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Liquid in accelerated motion

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Figure 1. A glass of cordial, resting on wooden triangle on a string

Abstract. What happens to a liquid in a swing or a loop? Accelerated motions lead to effects that may seem surprising. Consider a liquid in a glass placed on a wooden triangle, in turn attached to a string and moving fast in circle a vertical plane. The surface of the water remains parallel to the bottom of the glass - and orthogonal to the string - as long as gravity and the tension in the string are the only forces acting on the triangle. A key to the understanding is that the tangential acceleration of the liquid, the glass and the triangle, are all identical to the tangential component of the acceleration of gravity.

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1. Introduction

Have you ever swung a bucket full of water over your head, and marvelled about not getting wet. Most likely, you heard explanations from adults mentioning the "centrifugal force" as responsible for "pushing the water" away from the centre of the circle. Still, your physics textbook is more likely to insist that the centrifugal force does not exist. The water in the bucket is then instead described as accelerating towards the ground with an acceleration larger than the acceleration of gravity, g. The photo in Figure 1

Figure 2. During rotation - and other motions - the surface of the liquid remains parallel to the bottom of the glass, orthogonal to the string

shows a setup for a beautiful demonstration for the brave: A glass with red cordial, placed on a wooden triangle, suspended from a string in one of the corners. With a determined move of the arm, the triangle can be made to move a full circle, while the liquid remains parallel to the bottom of the glass, as in figure 2. For the less confident, an inch or so of coloured liquid in a closed bottle with a string will also do [1].

2. Force and acceleration

The only forces acting on the triangle with the glass are the force, $\mathbf{T}$, from the tension in the string and the force of gravity, $m\mathbf{g}$, as shown in Figure 3. The force from the string is always in the direction of the string, in the radial direction towards the centre of rotation, whereas the force of gravity can be divided into a tangential component $F_t = -mg\cos\theta$ and a radial component $F_r = -mg\sin\theta$, denoted by dashed yellow lines in figure 4, which also defines the angle θ. Similarly, the resulting acceleration can be divided into tangential and radial components, as in figure 4, giving $a_t = -g\cos\theta$ and $a_r = -g\sin\theta - T/m$.

For the glass to remain in the triangle, the string must be extended with some tension. In the highest point, this implies that the downward acceleration must be at least g. The centripetal acceleration is given by v^2/L, where v is the speed and L is the distance between the centre of the circle and the centre of mass of the triangle, implying a minimum speed at the top, $v \geq \sqrt{gL}$.

For an angle close to 30^o, as in the photo, the glass is lower by $\Delta H = L/2$. This implies an increased speed and the centripetal acceleration will be larger by $2g\Delta H/L$, giving, $a_c = g + 2g\Delta H/L = 2g$. The radial component of the the force of gravity can only provide an acceleration $gsin\theta \approx g/2$. The tension in the string in the photo must thus be at least $3mg/2$, to provide the required centripetal force. A faster rotation implies a larger centripetal force, and a larger tension in the string, while leaving the tangential components of force and acceleration unaffected.

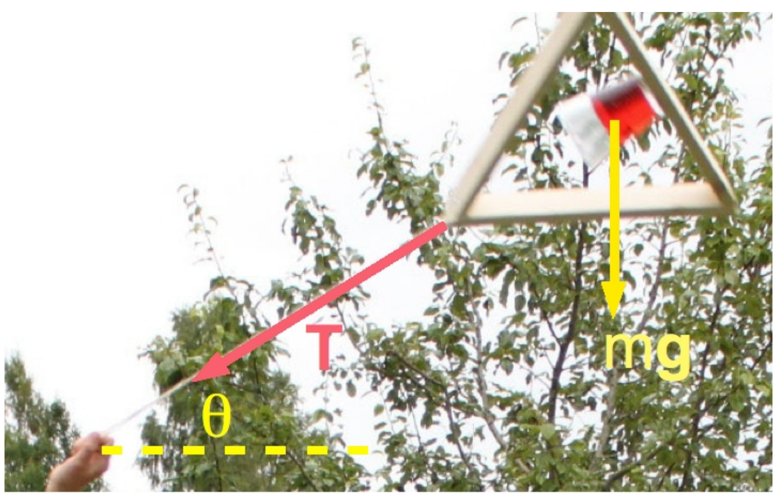

Figure 3. During a sufficiently fast rotation the surface of the liquid remains parallel to the bottom of the glass, orthogonal to the string

