PI/PID Control of Resonant Dynamics

Hang, Chang C.; Åström, Karl Johan

1992

Document Version:
Publisher's PDF, also known as Version of record

Link to publication

Citation for published version (APA):
Hang, C. C., & Åström, K. J. (1992). *PI/PID Control of Resonant Dynamics*. (Technical Reports TFRT-7501). Department of Automatic Control, Lund Institute of Technology (LTH).

Total number of authors:
2

ISSN 0280–5316
ISRN LUTFD2/TFRT--7501--SE

PI/PID Control of Resonant Dynamics

C. C. Hang
K. J. Åström

Department of Automatic Control
Lund Institute of Technology
December 1992

Department of Automatic Control
Lund Institute of Technology
P.O. Box 118
S-221 00 Lund Sweden

Document name
INTERNAL REPORT

Date of issue
December 1992

Document Number
ISRN LUTFD2/TFRT--7501--SE

Author(s)
C. C. Hang and K. J. Åström

Supervisor

Sponsoring organisation

Title and subtitle
PI/PID Control of Resonant Dynamics

Abstract

The performance of PI/PID controllers for a system with resonant (or oscillatory) dynamics is investigated. Simplified designs to compute controller settings are given where feasible. Possible limits of the effectiveness of PI and PID control when the resonant modes are almost outside or well within the servo bandwidth are established.

Key words
Resonant dynamics; PI/PID control; Controller zeros; Setpoint and local response

Classification system and/or index terms (if any)

Supplementary bibliographical information

ISSN and key title
0280–5316

ISBN

Language
English

Number of pages
21

Security classification

Recipient's notes

The report may be ordered from the Department of Automatic Control or borrowed through the University Library 2, Box 1010, S-221 03 Lund, Sweden, Fax +46 46 110019, Telex: 33248 lubbis lund.

PI/PID CONTROL OF RESONANT DYNAMICS

by C C Hang and K J Astrom

Abstract

The performance of PI/PID controllers for a system with resonant (or oscillatory) dynamics is investigated. Simplified designs to compute controller settings are given where feasible. Possible limits of the effectiveness of PI and PID control when the resonant modes are almost outside or well within the servo bandwidth are established.

1. Introduction

The proportional-integral-derivative (PID) controllers have been widely used in the process industry owing to its simplicity and robustness [Astrom and Hagglund, 1988]. For processes with well damped dynamics, PID or even PI control is found to be adequate for most applications. A well-known and well documented limitation of PI/PID control in process control is the presence of large dead time [Seborg et.al, 1989]. PID controllers are also widely used for control of mechanical systems such as motor drives, servos, disk drives, flight control systems and missile boosters, for the same reason of simplicity and robustness. A possible limitation in this case is the presence of resonant (also called oscillatory) modes [Astrom and Wittenmark, 1990]. However, more precise knowledge about the effectiveness of PID control of resonant dynamics is not well documented in the literature and is usually passed on by experienced designers or reinvented by younger ones. The inexperienced ones may even come to the wrong conclusion that PID controllers are not applicable to resonant dynamics, based on some attempts to use the Ziegler-Nichols type of controller tuning procedure which is more suitable for damped dynamics. Notice also that the series form [Astrom and Hagglund, 1988] of PID controller cannot produce complex zeros which may be needed for good control of resonant modes.

It is clear that more extensive knowledge of the effectiveness of PID control of resonant dynamics is useful in deciding when one should go beyond PID control to apply the more sophisticated, higher order control methods such as pole-placement [D'Azzo and Houpis, 1988; Wallenberg, 1987]. An attempt is made in this report to address this subject as part of our research in knowledge-based control [Astrom et.al, 1991].

The report is organised as follows. Section 2 describes two classes of resonant dynamics that are investigated: one in which the resonant modes are almost outside the servo bandwidth and one in which they are well inside. Sections 3 and 4 present design and performance evaluation of PI and PID control for these two classes of systems. Concluding remarks are given in Section 5.

2. Scope of Study

The parallel form of PID controller with suitable modifications for practical implementation [Astrom and Hagglund, 1988]. It is described by:

$$u_c(t)=K_c[(\beta y_r-y)+\frac{1}{T_i}\int edt-T_d\frac{dy_f}{dt}] \tag{1}$$

where

$$Y_f(s)=\frac{1}{1+sT_d/N_f}Y(s)$$

and y_r, y, u_c and e are the setpoint, process output, control and error respectively. The setpoint weighting factor β is assumed to be 1 unless otherwise stated. Without loss of generality, we use $N_f = 10$ in all the simulations. For the purpose of analysis and design, a simplified form with $\beta = 1$ and $N_f = 1$ is also used.

The PID controller thus has one pole at the origin of the s-plane and two zeros which are complex when $T_d > T_i/4$:

$$G_c(s) = \frac{K_c T_d}{s}\left(s^2 + \frac{1}{T_d}s + \frac{1}{T_i T_d}\right) \tag{2}$$

The PI controller has one pole at the origin and one simple zero:

$$G_c(s) = \frac{K_c}{s}\left(s + \frac{1}{T_i}\right) \tag{3}$$

Two simple but important classes of resonant dynamics will be studied. They have the following form of transfer function:

$$\frac{Y(s)}{U(s)} = G_p(s) = \left(\frac{\alpha}{s + \alpha}\right)\left(\frac{1}{s^2 + 2\zeta_p s + 1}\right) \tag{4}$$

An example of a mechanical system having this type of dynamics is the speed control of a motor driving two flexibly coupled inertia loads [Astrom and Wittenmark, 1990; Wallenborg, 1987]. The first class of systems is one where the resonant modes are outside or almost outside the servo (closed-loop system) bandwidth, a typical class being $\alpha = 0.3$ in eqn. (4). This also means that it is difficult for the controller to increase damping of the resonant modes. When α is very small, the resonant modes are either negligible or are uncontrollable. Note that the term "uncontrollable" is used in the sense that the system will filter out any high frequency control signal designed to influence the resonant modes. When α is in the region of 0.3, the resonant modes are no longer negligible especially when tight control is exerted; yet they are not easily controllable as they are almost outside the servo bandwidth. The second class is one where the resonant modes are well inside the servo bandwidth, a typical case being $\alpha = 3$ in eqn. (4). In this case the controller will be able to increase the damping of the resonant modes. The open-loop step responses of the two classes of process dynamics are shown in Fig. 1 for $\zeta_p = 0.3$.

