Axial asymmetry and the spectrum of 16O

Sheline, Raymond K; Nilsson, Sven Gösta; Ragnarsson, Ingemar; Larsson, S E; Leander, G

Published in:
NORDITA publications

1973

NORDITA *publications*

No. 526

S. E. Larsson, G. Leander, S. G. Nilsson, I. Ragnarsson and R. K. Sheline

Axial Asymmetry and the Spectrum of ^{16}O

Physics Letters 47B (1973) 422–426

NORDITA
Nordisk Institut for Teoretisk Atomfysik
Blegdamsvej 17
DK-2100 København Ø, Danmark

AXIAL ASYMMETRY AND THE SPECTRUM OF ^{16}O

S.E. LARSSON, G. LEANDER, S.G. NILSSON
Department of Mathematical Physics, Lund Institute of Technology, Lund, Sweden

I. RAGNARSSON
NORDITA, Copenhagen, Denmark

and

R.K. SHELINE*
Florida State University, Tallahassee, Florida, USA

Received 1 November 1973

The triaxial harmonic oscillator is found to be associated with a shell gap for nucleon number 8 corresponding to oscillator frequency ratios 3 : 2 : 1 and a deformation $\gamma = 30°$, $\epsilon = 0.87$. The experimental positive-parity spectrum of ^{16}O is interpreted in terms of an anharmonic ground state vibrational spectrum on which is superimposed 3 : 2 : 1 triaxial bands. The moment of inertia of the latter are found to agree well with expectations. There is also empirical evidence for an extremely deformed band at about 17 MeV, that might be associated with a 4 : 1 shell structure.

The spectrum of ^{16}O is one of the most studied ones experimentally and theoretically. A band based on the 6.05 MeV 0^+ excitation was early identified as a "coexistence" band by Morinaga [1], Bohr and Mottelson [2] and Brown [3]. In the latter reference the state is described in terms of four-particle-four-hole excitations with reference to a spherical "vacuum" with the important inclusion of core polarisation. Actually the four-particle-four-hole description of the second 0^+ state also exists in an even earlier version, without polarisation included, by Christy and Fowler [4]. The polarisation effect was first introduced qualitatively by Moringa. Important contributions to the development in the understanding of ^{16}O and neighbouring nuclei were made by Ellis and England [5] (cf. ref. contained therein).

The four-particle-four-hole excitation, being the basic component of the 6.05 MeV 0^+ state, involves in the oscillator description the promotion of four particles from the $N = 1$ into the $N = 2$ shell. As both the shell $N = 1$ and $N = 2$ are now half-full, there is a strong tendency to shape distortions away from spherical symmetry. Often, such n-particle-n-hole excitations can also be equivalently described as minima in the potential-energy surface in deformation space. Thus, as a function of spheriodal distortions ϵ, the orbital $[n_z = 2, n_\perp = 0]$ which is two-fold degenerate, comes below $[n_z = 0, n_\perp = 1]$, the latter four-fold degenerate, for large enough ϵ. Furthermore, there is a tendency for minima in the potential-energy surface to be established for those shapes where the nucleon number in question happens to correspond to a closed shell.

Geilikman [6] was the first to point out that, based on the harmonic oscillator, one expects to encounter shell structure for other shapes than that of the pure sphere. For the spheroidal case, a harmonic oscillator shell structure is associated e.g. with $\omega_\perp : \omega_3 = 2:1, 3:1$ etc. In the more general ellipsoidal case, as pointed out by Swiatecki, Tsang and Nix [7] and by Bohr and Mottelson [8], the condition on the energy eigenvalues for shell structure can be written

$$\frac{\partial e}{\partial n_x} = \frac{\partial e}{\partial n_y} = \frac{\partial e}{\partial n_z} = \omega_1 : \omega_2 : \omega_3 = i : j : k$$

where i, j and k are integers and n_x, n_y and n_z are the Cartesian nodal quantum numbers.

The fission shape isomers seem to be associated with a shell structure characteristic of a spheriodal shape [6–8] with $\omega_\perp : \omega_3 = 2:1$. Other nuclei in the medium mass range have recently [9] been shown to exhibit rotational bands of a character which

* Supported in part by the U.S. Atomic Energy Commission.