3. Conclusion

The key to understanding of the behaviour of the liquid in figure 2 is to note that the only force acting in the direction of the tangent is the force of gravity. This component, $F_t = mg\cos\theta$, leads directly to an acceleration $a_t = g\cos\theta$. From within the accelerating triangle, it seems like gravity has been turned off in the tangential direction. This is analogous to free fall causing an experience of weightlessness. An accelerometer measuring the vector $\mathbf{a} - \mathbf{g}$ will show zero in the tangential direction (see e.g. [2, 3]). It is a consequence of the equivalence between inertial and gravitational mass, which has been explored e.g. in [4]

The triangle, the glass and the liquid in the glass share the same acceleration, and the force to be added to gravity must point to the centre of rotation for all of them. The net force from liquid on a small droplet in the glass must also be orthogonal to the surface of the liquid - which thus remains orthogonal to the string and parallel to the bottom of the glass. The orthogonality between the liquid surface and the string holds also for other motion, e.g. in a glass taken along in a waveswinger amusement ride, or in a glass or bottle put on a swing [1]. That experiment has also been adapted for Science Days at the Bakken amusement park in Denmark [5].

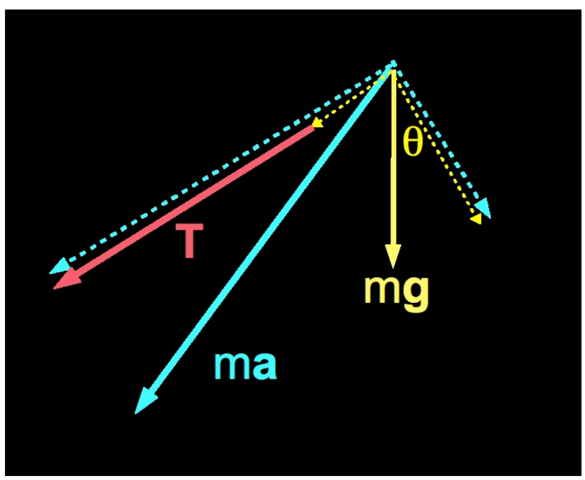

Figure 4. Radial and tangential components of force and acceleration for the triangle on the string

Acknowledgements

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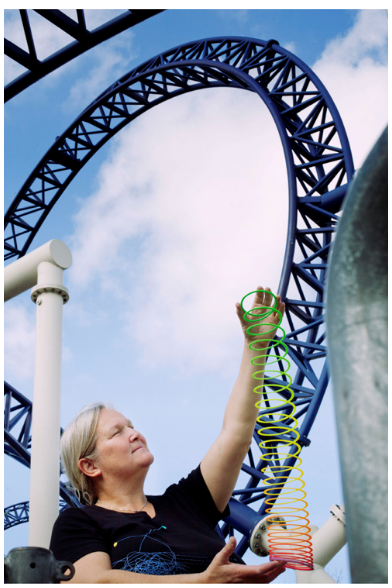
Biographies

Carl-Olof Fägerlind is a recently retired upper-secondary school physics teacher and also a singer. He enjoys creating physics demonstrations and is part of the annual Faraday Christmas lecture at Stockholm university.



Ann-Marie Pendrill is professor in Science Communication and Physics Education at Lund University, where she is also the director of the Swedish National Resource Centre for Physics Education. She enjoys using outdoor environments for physics education, e.g. to illustrate relations between force and acceleration. (Foto Maja-Kristin Nylander)