The damping factor ζ_p of the resonant poles is chosen to be 0.3 in the major part of this investigation. It is expected that the control performance will improve when ζ_p is increased and deterioriate when ζ_p is decreased. The extreme case of $\zeta_p = 0$ will be studied briefly to further explore the possible limits of PI/PID control. In all the simulations, step changes in the load introduced at the process input will be used to assess the controller performance in addition to setpoint responses.

3. Resonant Dynamics Outside The Servo Bandwidth

The control of the process of eqn. (4) with $\alpha = 0.3$ and $\zeta_p = 0.3$ is studied first. Fig. 1 shows that the resonant modes are just noticeable in the open step response. As the resonant modes should not be unduely excited in this case, an approximate analytical design will be developed.

3.1 PI Control

The transfer function of the PI controller in eqn. (3) has a zero at $s = -1/T_i$. It is tempting to simply use it to cancel the process pole at $s = -\alpha$. This is not advisible as the load response will be sluggish [Shinskey, 1988; Astrom and Hagglund, 1988]. For the purpose of comparison, we shall however examine the following controller based on pole-zero cancellation:

$$\text{Controller } (PI_1): \; k_c = 1; \; T_i = 3.3 .$$

The simulation results of Fig. 2 confirm that the load response is indeed sluggish. The setpoint response also has a large undershoot due to the large integral time resultant from this design.

It is known from design experience that for a good compromise between settling time and load attenuation, the controller zero on the s-plane should be placed two to three times away from the major process pole. Neglecting the presence of the resonant poles for the moment, we shall choose the controller zero at s = -3α. Thus,

$$T_i = \frac{1}{3\alpha} = 1.1.$$

Notice that this value is much lower than that obtained by cancelling the process pole. We then have

$$G_p(s)\,G_c(s) \approx \frac{\alpha K_c}{s}\left(\frac{s+3\alpha}{s+\alpha}\right) \tag{5}$$

which gives a closed-loop characteristic equation of :

$$s^2 + \alpha(1+K_c)\,s + 3\alpha^2 K_c = 0 \tag{6}$$

Comparing with a standard second order system of $s^2 + 2\zeta w_o s + w_o^2 = 0$, we obtain:

$$K_c^2 + (2-12\zeta^2)\,K_c + 1 = 0 \tag{7}$$

For positive values of k_c, we need $\zeta \geq 0.58$. At this value of ζ, we obtain the analytical solution of $k_c = 1$. If we instead choose a ζ of 0.707, we obtain $k_c = 0.27$ or 3.73. Taking into account the presence of the resonant modes which should not be unduely excited, a compromised value of the controller gain of 0.5 will be chosen. We thus have:

Controller (PI_2) : $k_c = 0.5$; $T_i = 1.1$.

The actual root loci and time responses are shown in Fig. 2 and 3. Notice that the damping of the resonant poles will also decrease with increasing gain. However, it is evident from Fig. 2 that the performance is superior to the design based on pole-zero cancellation in both setpoint and load responses.

3.2 PID Control

We shall not pursue the design based on cancelling the real or complex poles owing to its poorer performance [Astrom and Hagglund, 1988]. If we wish to have fast control with sufficient attenuation of the resonant modes, we have learnt from experience that the zeros of the PID controller in eqn. (2) should be chosen to be complex and placed some distance away from the process poles on the s-plane. A guideline is that their real parts should be about equal to the real parts of the process poles in order to place the root loci asymptotes near the imaginary axis; their imaginary parts should be about half of those of the resonant poles for this class of system. Note that the choice of such complex zeros is quite frequently made by experienced designers [Horowitz, 1963] or synthesized by the pole-placement design [D'Azzo and Houpis, 1988]. We shall thus proceed with a choice of controller zeros at $s = -0.5 \pm j\,0.5$, giving $T_i = 2$, $T_d = 1$ and a root locus plot as shown in Fig. 4. Following the simplified design procedure of Section 3.1, we obtain the characteristic equation:

$$s^2 + \alpha s + \alpha K_c T_d (s^2 + s + 0.5) = 0 \tag{8}$$

which will have positive values of k_c for $\zeta \geq 0.65$, at which $k_c = 2.5$. The gain will vary greatly around this region as indicated by the root loci of this design as shown in Fig. 4. A choice of $\zeta = 0.707$, for instance, gives $K_c = 0.75$. From the shape of the root loci originated from the resonant poles, we observe that a medium gain is a good compromise between speed and damping. We shall therefore propose two controllers, one with a more conservative controller gain of 1.25 and one with a larger gain of 2.5 which may be acceptable if a larger load attenuation is needed at the expense of more oscillatory response. We thus have:

Controller (PID_1) : $k_c = 1.25$; $T_i = 2$; $T_d = 1$.

Controller (PID_2) : $k_c = 2.5$; $T_i = 2$; $T_d = 1$.

Their performance is shown in Fig. 5. For the second controller, the larger attenuation of load effect is achieved with the higher gain but this is accompanied by an unacceptably oscillatory setpoint response. Fortunately, this may be reduced substantially by the use of the setpoint weighting factor β [Astrom and Hagglund, 1988; Hang et.al, 1990]. With a choice of $\beta = 0.2$, the setpoint response is improved as shown in Fig. 5.

3.3 The Extreme Case of $\zeta_p = 0$

From the shapes of the root loci shown in Fig. 3 and 4, it is evident that both PI and PID controllers will be effective for $\zeta_p > 0.3$. For $\zeta_p < 0.3$, their effectiveness will deterioriate since the root loci from the resonant poles will move towards the right-half of the s-plane. The closed loop system may be unstable when ζ_p very close to or equal to zero.

A possible solution for the case of PI control is to use a negative gain and negative integral time, thus providing a negative proportional action but a positive integral action. The controller zero is then placed on the right-half plane; again this type of controller zero is sometimes synthesized by the pole-placement controller [D'Azzo and Houpis, 1988], and has been suggested long ago [Evans, 1950]. A choice of $T_i = -4$ gives the root loci as shown in Fig. 6. Note that a larger T_i (remembering that T_i is negative) may lead to instability while a smaller T_i may lead to sluggish response. The approximate design similar to that of Section 3.1 will give a K_c of -0.4. We thus have:

Controller (PI_3) : $K_c = -0.4$; $T_i = -4$.

