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Fuzzy Anti-Reset Windup for Heater Control

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Abstract

In this report a fuzzy anti windup scheme for an observer for the reference value of the flow temperature of a heating installation is designed. The observer consists of an averager in series with a characteristic curve. A nominal design of the time constant of the averager is done by considering only the heater dynamics. When there are large load disturbances or set point changes, the reference value runs away from the heater due to state and actuator constraints of the heating installation. To avoid this fuzzy anti windup is considered. The resulting scheme has only one parameter to tune. To obtain a good value of this parameter simulations are performed. It is seen that the scheme is robust with respect to the parameter, and thus it is easy to find a good value. Further the simulations show that the performance is improved when anti windup is used. The resulting scheme has successfully been incorporated in the heating controller Sigmagyr RVP110 manufactured by Landis & Gyr AG in Switzerland.

Key words
Heater control, Anti-Reset Windup, Fuzzy Control

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1. Introduction

In this report part of the design procedure for the heating controller Sigmagyr RVP110 will be described, see Figure 1. This controller is manufactured by Landis & Gyr AG, and is used mainly for small gas and oil heaters which are installed in single family homes, residential flats and small commercial buildings. This product covers the lower range of temperature control applications. Simple hydraulics and few sensors are typical for this market segment. Figure 2 shows a standard heating installation for which the controller can be use. The controllers are delivered to the original equipment manufacturer of the heaters which build them into their products. The controller has several modes of operation: the setpoint of the water temperature of the heater can either be driven by the outdoor temperature or by the load or by both. The mode discussed in this report is the load driven one. This mode can only be used, if the radiators installed in each room are equipped with a temperature controller acting on the valve positions. For this case no outdoor sensor is needed.

High performance control is always of interest. Usually this is not accomplished by means of only sophisticated analysis and design, but also expensive measurements are needed. This is due to the fact that it is difficult to design observers whenever the process to be controlled is non-linear or has state or actuator constraints. One of the more apparent constraints in heater control is the maximal power that the burner can deliver. This is usually as small as 150% of the maximal power demand in stationarity. In Sigmagyr RVP110 an observer for the setpoint of the flow temperature is used. It will be seen that it is possible to obtain sufficiently high performance without expensive measurements. To this end fuzzy logic will be used. However it must be stressed that without traditional analysis tools such as the method of harmonic balance it would not have been possible to find a good observer.

The idea used in the observer is that the flow temperature needed in stationarity in order to keep the control valves acting on the radiators in operational range is related to the stationary power demand via a characteristic curve. Thus by putting this curve in series with an averager that computes the stationary power demand from the heater on-off signal an observer is obtained. The problem considered in this report is to choose the time constant in the averager.

Using the method of harmonic balance will give some guidelines on how to choose the time constant when the heating installation is operating in stationarity. However, when not operating in stationarity, the non-linear behavior of the heating installation, and primarily the hard constraints due to saturations of the hydraulics and due to limiting power of the heater, will cause windup of the averager, i.e. the reference value for the flow temperature will run away from the flow temperature due to the constraints. This has to be avoided in order to prevent an unstable behavior. To this end anti windup will be considered. Windup is here used in a more general sense than usual. Normally the notion of windup is used when the integrator state of a controller becomes large due to saturation of the control signal. Here the reference value for the flow temperature becomes large when the heating system cannot deliver the power needed, and this may thus be interpreted as windup of the reference value. It will be seen that a fuzzy generalization of the so called conditional integration methods is an appealing scheme for anti windup when there are constraints not only in the actuator signal but also other constraints within the process as is the case with the heating process.

The fuzzy scheme for the time constant of the averager has one tuning parameter. To find a good value of this parameter simulations have to be performed. It will be seen that the scheme is robust with respect to this parameter, and that it thus is easy to find a good value of the parameter. Further the simulations show that anti windup improves the performance.

Outline

The report is organized as follows. In Section 2 the control problem is stated. Constraints on performance due to power constraints of the heater as well as water flow constraints of the hydraulics are discussed. In Section 3 the model of the heater is investigated. The analysis is done both in time domain and frequency domain. It result in a lower bound on the time constant of the averager. In Section 4 first anti windup and fuzzy control are reviewed. Then the fuzzy anti windup scheme is proposed. In Section 5 the proposed scheme is simulated on a model. Finally, in Section 6, the results are summarized, and suggestions for further research are given.

Figure 1. Heating controller Sigmagyr RVP110.01.

2. Control Problem

In this section the control problem will be described. The goal is to find a cheap solution with few measurements but which still gives as high performance as possible.

In Section 2.1 some of the different existing solutions are reviewed. Their less good performance is the reason for this work. In Section 2.2 the control

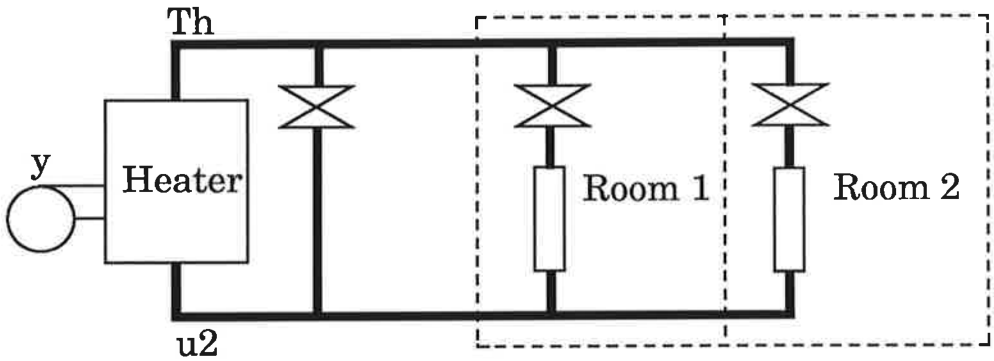

Figure 2. The heating installation without control.

objectives are discussed. In Section 2.3 the impact of the constraints of the controlled process are discussed. Finally, in Section 2.4 the results of this section are summarized.

