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Abstract

Inverted pendulums are very well suited to investigate friction phenomena and friction compensation because the effects of friction are so clearly noticeable. This paper analyses the effect of friction on the Furuta pendulum. It is shown that friction in the arm drive may cause limit cycles. The limit cycles are well predicted by common friction models. It is also shown that the amplitudes of the limit cycles can be reduced by friction compensation. Compensators based on the Coulomb friction model and the LuGre model are discussed. Experiments performed show that reduction of the effects of friction can indeed be accomplished.

1 Introduction

A pendulum on a cart is a classical control experiment which has been used for many different purposes. The Furuta pendulum [9] is a very nice implementation, see Figure 1. A particularly nice feature is that it permits infinite travel of the cart.

Friction was studied extensively in classical mechanical engineering and there has lately been a strong resurgence. Apart from intellectual curiosity this is driven by strong engineering needs in a wide range of industries from disc drives to cars. The availability of new precise measurement techniques has been a good driving force.

Friction is very important for the control engineer, for example in design of drive systems, high-precision servo mechanisms, robots, pneumatic and hydraulic systems and anti-lock brakes for cars. Friction is highly non-linear and may result in steady state errors, limit cycles, and poor performance. It is therefore important for control engineers to understand friction phenomena
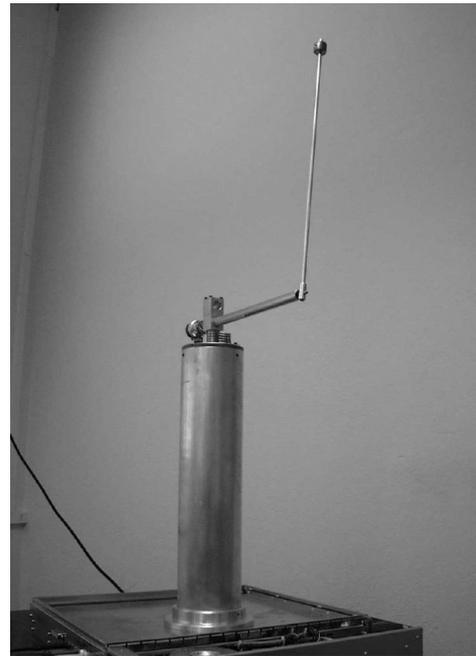
Figure 1: The Furuta pendulum.

and to know how to deal with them. With the computational power available today it is in many cases possible to deal effectively with friction. This has potential to improve quality, economy, and safety of a system. In this paper we are using the pendulum to illustrate the effects of friction and friction compensation. The effects of friction are clearly seen because friction causes limit cycles. Effects of friction compensation can be illustrated very nicely because the amplitude decreases significantly when friction compensation is applied.

Mathematical descriptions of the Furuta pendulum and the friction models are given in Section 2. A stabilizing controller with friction compensation in presented in the same section. The effects of friction on the Furuta pendulum is discussed in Section 3. In Section 4 the
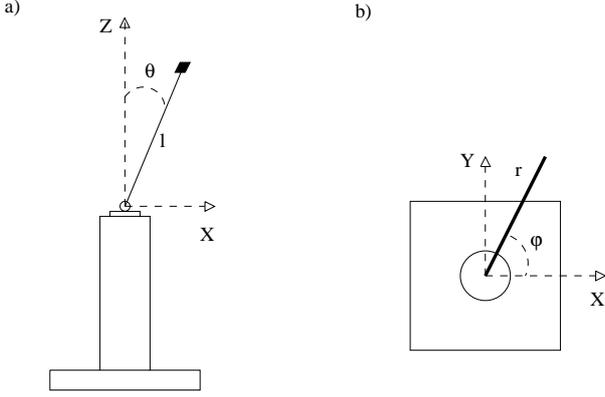
Figure 2: Schematic picture of the Furuta pendulum.

pendulum model is validated with experiments. The controller with friction compensation is implemented and evaluated on the real pendulum.

2 Mathematical Models

In this Section we will present the basic mathematical models of the system.