The setpoint and load responses of Fig. 7 show that the performance of this controller is reasonable considering the difficult situation. Note the vast improvement in setpoint response by using $\beta = 0$.

A slightly improved control in damping may be achieved using a PID controller with all negative settings (giving negative proportional action but positive integral and derivative actions). We do not recommend it as the improvement is only marginal.

3.4 Summary

When the resonant modes are almost outside the servo bandwith, a typical case being $\alpha = 0.3$ in eqn. (4), we have explored the effectiveness of PI and PID control.

An approximate analytical design procedure has been developed to achieve reasonably fast control response without undue excitation of the resonant modes. In the case of $\zeta_p \geq 0.3$, PID control offers improvement over PI control. Setpoint weighting has been found effective in improving the setpoint response when the PID controller is tuned for a tight load response. For convenience of comparison, their performances are shown together in Fig. 8. This advantage will diminish as ζ_p gets smaller. In the extreme case of $\zeta_p = 0$, only a PI/PID controller with negative settings is able to provide stable control of the system. This is an area where a more sophisticated, higher order controller should be considered [Astrom and Wittenmark, 1990].

4. Resonant Dynamics Inside The Servo Bandwidth

The process of eqn. (4) with $\alpha = 3$ and $\zeta_p = 0.3$ is studied in this section. As the resonant modes are well within the servo bandwidth, the controller has to ensure sufficient damping of the resultant oscillations. An approximate design procedure will also be developed.

4.1 PI Control

As a PI controller does not provide good damping action owing to the net phase lag it contributes, the focus of the controller design is on how to reduce the resonance effect without over-reduction of response speed. We have

$$G_p(s)\,G_c(s) = [\frac{K_c(s + 1/T_i)}{s}]\,(\frac{\alpha}{s + \alpha})\,(\frac{1}{s^2 + 2\zeta_p s + 1}) \qquad (9)$$

If we choose T_i sufficiently large, e.g. $T_i = 2$ or larger to reduce the net phase lag, we may for the moment neglect the effect of $(s + 1/T_i)/s$ and hence obtain the following characteristic equation as though only a proportional control is used:

$$s^3 + (\alpha+2\zeta_p)\,s^2+(1+2\zeta_p\alpha)\,s+(1+K_c)\,\alpha=0 \qquad (10)$$

It is straight forward to obtain the stability condition:

$$K_c \leq (1+2\zeta_p/\alpha)\,(1+2\alpha\zeta_p)-1$$

For $\zeta_p = 0.3$ and $\alpha = 3$, K_c should therefore be smaller than 2.36. From the root loci shown in Fig. 9, it is evident that we should choose a smaller gain in order to have acceptable damping. Choosing a gain margin of 4, we thus have:

Controller (PI_4) : $K_c = 0.6$; $T_i = 2$.

The performance of this design is demonstrated in Fig. 10. It has been found that not much may be gained by fine tuning this controller and its performance is indeed worse than in the case of $\alpha = 0.3$. A PI controller is thus not very effective in controlling the process.

4.2 PID Control

The PID controller has a net phase lead which may by used to damp the resonant modes. A guideline is that the real parts of the controller zeros should be small so that the asymptotes for the root loci will be located to the left side of the resonant poles; yet to obtain a high controller gain we should place the controller zeros to the left of the resonant poles. We thus choose the controller zeros at $s = -0.8 \pm j\,0.6$ which give $T_i = 1.6$ and $T_d = 0.625$, and the root loci are shown in Fig. 11. Neglecting for a moment the effect of the resonant poles, we have the following characteristic equation:

$$s^2 + \alpha s + K(s^2 + 1.6s + 1) = 0 \tag{11}$$

where $K = K_c T_d$. With $\alpha = 3$ and comparing with the standard second order system, we have:

$$(2.56 - 4\zeta^2)K^2 + (3.2\alpha - 4\zeta^2)K + \alpha^2 = 0 \tag{12}$$

With this choice of controller zeros, we already know the constraint $\zeta > 0.8$. The gain is infinite at this value of ζ. A convenient point to obtain a practical gain is at $\zeta = 1$, which gives a K of 5.11 and hence a K_c of 8.18. From the shapes of the root loci as shown in Fig. 11, we expect that the influence of the resonant poles will reduce at higher gains. But a high controller gain will be limited by the pole of the derivative filter (pole at T_d/N_f) which has been neglected in the analysis. A compromise is needed and a choice of $K_c = 5$

gives:

Controller (PID_3) : $K_c = 5$; $T_i = 1.6$; $T_d = 0.625$; $\beta = 0.2$.

The performance of this controller as shown in Fig. 12 is excellent. The role of the setpoint weighting factor β in improving the setpoint response is also evident.

4.3 The Extreme Case of $\zeta_p = 0$

From the shapes of the root loci shown in Fig. 9 and 11, it is clear that the PI and PID controllers will be effective when $\zeta \geq 0.3$. For a smaller value of ζ_p, the effectiveness of the PI controller will deterioriate quite rapidly as the roots from the resonant poles move towards the right-half of the s-plane. Unlike the earlier case of $\alpha = 0.3$, even the use of negative gain and negative integral time will not be able to produce a reasonable control performance. The shape of the root loci for PID control, however, indicates a good possibility of stable control if the controller zeros are placed sufficiently close to the imaginary axis. For the extreme case of $\zeta_p = 0$ while $\alpha = 3$, we choose the the controller zeros at ($-0.4 \pm j\,0.6$) resulting in the root loci asymptotes at $s = -1.1$. The root loci are shown in Fig. 13. Following the same approximate design procedure of Section 4.2, we obtain $K_c = 2.5$ and $\beta = 0.2$:

Controller (PID_4) : $K_c = 2.5$; $T_i = 1.54$; $T_d = 1.25$; $\beta = 0.2$.

The setpoint and load responses of Fig. 14 demonstrate the reasonably good performance of this controller.

4.4 Summary

When the resonant modes are well inside the servo bandwidth, a typical case being $\alpha = 3$ in eqn. (4) as studied above, PI control is found to be quite inadequate for high performance as the controller does not provide a net phase lead required for good damping. PID control in this case offers a drastic improvement. A comparison of their performance as shown in Fig. 15 clearly demonstrates the great advantage of PID over PI control. In the extreme case of $\zeta_p = 0$, good performance can still be achieved by a PID controller with positive settings. An approximate analytical design procedure has also been developed to design PI and PID controllers for this class of resonant dynamics.