Background

The background to this control problem is described in RI91. The process to be controlled is a heating installation in a house, see Figure 2. The heater water

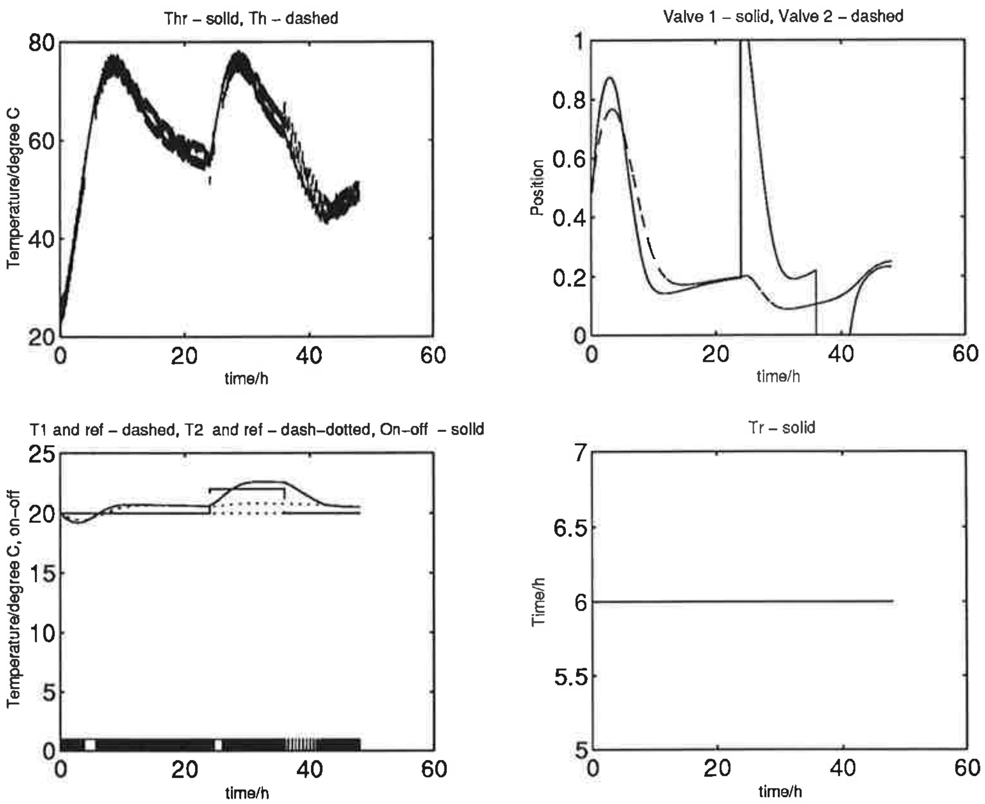

Figure 3. Simulation with a constant value of the time constant $T_r = 6$ h.

temperature is controlled by means of a relay acting on the burner, and each room is equipped with a temperature controller acting on the valve position of the radiators. There is no mixing of the return water from the radiators with the heater water. All return water goes directly back to the heater. Thus the temperature of the water to the radiators, i.e. the flow temperature, is controlled by controlling the temperature of the heater.

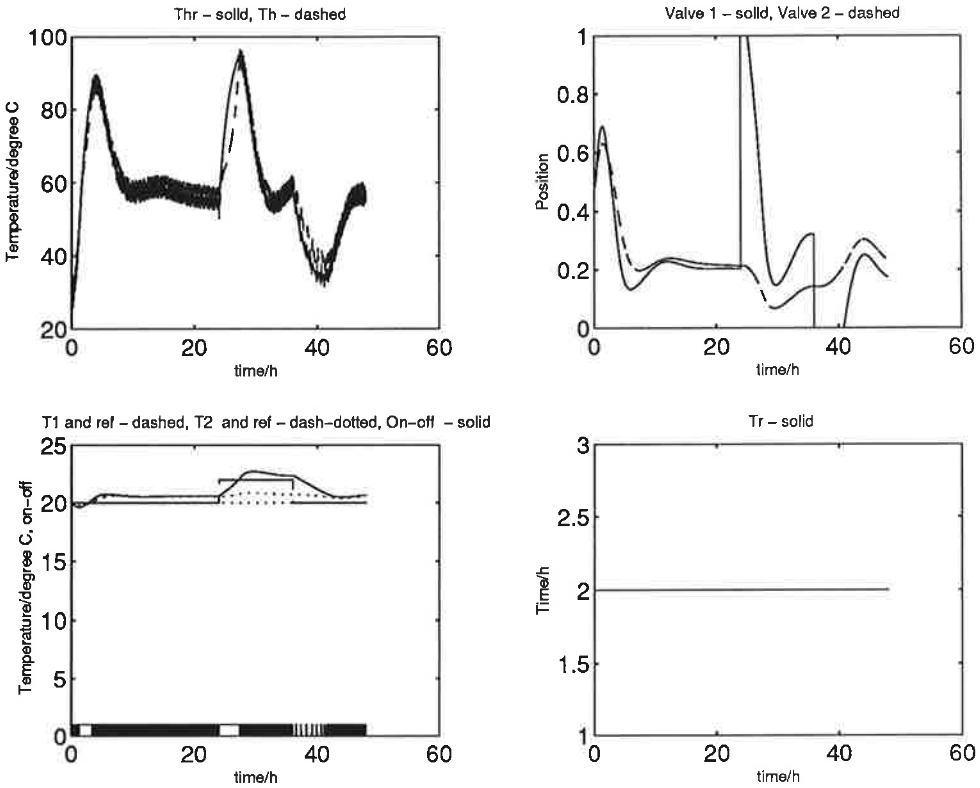

Figure 4. Simulation with a constant value of the time constant $T_r = 2$ h.

The reference value for the flow temperature can be obtained as a feedforward from the outdoor temperature via some characteristic curve, or it can be set to a constant value. The feedforward solution gives higher performance. Here a cheaper solution with no outdoor temperature measurement is considered which still gives high performance. The idea, as described in R89, is to build an observer for the flow temperature needed in order to keep the valves of the radiators in operational range as good as possible, i.e. to prevent them to saturate. To make the solution cheap only measurements of the flow temperature and thus indirectly also of the heater on-off signal are used in the observer.

The observer is composed of two parts. This is due to the fact that it is possible to prove that there is a certain relation, characteristic curve, between the power demand in stationarity and the flow temperature needed in order to keep the radiator valves 50 % open in average. Thus one part is the characteristic curve, and the other part is an averager for computing a signal proportional to the stationary power demand from the heater on-off signal. The characteristic curve is dependent on the house characteristics, and can be obtained in a similar way as the traditional curve from outdoor temperature, R89. The averager is a transfer function of first order with unit stationary gain and a time constant to be chosen.

Different values of the time constant have been considered. First a rather large value was used. This caused, as is seen in Figure 3, too slow a step response when changing the set point for the temperature in a room. Then adaptation was proposed. This caused faster step responses, but sometimes the reference value for the flow temperature started to oscillate. This is obviously not desirable. In this report a nominal design of the time constant will be done from the heater dynamics, and then fuzzy anti windup will be considered to avoid the oscillations.

Control Objectives

The primary control objective for the heater control is to keep the valves of the radiators in operational range, such that the valves are able to increase or decrease the water flow when there are control errors in the room temperatures. The secondary goal is to speed up the temperature control in the rooms by also increasing the flow temperature when the valves open, and decreasing it when the valves close.

This may of course be accomplished by choosing a small time constant of the averager. However, too small a time constant may, as was mentioned in the previous subsection, cause undesirable oscillations, see Figure 4.

Constraints

The nominal design of the averager will be done by only considering the dynamics of the heater. This is advantageous, since these dynamics are well known. Then some fix, i.e. anti windup, is needed in order to cope with the less well-known house dynamics whenever these interfere with the desired performance. This will be described in Section 4. Here only the motivation for the anti windup will be given.

