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Analysis of Energy Modes for MIMO Antennas

Casimir Ehrenborg, Mats Gustafsson, and Miloslav Capek
Abstract

The optimal spectral efficiency of MIMO antennas in an ideal line-of-sight channel is investigated when bandwidth requirements are placed on the channel. By posing the problem as a convex optimization problem restricted by the stored energy a semi-analytical expression is formed for its solution. It is shown that this solution is solely dependent on energy modes of the antenna. The ability to induce these modes by utilizing only a few sub-regions of the antenna is analyzed and compared to the full plate. The position of these regions is also investigated when they are raised above the ground plane. The performance of these cases is illustrated by calculating the optimal spectral efficiency of the unrestricted problem and plotting how much is lost when extra constraints are added. It is demonstrated that the spatial diversity of the controlled regions correlates with the number of significant energy modes.

1 Introduction

Design of multiple-input-multiple-output (MIMO) antennas is based on effectively exciting discrete communication channels with low correlation [18], in so doing the transmittable bit-rate, or capacity, is increased. A proposed strategy for accomplishing this is to design the antennas such that they effectively induce modes with orthogonal radiation patterns, such as characteristic modes, of the structure which they are embedded in [15, 16, 17]. Previously, a method for calculating the optimal performance bound of a MIMO antenna in an ideal channel was presented in [4, 5]. This upper bound serves as a measure of how well a certain configuration is utilizing the total available performance of the design region [5].

Electrically small antennas suffer from a degradation in possible performance as their size is reduced compared to the wavelength. Some of the parameters where this is most evident are radiation efficiency, directivity, and bandwidth [10, 12, 22, 23]. It was shown in [5] that restricting the radiation efficiency of the antenna does not necessarily restrict embedded regions from inducing the full available performance of the entire structure. This could be achieved by analyzing the modes that contribute the most to the performance of the antenna and exciting them effectively. This observation motivates further investigations of the maximum spectral efficiency for a channel with restricted bandwidth, a scenario typically valid in the electrically small regime.

Bandwidth can be estimated, for electrically small systems, through the quotient of the energy stored in a system over the energy dissipated by it [19]. This relation produces accurate estimations and is equal to the Q-factor for single feed, single resonance systems [25]. This classical relation does not hold for multi-port systems, such as MIMO antennas, however, for each individual port feeding a MIMO antenna we have a well defined Q-factor calculated from the stored energy of the current induced by that port [21]. All of the currents these ports induce create a total current distribution on the antenna. Restricting the stored energy calculated from the total current serves as a relaxed problem to limiting the stored energy of each
port. In this way limiting the stored energy of the total current density implicitly imposes a bandwidth requirement on the system.

The ports of a MIMO antenna induce a current distribution across the antenna, optimizing the inputs to those ports is a very restricted way of manipulating the currents on the antenna. Optimizing the current distribution directly is a relaxed problem to optimizing the ports. This means that the optimal value of that optimization will always bound the initial problems solution, therefore providing a performance bound for the antenna. Current optimization is a method used for calculating the optimal performance of a design region by optimizing over the possible currents within that region [10]. These currents have the ability to express all possible solutions that could be created within the considered volume. By formulating the optimization problem as a convex problem the optimality of the solution is guaranteed, as by definition all local minima of a convex problem are also global minima [1]. Previously this method has been used to determine performance bounds for antennas in terms of, e.g., Q-factor [10, 11, 13], efficiency [7, 14], directivity [9], and in multi-objective optimization such as spectral efficiency [4, 5] or trade-off between Q-factor and radiation efficiency [7]. For single feed, single resonance antennas these problems can be solved efficiently due to being expressed as quadratic forms. However, spectral efficiency (capacity), is evaluated based on the covariance of the current distribution and as such cannot be formulated as a quadratic form. The capacity expression creates a semi-definite optimization problem which has one order more of unknowns, making it computationally demanding to solve [4]. In [5] a method for solving such problems was introduced. By formulating a dual of the optimization problem [1] and utilizing the good properties of the matrices restricting it, it is possible to solve them semi-analytically.