5. Concluding Remarks

We have studied the effectiveness of PI/PID control for two major classes of resonant dynamics. In the first class where the resonant dynamics are almost outside the servo bandwidth, PI control may be sufficient while PID control may be used to improve load attenuation at the expense of reduced damping. In the second class where the resonant dynamics are well inside the system bandwidth, the performance of the PI controller may be inadequate and a great improvement in both speed of response and damping will be provided by a PID controller.

Approximate analytical design procedures have been developed to compute reasonable settings of the controllers. There was no intention to provide optimal solutions and the root loci plots being shown were only used to substantiate the design choices. The application of setpoint weighting to improve the setpoint responses at high controller gains required for good load rejection is shown to be an useful option to the designers.

There remain other topics not being studied in this report. For instance, the presence of anti-resonance zeros [Wallenborg, 1987] will make the control problem more complex but its stabilising effect should assist the controller. The sensitivity of the designs to process parameter uncertainties and the auto- and self-tuning features for PI/PID control of resonant dynamics are also important topics to be investigated.

<u>References</u>

Astrom, K.J. and Hagglund, T. (1988), Automatic Tuning of PID Controllers, Instrument Society of America.

Astrom, K.J., Hang, C.C., Persson, P. and Ho, W.K. (1991), Towards Intelligent PID Control, Automatica, 28, pp 1-9.

Astrom, K.J. and Wittenmark, B. (1990), Computer Controlled Systems, Prentice Hall, 2nd Edition.

D'Azzo, J.J. and Houpis, C.H. (1988), Linear Control System Analysis and Design, McGraw-Hill, 3rd Edition.

Evans, W.R. (1950), "Control System Synthesis by Root Locus Method", Trans. AIEE, 69, pp 1-4.

Horowitz, I.M. (1963), Synthesis of Feedback Systems, Academic Press.

Shinskey, F.G. (1988), Process Control Systems, McGraw-Hill, 3rd Edition.

Wallenborg, A. (1987), Control of Flexible Servo Systems, Master thesis (Report TFRT-3188), Lund Institute of Technology, Sweden.

