A Distributed Power Coordination Scheme for Fatigue Load Reduction in Wind Farms

Madjidian, Daria; Mårtensson, Karl; Rantzer, Anders

2011

2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A Distributed Power Coordination Scheme for Fatigue Load Reduction in Wind Farms

Daria Madjidian, Karl Mårtensson, and Anders Rantzer

Abstract— **We consider a scenario where a wind farm is given a power set point below its actual power production capacity. The objective is to dynamically redistribute power in order to minimize the fatigue loads experienced by the turbines, while maintaining the desired power production at all times. We show that this can be done in a distributed way by coordinating neighboring turbines. The result is a control scheme where both the synthesis and the resulting control law only require each turbine to communicate with a limited set of neighboring turbines.**

I. INTRODUCTION

In the past decade, wind power capacity has continued to grow at an annual rate of 30% [1]. While economy of scale makes it attractive to position turbines close to each other, forming large wind farms, such a placement causes problems due to wake effects. The wind in a turbine wake is characterized by a mean speed deficit and an increased turbulence level. While the deficit reduces mean power production, the increased turbulence levels increase fatigue loads, and thereby maintenance costs [2].

Still, wind farms also offer an opportunity to mitigate the loads experienced by the turbines. As wind farms become more common, they will be expected to contribute to the stability of the electrical grid [3], [4]. This means that wind farms should be able to receive and maintain power set points. In cases where a farm is asked to produce less than maximum power, some turbines will need to limit their power production. This implies that they have the freedom to vary their power production in response to wind speed fluctuations, as long as the total power production of the farm meets the demand. Thus power can be redistributed between turbines according to local wind conditions. Note that since wind conditions across the farm are not uniform, at any given time, one turbine might benefit from increasing its power production while another from decreasing it.

To the authors' best knowledge, most work on load mitigation is devoted to individual turbine control (see e.g. [5], [6] as well as [7], [8] and references therein). One exception is [9], where the idea to exploit the freedom in power distribution to reduce fatigue loads was first presented in the context of disturbance rejection. There, the problem is divided into two parts. First, optimal power set points are computed explicitly offline for each turbine using a receding horizon strategy. These set points are based on other auxiliary power variables that are then used for online coordination of the turbines in order to meet the total power demand.

The authors are with the Department of Automatic Control, Lund University, Sweden. Email: daria@control.lth.se, karl@control.lth.se, rantzer@control.lth.se

In this paper, we take a different approach and consider a stochastic problem formulation. The goal is to an find a sparse state feedback controller that minimizes a measure of fatigue loading while maintaining the global power set point. We solve the problem by extending the distributed control synthesis scheme presented in [10]. We then show how the optimal feedback matrix can be obtained iteratively based only on information from a limited set of neighboring turbines.

The outline is as follows: we begin by presenting the wind farm model in Section II. The problem formulation is given in Section III, and Section IV describes the synthesis scheme. Section V presents simulation results, and finally we summarize our findings in Section VI.

II. MODELING

We consider a wind farm consisting of N turbines in a discrete time setting with a sampling time of 1 second. The turbines are numbered $1, \dots, N$. When describing two turbines i and j, we say that i is to the left of j (and j to the right of i) if $i < j$. Note that this numbering does not reflect the geographical position of the turbines. It is only used in describing the synthesis scheme in Section IV.

Wind Turbine Interaction

Wind turbines in wind farms are coupled by the wind flow. This coupling is due to wake effects (the effect on the wind field caused by turbines extracting power), and the natural wind propagation through the farm.

Wake effects play an important role in the static analysis of wind farms [11], [12]. However, when the dynamic behavior of a turbine is considered in the vicinity of an operating point, this coupling becomes less relevant. Empirical studies suggest that the wind speed variation caused by pitch activity at upwind turbines is small compared to the natural variation of the wind [13]. We therefore neglect wake effects in the model. We also neglect the wind propagation for two reasons. First, we do not have a good model describing the flow between adjacent turbines, and obtaining such a model is not a trivial task [13]. Adding this part to the model at this stage might increase complexity without contributing to accuracy. Second, by neglecting the propagation, the model becomes independent of wind direction and turbine positioning.

Power Controlled NREL Turbines

We consider a farm consisting of NREL 5 MW turbine models. The turbines are variable speed, (collective) pitch controlled, and equipped with standard internal controllers.

978-1-4577-0079-8/11/$26.00 ©2011 AACC

Here, we only provide a brief overview of the internal controller. More details can be found in [14] and [9].

The internal turbine controller manipulates generator torque and pitch angle, and has three main regions of operation, illustrated in Figure 1. In region 1, the wind speed is too low to produce power. In region 2 the controller tries to extract maximum power by fixing the pitch angle to the optimal angle for power capture. The controller then varies generator torque to track the optimal rotor speed. In region 3, the controller strives to maintain a power reference P_{ref}. This is achieved by keeping the rotational speed close to its rated value by varying the pitch angle. The generator torque M_g is used to produce the desired power according to

$$P_{\text{ref}} = M_g \omega_g$$

where w_g is the generator speed.