The Furuta Pendulum

A schematic picture of the Furuta pendulum is shown in Figure 2. The pendulum consist of a motor driven vertical axis with an arm. This replaces the cart in the conventional pendulum. The pendulum is mounted at the tip of the arm. The orientation of the arm is represented by the angle φ, and the orientation of the pendulum by θ. The angle θ is defined to be zero when the pendulum is upright. The equations of motion of the system can be written as.

$$
\begin{aligned}
& \left(\alpha + \beta \sin^2 \theta\right) \frac{d^2\varphi}{dt^2} + \gamma \cos\theta \frac{d^2\theta}{dt^2} \\
& + 2\beta \cos\theta \sin\theta \frac{d\varphi}{dt} - \gamma \sin\theta \frac{d\theta}{dt}^2 = u - F \\
& \gamma \cos\theta \frac{d^2\varphi}{dt^2} + \beta \frac{d^2\theta}{dt^2} - \beta \cos\theta \sin\theta \frac{d\varphi}{dt}^2 \\
& \hspace{4cm} - \delta \sin\theta = 0
\end{aligned}
\tag{1}
$$

where α, β and γ are inertial coefficients, and δ a gravitational coefficient. The pendulum is driven by the torque input u on the horizontal arm. The friction torque on the arm joint is F. The friction on the pendulum joint is assumed to be zero. Introduce the state vector $x = (\varphi, \frac{d\varphi}{dt}, \theta, \frac{d\theta}{dt})^T$ and linearize (1) around the

unstable equilibrium $x_0 = (0, 0, 0, 0)^T$. This gives

$$
\frac{dx}{dt} = Ax + B\left(u - F\right)
\tag{2}
$$

with

$$
A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_{43} & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ b_{21} \\ 0 \\ b_{41} \end{pmatrix}, \tag{3}
$$

where

$$
a_{23} = -\frac{\delta\gamma}{\alpha\beta - \gamma^2} \qquad a_{43} = \frac{\alpha\delta}{\alpha\beta - \gamma^2}
$$
$$
b_{21} = \frac{\beta}{\alpha\beta - \gamma^2} \qquad b_{41} = -\frac{\gamma}{\alpha\beta - \gamma^2}.
$$

The poles of the system are $\{0, 0, \pm\sqrt{a_{43}}\}$. Most of the simulations in this paper are based on the linearized model although the nonlinear model is used for some simulations.

2.1 Friction

Many models have been proposed to describe friction, see [2], [13]. Coulomb friction [8]

$$
F = F_c \, \mathrm{sgn}(\frac{d\varphi}{dt}),
\tag{4}
$$

is a very simple model. This model does not describe what happens when the velocity is zero. A popular way to deal with this is to introduce stiction. It is then assumed that if the velocity is zero and the total force acting on the system is less than the stiction force F_s then the the velocity will remain zero. A motion will occur when the applied force is larger than the stiction.

In this paper we will use the LuGre model [6], [13]. This can be viewed as an attempt to regularize the model for Coulomb model with stiction. The model also captures several other friction characteristics, such as increased friction torque at lower velocities, see e.g. [12]. The LuGre model is described by

$$
\begin{aligned}
\frac{dz}{dt} &= v - \sigma_0 \frac{|v|}{g(v)} z \\
g(v) &= \alpha_0 + \alpha_1 e^{-(v/v_s)^2} \\
F &= \sigma_0 z + \sigma_1 \frac{dz}{dt}
\end{aligned}
\tag{5}
$$

2.2 The Control Law

With the particular experimental setup it was possible to extract velocity signals from the angular measurements. Stabilization could then be accomplished with the simple control law

$$
u = -Lx.
\tag{6}
$$
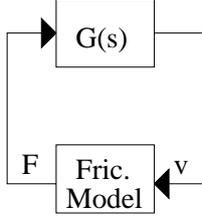
Figure 3: Representation of the inverted pendulum with friction.

The matrix L was determined to give the following characteristic equation of the closed loop system

$$(s^2 + 2\zeta_1\omega_1 s + \omega_1^2)(s^2 + 2\zeta_2\omega_2 s + \omega_2^2) = 0 \quad (7)$$

2.3 Friction Compensation

The idea of friction compensation is simple. Consider the model given by Equation 1. Determine an estimate $\hat{F}$ of the friction force F and choose the control signal

$$u = -Lx - \hat{F} \quad (8)$$

For the Coulomb friction model the estimate is simply obtained from (4) and the measured velocity. The situation is a little more complicated for the LuGre model because the state z in the model is not directly measurable. Friction compensation therefore requires and observer. Different observers have been suggested in [6, 11, 12]. Passivity theory has been used to analyze the convergence of the estimated states, see [4, 10]. In this paper we will use the following simple friction observer.