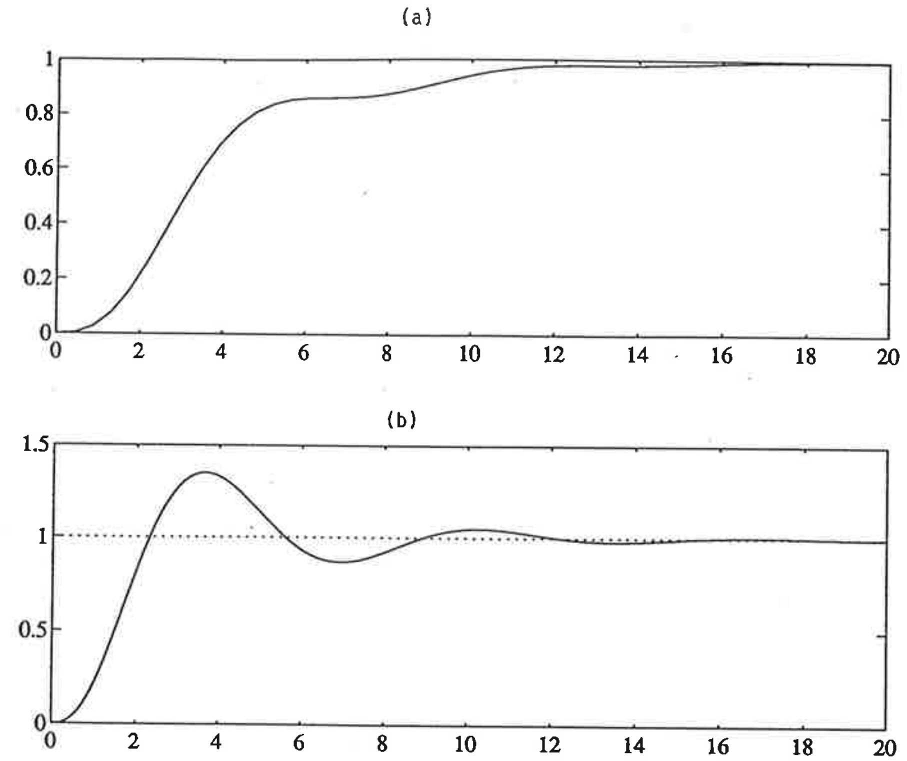

Fig. 1 Open-loop Step Responses
(a) α = 0.3; (b) α = 3

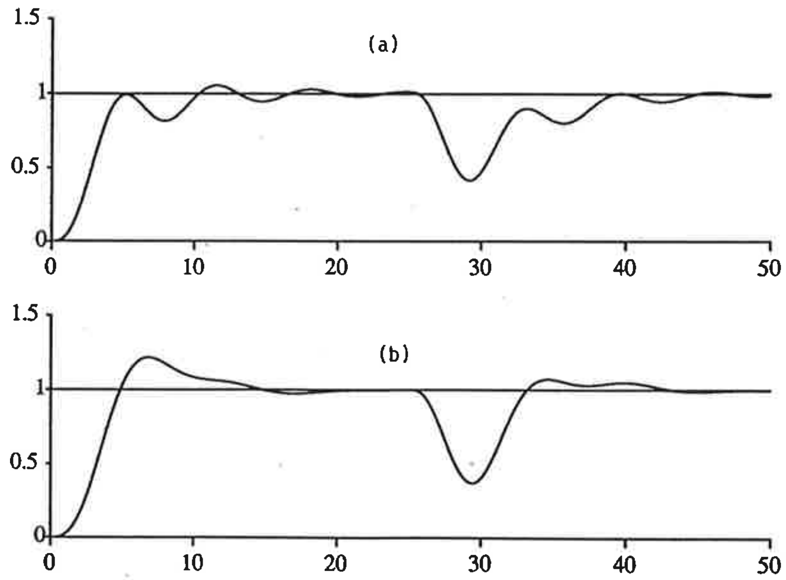

Fig. 2 Performance of PI Control (α = 0.3 ; ζ_p = 0.3)
(a) PI_1 ; (b) PI_2

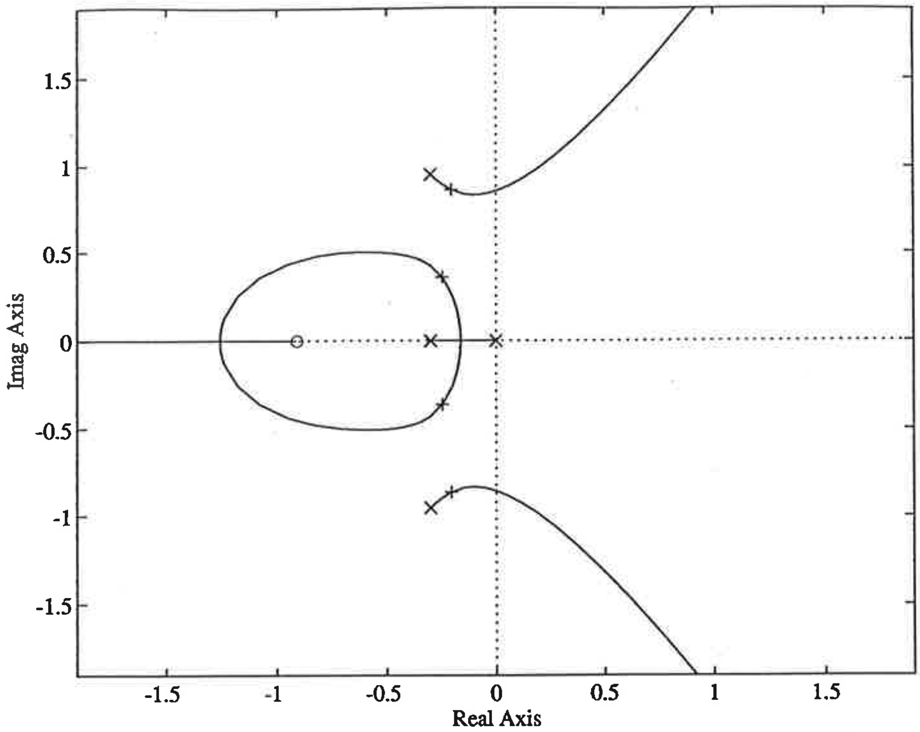

Fig. 3 Root Loci For PI Control (α = 0.3 ; T_i = 1.1)
('+' showing roots at K_c = 0.5)

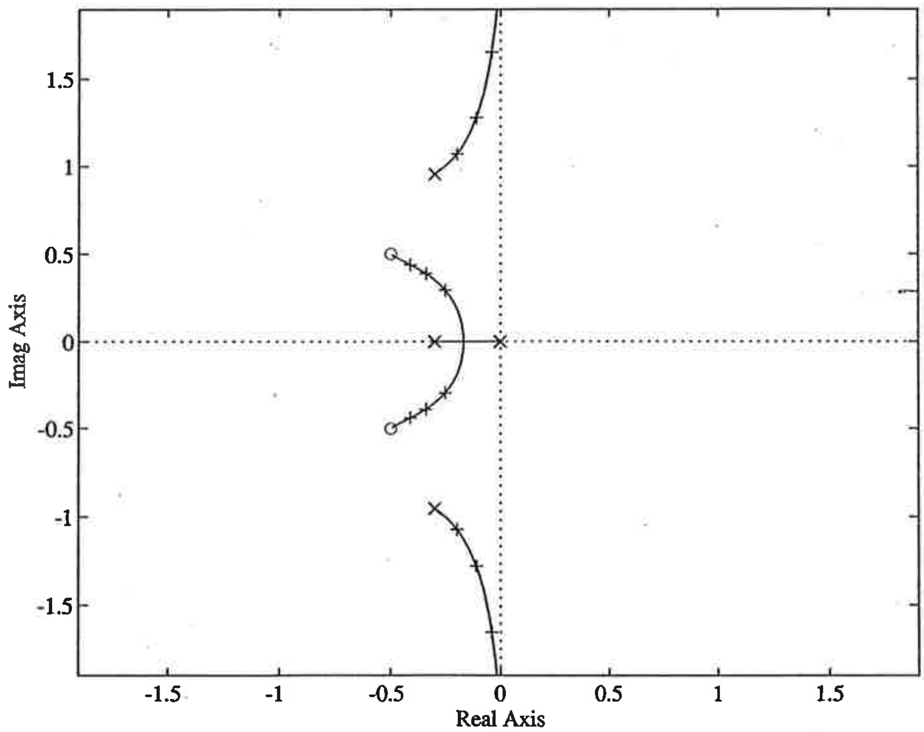

Fig. 4 Root Loci For PID Control (α = 0.3 ; T_i = 2 ; T_d = 1)
('+' showing roots at K_c of 1.2, 2.9 and 6.6)

