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An Upper Bound on Decoding Bit-Error Probability with Linear Coding on Extremely Noisy Channels

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Abstract — When concatenated coding schemes operate near channel capacity their component encoders may operate above capacity. The decoding bit-error performance of binary convolutional codes near and above capacity is investigated.

Let \(G(D)\) be a \(b \times c\) generator matrix of a rate \(R = b/c\) convolutional code. We define a tap-minimal right pseudo inverse of the generator matrix \(G(D)\) to be a right pseudo inverse of \(G(D)\) with the minimum number of taps among all right pseudo inverses. By the number of "taps" in a right pseudo inverse we mean the total number of nonzero coefficients in the power series that are entries of this \(c \times b\) matrix.

We now define the pseudo-inverse decoder (\(\pi\)-decoder) for convolutional codes. Assume that we use a convolutional code \(C\) encoded by the generator matrix \(G(D)\) for transmission over a binary symmetric channel (BSC) with crossover probability \(\epsilon\). The decoding technique is as simple as it gets: The received sequence \(r\) is fed directly to a tap-minimal right pseudo inverse of \(G(D)\) whose output is the decoded information sequence.

The exact decoding bit-error probability using the \(\pi\)-decoder is 
\[
P_b = \frac{1}{2} \sum_{i=1}^{b} \frac{1}{2} \left(1 - \left(1 - 2 \epsilon\right)^{2i}\right).
\]
Clearly this is an upper bound on the decoding bit-error probability with minimum bit-error probability decoding. We call it the \(\pi\)-bound. For probabilities \(\epsilon < 0.5\), it suggests that systematic encoders, which have the fewest taps in their tap-minimal right pseudo inverse, give lower bit-error probability than nonsystematic ones. Fig. 1 shows that for large \(\epsilon\), the \(\pi\)-bound is very tight.

When we transmit over a binary erasure channel (BEC), either a zero or a one is assigned randomly to the erased digits in the channel output sequence which thereafter is fed to a tap-minimal right pseudo inverse of \(G(D)\) whose output is the decoded information sequence. Then, 
\[
P_b = \frac{1}{2} \sum_{i=1}^{b} \frac{1}{2} \left(1 - \left(1 - p\right)^{2i}\right)
\]
for this "\(\pi\)-decoder", where \(p\) is the erasure probability of the BEC. This \(P_b\) is again an upper bound on the bit-error probability with minimum bit-error probability decoding.

Using Ancheta's bound on linear source coding [1], we can show that the minimum bit-error rate that can be achieved with rate \(R\) linear coding for a BEC is
\[
R = \frac{C}{1 - (1 - C)h\left(\frac{P_b}{1 - C}\right)}
\]
where \(C = 1 - p\) is the capacity of the BEC.

Shamai et al. [2] have given a general formulation for the minimum code rate required to approach a specified bit-error probability, showing that nonsystematic codes are inherently superior to systematic codes. For systematic coding on the BEC, this minimum code rate can be explicitly written as

\[ R = \frac{C}{1 - (1 - C)h\left(\frac{P_b}{1 - C}\right)} \]

REFERENCES