There are different types of constraints that can cause detoriation of performance. These can be classified into two groups—hard constraints and soft constraints. Examples of the former ones are saturations of control signals and limitations of state variables within the process. Examples of the latter ones are non-linearities, less abrupt than the previous ones, and unmodeled dynamics.

Both types mentioned above are present in the heater control problem. The limitation of the heater power and the limitation of the water flow are hard constraints, whereas the non-linearities of the heater as well as the unmodeled house dynamics are soft constraints.

As is seen in Figure 4 these constraints do not interfere with the performance in stationarity. However, as soon as there is a load disturbance or a change of reference value sufficiently large, the constraints result in bad performance. In Section 3 the nominal design considering only the heater dynamics will be done, and then in sections 4 and 5 an anti windup scheme will be designed to cope with the constraints.

Summary

In this section the control problem has been described. The idea is to design a steady state observer for the reference value of the flow temperature. If more measurements had been to disposal, it would most certainly have been possible to design a more sophisticated observer giving higher performance of the overall control. However, this would have been more expensive due to the need of e.g. a sensor for the return temperature.

The results from previous designs have been reviewed. The control objectives as well as the impact of the constraints within the process have been discussed. It seems to be a good idea to design a nominal observer based on only the well-known heater dynamics, and then to make a modification of this design by means of fuzzy anti windup to cope with the in the nominal design unmodeled dynamics and constraints of the house and hydraulics.

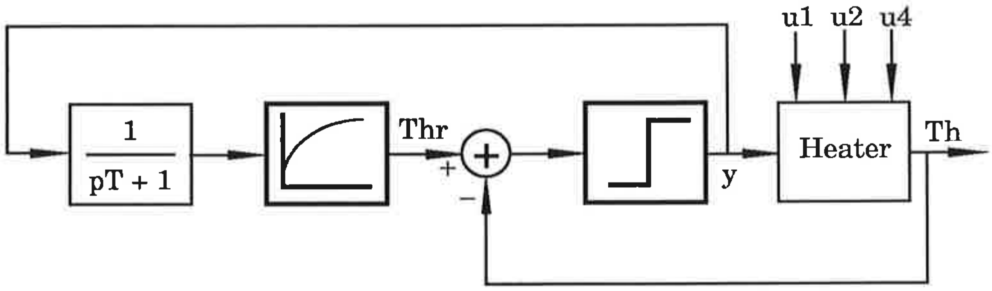

Figure 5. The controlled heater with reference value generated via the characteristic curve.

3. Analysis

In this section analysis of the controlled heater will be done. Some guidelines for how to chose the averager will be given. Since these guidelines are only based on the heater model which does not include the house dynamics, the final design of the averager has to be done iteratively using simulations. This will be described in Section 5.

In Section 3.1 the differential equations for the heater are given. A transfer-function description is derived, and some simple approximations are made. The averager in series with the characteristic curve is also described. It will be seen that this part can also be approximated with a transfer-function description. Finally the controlled heater with reference value generated via the characteristic curve is rewritten into a standard form that will be used later on in Section 3.3. In Section 3.2 the transient or high-frequency behavior of the heater is investigated. This will give some guidelines on how to chose the averager. In Section 3.3 the more interesting behavior around the ultimate frequency of oscillation is investigated by means of the method of harmonic balance. Further an averager is designed using the observations made. Finally, in Section 3.4 the results are summarized.

Model

The controlled heater with reference value generated via the characteristic curve is described in Figure 5. The idea is that the output $\bar{y}$ of the averager

$$\frac{1}{pT_r + 1} \tag{1}$$

is proportional to the static power demand. Using the characteristic curve

$$T_h^r = 22 + 48\left(\frac{\bar{y}}{1-\xi}\right)^{1/1.3}, \quad \xi \in [0.02, 0.7] \tag{2}$$

for the heater temperature will give a reference value T_h^r for the heater related to this power demand, which then is used in the on-off control of the heater. The parameter ξ is used to adjust the curve to the specific house that is heated. In Figure 6 the curve is plotted for $\xi = 0.5$. It is seen that the curve is almost linear. A good linear approximation is given by

$$T_h^r = T_0 + K_r\bar{y} \tag{3}$$

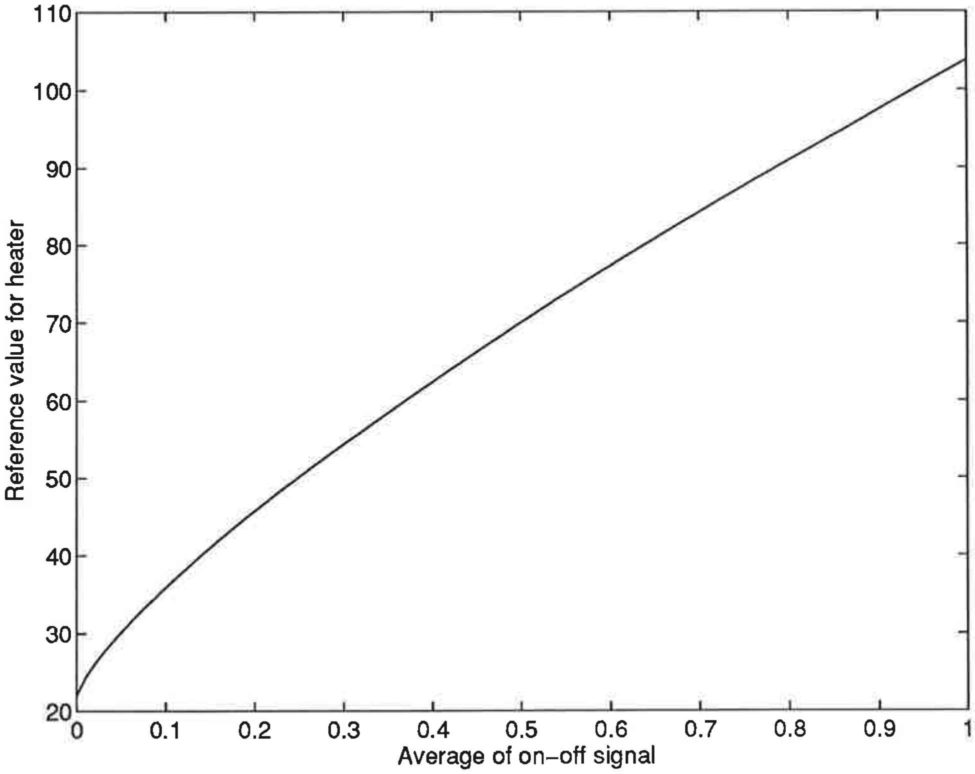

Figure 6. Plot of the characteristic curve relating the averaged on-off signal to the static power demand for the adjustment $\xi = 0.5$.

where

$$\begin{aligned} T_0 &= T_h^r(0.5) - 0.5\frac{dT_h^r(0.5)}{d\bar{y}} = 70 - 0.5 \cdot 74 = 33 \\ K_r &= \frac{dT_h^r(0.5)}{d\bar{y}} = 74 \end{aligned} \tag{4}$$

The averager in series with the characteristic curve may then be approximated by

$$T_h^r = T_0 + \frac{K_r}{pT_r + 1} y \tag{5}$$

where y is the heater on-off signal—0 for off and 1 for on. This approximation will be used in the sequel. If another user adjustment than $\xi = 0.5$ is used, then other values of T_0 and K_r will be obtained. It can be shown that $T_0 \in [28, 39]$ and that $K_r \in [44, 110]$ for $\xi \in [0.02, 0.7]$.