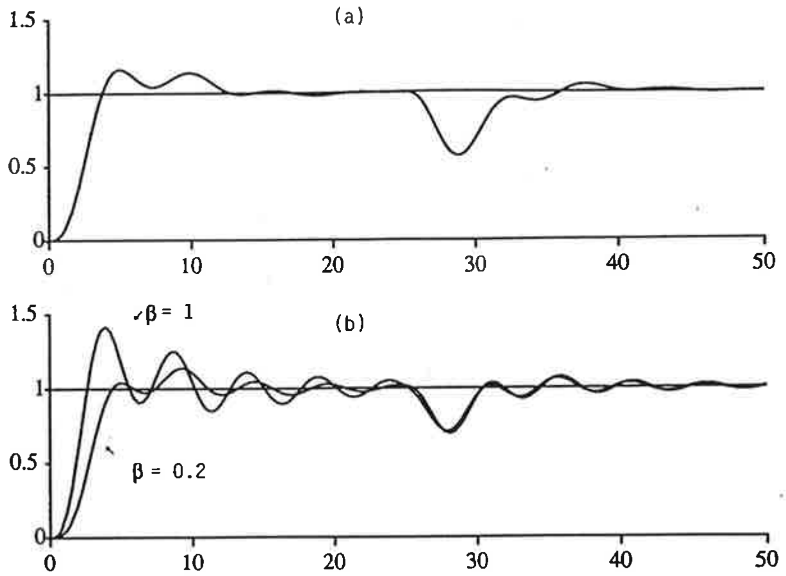

Fig. 5 Performance of PID Control (α = 0.3 ; ζ_p = 0.3)
(a) PID_1 ; (b) PID_2

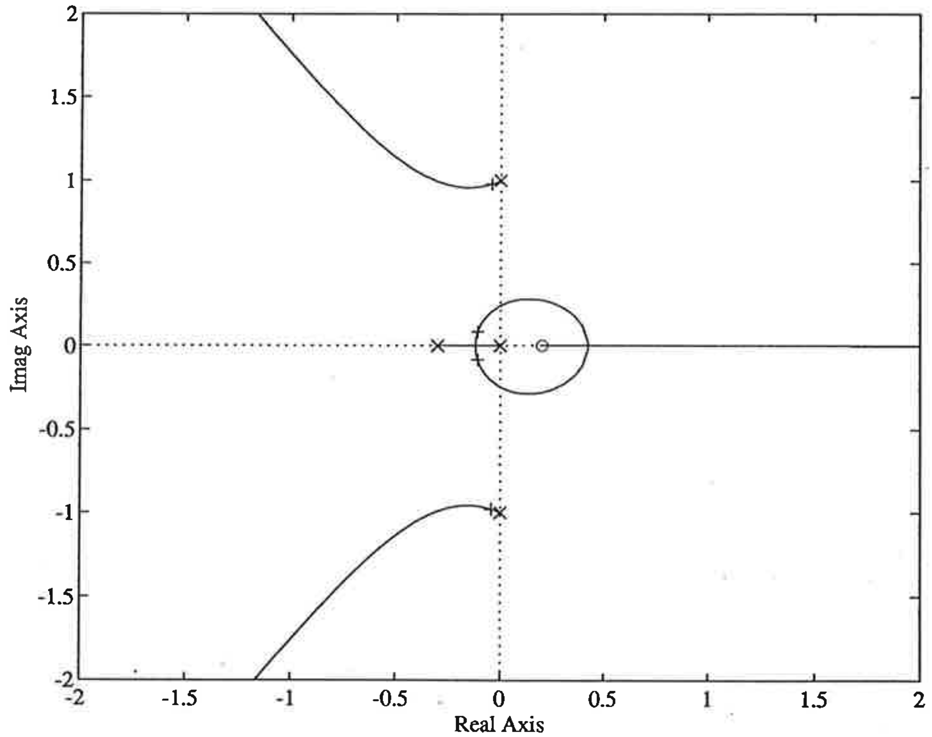

Fig. 6 Root Loci with PI Controller Having a Negative Gain
(α = 0.3 ; ζ_p = 0)
('+' showing roots at K_c of 0.3)

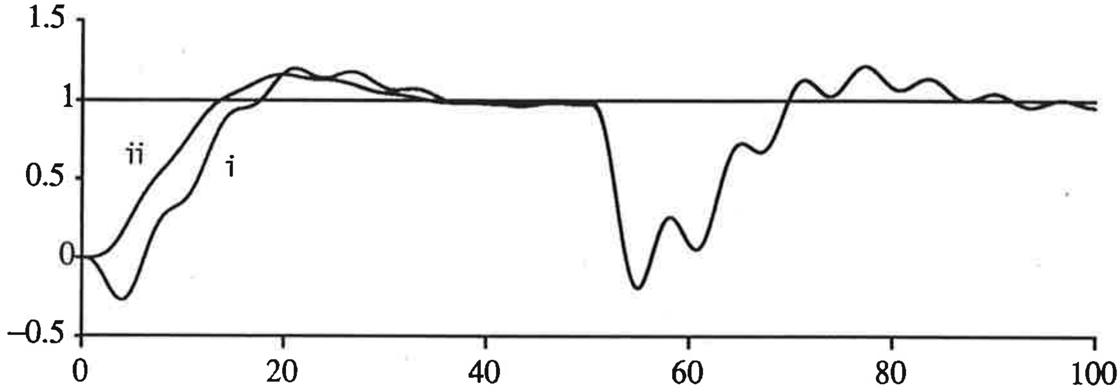

Fig. 7 Performance of Controllers With Negative Gains
(α = 0.3 ; ζ_p = 0) : (i) β = 1 ; (ii) β = 0

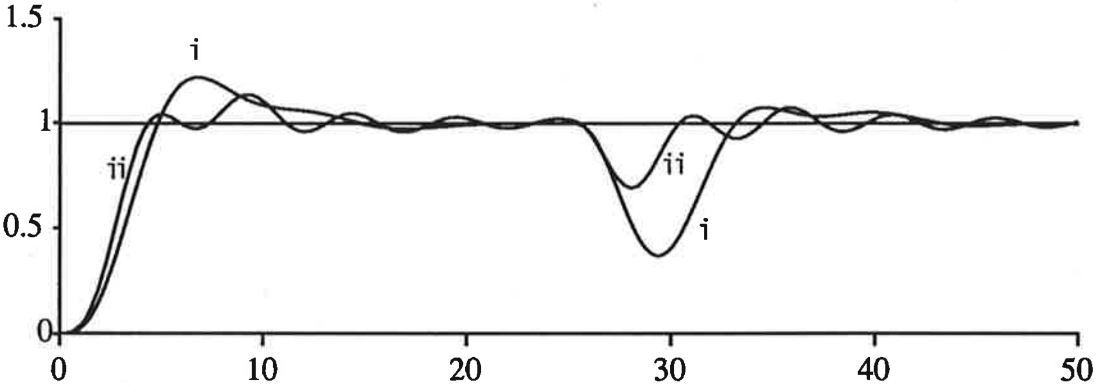

Fig. 8 Performance Comparison (α = 0.3 ; ζ_p = 0)
(i) PI Control ; (ii) PID Control