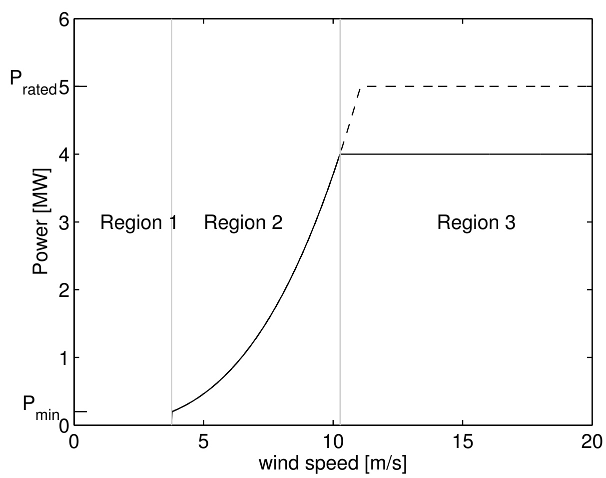

Fig. 1. Power curve of the NREL 5 MW turbine. $P_{\text{ref}} = 4$ MW (solid), maximum power production (dashed). The gray dashed lines show the operating regions for the controller.

Note that on a farm level, the control input to a turbine is the desired power P_{ref}. Therefore, a turbine will only respond to farm control in region 3, where the desired power can be attained. Since this is the only region of interest, the remainder of the paper assumes that all turbines operate in region 3.

As previously mentioned, we assume that the incoming wind speed to each turbine can be modeled independently. We follow common practice, and model the incoming wind speed at a turbine as a mean wind speed with turbulent fluctuations superimposed. The fluctuations will have zero mean when averaged over a period of about 10 minutes, and are roughly Gaussian [11]. Thus, the incoming wind speed v_i to turbine i will be modeled as:

$$v_i(t) = \bar{v}_i + w_i(t) \tag{1}$$

where $\bar{v}_i$ denotes the mean wind speed over a 10 minute interval, and w_i is a stationary Gaussian process with zero mean, and variance σ_i^2. σ_i is usually defined implicitly through the *turbulence intensity* $t_i = \frac{\sigma_i}{\bar{v}_i}$. To account for the frequency content of turbulence, we model w_i as the output of an LTI system, $(\Phi_{w,i}, \Gamma_{w,i}, C_{w,i})$, driven by Gaussian white noise, e_i with variance σ_i^2:

$$\zeta_i(t+1) = \Phi_{w,i}\zeta_i(t) + \Gamma_{w,i}e_i(t), \quad w_i = C_{w,i}\zeta_i \tag{2}$$

The matrices $(\Phi_{w,i}, \Gamma_{w,i}, C_{w,i})$ are obtained by fitting a Kaimal spectrum (see [11]) to a rational function and performing spectral factorization.

We will use the third order model used in [9], augmented with the turbulence filter in (2), to describe turbine i around its operating point $(\bar{v}_i, \bar{P}_{\text{ref},i})$. Thus, given operating points $(\bar{v}_i, \bar{P}_{\text{ref},i})$, $i = 1, \dots, N$, the complete wind farm is described by:

$$\begin{aligned} x_i(t+1) &= \Phi_i x_i(t) + \Gamma_i u_i(t) + \Gamma_{e,i} e_i(t) \\ y_i &= C_i x_i + D_i u_i \end{aligned} \tag{3}$$

The state vector $x_i^T = [\beta_i \ \omega_i \ z_i \ \zeta_i^T]$, where β_i, and ω_i are deviations from nominal pitch angle and rotor speed, respectively, z_i is a state of the internal controller, and ζ_i is the state of the turbulence filter (2). The input u_i is the deviation from nominal power $\bar{P}_{\text{ref},i}$, and e_i is white noise with variance σ_i^2. The output $y_i^T = [T_i \ M_i]$, where T_i and M_i are the deviations in tower bending moment and shaft bending moment, respectively. It is important to mention that the model neglects generator dynamics. This implies that u_i is the actual deviation in power production.

III. PROBLEM FORMULATION

Consider a scenario where a farm of N turbines is asked to produce a certain power, P_d, which is less than what the farm is capable of producing. We assume that a nominal power distribution, P_i, $i = 1, \dots, N$, is given such that, for each i, $P_i < P_{\text{rated}}$, and that all turbines are operating in region 3 where they can be controlled on a farm level. This means that there is freedom in distributing the desired power over the turbines. The objective is to dynamically redistribute power in order to minimize fatigue loads on the turbines, while maintaining the desired power production P_d.