The differential equations for the heater, given in R92c, can be summarized in

$$\begin{aligned} \frac{dx}{dt} &= A(u_1)x + B(u_1)u \\ T_h &= Cx \end{aligned} \tag{6}$$

where

$$\begin{aligned} A &= \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \\ B &= \begin{pmatrix} 0 & b_{12} & 0 \\ b_{21} & 0 & b_{23} \end{pmatrix} \\ C &= \begin{pmatrix} 0 & 1 \end{pmatrix} \end{aligned} \tag{7}$$

and where

$$
\begin{aligned}
a_{11} &= -h_6(u_1)/\left[450p_8\right] \\
a_{12} &= h_6(u_1)/\left[450p_8\right] \\
a_{21} &= h_6(u_1)/\left[4180p_9\right] \\
a_{22} &= -\left[h_6(u_1) + 4180u_1 + \gamma\right]/\left[4180p_9\right] \\
b_{12} &= p_1(1-p_6)/\left[450p_5p_8\right] \\
b_{21} &= u_1/p_9 \\
b_{23} &= \gamma/\left[4180p_9\right] \\
h_6(u_1) &= p_{10}\left(1 + 9\sqrt{4180p_3u_1/p_1}\right) \\
\gamma &= p_1p_7/\left[p_5(p_2-p_4)\right]
\end{aligned}
\tag{8}
$$

The states $x = (x_1\ x_2)^T$ are the temperatures of the heater wall, and the heater water, the same as the flow temperature. The inputs u_1 and $u = (u_2\ u_3\ u_4)^T$ are the water flow, the return temperature, the heater on-off signal, and the temperature outside the heater. Note that $u_3 = y$. The parameters p_1 to p_{10} are constants depending on the specific heater. The values that will be used in the numerical examples as well as in some of the simulations are

$$
\begin{aligned}
p_1 &= 18000\text{W} \\
p_2 &= 80\text{C} \\
p_3 &= 15\text{K} \\
p_4 &= 20\text{C} \\
p_5 &= 0.915 \\
p_6 &= 0.070 \\
p_7 &= 0.015 \\
p_8 &= 100\text{kg} \\
p_9 &= 40\text{kg} \\
p_{10} &= 100\text{W/K}
\end{aligned}
\tag{9}
$$

Assuming that the water flow u_1 is constant, the differential equations describing the heater dynamics are linear, and it is possible to define the transfer functions relating the inputs u to the output T_h, i.e. to the heater water temperature. Simple calculations will give

$$T_h = G_2(p;u_1)u_2 + G_3(p;u_1)u_3 + G_4(p;u_1)u_4 \tag{10}$$

where

$$
\begin{aligned}
G_2(p;u_1) &= b_{21}(p-a_{11})/D(p;u_1) \\
G_3(p;u_1) &= b_{12}a_{21}/D(p;u_1) \\
G_4(p;u_1) &= b_{23}(p-a_{11})/D(p;u_1) \\
D(p;u_1) &= p^2 - (a_{11}+a_{22})p + a_{11}a_{22} - a_{12}a_{21}
\end{aligned}
\tag{11}
$$

To get some feel for how the water flow u_1 influences the transfer functions, the stationary dependence will be investigated. Some calculations will give

$$
\begin{aligned}
G_2(0;u_1) &= 4180u_1/\left[4180u_1 + \gamma\right] \\
G_3(0;u_1) &= p_1(1-p_6)/\left[p_5(4180u_1+\gamma)\right] \\
G_4(0;u_1) &= \gamma/\left[4180u_1 + \gamma\right]
\end{aligned}
\tag{12}
$$

Plots of these stationary values as functions of u_1 for values in the range $[0.00067, 0.033]$ are shown in Figure 7, where it is seen that $G_2(0; u_1)$ is ap-

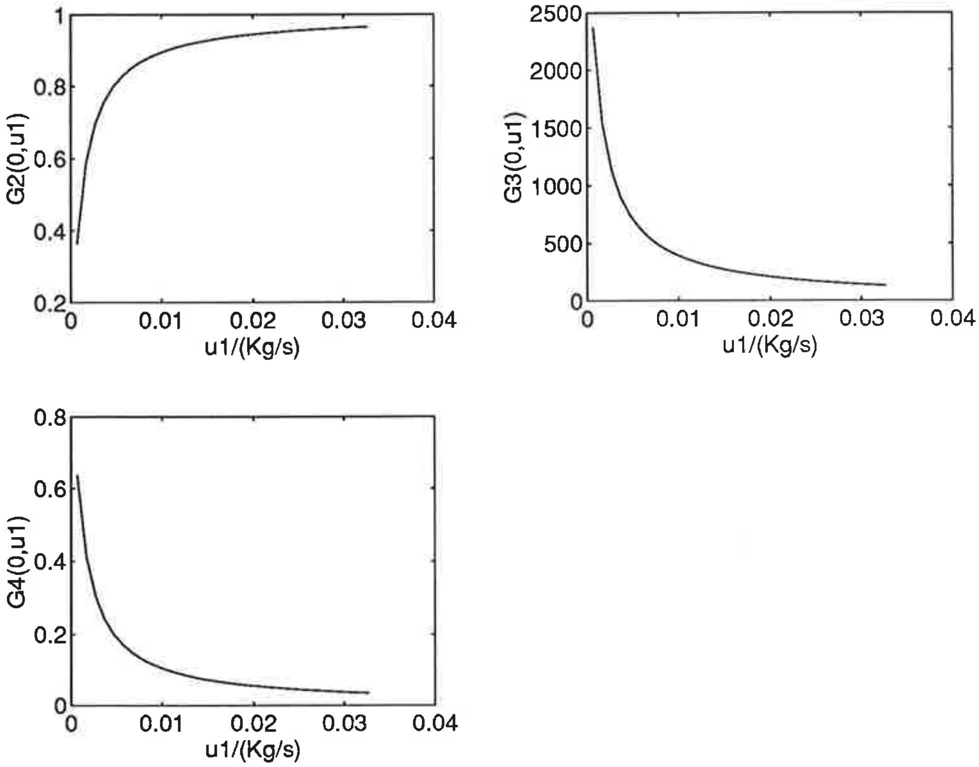

Figure 7. Plots of the stationary gains of the transfer functions relating u to T_h for different values of the flow u_1.

proximately 1 , that $G_3(0; u_1)$ is varying drastically with the flow, and that $G_4(0; u_1) < 0.6$. Thus, since u_2 and u_4 are normally varying slowly, a good approximation to Equation 10 for values of u_1 in the range above is

$$T_h = u_2 + G_3(p; u_1)u_3 \tag{13}$$

This approximation will be used in the following analysis. Notice that the range for u_1 considered above is the one that is imposed by the hydraulic characteristics.