In this letter the method from [5] is applied to an optimization problem restricted by the stored energy of an antenna. A convex optimization problem for the maximization of the spectral efficiency in the covariance of the current distribution is stated and solved in Section 2 restricted by the stored energy and radiated power of the structure. This optimization problem is a multi-criteria optimization problem, that forms a Pareto frontier between the spectral efficiency, stored energy, and signal-to-noise ratio (SNR). The MIMO antenna is optimized in an ideal line-of-sight channel consisting of the spherical modes in the far-field [6, 8]. It is shown that the optimal channel is constructed from the set of modes maximizing radiated energy while minimizing stored energy, denoted here as energy modes [10]. In Section 3 this set of modes is used to analyze the ability of embedded antennas to effectively induce the available performance of the entire structure. The relationship between stored energy restriction and SNR is investigated for these antennas. Finally, regions situated above the ground plane region are considered, mimicking the way common cellphone antennas are designed and fed [24]. The position of these is considered in relation to the edge of the structure.
2 Theory

A MIMO system is described by

\[ y = Hx + n, \]  

(2.1)

where \( y \) are the received signals, \( H \) is the channel matrix, \( x \) are the transmitted signals, and \( n \) is the noise density in each of the receivers [18]. In order to calculate optimal performance bounds for a MIMO antenna in an arbitrary scenario either the transceiver or receiver needs to be idealized. Here we choose to model the receiver as an idealized absorber perfectly matched to all spherical waves reaching the far-field, characterized as the spherical modes in the far-field. The diversity of the transmitted signal is measured as the diversity in the spherical modes. This configuration lets us bound the performance of one MIMO antenna in terms of its total transmitted power to the far-field [4, 5, 6, 8]. The currents induced across the antenna are modeled by method of moments (MoM) [2]. Instead of optimizing the ports inducing these currents, the currents themselves serve as the inputs to the MIMO system in (2.1).

The optimal spectral efficiency of a MIMO channel can be calculated by solving the optimization problem [5],

\[
\begin{align*}
\text{maximize} & \quad \log_2 \det(1 + \frac{1}{N_0} \mathbf{SPS}^H) \\
\text{subject to} \quad & \text{Tr}(\mathbf{R}_r, \mathbf{P}) = P_r \\
& \mathbf{P} \succeq 0,
\end{align*}
\]

(2.2)

where \( N_0 \) is the noise spectral density, \( P_r \) is the radiated power, \( \mathbf{S} \) is the channel matrix connecting the antenna to the spherical modes [6, 20], \( \mathbf{P} \) is the covariance of the currents [4], and \( \mathbf{R}_r = \mathbf{S}^H \mathbf{S} \) is the radiation matrix [2, 20]. Because none of the channels are penalized in this formulation, the optimal solution has equally allocated power [18]. Therefore, the optimal spectral efficiency converges to

\[
\log_2 \det(1 + \gamma \mathbf{SPS}^H) = M \log_2 \left( 1 + \frac{P_r}{MN_0} \right) \approx \frac{\gamma}{\log(2)},
\]

(2.3)

where \( M \) is the number of channels, \( \log \) is the natural logarithm, and \( \gamma = P_r/N_0 \) is the total SNR. This is the optimal unconstrained spectral efficiency. When adding any constraints or penalties to the channel or antenna we can measure how much is lost in comparison with this ideal spectral efficiency.

Additional constraints must be added to (2.2) in order to reflect realistic requirements put on the antenna design [4, 5]. Here we want to investigate how the bandwidth affects the spectral efficiency and thus formulate the problem,

\[
\begin{align*}
\text{maximize} & \quad \log_2 \det(1 + \gamma \mathbf{SPS}^H) \\
\text{subject to} \quad & \text{Tr}((\mathbf{X}_e + \mathbf{X}_m)\mathbf{P}) \leq 2 \frac{\omega W_{\text{tot}}}{P_r} \\
& \text{Tr}(\mathbf{R}_r, \mathbf{P}) = 1 \\
& \mathbf{P} \succeq 0,
\end{align*}
\]

(2.4)
where $X_e$ and $X_m$ are the stored electric and magnetic energy matrices, respectively, all equations have been normalized to the radiated power, unit radiated power is considered, $\omega$ is the angular frequency, and $W_{tot}$ is the allowed stored energy. This stored energy converges to the Q-factor for single feed, single resonance systems at resonance [10]. However, for a multi-port system, such as a MIMO antenna, this problem is a semi-definite relaxation [1] of the problem where the stored energy is limited for each port feeding the antenna. This means that the solution to (2.4) will always be an upper bound to the problem limited by the individual stored energies of the ports.

