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Published in:
[Host publication title missing]

DOI:
10.1109/ISIT.1995.535803

1995

Link to publication

Citation for published version (APA):
Bratt, G., Johannesson, R., & Zigangirov, K. (1995). Decoding procedure capacities for the Gilbert-Elliott channel. In *[Host publication title missing]* (pp. 288) https://doi.org/10.1109/ISIT.1995.535803

Total number of authors:
3

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Decoding Procedure Capacities for the Gilbert-Elliott Channel [1]

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Abstract — **Sequential decoding for the Gilbert-Elliott channel is considered. The decoding procedure capacity C_D is defined to be the supremum of the rates for which there exists a code that gives arbitrarily small decoding error probability. For different assumptions of the decoder's knowledge of the channel states expressions for C_D are derived.**

I. Introcuction

Assume that a tree code is used together with sequential decoding to communicate over the Gilbert-Elliott channel. Let $P(\mathcal{E})$ denote the average probability of decoding error over the ensemble of random, infinite depth tree codes. In this paper we address the question: "When will $P(\mathcal{E}) \to 0$?".

Consider the Gilbert-Elliott channel model and denote the error probabilities in the Good and Bad states by e_G and e_B, respectively. Furthermore, let P_G and P_B denote the fraction of time spent in the Good and Bad states, respectively.

II. Decoding procedure capacity

Let us define the *decoding procedure assumptions*, D. The optimistic assumption, $D = o$, assumes that the decoder has a complete knowledge of the channel state, which could be given by a genie. The pessimistic assumption, $D = p$, assumes that the decoder neither is given any channel state information nor tries to make any estimate of it. Given the decoding procedure assumption D and the use of the Gilbert-Elliott channel, let C_D denote the supremum of the rates for which we can guarantee that there exists a code that gives an arbitrarily small decoding error probability $P(\mathcal{E})$. We will call C_D the *decoding procedure capacity.*

We have proved that the decoding procedure capacities are given by

$$C_o = P_G \cdot C_{BSC}(e_G) + P_B \cdot C_{BSC}(e_B)$$

and

$$\begin{aligned} C_p &= P_G \cdot (C_{BSC}(e_G) - h(b)) + P_B \cdot (C_{BSC}(e_B) - h(g)) \\ &= C_o - (P_G \cdot h(b) + P_B \cdot h(g)), \end{aligned}$$

where b and g denote the transition probabilities from Good to Bad and from Bad to Good, respectively, in the channel model.

Theorem 1 *Given the Gilbert-Elliott channel and the decoding procedure assumptions, the use of a rate R random, infinite depth tree code with the stack decoder, then for any rate $R < C_D$ and $\eta \in \mathbb{Z}^+$,*

$$P(N \geq \eta) \to 0 \quad \text{if } \eta \to \infty,$$

where N is the number of computations in an incorrect *subtree.*

When we wish to transmit over an ordinary Discrete Memoryless Channel at rates (above R_{comp} and) close to its capacity, it is sufficient to allow the number of computations of sequential decoding to go to infinity to be able to guarantee that $P(\mathcal{E})$ can be chosen arbitrarily small. We will show that this is also sufficient for transmission close to rates C_D, which is the motivation why we call these rates "decoding procedure *capacities*".

Theorem 2 *Given the assumptions of Theorem 1, then for any rate $R < C_D$ the average probability of decoding error*

$$P(\mathcal{E}) \to 0,$$

if the number of computations, N, is allowed to go to ∞.

Since the important condition in Theorem 2 is that $R < C_D$, it is clear that the theorem's statement, *given* the decoding procedure assumptions, is equivalent to stating that the maximal transmission rate over the Gilbert-Elliott channel is at least the rate C_D.

In the pessimistic case we can interpret this as follows. For arbitrarily small $P(\mathcal{E})$, there exists a code such that the transmission rate will be (at least) C_p, *even without* any knowledge of the channel state or any attempt to estimate it.

III. Channel capacity

A common method to lowerbound C_{GE} is to calculate $C_{BSC}(\bar{e})$, where $\bar{e} = P_G \cdot e_G + P_B \cdot e_B$, but it turns out that C_p is a better lower bound for channels with a stable behaviour. The optimistic case helps us to find a stronger result:

Theorem 3 *Given that the receiver has a complete channel state knowledge, then the channel capacity for the Gilbert-Elliott channel C_{GE}^R is equal to*

$$C_{GE}^R = C_o.$$

From the proof of Theorem 3 follows immediately

Corollary 4 *Given that* both *transmitter and receiver have complete knowledge of the channel state* sequence *then for the channel capacity of the Gilbert-Elliott channel C_{GE}^{TR} we have*

$$C_{GE}^{TR} = C_{GE}^R.$$

It should be noted that the capacities C_{GE}^{TR} and C_{GE}^R, in contradiction to what is the case for C_D, are parameters purely dependent of the channel's properties and that nothing is assumed about the decoding method. In the derivations of C_D we assume sequential decoding, but by deriving them we show that they are achievable rates as such, given the decoding procedure assumptions.

References

[1] Gunilla Bratt: "Sequential Decoding for the Gilbert-Elliott Channel — Strategy and Analysis". Ph.D. Thesis. Dept. of Information Theory, Lund University, Lund, Sweden, 1994.

[1]This research was supported in part by the Royal Swedish Academy of Sciences in liaison with the Russian Academy of Sciences, and in part by the Swedish Research Council for Engineering Sciences under Grant 91–91.