$$\frac{d\hat{z}}{dt} = v - \sigma_0 \frac{|v|}{g(v)}\hat{z}$$
$$g(v) = \alpha_0 + \alpha_1 e^{-(v/v_s)^2} \quad (9)$$
$$\hat{F} = \sigma_0\hat{z} + \sigma_1\frac{d\hat{z}}{dt}$$

The estimate $\hat{z}$ will converge to z when there are no modeling errors. The observer depends critically on high quality measurement data.

3 Effects of Friction

There are many techniques that can be used to explain the effects of friction qualitatively. To start with we observe that the system can be represented as in Figure 3. To obtain this representation the pendulum dynamics is represented by linearized models. The only nonlinearity is represented by friction. The transfer function $G(s)$ is given by

$$G(s) = \frac{s(b_0 s^2 + b_2)}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4} \quad (10)$$
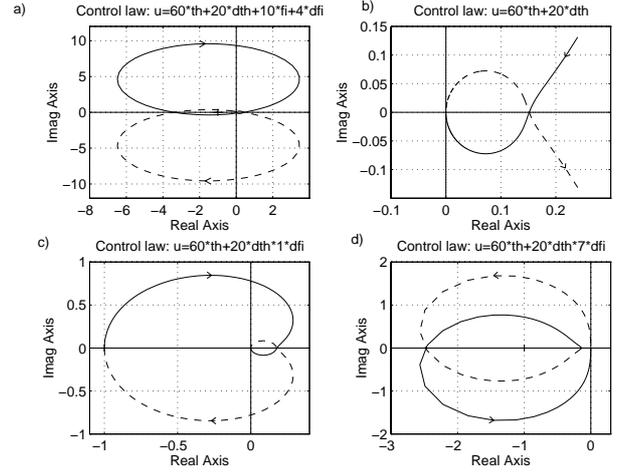
Figure 4: Nyquist plots for different feedback. Figure a) shows when the feedback $u = 60*\theta+20*\dot{\theta}+10*\varphi+4*\dot{\varphi}$ is used. In Figure b) is the feedback $u = 60*\theta+20*\dot{\theta}$ and in Figure c) and d) $u = 60*\theta+20*\dot{\theta}+\dot{\varphi}$ respectively $u = 60*\theta + 20*\dot{\theta} + 7*\dot{\varphi}$.

with $b_0 = \beta$, $b_2 = -\delta$, $a_0 = \zeta$, $a_1 = (\gamma l_4 - \beta l_2)$, $a_2 = (\gamma l_3 - \beta l_1 - \delta\alpha)$, $a_3 = \delta l_2$ and $a_4 = \delta l_1$. The properties of $G(s)$ depends on the properties of the feedback. Nyquist plots for the transfer function $G(s)$ for a few different cases are is shown in Figure 4. Some aspects of the system can be understood qualitatively using the describing functions, [3]. The describing function for the nonlinearity represented by Coulomb friction is simple the negative real axis. Describing function theory thus predicts that limit cycles may occur for the cases shown in a), c) and d). A detailed study of the parameter values that may lead to limit cycle oscillations is given in [15].

4 Experiments

The pendulum is shown in Figure 1. The horizontal arm of the pendulum is driven by a DC motor with an amplifier. The drive voltage is between ± 10 V. The arm angle φ is measured with a decoder which has a resolution of 250 pulses per revolution. The angular velocity of the arm $\frac{d\varphi}{dt}$ is measured with a tachometer. The pendulum angle θ is measured with a potentiometer. The resolution is $3.8 \cdot 10^{-4}$ [rad] in the range $-\pi/4$ [rad] to $\pi/4$ [rad]. A low-pass filter is used to reduce the measurement noise. The angular velocity $\frac{d\theta}{dt}$ is accessible because it is an internal state in the filter, see [15].

The coefficients of the linearized pendulum dynamics (3) are identified by means of least-squares estimation to be $a_{23} = -11$, $a_{43} = 33$, $b_{21} = 45$ and $b_{22} = -29$. The identification procedure also gives a measure of the

Coulomb friction $F_c = 0.21$, normalized to the control signal. The friction is significantly large.