Using the approximate model in (13) and assuming that the characteristic curve is given by (5) the controlled heater with averager can be described as in Figure 8 where

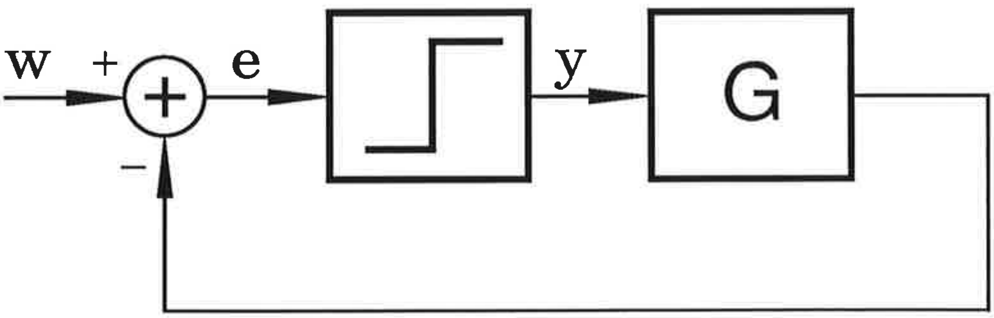

Figure 8. Approximate model of the controlled heater.

$$G(p) = G_3(p; u_1) - \frac{K_r}{pT_r + 1} \tag{14}$$

and where

$$w = T_0 - u_2 \tag{15}$$

This description will be used later on in Section 3.3.

Transient Behavior

It is obvious that for the averaging scheme to work, the averager must in some sense be slower than the heater, otherwise it will run away from the heater. In Figure 9 the step responses for $G_3(p; u_1)$ have been plotted for values of u_1 in the range $[0.001, 0.01]$ with steps of 0.001. It is seen that the lower u_1 is

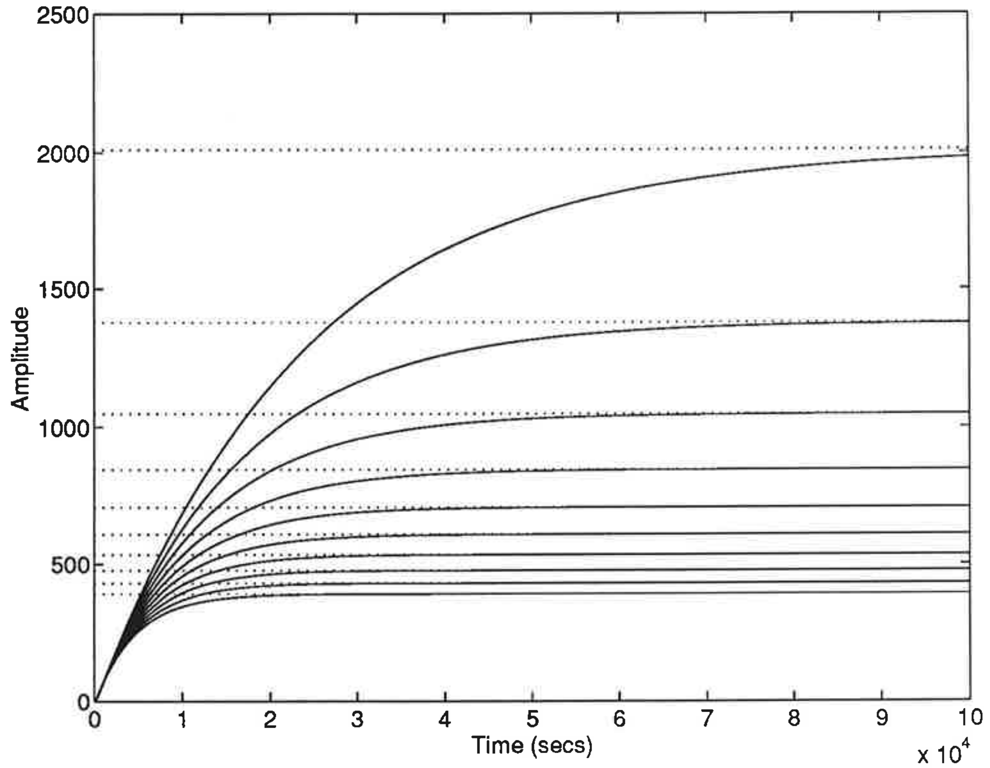

Figure 9. Plots of step responses for $G_3(p; u_1)$ for values of u_1 in the range $[0.001, 0.01]$ with steps of 0.001. The higher stationary values of the step responses corresponds to the lower values of u_1.

the higher is the stationary gain. This was noted already in Figure 7. Further it is seen that the slope of the step response for small values of the time is independent of u_1. Thus it seems to be that it is only the low-frequency characteristics that are dependent on u_1.

The slope $S_h(u_1)$ of the step response for small values of the time is approximately given by

$$S_h(u_1) = \frac{K(u_1)}{T(u_1)} \tag{16}$$

where $K(u_1) = G_3(0; u_1)$, $T = \tau_1 + \tau_2$, and where τ_1 and τ_2 are the time constants of the heater. Some calculations will give