We will consider fatigue loads on tower and low speed shaft. As in [9], we use tower bending moment, and shaft bending moment, to model fatigue loading. The measure of fatigue for turbine i is taken as

$$\mathbf{E}(\rho_{T,i}T_i^2 + \rho_{S,i}M_i^2) \tag{4}$$

where $\rho_{T,i}$, and $\rho_{S,i}$ are weights reflecting the relative importance of tower and shaft, respectively. Note that although this is not a standard measure of fatigue loading, simulation studies suggest that (4) correlates well with results obtained based on rainflow counting algorithms [15].

The goal is to find a linear state feed back law, $u_i = -\sum_j (L)_{ij} x_j$, that minimizes

$$\sum_{i=1}^{N} \mathbf{E} x_i^T Q_i x_i + 2x_i^T S_i u_i + u_i^T R_i u_i$$

subject to (3) and

$$\sum_{i=1}^{N} u_i = 0 \tag{5}$$

where $Q_i = C_i^T O_i C_i$, $S_i = C_i^T O_i D_i$, $R_i = D_i^T O_i D_i + \rho_i$, and $O_i = \mathrm{diag}(\rho_{T,i}, \rho_{S,i})$. The positive scalar ρ_i is added to penalize large deviations from nominal power, $\bar{P}_{\mathrm{ref,i}}$. Recall that since the model neglects generator dynamics, (5) states that the deviation from nominal farm power should be zero.

In addition, we impose an information constraint on the control:

$$L \in \mathcal{L} \tag{6}$$

where $\mathcal{L}$ defines a structural restriction, that determines the neighbors that each turbine is allowed to communicate with. This will be defined and further explained in Section IV.

IV. DISTRIBUTED SYNTHESIS

For each stabilizing feedback matrix, L, define:

$$J(L) = \mathbf{E}x^T Qx + 2x^T Su + u^T Ru \tag{7}$$

We restate the optimal control problem in a more compact form. Minimize (7) subject to (6), $\mathbf{1}^T u = 0$, and

$$x(t+1) = \Phi x(t) + \Gamma u(t) + e(t) \tag{8}$$

where $x = [x_1^T \ \ldots \ x_N^T]^T$, $u = [u_1 \ \ldots \ u_N]^T$, $e = [e_1 \ \ldots \ e_N]^T$, and the matrices $Q = Q^T \succeq 0$, $R = R^T \succ 0$, S, Φ, and Γ are all block diagonal.

Note that without the global power constraint, (5), the problem is completely decoupled, and that this constraint creates a direct coupling between all turbines. To satisfy (6), the scheme presented in [10] will now be extended in order to handle the power constraint.

The power constraint in (5) can be eliminated by constraining the control. Introduce auxiliary control variables, $\hat{u}_i, \ i = 1, \ldots, N-1$, and coordinate neighboring turbines according to:

$$u_1 = \hat{u}_1 \tag{9}$$

$$u_i = \hat{u}_i - \hat{u}_{i-1}, \quad i = 2, \ldots, N-1 \tag{10}$$

$$u_N = -\hat{u}_{N-1} \tag{11}$$

Note that u_i is the actual control applied by turbine i, and that (9)-(11) results in $\sum_i u_i = 0$.

Define $\hat{u} = [\hat{u}_i \ \ldots \ \hat{u}_{N-1}]^T$, and let $T = \{t_{i,j}\}$ be a $N \times N-1$ matrix, with $t_{i,i} = 1$, $t_{i,i-1} = -1$, and 0 elsewhere. The constrained system becomes:

$$x(k+1) = \Phi x(k) + \hat{\Gamma}\hat{u}(k) + e(k) \tag{12}$$

where $\hat{\Gamma} = \Gamma T$.

Let $\hat{R} = T^T RT$, and $\hat{S} = ST$. For each control law, $\hat{u} = -\hat{L}x$, that stabilizes (12), define

$$J(\hat{L}) = \mathbf{E}x^T Qx + 2x^T \hat{S}\hat{u} + \hat{u}^T \hat{R}\hat{u} \tag{13}$$

The *auxiliary* control problem can be formulated as: minimize (13), subject to (12) and

$$\hat{L} \in \hat{\mathcal{L}}_{l_1}^{l_2} \tag{14}$$

where

$$\hat{\mathcal{L}}_{l_1}^{l_2} = \{X \mid (X)_{ij} \neq 0 \quad \text{only if} \quad i - l_1 \leq j \leq i + l_2\}$$

and $(X)_{ij}$ is defined as the i,j :th block of X. This means that each $\hat{u}_i$ may only depend on measurements from l_1 neighbors to the left, and l_2 neighbors to the right. For notational purposes, from now on we will refer to $\hat{\mathcal{L}}_{l_1}^{l_2}$ simply as $\hat{\mathcal{L}}$.

Once the optimal $\hat{L}$ has been found, L can be retrieved through $L = T\hat{L}$. This means that each u_i will depend on $l_1 + 1$ turbines to the left and l_2 turbines to the right. Note that if $l_2 = 0$ the will be no feedback from turbine N in the final control law $u = -Lx$.