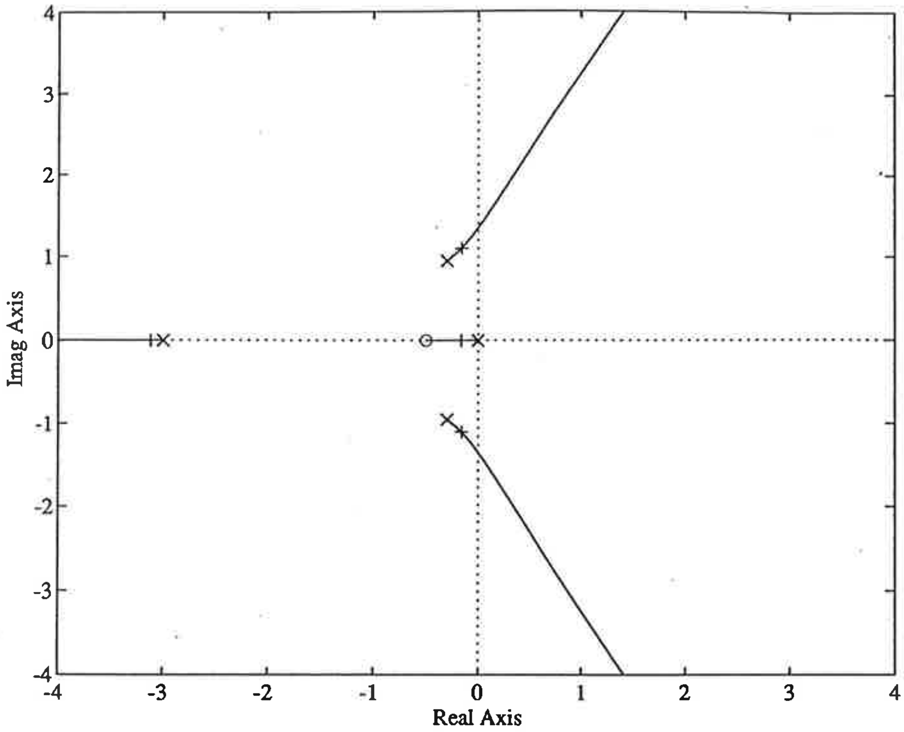

Fig. 9 Root Loci For PI Control (α = 0.3 ; T_i = 2)
('+' showing roots at K_c of 0.4)

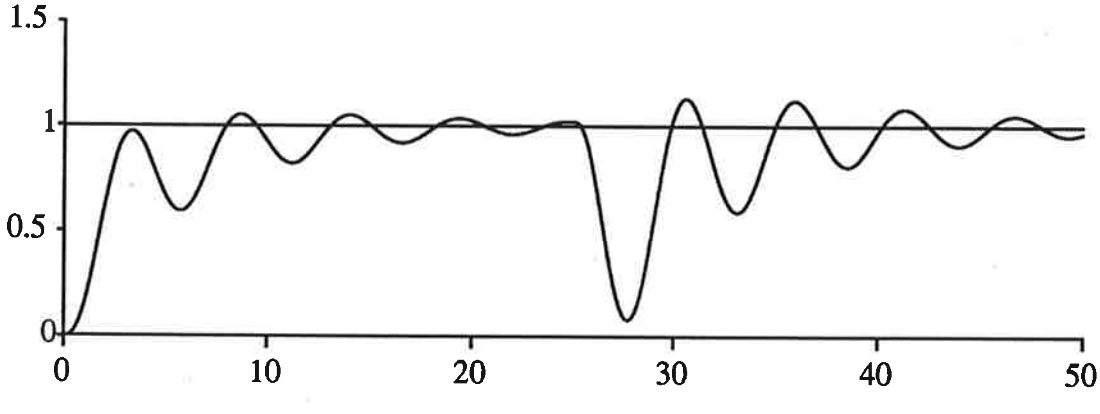

Fig. 10 Performance of PI Control (α = 0.3 ; ζ_p = 0.3)

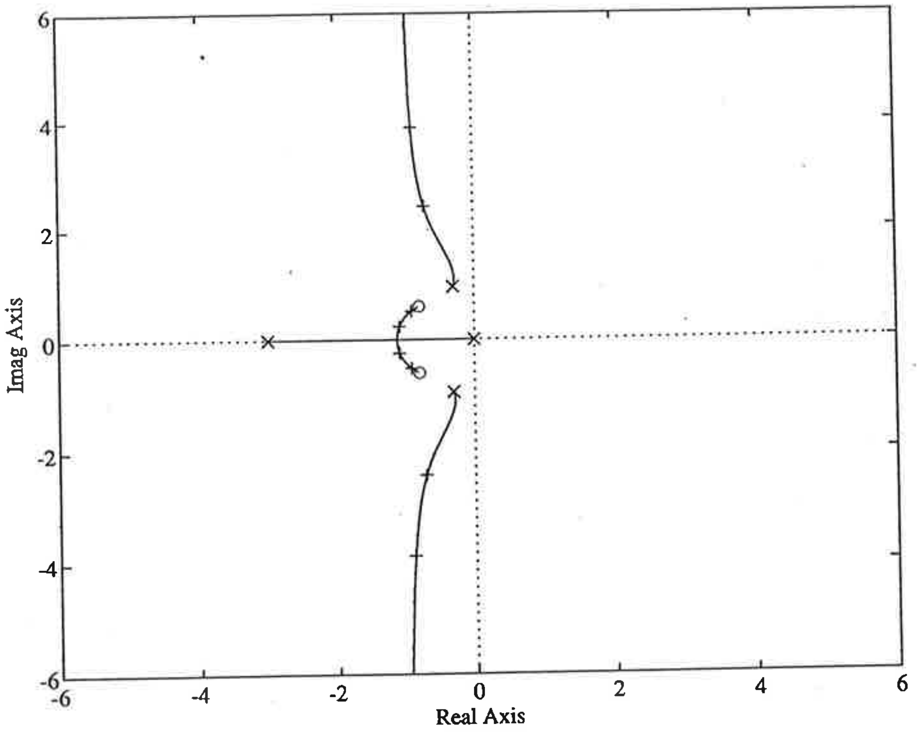

Fig. 11 Root Loci For PID Control (α = 3 ; T_i = 1.6 ; T_d = 0.625)
('+' showing roots at K_c of 4.3 and 9.3)