$$T = \frac{|a_{11}| + |a_{22}|}{a_{11}a_{22} - a_{12}a_{21}} \tag{17}$$

A plot of S_h is seen in Figure 10. It is seen that the slope is approximately

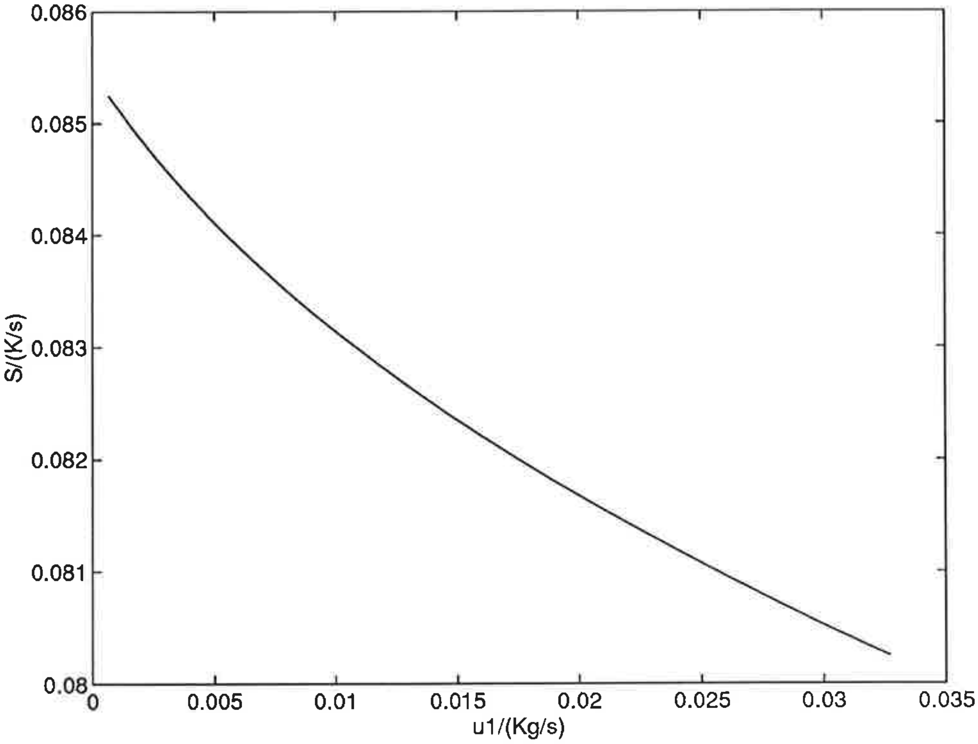

Figure 10. The slope of the step response of $G_3(p, u_1)$ as a function of u_1.

constant and equal to 0.082 K/s. So for the heater to be faster than the averager, the slope

$$S_r = K_r/T_r \tag{18}$$

of the averagers step response has to be smaller than S_h. This implies the following inequality for choosing T_r for a given K_r

$$T_r > \frac{K_r}{K}T = \frac{K_r}{0.082} = 12.2K_r \tag{19}$$

As was pointed out in the previous section K_r may vary in the interval $[44, 110]$ with the user adjustment. Thus it is obvious that T_r also should vary as the user adjusts the characteristic curve. How much larger T_r should be made as compared to the right hand side in the inequality above will be investigated in the next subsection. This is related to the desired period and amplitude of the oscillation of the heater temperature. The actual value of T_r will be obtained by chosing a certain κ-value in the formula below.

$$T_r = \kappa\frac{K_r}{K}T = \kappa \cdot 12.2K_r \tag{20}$$

where κ is some constant strictly larger than 1.

Harmonic Balance

To further investigate the behavior of the controlled heater when the reference value is given as the output from the characteristic curve, the method of harmonic balance will be used. The idea of this method is to assume that there is an oscillation in the closed loop in Figure 8, and that

$$e(t) = E_0 + E\sin\omega t \tag{21}$$

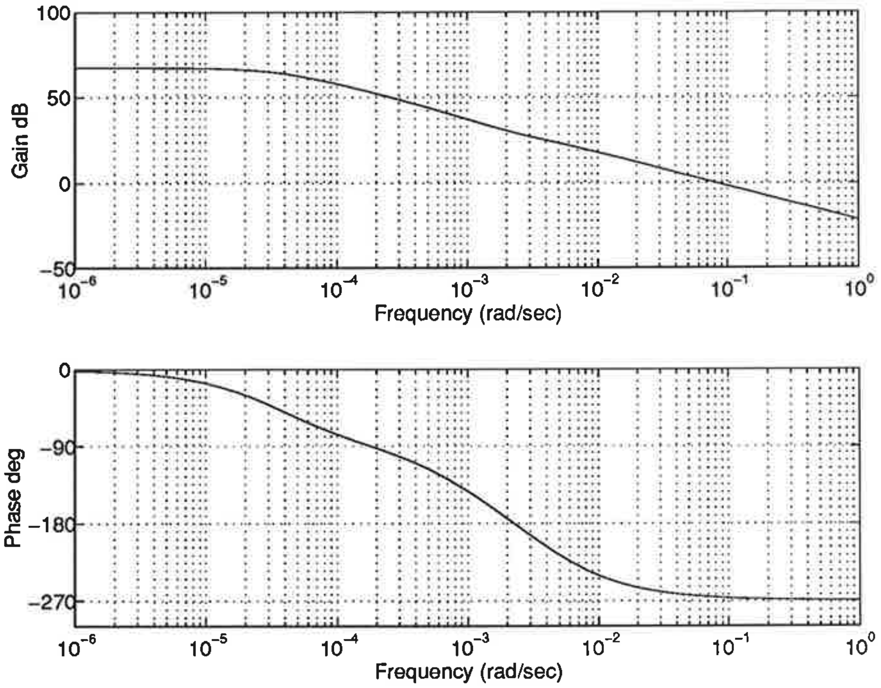

Figure 11. The Bode-diagram of G for $u_1 = 0.00067$ and $\kappa = 1$.

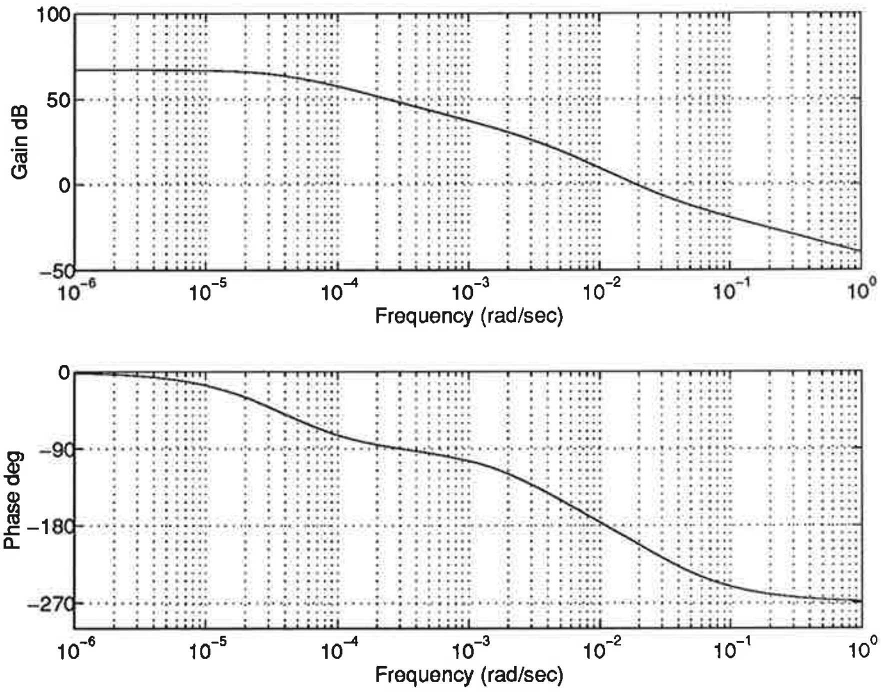

Figure 12. The Bode-diagram of G for $u_1 = 0.00067$ and $\kappa = 8$.

