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Abstract

A low frequency algorithm designated for the calculation of the magnetic and the electric energies and the ohmic losses of passive components has been developed. These quantities can be used to achieve the inductance, the capacitance, the resistance, and the Q-value of the inductor at low frequencies. The method is based on an iterative scheme in which the different fields, such as the electric potential, Φ, and the vector potential, $\boldsymbol{A}$, are expanded in power series in ω. When the power series expansions are inserted into the quasi static Maxwell equations, *i.e.*, a low frequency approximation, it is possible to formulate the full problem in a set of differential equations in which the power series coefficients are the unknowns.

1 Introduction

Inductors on silicon are today important components in integrated high frequency circuits (*i.e.*, radio circuits). In comparison with off-chip technology this technology reduces size and cost. However, since inductors are very size consuming it is still expensive to manufacture them. Hence it is important to design them in a way that maximum performance is gained from a small area.

Since inductors are complicated structures efficient EM simulators are necessary design tools. There are several methods based on different approximations, *e.g.*, FastHenry [3], Asitic [4], and Momentum (see http://eesof.tm.agilent.com/docs/adsdoc2002/mom/ for more information).

For high frequencies, when the skin depth is much smaller than the thickness of the inductor, the method of moment (MoM) based softwares are suitable. It is here possible to assume surface currents and a surface integral equation rather than a volume integral equation (one can also assume infinite conductivity [1]). The coefficient matrix, in the linear system, is still dense but the number of unknowns are greatly reduced. At lower frequencies it is harder to predict the behavior of the current density and the conductive regions has to be included. This leads to a volume integral formulation and a less efficient method due to the increased number of unknowns.

Methods based on the finite-difference time-domain method (FDTD) have also been used to analyze inductors in lossy media [5]. Although they are very flexible, they require a significant effort in computational time.

In this paper we describe an iterative algorithm that uses a low frequency approximation to solve the Maxwell equations. The iteration is based on a power series expansion of the different fields, which leads to a set of scalar partial differential equations. The zeroth order problem corresponds to the static case which means that no reactive effects occur. The inductive and capacitive effects appears in the first and higher order terms.

The differential equations are solved using a finite element method (FEM). FEM generates a linear system with a sparse matrix that has the advantage of being solvable in a few number of operations in comparison to a dense matrix. Since an unstructured grid is used, good agreement with the real structure can be achieved.

The problem formulation is divided into two parts, one that handles the inner conductive region and one that handles the outer non-conductive region. The division leads to a fast convergence of the equation system and a well behaved coefficient matrix. This is a way to overcome the problem at the boundary between the inner and outer region, where $\nabla\Phi$ is discontinuous.

For low frequencies, the capacitive effects are negligible and only ohmic and inductive effects have to be considered. When the frequency approaches the self-resonant frequency of the inductor, the contributions from the surface charges become considerable, which leads to degraded effective inductance, L_{eff}. In order to include the capacitive effects, the displacement current has been included in the boundary conditions. Inside the conductor the displacement current can be neglected. Losses due to radiation have been assumed to be negligible since the structure is much smaller than the wavelength.

The method in this paper covers the low frequency spectrum. Similar methods for higher frequencies are currently under investigation.

2 Preliminaries

There are three different length scales involved, defined by the wavelength, the geometry and the skin-depth. The wavelength is assumed to be much longer than the structure, whereas the inductor is much longer than the skin depth. The assumption is that the frequency is low enough for the quasi-static approximation to be valid outside and inside the conductor. In the conductor the displacement current can be neglected since it is much smaller than the current density.

The conductor has a conductivity σ and is confined in the volume V_{C}. The surface that encloses V_{C} is divided into three parts; the two surfaces of the terminals and the surface covering the rest of the conductor. The surfaces of the terminals are denoted S_0 and S_1, and the surface covering the rest of the conductor is denoted S_{C}. The normal vector to the surface S_{C} is denoted $\hat{\boldsymbol{n}}$. The dielectric region outside the conductor has a relative permittivity ϵ_{r} and a volume V_{out}.