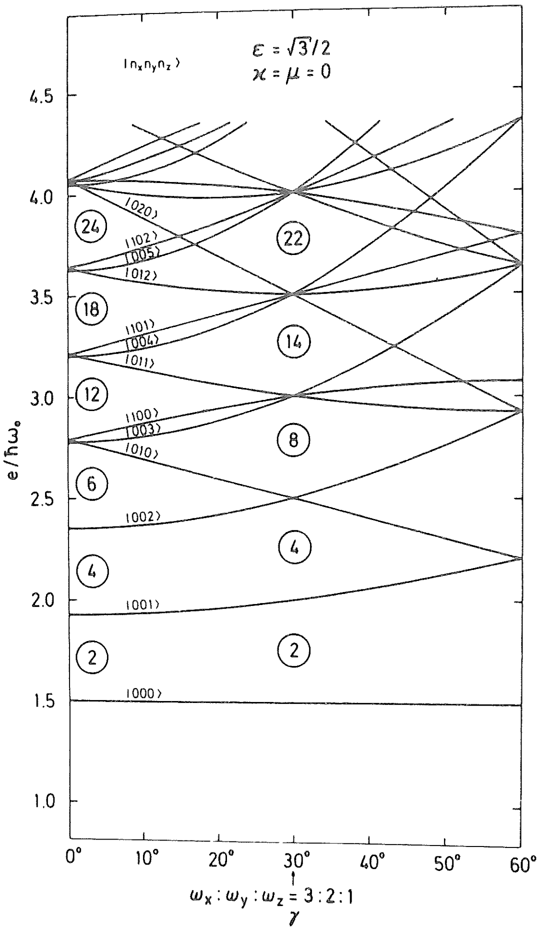

Fig. 1. Single-particle levels of the harmonic oscillator, neglecting spin-orbit coupling, as a function of the gamma coordinate for $\epsilon = \sqrt{3}/2$. Note in particular the shell at nucleon number 8 at $\omega_x : \omega_y : \omega_z = 3:2:1$. The shell structure to the left in the figure approximately corresponds to the frequency ratios $\omega_x : \omega_y : \omega_z = 3:3:1$. The levels are labeled by their Cartesian quantum numbers.

might be explained as reflecting shell energy minima corresponding to 2:1 spheroidal shape. However, due to the large liquid-drop stiffness in medium-energy nuclei, the 2:1 shape ratios are only approximately approached in this intermediate mass region. For lighter nuclei, however, the surface energy is of less significance relative to the shell energy. In ^{16}O we find a shell structure effect large enough to explain the occurence of a shape isomer, provided we also allow distortion away from axial symmetry. Thus, for $\gamma = 30°$ we obtain, as shown in fig. 1, the single-particle

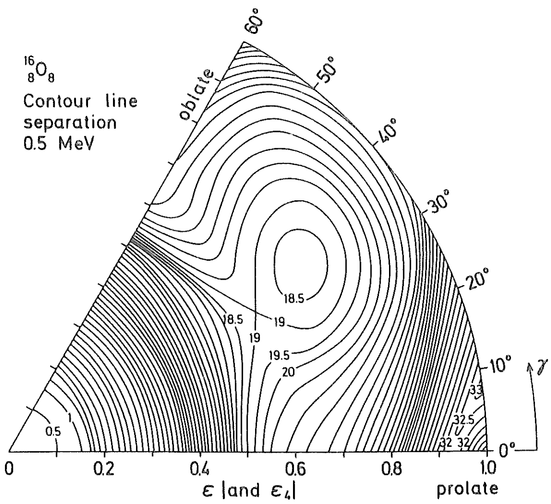

Fig. 2. The potential-energy surface for ^{16}O in terms of ϵ (elongation), ϵ_4 (necking-in) and γ (axial asymmetry) obtained by the Strutinsky normalisation method. Note the triaxial minimum obtained at $\gamma \approx 30°$.

diagram corresponding to the pure oscillator as function of the asymmetry coordinate γ for $\epsilon = \sqrt{3}/2$. The relation of ω_1, ω_2 and ω_3 to ϵ and γ is given as [10] (cf. ref. [8])

$$\omega_k = \omega_0 \cdot [1 - \tfrac{2}{3}\epsilon \cdot \cos(\gamma + k\,\tfrac{2}{3}\pi)] , \quad k = 1, 2, 3 .$$

In fig. 1 a large gap is found corresponding to neutron or proton number 8 for $\gamma = 30°$ and $\epsilon = \sqrt{3}/2$ with the simple ω-ratios $\omega_1 : \omega_2 : \omega_3 = 3:2:1$.

