



LUND UNIVERSITY

On interleaver design for serially concatenated convolutional codes

Jordan, Ralph; Höst, Stefan; Johannesson, Rolf

Published in:
[Host publication title missing]

DOI:
[10.1109/ISIT.2001.936075](https://doi.org/10.1109/ISIT.2001.936075)

2001

[Link to publication](#)

Citation for published version (APA):

Jordan, R., Höst, S., & Johannesson, R. (2001). On interleaver design for serially concatenated convolutional codes. In [Host publication title missing] (pp. 212) <https://doi.org/10.1109/ISIT.2001.936075>

Total number of authors:
3

General rights

Unless other specific re-use rights are stated the following general rights apply:

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Read more about Creative commons licenses: <https://creativecommons.org/licenses/>

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

LUND UNIVERSITY

PO Box 117
221 00 Lund
+46 46-222 00 00

On Interleaver Design for Serially Concatenated Convolutional Codes¹

Ralph Jordan

Department of Telecommunications and
Applied Information Theory
University of Ulm

Albert-Einstein-Allee 43, D-89081 Ulm, Germany
ralph.jordan@e-technik.uni-ulm.de

Stefan Höst and Rolf Johannesson

Department of Information Technology
Information Theory Group
Lund University

P.O. Box 118, SE-221 00 Lund, Sweden
{stefan.host,rolf}@it.lth.se

Abstract—Serially concatenated convolutional codes are considered. The free distance of this construction is shown to be lower-bounded by the product of the free distances of the outer and inner codes, if the precipices of the interleaver are sufficiently large. It is shown how to construct a convolutional scrambler with a given precipice.

I. INTRODUCTION

An interleaver is a single input, single output, causal device which produces the output sequence $\mathbf{y} = \dots y_{-1}y_0y_1\dots = \dots x_{\pi(-1)}x_{\pi(0)}x_{\pi(1)}\dots$, that is, a permutation of the input sequence $\mathbf{x} = \dots x_{-1}x_0x_1\dots$. The invertible function π denotes the permutation on the input sequence indices, i.e., the output symbol y_j at depth j is the $\pi(j)$ th symbol $x_{\pi(j)}$ of the input sequence. The interleaver delay is given by $\delta = \max_j \{j - \pi(j)\}$.

The set of separations [1] (s, t) of an interleaver with permutation π is given by

$$\{(s, t) \mid |\pi(j) - \pi(j')| < s \Rightarrow |j - j'| \geq t, \forall j \neq j'\}$$

That is, two symbols positioned within an interval of length s in the input sequence are guaranteed to be separated by at least $t - 1$ positions in the output sequence. Clearly, if the interleaver has the separation (s, t) , then the corresponding deinterleaver has the separation (t, s) . Furthermore, the precipice $(s, t)_p$ is a separation (s, t) such that neither $(s+1, t)$ nor $(s, t+1)$ do exist in the set of separations. In general, an interleaver can have several precipices.

We use the concept of *convolutional interleaving* to describe the interleaver by a *convolutional scrambler* [2].

Definition 1 An infinite matrix $\mathbf{S} = (s_{ij})$, $i, j \in \mathbb{Z}$, that has one 1 in each row and one 1 in each column and that satisfies $s_{ij} = 0$, $i > j$ is called a *convolutional scrambler*.

The interleaved sequence is then given by $\mathbf{y} = \mathbf{x}\mathbf{S}$.

II. SERIALY CONCATENATED CONVOLUTIONAL CODES

Consider a serial concatenation of two convolutional encoders with a convolutional scrambler in between.

Theorem 1 Let d_{free} be the free distance of a serially concatenated convolutional code. If the interleaver has at least one precipice $(s, t)_p$ that satisfies the inequalities

$$s \geq \min\{(j_{\text{free}}^{\text{co}} + 1)c_o, (j_{\text{free}}^{\text{rco}} + 1)c_o\}$$

$$t \geq j_{2\text{free}}^{\text{bi}} b_i$$

¹This work was supported in part by the Swedish Research Council for Engineering Sciences under Grant 97-723 and in part by the German Research Council Deutsche Forschungsgemeinschaft under Grant Bo 867/8.

then

$$d_{\text{free}} \geq d_{\text{free}}^o d_{\text{free}}^i$$

where d_{free}^o and d_{free}^i denote the free distance of the outer and inner convolutional codes, respectively, and $j_{\text{free}}^{\text{co}}$, $j_{\text{free}}^{\text{rco}}$, and $j_{2\text{free}}^{\text{bi}}$ are derived from the active distances [3].

III. THE (q, r) CONVOLUTIONAL SCRAMBLER

A (q, r) convolutional scrambler is a convolutional scrambler $\mathbf{S}_{(q,r)} = (s_{ij})$ with

$$s_{ij} = 1, \quad j = i + R_r(iq), \quad q + 1 < r$$

where $\gcd(q+1, r) = 1$ and $R_{(q+1)}(r) = 1$. The period of this scrambler is $T = r$ and the delay is $\delta = r - 1$.

Theorem 2 Given a (q, r) convolutional scrambler, then

$$(s, t) = \left(\frac{r-1}{q+1}, q+1 \right)$$

is a precipice.

Example 1 Consider the $(3, 13)$ convolutional scrambler. It has period $T = 13$ and delay $\delta = 12$. There is one precipice at $(s, t)_p = (3, 4)$. Thus, all symbols within a segment of size three in the input sequence are separated by at least three bits in the output sequence.

The (q, r) convolutional scrambler provides the possibility to realize a convolutional scrambler for a given precipice $(s, t)_p$ by letting $q = t - 1$ and $r = st + 1$. This gives an interleaver with interleaver delay $\delta = st$, which is the minimal required interleaver delay for the considered precipice.

Example 2 Consider a serially concatenated convolutional code generated with one inner rate $R_i = 1/2$ encoder with $d_{\text{free}}^i = 5$ and one outer rate $R_o = 2/3$ encoder with $d_{\text{free}}^o = 3$, both with overall constraint length $\nu = 2$. This construction gives a free distance of $d_{\text{free}} = 15$. The required precipice is $(s, t)_p = (13, 12)$, hence, an interleaver with delay of $\delta = 156$ is required. This corresponds to 78 information bits.

REFERENCES

- [1] J. L. Ramsey, "Realization of optimum interleavers", *IEEE Trans. on Inform. Theory*, vol. IT-16, pp. 338-345, Sep. 1970.
- [2] R. Johannesson and K. Sh. Zigangirov, *Fundamentals of Convolutional Coding*, IEEE Press, Piscataway, N.Y., 1999. ISBN 0-7803-3483-3.
- [3] S. Höst, R. Johannesson, K. Sh. Zigangirov, and V. V. Zyablov, "Active distances for convolutional codes", *IEEE Trans. on Inform. Theory*, vol. IT-45, pp. 658-669, March 1999.