Several different pendulums and computer systems have been used in the experiments, see [1], [10] and [15]. All control laws have been implemented on a PC. The latest experiments used the software RealLink/32, which is an extension of Simulink and Matlab, see [14]. It makes it possible to implement the algorithms as Simulink blocks. They are translated to C-code using the Real Time Workshop, and a Windows NT executable is built. Reallink/32 provides a small real-time kernel that is used to run the executable code. A standard Pentium PC with a 12 bit AD/DA-converter board is booted with the real-time kernel. The control program is loaded. Experimental data can be collected and analyzed in Matlab.

Control design is made for the linearized continuous model. The feedback gain L in (8) is chosen such that $\omega_1 = 7$, $\omega_2 = 5$ and $\zeta_1 = \zeta_2 = 0.7$ in (7). The control law is implemented in sampled digital form with fast sampling. A sampling rate of 1 kHz was used in the experiments.

4.1 Model Validation

The nonlinear model of the pendulum with friction has been validated against experiments. Open loop experiments were first performed. The system was initialized in the unstable equilibrium when the pendulum is upright. When released there is a highly irregular motion. Experiments were then performed with the stabilizing controller (6).

Figure 5 shows a typical result of such an experiment. There is a reasonable agreement between simulation and experiments. The limit cycle caused by friction is clearly visible in the figure. Notice in particular that there is stiction both in the experiments and in the simulation. The time the arm is stuck is a little larger in the simulation. The similarity between simulation and experiment can be improved by adjusting the parameters of the friction model.

4.2 Friction Compensation

A number of experiments with friction compensation have been performed. The parameters of the friction models varies with time. Therefore we will present a sequence of curves that are taken in sequence. Figure 6 shows results of an experiment with the linear control law (8) without friction compensation. A limit cycle caused by the friction is clearly visible. Notice that the arm gets stuck when the velocity changes sign.
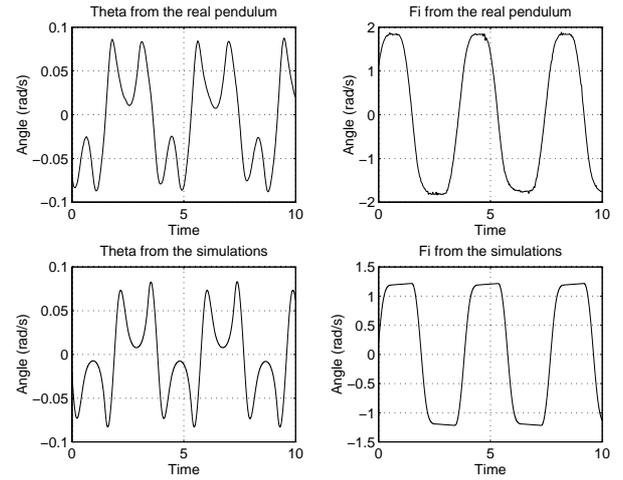
Figure 5: Comparison of experiments with stabilization of the pendulum with simulations.
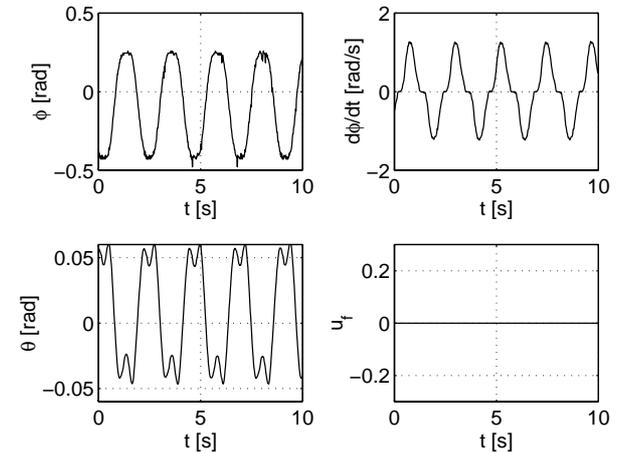
Figure 6: Experiments of stabilization of the pendulum without friction compensation

The oscillation is asymmetry because of the asymmetric friction characteristics. Such an asymmetry is quite common in motors, see [5].