The optimization problem (2.4) is solved by reformulating it in such a way that the constraints are included into the channel matrix [5]. If this is done (2.4) can be written in the same form as (2.2), with the difference that the channels are still penalized by their stored energy. A problem of this form can be solved by taking the singular value decomposition of its channel matrix, and performing water filling to find the optimal energy allocation over those singular values [18]. The water filling optimization is formulated as,

$$ \text{maximize} \sum_{n=1}^{M} \log_2 \left( 1 + a_n \gamma \sigma_n^2 \right) $$

subject to $$ \sum_{n=1}^{M} a_n = 1 $$

where $a_n \geq 0$ is the power allocation fraction in each channel, and $\sigma_n$ is the singular value of the corresponding channel. The maximum to this problem is easily found by iteratively filling each channel until it is more beneficial to fill the next instead [18].

The singular value decomposition of the channel matrix in (2.4) can be found by formulating a dual problem which is only restricted by one condition formed from an affine combination of the first two [1, 5]. This solution method is reliant on the matrices present in (2.4) being positive semi-definite which ensures the problem to be convex. The matrices $X_e$ and $X_m$ are in general indefinite, however, it was shown in [10, 19] that they are positive semi-definite when the electrical size of the considered structure is less than half-a-wavelength. By restricting our problems to that size it allows us to use the method presented in [5] to compute the singular values of the optimal channel matrix. That method gives the singular values

$$ \sigma_n^2 = \frac{w_n(1 + \nu)}{2\omega W_{tot}} + w_n \nu $$

where $\nu$ is the scalar combining the two conditions in (2.4), and $w_n$ are the eigenvalues of a set of modes we call energy modes [10]. The energy modes are calculated through the generalized eigenvalue problem

$$ R_e I_n = w_n (X_e + X_m) I_n, $$

where $I_n$ are the mode currents. These modes are similar to characteristic modes in the sense that they have the property of orthogonal radiation patterns. However,
Figure 1: The eigenvalues $w_n$ of a spherical shell, a rectangular plate with aspect ratio 2 : 1, and several of its sub-region arrangements. All the eigenvalues have been normalized to the first energy mode of the full plate, the electrical size is $ka = 1$ with $k$ being the wavenumber and $a$ being radius of the smallest sphere circumscribing all the sources.

Instead of being resonant across the structure they minimize the total stored energy in it. This has the effect of implicitly maximizing the bandwidth of the modes. Effectively inducing these modes improves the optimal channel (2.6), therefore increasing the optimal spectral efficiency. The amplitude of this set of modes is solely dependent on the geometry of the structure, i.e., the design of the antenna.

3 Results

Antennas inside communication devices are in general much smaller than the total device size [24]. This means that only a sub-region is utilized to excite currents over the entire device that are used for communication. It is possible to solve for the optimum solution of (2.4) when controlling only a sub-region of the device by reformulating the problem in only the controlled currents of the sub-region [5]. This is interesting to investigate since bandwidth and stored energy are usually harshly punished by reducing the size of the structure. By studying the eigenvalues $w_n$ for a few sub-region cases relative to the energy modes of the entire plate we can determine
Figure 2: The optimal spectral efficiency loss in comparison to the ideal (2.3) for the $ka = 1$ spherical shell, plate with aspect ratio 2 : 1, and its sub-regions presented in Fig. 1 for different stored energy restrictions. The SNR has been set to $\gamma = 10$.

if it is possible to induce the full performance of the plate through them. In Fig. 1 we see that there is a considerable gap between the eigenvalue of the full plate compared to the sub-regions. This contrasts to the results in [5] where it was shown that, when restricting (2.4) by efficiency, it is possible to induce the entire structures available performance while only feeding a couple of small sub-regions. We can also see that the two diagonally placed elements in case B outperform the two elements in case C for all mode indexes except the second. This is similar to the case in [5] due to the first and second order modes being induced diagonally across the plate. Therefore the two diagonally situated sub-regions do not effectively induce the second mode across the opposite diagonal. The eigenvalues of the circumscribing spherical shell have been included as a reference, and we can see that they are significantly higher than those of the plate.

