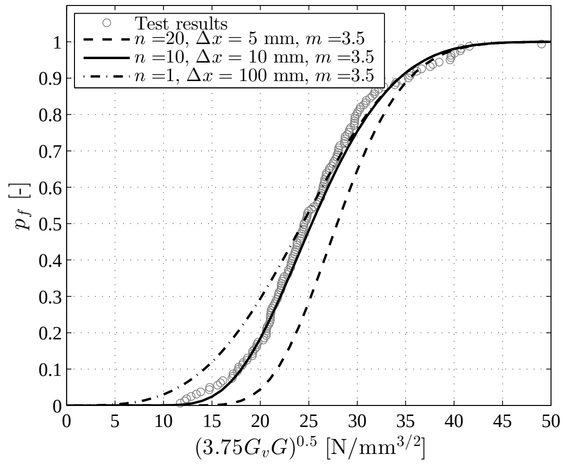

Figure 6: Representation of test results on notched beams (Jockwer, 2014) by the model in Eq. 5 for number of links n of length Δx.

5.3. *Discussion*

The application of the strongest link model in Eq. 5 and the comparison with test data from literature (Jockwer, 2014) is shown in Fig. 6. The material properties in Tab. 2 were used with $G_c = G_{f,I}$ and a crack propagation length $L = 100\ mm$. A good fit of the model with the test data is achieved for $m = 3.5$ and $n = 10$ links each with a length of $\Delta x = 10\ mm$. For a decreasing number of links both mean and 5$^{\text{th}}$–percentile values decrease whereas for more links of smaller length these values increase. The 95$^{\text{th}}$–percentile values are affected only marginally. The plausibility of L as a reference length for fracture mechanic problems has to be evaluated more extensively.

6. SUMMARY

Growth characteristics have an important impact on the strength and stiffness properties of timber. In this paper their impact on the fracture perpendicular to grain of timber was studied by means of different models. Changes in fracture energy and crack propagation load due to growth characteristics were evaluated by means of numerical models. The results serve as reference for an analytical model in which the crack path is separated into knot and clear wood sections. In this model the estimated strength of the notched beam increases with increased occurrence of knot sections. This statistical effect is described by a strongest link model representing a parallel system of failure events. The behavior of the strongest link model is discussed in a comparison with test data of notched beams and a good fit is found. Nevertheless, for further applications of the strongest link model to other situations additional calibration is necessary.

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