We now show how to adaptively change $\hat{L}$ to improve the performance $J(\hat{L})$ by using only local measurements. At every iteration, we wish to change $\hat{L}$ in a descent direction of J. If the gradient $\nabla_{\hat{L}} J$ of the cost is non-zero, we can change the feedback matrix in the negative gradient direction to get a lower cost. Since there is no structural restriction on the gradient, the updated $\hat{L}$ would not satisfy the structure of $\hat{\mathcal{L}}$. But the gradient projected to the structural subspace, $\nabla_{\hat{L}} J\big|_{\hat{\mathcal{L}}}$, is also a descent direction. Hence, the cost J is reduced when changing $\hat{L}$ by

$$\hat{L}_{\mathrm{new}} = \hat{L} - \gamma \ \nabla_{\hat{L}} J\big|_{\hat{\mathcal{L}}}$$

where γ is sufficiently small. With this new feedback matrix, we recompute a descent direction and continue iterating. The next proposition gives an expression for the gradient of $J(\hat{L})$, which is suitable for distributed calculations.

Proposition 1: Given matrices Φ and $\hat{\Gamma}$, consider $\hat{L}$ such that $\Phi - \hat{\Gamma}\hat{L}$ has all eigenvalues inside the unit circle. Let the solutions to (12), with $\hat{u} = -\hat{L}x$, be stationary stochastic processes, where e is white noise with covariance W. Consider the stationary stochastic process λ defined by the backwards iteration

$$\begin{aligned}\lambda(t-1) = &(\Phi - \hat{\Gamma}\hat{L})^T \lambda(t) - (Q - \hat{S}\hat{L} - (\hat{S}\hat{L})^T \\ &+ \hat{L}^T \hat{R}\hat{L})x(t)\end{aligned} \tag{15}$$

where x are the states of the original system. Then $J(\hat{L})$, defined by (13), has the gradient

$$\nabla_{\hat{L}} J = 2\left((\hat{R}\hat{L} - \hat{S}^T)\mathbf{E}xx^T + \hat{\Gamma}^T \mathbf{E}\lambda x^T\right)$$

Remark 1. The proof of Proposition 1 will show that the gradient can also be expressed as

$$\nabla_{\hat{L}} J = 2\left(\hat{R}\hat{L} - S^T - \hat{\Gamma}^T P(\Phi - \hat{\Gamma}\hat{L})\right)X \tag{16}$$

where P, and X are solutions to Lyapunov equations (17), and (18) below. This expression can be useful for offline computations.

Proof. First, define the following matrices

$$\Phi_{\hat{L}} = \Phi - \hat{\Gamma}\hat{L}$$
$$Q_{\hat{L}} = Q - \hat{S}\hat{L} - (\hat{S}\hat{L})^T + \hat{L}^T \hat{R}\hat{L}$$

The solutions X and P for the Lyapunov equations

$$X = \Phi_{\hat{L}} X \Phi_{\hat{L}}^T + W \tag{17}$$

$$P = \Phi_{\hat{L}}^T P \Phi_{\hat{L}} + Q_L \tag{18}$$

are used in the proof. We know that $J(\hat{L}) = \mathrm{tr}(PW)$. By calculating the differential of P with respect to $\hat{L}$, we can determine an expression for the gradient of J with respect to $\hat{L}$. Let $M = d\hat{L}^T[\hat{R}\hat{L} - \hat{S}^T - \hat{\Gamma}^T P \Phi_{\hat{L}}]$

$$dP = \Phi_{\hat{L}}^T dP \Phi_{\hat{L}} + M + M^T \iff$$
$$dP = \sum_{k=0}^{\infty} (\Phi_{\hat{L}}^T)^k (M + M^T) \Phi_{\hat{L}}^k$$

Hence, since $dJ = \mathrm{tr}(dP \cdot W)$, we get that

$$\begin{aligned} dJ &= 2\mathrm{tr}\left(\sum_{k=0}^{\infty} (\Phi_{\hat{L}}^T)^k M \Phi_{\hat{L}}^k W\right) = \\ &= 2\mathrm{tr}\left(M \sum_{k=0}^{\infty} \Phi_{\hat{L}}^k W (\Phi_{\hat{L}}^T)^k\right) \\ &= 2\mathrm{tr}\left(d\hat{L}^T[\hat{R}\hat{L} - \hat{S}^T - \hat{\Gamma}^T P \Phi_{\hat{L}}] X\right) \end{aligned}$$

By using the relation about differentials, $dZ = \mathrm{tr}(dX^T Y) \Rightarrow \nabla_X Z = Y$ for matrices $X^T, Y \in \mathbb{R}^{n \times p}$, we conclude that

$$\nabla_{\hat{L}} J = 2[\hat{R}\hat{L} - \hat{S}^T - \hat{\Gamma}^T P \Phi_{\hat{L}}] X \tag{19}$$