The limit cycle can be reduced with friction compensation. Figure 7 shows the results with a compensator based on Coulomb friction with $\hat{F}_c = 0.21$. It is reduced even further when the friction compensator is based on the LuGre model. This is illustrated in Figure 8. The parameters used in this experiment are $\alpha_0 = 0.21$, $\alpha_1 = 0.022$, $\sigma_0 = 80$, $\sigma_1 = 1.5$ and $K = 0.01$. The parameters of the friction model were found by manually tuning the compensator. With careful tuning of the parameters it is possible to obtain even better results as shown in Figure 9. In this case the limit cycle is reduced to the sensor resolution. It is of course not practical to tune parameters manually, an alternative
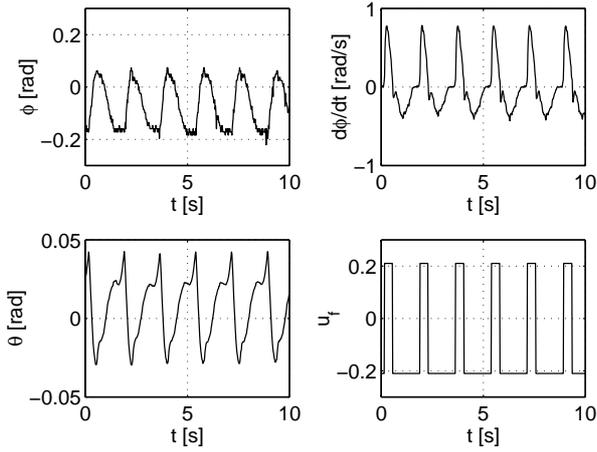
Figure 7: Experiments of stabilization of the pendulum with friction compensation based on Coulomb's model.
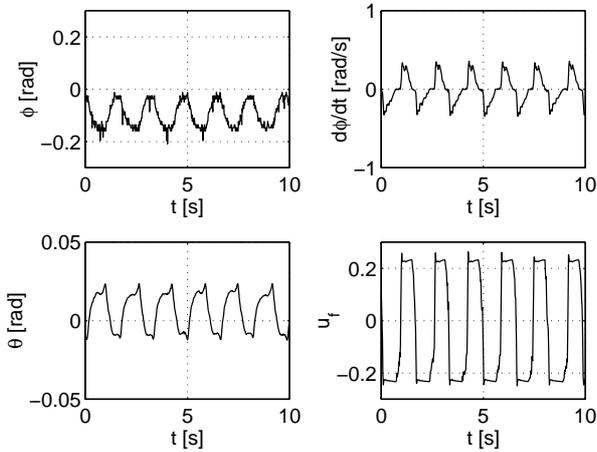
Figure 8: Experiments of stabilization of the pendulum with friction compensation based on the LuGre model.

is to use adaptive friction compensation as discussed in [7] and [11].

5   Conclusions

The effects on friction on the Furuta pendulum has been investigated. The limit cycles obtained is predicted by several friction models. Good agreement with experiments is obtained with models that capture stiction. Different friction compensators were also investigated. The pendulum is an excellent device for demonstrating friction compensation because the changes in the amplitudes of the limit cycle are so clearly visible. In the paper we show results for compensators based on the Coulomb model and the LuGre model.
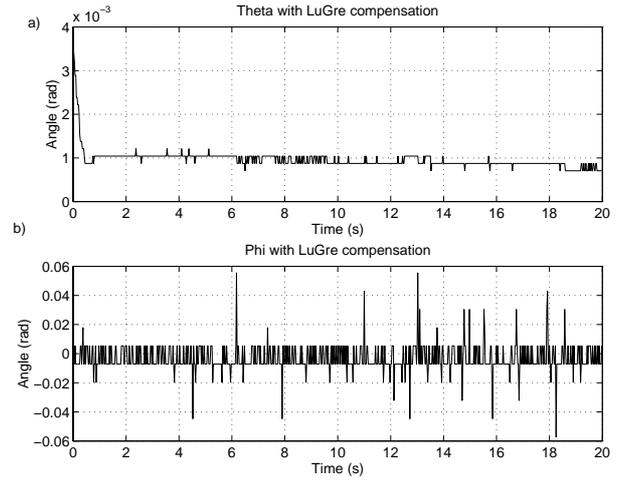
Figure 9: Experiment with friction compensation based on the LuGre model with very careful tuning of the parameters.

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