The current density, $\boldsymbol{J}$, the scalar potential, Φ, and the vector potential, $\boldsymbol{A}$, are expanded in a power series in ω of the form

$$\Psi(\boldsymbol{r}) = \sum_{n=0}^{\infty} \Psi^n(\boldsymbol{r}) = \sum_{n=0}^{\infty} \omega^n \psi^n(\boldsymbol{r}). \tag{2.1}$$

3 The quasi static Maxwell equations

In the low frequency approximation Ampères law is reduced to $\nabla \times \boldsymbol{H} = \boldsymbol{J}$, inside a conductive homogeneous region, and to $\nabla \times \boldsymbol{H} = j\omega \boldsymbol{D}$ in a non-conductive homogeneous region. The contribution from the displacement current in the outer region is assumed to be negligible, which leads to $\nabla \times \boldsymbol{H} = \boldsymbol{0}$. Since all regions are source-free the electric field in each region fulfills $\nabla \cdot E = 0$. The remaining two equations are unchanged. They read

$$\nabla \times \boldsymbol{E} = -j\omega\mu_0 \boldsymbol{H}, \qquad \nabla \cdot H = 0.$$

4 Boundary conditions

The electric potential takes the value $\Phi = 0$ at the terminal S_0 and $\Phi = \Phi_1$ at S_1. The boundary condition for the current density is

$$\hat{\boldsymbol{n}} \cdot \boldsymbol{J}(\boldsymbol{r}) = j\omega\rho_{\mathrm{S}}(\boldsymbol{r}), \quad \boldsymbol{r} \in S_0 \cup S_1 \cup S_{\mathrm{C}}, \tag{4.1}$$

where $\hat{\boldsymbol{n}}$ is the unit normal vector directed from V_{C} to V_{out}. If the general expression for the current density,

$$\boldsymbol{J} = -\sigma(\nabla\Phi + j\omega\boldsymbol{A}),$$

and the boundary condition for the displacement current,

$$\rho_{\mathrm{S}} = \hat{\boldsymbol{n}} \cdot \boldsymbol{D} = -\hat{\boldsymbol{n}} \cdot \epsilon_0\epsilon_{\mathrm{r}}(\nabla\Phi + j\omega\boldsymbol{A}),$$

are inserted into Eq. (4.1) we get

$$\hat{\boldsymbol{n}} \cdot \nabla\Phi(\boldsymbol{r}) = j\frac{\omega}{\sigma}\epsilon_0\epsilon_{\mathrm{r}}\hat{\boldsymbol{n}} \cdot \nabla\Phi(\boldsymbol{r}) - j\omega\hat{\boldsymbol{n}} \cdot \boldsymbol{A}(\boldsymbol{r}) - \frac{\omega^2}{\sigma}\epsilon_0\epsilon_{\mathrm{r}}\hat{\boldsymbol{n}} \cdot \boldsymbol{A}(\boldsymbol{r}), \quad \boldsymbol{r} \in S_0 \cup S_1 \cup S_{\mathrm{C}}.$$

5 Solution by iteration

To separate the fields in the two regions the subscript "C" is used for the fields inside the conductor and the subscript "out" for the fields outside the conductor.

The power series expansions, Eq. (2.1), of $\boldsymbol{J}, \Phi$ and $\boldsymbol{A}$ are inserted into the quasi static Maxwell equations. The terms with zeroth order power of ω are first identified. These results are thereafter used to generate higher order terms.

A general expression for the boundary value problem in the conductor yields

$$\begin{cases} \nabla^2\Phi_{\mathrm{C}}^n(\boldsymbol{r}) = 0, & \boldsymbol{r} \in V_{\mathrm{C}}, \quad n = 0, 1, \dots, \\ \Phi_{\mathrm{C}}^n(\boldsymbol{r}) = 0, & \boldsymbol{r} \in S_0, \quad n = 0, 1, \dots, \\ \Phi_{\mathrm{C}}^n(\boldsymbol{r}) = \Phi_1, & \boldsymbol{r} \in S_1, \quad n = 0, \\ \hat{\boldsymbol{n}} \cdot \nabla\Phi_{\mathrm{C}}^n(\boldsymbol{r}) = 0, & \boldsymbol{r} \in S_{\mathrm{C}}, \quad n = 0, \\ \hat{\boldsymbol{n}} \cdot \nabla\Phi_{\mathrm{C}}^n(\boldsymbol{r}) = \frac{1}{\sigma}f^n(\boldsymbol{r}), & \boldsymbol{r} \in S_1 \cup S_{\mathrm{C}}, \quad n = 1, 2, \dots \end{cases} \tag{5.1}$$