At the large distortions in question, the liquid-drop energies entering through the Strutinsky shell correction method employed, are of considerable significance in teh determination of the equilibrium nuclear shape. Thus, it is essential also to include the deformation coordinate ϵ_4, and for large deformations also ϵ_6, together with ϵ and γ. A method to include all these degrees of freedom simultaneously has recently been developed in ref. [11] for the calculation of the Coulomb energy.

In the calculations we have also employed a spin-orbit strength defined by $\kappa = 0.08$ and $\mu = 0$, as usually assumed in this region of nuclei.

The resulting total-energy surface is given in fig. 2. It is shown there that the liquid-drop terms push the minumum somewhat away from the point $\epsilon = \sqrt{3}/2$, $\gamma = 30°$, towards a somewhat smaller ϵ-value.

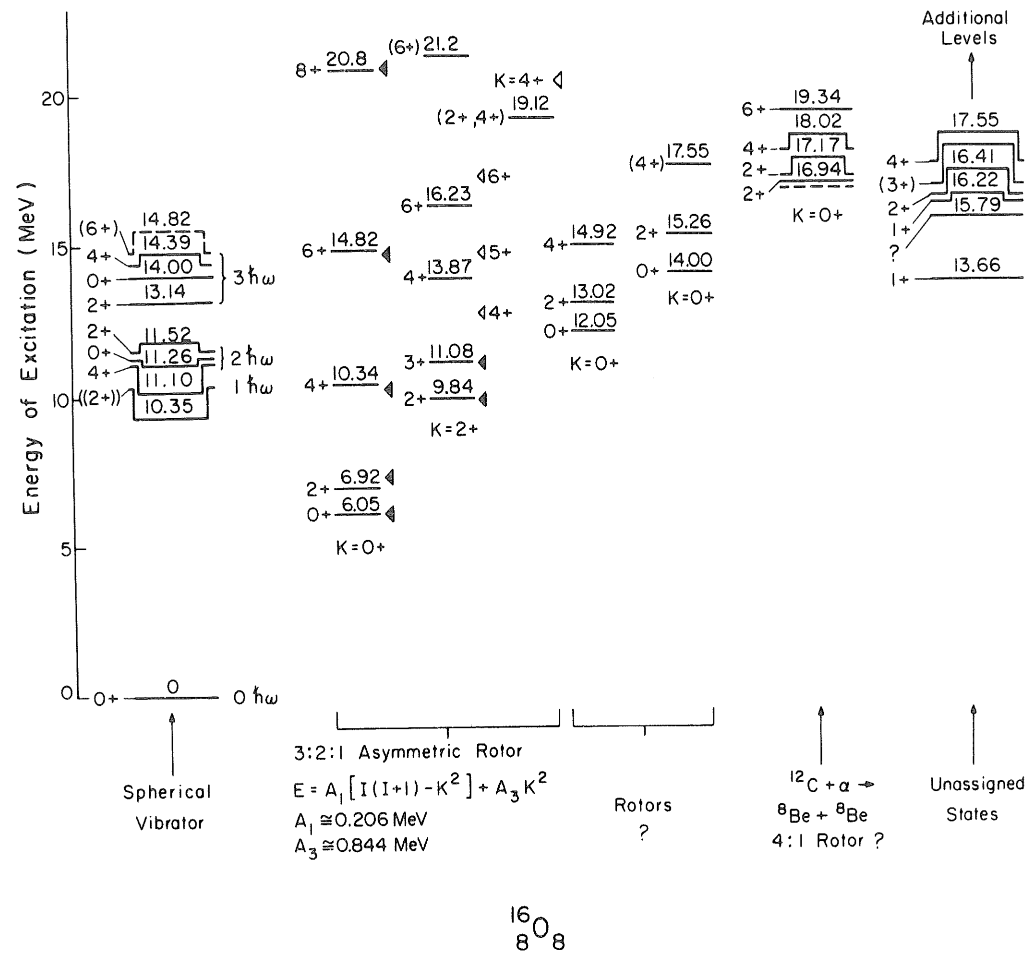

Fig. 3. Low-lying positive-parity states of ^{16}O experimentally observed, group into systematics rotational and vibrational states as described in the text. In particular note the three 3 : 2 : 1 asymmetric rotor-bands.

The wave function for the coexistent 6 MeV band, obtained by Brown and coworkers, is largely $[n_z = 0, n_\perp = 1]^{-4} \cdot [n_z = 2, n_\perp = 0]^4$. A still stronger alignment of the particles can be achieved by aligning the holes along the x-axis $[n_x = 1, n_y = 0, n_z = 0]^{-4}$. In this way the total nuclear wave function can be written $[0,0,0]^4 \cdot [0,1,0]^4 \cdot [0,0,1]^4 \cdot [0,0,2]^4$ in an $[n_x, n_y, n_z]$-representation. Such an axially asymmetric description has earlier been considered by Hayward [12] and by Stephenson and Banerjee [13].