In Fig. 2 the optimal spectral efficiency loss, in comparison with the ideal (2.3), for the plate and its sub-regions presented in Fig. 1 is depicted as a function of the maximum allowed stored energy. This Pareto-type curve delimits the feasible region of the problem and reveals that the capacity and the stored energy are strictly conflicting parameters. Here, it is evident that even with four sub-regions placed in the corner of the plate, as in case E, we are far from achieving the optimal spectral efficiency available to the entire plate. In contrast to the case studied in [5] that was restricted by efficiency, this effect remains the same when the size of the plate is reduced. Instead of narrowing the gap between the full plate and its sub-regions, as in [5], a reduction of size makes the sub-region solution unfeasible, due to their lower bound for stored energy increasing [3]. Therefore, we can conclude that the optimal performance of the embedded MIMO antennas is more restricted by the limited bandwidth than the requirements on radiation efficiency.
Figure 3: The optimal spectral efficiency loss in comparison to the ideal (2.3) for a spherical shell, a plate with aspect ratio 2 : 1 and its sub-regions presented in Fig. 1 for different SNR. The stored energy has been chosen as $\omega W_{tot}/P_r = 50$ and electrical size as $ka = 1$.

By picking a value of $\omega W_{tot}/P_r = 50$ where all sub-region solutions are feasible we can instead study the effect of the SNR on the optimal spectral efficiency loss. In Fig. 3 we see that all cases studied converge towards the ideal optimal spectral efficiency when the SNR $\gamma$ is very small. This is due to the formulation of problem (2.4) where the radiated power is normalized. All different solutions radiate the same small amount of power at these values without any discernible difference. As SNR increases we see how the cases start to deviate. It is interesting to note that case C, from Fig. 1, outperforms case B, even though case B has a higher mode strength for all modes except the second one, indicating that the two first modes are the two most heavily utilized in this simulation. This has been verified by viewing the power allocation after optimization.

For many applications where embedded antennas are used, such as mobile phones, the antennas are not truly embedded inside the ground plane. Normally there is a substrate layer, or edge that the antennas have been designed upon [24]. In the top of Fig. 4 such a geometry, with regions raised above the ground plane, is illustrated. Here the gap between the regions and the ground plane has not been filled with any material. The placement of the regions has been done in accordance with the intuitive understanding for minimizing stored energy, i.e., maximal charge separation, see Fig. 1. In Fig. 4 the performance of case E, with 4 regions, has been considered when the placement of the regions is shifted in from the edge. Here, it is possible to see the cost of moving away from the edge position considered in the other examples. By shifting incrementally we can see that we quickly lose a significant amount of performance. This effect may be due to the fact that currents need to flow around the outer edges of the regions rather than being fed directly along the
Figure 4: The optimal spectral efficiency loss compared to the ideal case for the $ka = 1$ spherical shell, plate with aspect ratio 2 : 1, and case E, present in Fig. 1 with its regions raised $L/20$ above the ground plane, as well as when those regions are shifted in a number of mesh cells towards the center as shown above. The restricting stored energy $W_{\text{tot}}$ has been swept and the SNR has been set to $\gamma = 10$.

plate. In this case the regions are only connected to the ground plane on two edges, if all edges are connected the loss is even greater. Such a case is similar to when the regions are fully embedded within the ground plane. The raised regions can realize their solution for regions shifted more from the edge, but upon comparison of the case when the regions are placed on the edge, in Figs. 2 and 4, it is clear that the performance does not increase dramatically.

4 Conclusions

The optimal spectral efficiency bound of a MIMO antenna in an ideal channel has been considered when restricted by the stored energy. The Pareto-type bound has been illustrated in comparison to the degradation of the optimal spectral efficiency of the unrestricted problem. It has been shown that a set of modes known as energy modes serves as a useful design tool when analyzing performance of such antennas. However, due to the harsh penalties on stored energy when reducing the design region, it has been concluded that it is not possible to reach the full potential of the
plate while only feeding it with a set of small sub-regions. The placement of these regions has also been analyzed, and it could be seen that the performance quickly deteriorated when they were moved away from the edge of the structure.

While out of the scope of this letter, this method has the potential to include statistical channel models and more realistic scenarios. The modal analysis could also be carried out on designed antennas to evaluate their adherence to the principals suggested here. Generalizing this method to include several different design parameters remains an interesting future prospect.

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