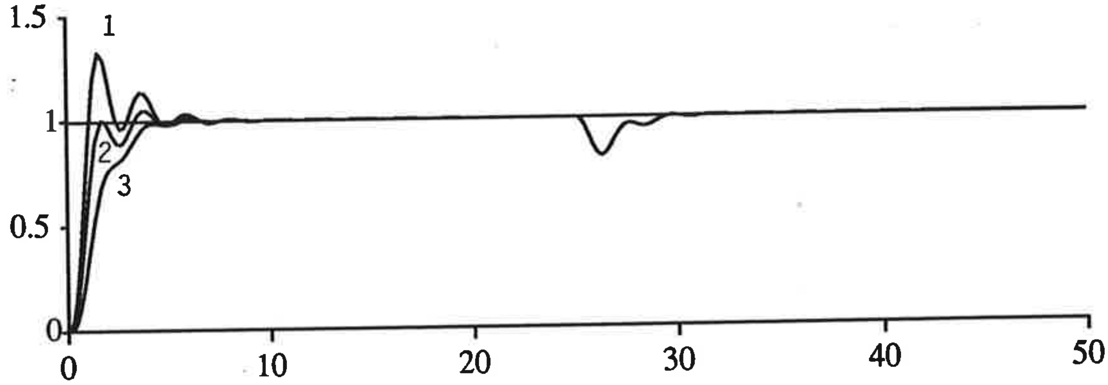

Fig. 12 Performance of PID Control (α = 3 ; ζ_p = 0.3);
Controller PID_3 with (1) β = 1 ; (2) β = 0.6 ; (3) β = 0.2

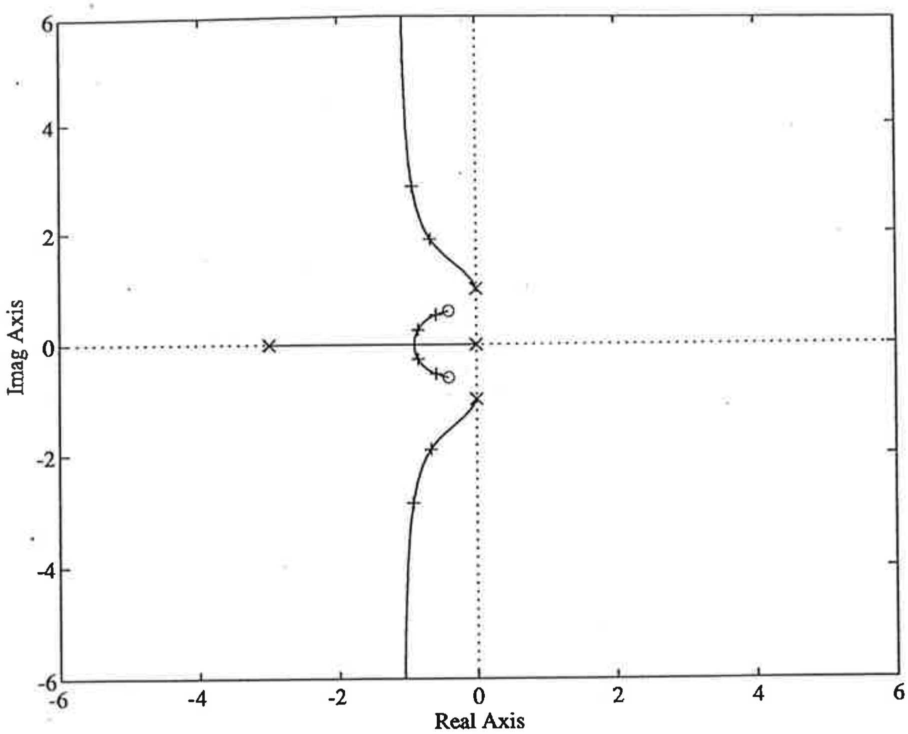

Fig. 13 Root Loci For PID Control
(α = 3 ; ζ_p = 0; T_i = 1.54 ; T_d = 1.25)
('+' showing roots at K_c of 1.6 and 3)

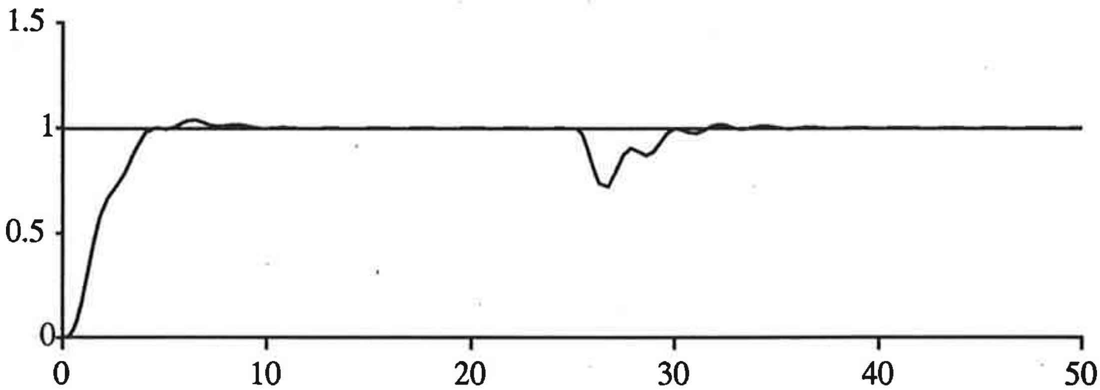

Fig. 14 Performance of Control (α = 3 ; ζ_p = 0)
With PID_4

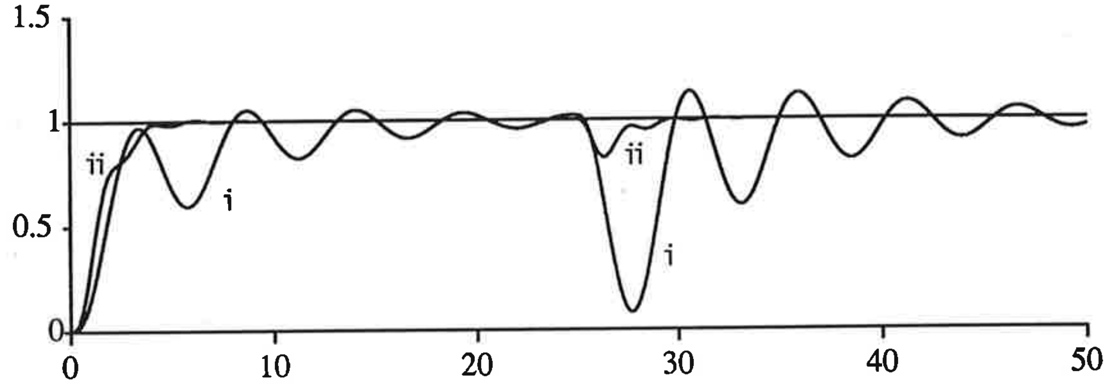

Fig. 15 Performance Comparison (α = 3 ; ζ_p = 0.3)
(i) PI Control ; (ii) PID Control