These base states are futhermore assumed to be polarized by having different oscillator frequencies $\omega_x \neq \omega_y \neq \omega_z$.

Actually the calculated four-particle-four-hole minimum of ref. [12] appears to correspond to $\epsilon \approx 0.88, \gamma \approx 20°$ in our parametrisation. Also the analysis of the electromagnetic excitation data of the 6.92 MeV state leads the authors of ref. [14] to conclude for these states a large non-axial deformation with $\gamma = 25°$.

In fig. 3 all of the low-lying positive-parity levels experimentally observed [15] for ^{16}O have been plotted. The most dominant shell structure for the double magic ^{16}O involves the doubly-closed spherical shell with 8 nucleons. The corresponding minimum can be associated with the ground state. We have

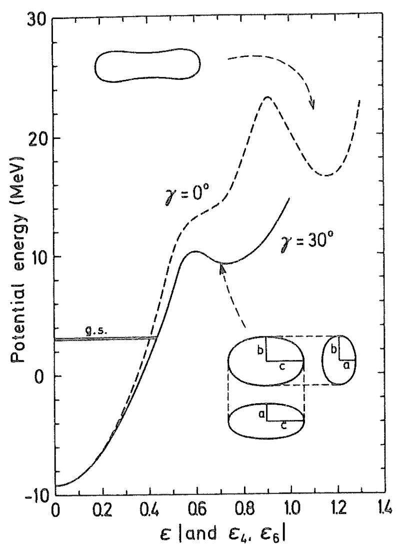

Fig. 4. The potential energy plotted along the two paths, one with $\gamma = 30°$ (full line) and the other with $\gamma = 0°$ (dashed line) as a function of ϵ for the nucleus ^{16}O. In addition the energy has been minimized with respect to ϵ_4 and ϵ_6. Note the shallow second minimum at frequency ratios 3 : 2 : 1, and the deep minimum at 4 : 4 : 1. The nuclear shapes corresponding to the potential-energy minima are also plotted. The excitation energy of the ground state above the spherical minimum has been calculated as the experimental mass correction relative to the liquid-drop value.

furthermore tentatively assigned 1, 2 and 3 phonon vibrations built on the ground state.

The spacing of these phonon states is grossly consistent with the very anharmonic character of the calculated potential-energy surface (see fig. 2 and 4). We furthermore posit that the triaxail band beginning at 6.05 MeV is directly connected with the 3 : 2 : 1 shape minimum seen in fig. 2.

To explore this possibility the triaxial band has been fitted neglecting the coupling to the phonons. In the fit we have assumed for the energy moments that $A_1 = A_2$ but A_3 finite ($A_k = K^2/2\mathcal{J}_k$, $k = 1, 2, 3$). Black triangles show the fitted levels, and open triangles levels which were not fitted. It is probable that the deviation from experiment of the fitted 2^+ level at 6.92 MeV and the 4^+ unfitted level at 13.87 MeV may be understood in terms of a neglected interaction with the level of assumed 2^+ character at 10.35 MeV and the 4^+ level at 11.10 MeV, respectively. The rotor level fits correspond to moments of inertia $\mathcal{J}_1$ and $\mathcal{J}_3$ which are 1.10 and 0.62 of the respective $\mathcal{J}_{\text{rigid}}$ for rotation around the 1- and 3-axes, respectively. In these rigid-inertia estimates also effects of ϵ_4 are included.

A few comments should be made on these inertias $\mathcal{J}_1$ and $\mathcal{J}_3$ obtained in the fit. With the axis ratios being $\omega_x^{-1} : \omega_y^{-1} : \omega_z^{-1} = 2 : 3 : 6$, the inertias around the 1- and 2-axis are expected to be very similar. Furthermore, in view of the large deformation we expect an inertia very near to the rigid one independent of any correlation, which latter in this region of nuclei is anyway rather weak. For the inertia around the 3-axis correlations are expected to play a larger role, as for $\gamma = 0°$ the inertia vanishes entirely, prohibiting rotations around this axis in the axially symmetric case.

The position of the $K = 2^+$ band, beginning at 9.84 MeV, is of particular interest, since it is sensitive to the magnitude of the triaxial deformation (the difference between A_1 and A_3 gives a measure of the separation between the $K = 0$ and the $K = 2$ band).