The function f^n is defined as

$$f^n := \begin{cases} j\omega\epsilon\hat{\boldsymbol{n}} \cdot \nabla\Phi_{\mathrm{out}}^{n-1} - j\omega\sigma\hat{\boldsymbol{n}} \cdot \boldsymbol{A}^{n-1}, & n = 1, \\ j\omega\epsilon\hat{\boldsymbol{n}} \cdot \nabla\Phi_{\mathrm{out}}^{n-1} - j\omega\sigma\hat{\boldsymbol{n}} \cdot \boldsymbol{A}^{n-1} - \omega^2\epsilon\hat{\boldsymbol{n}} \cdot \boldsymbol{A}^{n-2}, & n = 2, 3, \dots, \end{cases}$$

where $\epsilon = \epsilon_0\epsilon_{\mathrm{r}}$. Outside the conductor the boundary value problem, for $n = 0, 1, \dots$, becomes

$$\begin{cases} \nabla^2\Phi_{\mathrm{out}}^n(\boldsymbol{r}) = 0, & \boldsymbol{r} \in V_{\mathrm{out}}, \\ \Phi_{\mathrm{out}}^n(\boldsymbol{r}) = \Phi_{\mathrm{C}}^n(\boldsymbol{r}), & \boldsymbol{r} \in S_0 \cup S_1 \cup S_{\mathrm{C}}, \\ \Phi_{\mathrm{out}}^n(\boldsymbol{r}) \to 0, & |\boldsymbol{r}| \to \infty, \end{cases}$$

where $\Phi_{\mathrm{C}}^{n}(\boldsymbol{r})$ is given from Eqs. (5.1). The current density and the vector potential is given by

$$\boldsymbol{J}^{n}(\boldsymbol{r}) = -\sigma \begin{cases} \nabla\Phi_{\mathrm{C}}^{n}(\boldsymbol{r}), & n = 0, \\ \nabla\Phi_{\mathrm{C}}^{n}(\boldsymbol{r}) + j\omega\boldsymbol{A}(\boldsymbol{r})^{n-1}, & n = 1.2.\dots, \end{cases} \qquad \boldsymbol{A}^{n}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int_{V_{\mathrm{C}}} \frac{\boldsymbol{J}^{n}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} dV'.$$

6 Inductor parameters

The input impedance of a general, two-terminal, linear, passive electromagnetic system [2] can be written as

$$Z\,|I|^{2} = \int_{V_{\mathrm{C}}} \boldsymbol{E}(\boldsymbol{r})\cdot\boldsymbol{J}(\boldsymbol{r})dV + j\omega\left[\int_{\mathbb{R}^3} \boldsymbol{H}^{*}(\boldsymbol{r})\cdot\boldsymbol{B}(\boldsymbol{r})dV - \int_{\mathbb{R}^3} \boldsymbol{E}(\boldsymbol{r})\cdot\boldsymbol{D}^{*}(\boldsymbol{r})dV\right] \tag{6.1}$$

where I is the complex harmonic input current and is given by

$$I = \int_{\Omega} \hat{\boldsymbol{n}}_t \cdot \boldsymbol{J}(\boldsymbol{r})dS.$$

Here Ω is the cross section surface of the conductor at the terminal S_1 and $\hat{\boldsymbol{n}}_t$ the normal unit vector to this surface. From this formulation it is possible to calculate the inductance, the capacitance and the resistance of an arbitrary lumped element model of an inductor. For a simple series LR-circuit the second term in Eq. (6.1) corresponds to $j\omega L_{\mathrm{eff}}$. From Eq. (6.1) it is also possible to calculate the Q-value of the inductor. We find

$$Q = \omega\frac{\operatorname{Im} Z}{\operatorname{Re} Z}.$$

The self-resonant frequency can be achieved by investigating when $Q = 0$.

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