Then it is assumed that all other signals in the loop are well described by their bias and first harmonics due to the low-pass characteristics of G, and that $w = W_0$ is constant. This will then imply the following set of equations

$$\begin{cases} W_0 - E_0 = G(0)Y_0(E, E_0) \\ 1 = |G(i\omega)|\,|N(E, E_0)| \\ (2k+1)\pi = \arg G(i\omega) + \phi_1(E, E_0) \end{cases} \tag{22}$$

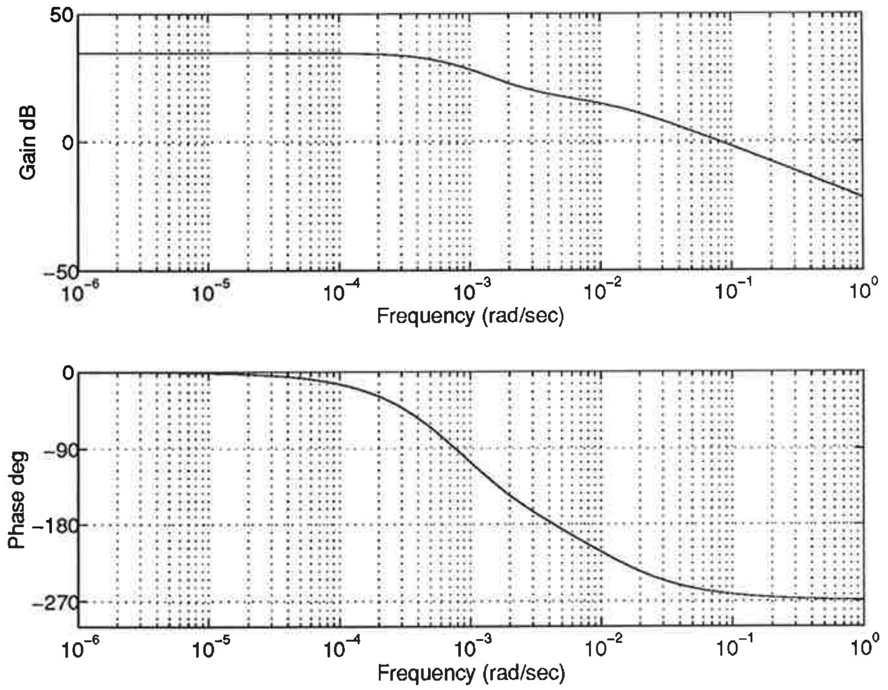

Figure 13. The Bode-diagram of G for $u_1 = 0.033$ and $\kappa = 1$.

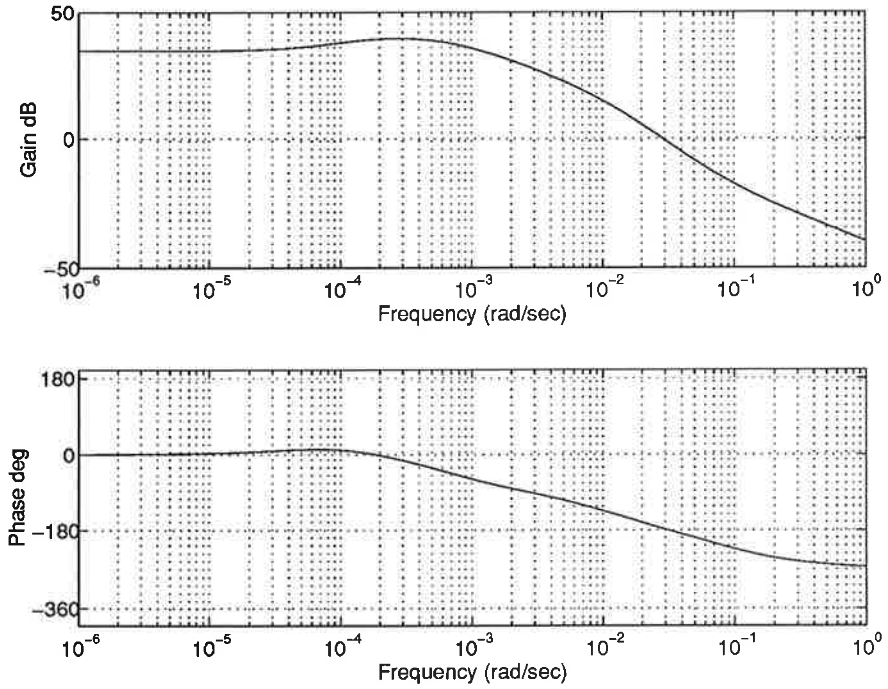

Figure 14. The Bode-diagram of G for $u_1 = 0.033$ and $\kappa = 8$.

where

$$N(E, E_0) = \frac{Y_1(E, E_0)}{E} e^{i\phi_1(E,E_0)} \tag{23}$$

and where Y_0, Y_1 and ϕ_1 are the bias of y, the amplitude of the first harmonic of y, and the phase of the first harmonic of y respectively. Notice that $y = u_3$.

Some tedious calculations give that

$$Y_0(E,E_0) = \begin{cases} \frac{1}{2\pi}\left[\pi + 2\arcsin(E_0/E)\right], & |E_0| \le E \\ 1, & E_0 > E \\ 0, & E_0 < -E \end{cases} \tag{24}$$

that $\phi_1(E,E_0) = 0$, and that

$$Y_1(E,E_0) = \begin{cases} \frac{2}{\pi}\sqrt{1-(E_0/E)^2}, & |E_0| \le E \\ 0, & |E_0| > E \end{cases} \tag{25}$$

Since $G(i\omega) < \infty$ it follows that $|E_0| < E$ for (22) to have a solution. Under this condition the equations in (22) may be written

$$\begin{cases} W_0 - E_0 = (K - K_r)\frac{1}{2\pi}\left[\pi + 2\arcsin(E_0/E)\right] \\ 1 = |G(i\omega)|\frac{2}{\pi E}\sqrt{1-(E_0/E)^2} \\ (2k+1)\pi = \arg G(i\omega) + 0 \end{cases} \tag{26}$$

Since $-\pi/2 \le \arcsin(\cdot) \le \pi/2$, it must hold that

$$|W_0 - E_0| \le |K - K_r| \tag{27}$$

for the first equation to have a solution. Thus it follows that it is easier to make E_0 small the larger $|K - K_r|$ is made. Further the ultimate frequency ω is obtained as the solution to the third equation in (26), i.e.

$$\arg G(i\omega) = -\pi \tag{28}$$

Since the second equation in (26) implies

$$E = \frac{2}{\pi}|G(i\omega)|\sqrt{1-(E_0/E)^2} \le \frac{2}{\pi}|G(i\omega)| \tag{29}$$

the amplitude of the error signal may be chosen by a proper selection of G. Remember that G is dependent on K_r as well as on T_r. This dependence will now be investigated.