Now, by (15) we have that

$$\begin{aligned} \lambda(k) &= \sum_{j=k+1}^{\infty} (\Phi_{\hat{L}}^T)^{j-k-1} Q_{\hat{L}} x(j) = \\ &= \sum_{j=0}^{\infty} (\Phi_{\hat{L}}^T)^j Q_{\hat{L}} \Phi_{\hat{L}}^{j+1} x(k) + \\ &+ \Psi\{e(k), e(k+1), \ldots\} \end{aligned}$$

where Ψ is the appropriate linear operator of how the noise $e(j)$, $j \geq k$ affects $\lambda(k)$. Since $x(k)$ and $e(j)$ are independent for all $j \geq k$, we have that

$$\begin{aligned} \mathbf{E}\lambda(k)x(k)^T &= -\mathbf{E}\left[\sum_{j=0}^{\infty} (\Phi_{\hat{L}}^T)^j Q_{\hat{L}} \Phi_{\hat{L}}^{j+1} x(k) x(k)^T\right] = \\ &= -P\Phi_{\hat{L}} X \end{aligned}$$

Using this relation in (19) concludes the proof. □

Let $\Phi_{\hat{L}} = \Phi - \hat{\Gamma}\hat{L}$. To simulate (15) locally, each turbine needs to run:

$$\begin{aligned} \lambda_i(t-1) = &[\Phi_L^T \lambda(t)]_i \\ &+ [(Q - \hat{S}\hat{L} - (\hat{S}\hat{L})^T + \hat{L}^T \hat{R}\hat{L}) x(t)]_i \end{aligned} \tag{20}$$

Using the structure of $\hat{L}$, Φ, $\hat{R}$, $\hat{\Gamma}$, and $\hat{S}$ we have

$$\left[\Phi_L^T \lambda(t)\right]_i = \sum_{j=i-l_2}^{i+(l_1+1)} (\Phi_L)_{ji}^T \lambda_j(t) \tag{21}$$

$$\left[(Q - (\hat{S}\hat{L})^T) x\right]_i = Q_i x_i - \sum_{j=i-l_2}^{i+(l_1+1)} (\hat{L}^T \hat{S}^T)_{ij} x_j \tag{22}$$

$$\begin{aligned} \left[(\hat{L}^T \hat{R}\hat{L} - \hat{S}\hat{L}) x\right]_i &= S_i u_i - S_i u_{i-1} \\ &- \sum_{j=i-(l_2+1)}^{i+(l_1+1)} (\hat{L}^T \hat{R})_{ij} u_j \end{aligned} \tag{23}$$

For each turbine i, the factors $(\Phi_L)_{ji}$, and $(\hat{S}\hat{L})_{ij}^T$, for $j \in [i - l_2, i + l_1 + 1]$, and $(\hat{L}^T \hat{R})_{ij}$, for $j \in [i - (l_2 + 1), i + l_1 + 1]$, can be obtained by allowing the turbine to communicate with $l_2 + 1$ turbines to the left, and $l_1 + 1$ turbines to the right. Hence, by (21)-(23), the adjoint system (15) can be simulated locally (provided that each turbine may communicate as explained above).

Since $\hat{L} \in \hat{\mathcal{L}}$, the actual direction that we update the feedback matrix in, must also belong to $\hat{\mathcal{L}}$. Hence, we project the gradient $\nabla_{\hat{L}} J$ on $\hat{\mathcal{L}}$. Letting G be the update direction, we have that

$$\begin{aligned} G_{ij} &= \left(\nabla_{\hat{L}} J\right)_{ij} \quad \text{if } j \in [i - l_1, i + l_2] \\ G_{ij} &= 0 \quad \text{otherwise} \end{aligned}$$

Assuming that the projected gradient G is non-zero, $-G$ is a descent direction of $J(\hat{L})$. Now, this means that to update the feedback matrix, turbine i only needs to determine the gradient in the blocks corresponding to the l_1 turbines to the left, and l_2 turbines to the right. This requires that both $\left((\hat{R}\hat{L} - \hat{S}^T)\mathbf{E}xx^T\right)_{ij}$ and $\left(\hat{\Gamma}^T \mathbf{E}\lambda x^T\right)_{ij}$ can be estimated locally. Due to the structure of $\hat{\Gamma}$, $\hat{S}$, $\hat{L}$, and $\hat{R}$, the terms can be expressed as

$$\begin{aligned} \left((\hat{R}\hat{L} - \hat{S}^T)\mathbf{E}xx^T\right)_{ij} &= -\sum_{k=-1}^{1} \hat{R}_{i,i+k} \mathbf{E} u_{i+k} x_j^T \\ &- S_i^T \mathbf{E} x_i x_j^T + S_{i+1}^T \mathbf{E} x_{i+1} x_j^T \end{aligned} \tag{24}$$

$$\left(\hat{\Gamma}^T \mathbf{E}\lambda x^T\right)_{ij} = \Gamma_i^T \mathbf{E}\lambda_i x_j^T - \Gamma_{i+1}^T \mathbf{E}\lambda_{i+1} x_j^T \tag{25}$$

which can be estimated locally, given that we allow communication with $\max(1, l_1)$ turbines to the left, and $\max(1, l_2)$ turbines to the right (since only $j \in [i - l_1, i + l_2]$ needs to be taken into account).