Sets of states that may be interpreted as two additional rotational bands with $K = 0^+$ are observed at 12.05 and 14.00 MeV. These are left without further assignments. Of particular interest is an extremely deformed rotational band assumed to have $K = 0^+$. The band head has not yet been definitely observed, although its extrapolated value is 16.76 MeV. However, 2^+, 4^+ and 6^+ members are observed with unusual clarity utilizing the reaction $^{12}C + \alpha \rightarrow {}^8Be + {}^8Be$ [16]. This band finds a ready approximate interpretation in terms of an axially asymmetric 4 : 1 prolate rotor, for which spheroid shape the nucleon number 8 is again magic.

In fact the inclusion of ϵ_4 and ϵ_6 in the calculations for $\gamma = 0$ deeppens the minimum and displaces it to $\epsilon = 1.15$, $\epsilon_4 = 0.22$ and $\epsilon_6 = -0.04$, which incidentally happens to correspond almost exactly to an axis ratio of 4 : 1 (see fig. 4). The corresponding shape is thus somewhat removed from that of a pure 4 : 1 spheroid and the rigid inertia is increased by about 10% relative to that of the spheroid. In fact the predicted value of $K^2/2\mathcal{J}$ comes out to be 0.10 MeV compared to the average empirical value for the band of 0.08 MeV. This is in turn in excess of the four-alphas-in-a-string limit considered by the authors of ref. [15], and of ref. [1] for the 6 MeV band.

In conclusion we add a few comments on the energy differences between the three main configuration states involved. A fit of the mass of ^{16}O based on the liquid-drop parameters of Meyers and Swiatecki [17] requires a zeropoint energy of 12 MeV, which is of the same order as expected from a harmonic vibration in the spherical well [18]. The energy of the second and third minima, as seen from fig. 4, are then in fairly good agreement with the excitation energy of the experimental levels asigned to them, especially in view of the large uncertainties of the liquid-drop background. (The zero-point energies of the deformed states are expected to be negligibly small compared to the spherical vibrational energy.)

The interpretation of the rotational bands at 6 and 17 MeV in ^{16}O is not new. The potential-energy surface picture mainly supports and sheds new light on an already accepted coexistence interpretation. Its main advantage is that it brings together the 6 MeV and 17 MeV bands into a unified picture.

We are grateful to Professors B. Mottelson, G. Brown and H. Morinaga for kind suggestions and comments and to Dr. P. Möller for his help with some of the computations.

References

[1] H. Morinaga, Phys. Rev. 101 (1956) 254.
[2] A. Bohr and B.R. Mottelson, 1960, private communication.
[3] G.E. Brown and A.M. Green, Nucl. Phys. 75 (1966) 401.
[4] R.F. Christy and W.A. Fowler, Phys. Rev. 96 (1954) 851.
[5] P.S. Ellis and T. Engeland, Nucl. Phys. A144 (1970) 161.
[6] B.T. Geilikman, Yad. Fiz. 9 (1969) 894; Sov. J. Nucl. Phys. 9 (1969) 521.
[7] Considerations of the group of Swiatecki, Tsang and Nix, see e.g. C.F. Tsang, Ph. D. thesis, Lawrence Radiation Laboratory Report, UCRL-18899 (1969).
[8] A. Bohr and B.R. Mottelson, Nucl. Structure, vol. 2 (W.A. Benjamin, Inc., New York), to be published.
[9] R.K. Sheline, I. Ragnarsson and S.G. Nilsson, Phys. Lett. 41B (1972) 115.
[10] S.E. Larsson, I. Ragnarsson and S.G. Nilsson, Phys. Lett. 38B (1972) 269.
[11] S.E. Larsson and G. Leander, Proc. Third IAEA Symp. on Phys. and Chem. of Fission, Rochester, 1973, to be published.
[12] J. Hayward, Nucl. Phys. 81 (1966) 193.
[13] G.J. Stephenson and M.K. Banerjee, Phys. Lett. 24B (1967) 209.
[14] J.C. Bergstrom et al., Phys. Rev. Lett. 24 (1970) 152.
[15] F. Ajzenberg-Selove, Nucl. Phys. A166 (1971) 1.
[16] P. Chevallier et al., Phys. Rev. 160 (1967) 827.
[17] W.D. Myers and W.J. Swiatecki, Ark. Fys. 36 (1967) 343.
[18] J.M. Eisenberg and W. Greiner, Nucl. theory, vol. 3 (North-Holland, Amsterdam, 1972) p. 420.