The method for updating L is summarized below:

Algorithm 1: At time t_k, let the state feedback law be $u(t) = -\hat{L}^{(k)} x(t)$, where $\hat{L}^{(k)} \in \hat{\mathcal{L}}$. To update the feedback matrix in turbine i:

1) For all neighbors j, compute the entries $(\Phi_L)_{ji}^T$, $(\hat{S}\hat{L})_{ij}^T$, and $(\hat{L}^T \hat{R})_{ij}^T$, by communicating with neighboring turbines.
2) Measure the states $x_i(t)$ of the system (8) for times $t = t_k, \ldots, t_k + M$.
3) Simulate the adjoint states $\lambda_i(t)$ according to (20), for times $t = t_k, \ldots, t_k + M$ in the backwards direction, by communicating states from and to neighboring turbines.
4) For each neighboring turbine j, calculate the estimates of $\mathbf{E} u_i x_j^T$ and $\mathbf{E}\lambda_i x_j^T$ by

$$\left(\mathbf{E} u_i x_j^T\right)_{\text{est}} = \frac{1}{M} \sum_{t=t_k}^{t_k+M} u_i(t) x_j(t)^T$$
$$\left(\mathbf{E}\lambda_i x_j^T\right)_{\text{est}} = \frac{1}{M} \sum_{t=t_k}^{t_k+M} \lambda_i(t) x_j(t)^T$$

5) Use (24) and (25) to compute the estimate G_{ij} of the i,j-block of the gradient.
6) For each neighboring turbine j, update $\hat{L}_{ij}^{(k+1)} = \hat{L}_{ij}^{(k)} - \gamma G_{ij}$ for some step length γ.
7) Let $t_{k+1} = t_k + M$, increase k by one and go to 1).
8) At the last iteration, communicate local feedback vectors, $\hat{L}_i$, upwind. Obtain the feedback vector, L_i, for the original system from (9)-(11).

We denote M by the iteration time, i.e. the length of the time interval where the system is controlled using a constant feedback matrix.

In order to obtain L, each turbine needs to communicate with $\max(l_1, l_2+1)$ turbines to the left, and $\max(l_1+1, l_2)$ turbines to the right. To apply $u = -Lx$, each turbine needs to communicate with l_1+1 turbines to the left, and l_2 turbines to the right. In total, each turbine needs to communicate with $\max(l_1, l_2)+1$ turbines to the left, and $\max(l_1+1, l_2)$ turbines to the right.

V. SIMULATION RESULTS

We consider an example where a farm with $N = 5$ turbines receives a power set point of $P_d = 17$ MW. For the sake of simplicity, we assume that each turbine experiences a mean wind speed of 20 m/s and a turbulence intensity of 0.1. The turbulence filter for each turbine is given by:

$$\Phi_w = \begin{pmatrix} 0.34 & -0.11 \\ 0.15 & 0.98 \end{pmatrix} \quad \Gamma_w = \begin{pmatrix} 0.62 \\ 0.09 \end{pmatrix} \quad C_w = \begin{pmatrix} 1.3 & 0.57 \end{pmatrix}$$

The nominal power distribution is as follows: turbines 1 and 2 produce 4 MW each, and turbines 3, 4, and 5 produce 3 MW each. The weights are set to $\rho_{T_i} = 1$, $\rho_{S,i} = 0.1$ and $\rho_i = 0.001$, $i = 1, \dots, 5$. The step size is chosen to $\gamma = 0.1$.

Figure 2 shows the result of running algorithm 1 for a large number of iterations with $M = 30$ s and different combinations of l_1 and l_2.

We now focus more specifically on the case $(l_1, l_2) = (0, 1)$. Figure 3 shows the cost J, as well as J_t, J_s, and J_u related to cost components of tower, shaft, and power during the first 50 iterations:

$$J_t = \sum \mathbf{E} T_i^2, \quad J_s = \sum \mathbf{E} M_i^2, \quad J_u = \sum \mathbf{E} u_i^2$$

Figure 4 shows the deviation from nominal power for each turbine, as well as for the whole farm for the first 150 seconds of running algorithm 1.

Next, we examine the relation between farm size and the relative performance, $J(L)/J(0)$. Table I shows the result of running algorithm 1 for 500 iterations with $M = 30$, and $\gamma = 0.1$ on different farm sizes. The last line in the table shows the result of applying optimal full state feedback. The result shows that the number of turbines, N, has a negligible effect on the performance. One possible explanation is that without the power constraint in (5), the turbines would no longer be coupled, and the farm size would therefore not have any influence on the performance. By constraining the control, we reduce the degrees of freedom to $N-1$. But for large farms $\frac{N-1}{N} \approx 1$. Another interpretation that is closely related is that when the farm is above a certain size, the likelihood for a turbine to find other turbines that they can exchange power with is high, and increasing N only increases that likelihood marginally.

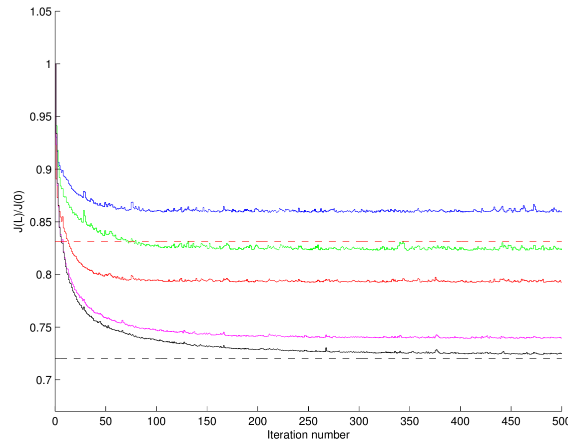

Fig. 2. Relative decrease of $J(L)$ for five different cases of (l_1, l_2) when $M = 30$ s: blue — $(l_1, l_2) = (0, 0)$; red — $(l_1, l_2) = (0, 1)$; green — $(l_1, l_2) = (1, 0)$; magenta — $(l_1, l_2) = (2, 2)$; black — $(l_1, l_2) = (4, 4)$; Note that in case of $l_2 = 0$, no measurements from turbine 5 are used in the final control law. This explains why $(l_1, l_2) = (0, 1)$ gives better performance than $(l_1, l_2) = (1, 0)$. The dashed black line shows the result of applying optimal full state feedback, and the dashed red line shows the result of applying the optimal state feedback vector projected onto the structural subspace $\hat{\mathcal{L}}$ with $(l_1, l_2) = (0, 1)$. As seen this is not the optimal solution as the result of algorithm 1 decreases the cost even further.

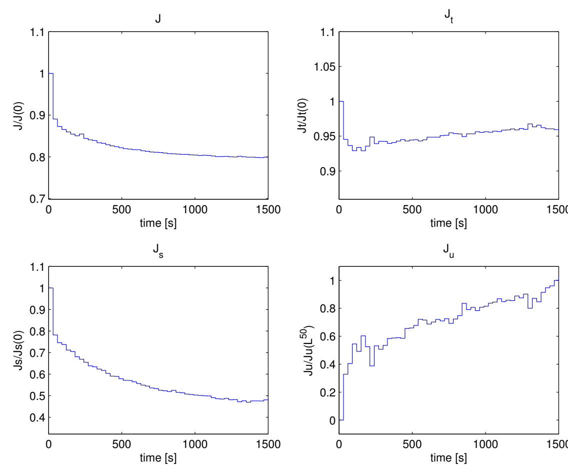

Fig. 3. Relative decrease of $J(L)$ and its components when $(l_1, l_2) = (0, 1)$ and $M = 30$ s. We observe that by allowing variations in individual turbine power, both tower and shaft loads are reduced simultaneously. The plots of J, J_t, and J_s have been normalized with respect to the case of $L = 0$. The plot of J_u was normalized with respect to the case of $L^{(50)}$.

VI. CONCLUSION

We have presented a method for reducing fatigue loads in wind farms, while maintaining a global power demand. The method has two properties. First, the resulting feedback matrix can be made sparse and the level of sparsity can be specified by choosing two parameters. This can be beneficial since the SCADA systems used in wind farm control often have low computational capacity and memory. The second property is that the control law for each unit can iteratively

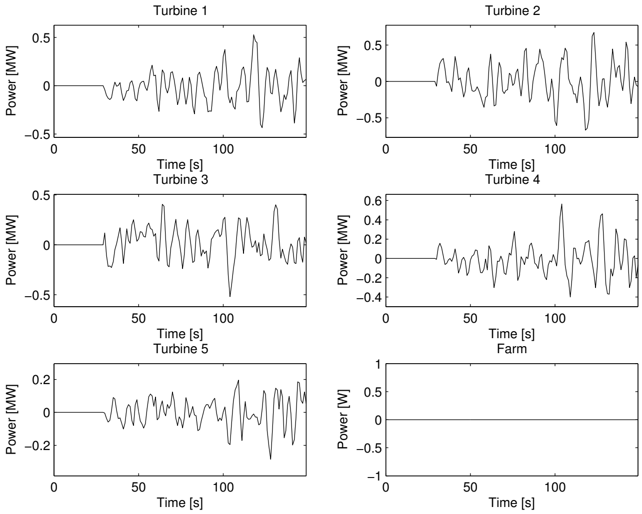

Fig. 4. Deviation from nominal power during the first 150 seconds. $(l_1, l_2) = (0, 1)$ and $M = 30$. As more iterations are performed the variation in individual turbine power increases. However, the total farm power is kept constant. Note that the power deviation for the farm is is shown in Watts.

TABLE I

COST REDUCTION FOR DIFFERENT VALUES OF N

N	5	10	20	50	80
$l_1 = 0$ $l_2 = 0$	0.850	0.831	0.821	0.815	0.814
$l_1 = 0$ $l_2 = 1$	0.773	0.760	0.753	0.749	0.748
$l_1 = 1$ $l_2 = 0$	0.823	0.793	0.777	0.769	0.766
$l_1 = 2$ $l_2 = 2$	0.731	0.715	0.707	0.701	0.700
$l_1 = 5$ $l_2 = 5$	0.722	0.701	0.693	0.689	0.687
Full optimal	0.690	0.651	0.631	0.620	0.617

be designed online, based on only on information from neighboring units. In general, the main advantages of the second property are:

- Modularity: All update laws are identical, and each unit only needs to know which units to communicate with.
- Scalability: Adding or removing a unit from operation does not change the computational effort of units that are not its neighbors.

However, these advantages might have a limited impact in a wind farm application. One reason is that they come at the expense of convergence time, as can be seen in Section V. Second, Table I suggests that it is possible to split the farm into smaller groups and consider each group separately without any noticeable loss in performance. This means that scalability is not important. Also, the benefits of modularity are not obvious. However, as suggested in Section IV (after Proposition 1), the update direction for the feedback matrix can also be obtained by iteratively solving two Lyapunov equations. This provides a way of predesigning gain scheduled sparse feedback controllers offline. In general, the iterative scheme gives better performance than solving the associated Riccati equation and projecting the resulting feedback matrix onto the structural subspace $\hat{\mathcal{L}}$ (see Figure 2).

VII. ACKNOWLEDGMENTS

This work has been funded by the European Commissions' Seventh Framework Programme under grant agreement 224548, acronym AEOLUS, and the Swedish Research Council through the Linnaeus center LCCC.

REFERENCES

[1] Global Wind Energy Council, "Wind is a global power source," http://www.gwec.net/index.php?id=13, accessed 06/04/2011.

[2] S. T. s Frandsen, "Turbulence and turbulence generated structural loading in wind turbine clusters," Ph.D. dissertation, Risø National Laboratory, 2007.

[3] Elkraft System and Eltra, "Wind turbines connected to grids with voltages above 100 kv – technical regulation for the properties and the regulation of wind turbines, Tech. Rep. 214439 v3, 2003.

[4] T. Bublat and T. Gehlhaar, "Comparison of high technical demands on grid connected wind turbines defined in international grid codes," in *7th International Workshop on Large Scale Integration of Wind Power and on Transmission Networks for Offshore Wind Farms*, May 2008.

[5] E. A. Bossanyi, "Wind turbine control for load reduction," *Wind Energy*, vol. 6, pp. 229–244, 2003.

[6] K. Hammerum, P. Brath, and N. K. Poulsen, "A fatigue approach to wind turbine control," *Journal of Physics: Conference Series*, vol. 75, 2007.

[7] J. H. Laks, L. Y. Pao, and A. D. Wright, "Control of wind turbines: Past, present, and future," in *American Control Conference*, St. Louis, MO, June 2009.

[8] L. Y. Pao and K. E. Johnson, "A tutorial on the dynamics and control of wind turbines and wind farms," in *American Control Conference*, June 2009.

[9] V. Supić, M. Jelavić, M. Baotić, and N. Perić, "Hierarchical wind farm control for power/load optimization," in *Torque 2010 The science of making torque from wind*.

[10] K. Mårtensson and A. Rantzer, "Gradient methods for iterative distributed control synthesis," in *Conference on Decision and Control*, 2010.

[11] T. Burton, D. Sharpe, N. Jenkins, and E. Bossayani, *Wind Energy Handbook*, 2nd ed. John Wiley and Sons, 2008.

[12] D. Madjidian and A. Rantzer, "A stationary model for control of wind farms," in *18th IFAC World Congress*, Sept. 2011, (to appear).

[13] T. Knudsen, "Validated dynamic flow model - aeolus - wp2," University of Aalborg, Tech. Rep., Dec. 2010, internal project report.

[14] J. Jonkam, S. Butterfield, W. Musial, and G. Scott, "Definition of a 5-mw reference wind turbine for offshore system development," National Renewable Energy Laboratory, Golden, Colorado, Tech. Rep. NREL/TP-500-38060, Feb. 2010.

[15] B. Biegel, "Distributed control of wind farm," Master's thesis, Aalborg Univeristy, June 2011, (expected date of publication).