If $K \ne K_r$, some calculations will give that

$$G(p) = \frac{N(p)}{D(p)} \tag{30}$$

where

$$\begin{aligned} N(p) &= (K - K_r)\left[1 - p/\left(\sqrt{a^2+b}+a\right)\right]\left[1 + p/\left(\sqrt{a^2+b}-a\right)\right] \\ D(p) &= (p\tau_1 + 1)(p\tau_2 + 1)(pT_r + 1) \\ a &= \frac{1}{2}\left[KT_r - K_r(\tau_1+\tau_2)\right] / \left[K_r\tau_1\tau_2\right] \\ b &= \left[K - K_r\right] / \left[K_r\tau_1\tau_2\right] \end{aligned} \tag{31}$$

The behavior of the transfer function is qualitatively different for different signs of the parameters a and $K - K_r$. For positive values of a, which is equivalent to the inequality in (19), the nominator N will contribute with phase advancement, while for negative values of a it will contribute with phase retardation. Since the transfer function G has a decreasing magnitude for high frequencies, it is advantageous to have a positive in order to get a high value of the ultimate frequency given by (28), and thus small values of the amplitudes $|G(i\omega)|$ and E. Further the larger a is made, the larger will T_r be, and the smaller will $|G(i\omega)|$ be due to the denominator D being dependent on T_r. Thus it is also seen from this analysis that T_r should be chosen to fulfill the inequality in (19).

Further, since a positive value of $K - K_r$, which is obtained for low values of the flow u_1, will give lower value of the argument of G, and thus a lower value of the ultimate frequency and a higher value of $|G(i\omega)|$ and E, as compared to a negative value of $K - K_r$, it is the lower values of the flow u_1 that are the critical ones when chosing T_r. In figures 11–14 the Bode-diagrams of G are shown for $K_r = 74$—corresponds to $\xi = 0.5$—and different values of u_1 and κ. It is seen that chosing $\kappa = 8$ will in this example imply an amplitude of G that is lower than 10 dB for the ultimate frequency and all values of the flow u_1. This value of κ corresponds to a time constant T_r of 2 hours. It is also seen that the ultimate period of the oscillation is about 3 to 10 minutes. This is of course too small a period from a practical point of view. It will be seen later in the simulations that a hysteresis of ± 1 in the relay will increase the period to about 15 minutes while still keeping the amplitude E of the oscillation below the maximum allowed value of 5 K.

Summary

In this section the controlled heater with its reference value given by the characteristic curve has been analyzed. This has given some guidelines on how to chose the averager.

It has been seen that the time constant of the averager should be made dependent of the user adjustment of the characteristic curve. Further a lower bound on the time constant has been obtained. By means of the method of harmonic balance the amplitude and frequency of the control error signal has been related to the averager time constant.

In an example it has been found that a reasonable value of the time constant is about 2 hours. This choice will cause the heater to turn on and off with a period of about 3 to 10 minutes on average. It must be stressed that the value of the time constant is dependent on the heater characteristics. It is easily seen that the higher the power and the smaller the water volume and mass of the heater is made, the smaller may the time constant of the averager be made. Further by introducing hysteresis the period of oscillation may be increased in order to find a practical value.

The approximations made in this section are not to be forgotten. The analysis is only valid in stationarity. This makes it possible to extrapolate the validity only to small variations in the back-water temperature. For larger variations the time constant of the averager may very well have to be made larger. This will be discussed in the next section.

4. Anti Windup Scheme

The constraints mentioned in Section 2.3 will cause windup in the averager scheme for sufficiently large disturbances and set-point changes. Windup is here used in a more general sense than usual. Normally the notion of windup is used when the integrator state of a controller becomes large due to saturation of the control signal. Here the reference value for the flow temperature becomes large when the heating system cannot deliver the power needed, and this may thus be interpreted as windup of the reference value.

In Section 4.1 different solutions proposed for avoiding windup— anti-windup—are reviewed. It will be seen that the so called conditional integration methods are appealing schemes for anti-windup when there are constraints not only in the actuator signal but also other constraints within the process as is the case with the heating process. In Section 4.2 fuzzy control will be briefly reviewed, and in Section 4.3 fuzzy anti windup for the heater control problem will be proposed. Finally, in Section 4.4 the results are summarized.

Review of Anti Windup

Windup is an inherent problem in design of controllers for industrial processes. The reason for this are not only actuator constraints, but also other kinds of constraints in the processes. In order to cope with these type of non-linearities, the design-method must either consider the non-linearities directly—non-linear design, or the design method has to be adjusted with some more or less ad hoc fix—anti windup.

As was mentioned above, one way to deal with the non-linearities is to use some non-linear design such as e.g. optimal non-linear control. However, this is often computationally expensive, model-demanding and gives little insight into the relation between model and controller, i.e. the mapping between model and controller is usually not easy to understand. Thus most of the schemes used are linear. Common to most of these schemes are that the controller is tuned around a certain operational point such that the performance is reasonable for small changes in the reference value and load disturbances. Most controllers are also equipped with some sort of anti windup. This anti windup is normally designed to prevent the integrator state in the controller to become too large when large disturbances or changes in reference value causes the actuator to saturate, see R91 for a good survey. However, this saturation of the actuator is usually only one of many non-linearities in an industrial process that can interfere with the linear controller and result in windup detoriation of performance. It is therefore of importance to have anti windup schemes which consider not only actuator non-linearities but also other process constraints.

In the heater control problem, as was described in Section 2.3, the constraints are not only the limitation in heater power but also the unmodeled dynamics with saturation of the valves on the radiators and of the water flow. Since there are no measurement signals relating directly to the latter constraints, this is a control problem with constraints within the process. For this type of problems it is not possible to use the classical anti windup schemes such as e.g. tracking, GST83. However, recently fuzzy generalizations and combinations of some of the classical methods have been proposed for these windup problems, and they seem to work well, HGT92.

The proposed scheme is also a fuzzy generalization of one of the classical methods—one of the so called conditional integration methods, see R91. There the rate of change of the integrator state is set to zero when some condition such as control error is large or control signal is saturated is fulfilled. Here the idea is to decrease the rate of change of the reference value for the heater temperature when the heater on-off signal is saturated, i.e. has been on or off for a longer period of time.

Fuzzy Control

The impact of fuzzy logic on design of controllers has increased during the last ten years. The first successful applications are described in M74, KNL76, KM77, TBL80, and the first industrial application in HO82. Fuzzy logic was however proposed already in Z65, but the applications to control were made possible first by the publication of Z73, where the so called inference-rules of fuzzy logic where presented. These rules make it possible to describe the control action in terms of `if ... then ... else`-constructions that mimics the human way of doing manual control. This seems to be one of the reasons why fuzzy control has become so popular. Since conditional integration methods, as the name says, are in this condition-form, it seems to be a natural idea to propose fuzzy versions of these methods.

Fuzzy Anti Windup

Fuzzy anti windup for PID controllers has been described in HGT92. The development of the fuzzy scheme for the heater temperature reference value will be similar to the development of the anti windup schemes for PID controllers given there.

As was mentioned above the idea is to decrease the rate of change of the reference value for the flow temperature when the on-off signal to the heater is saturated, i.e. has been on or off for a longer period of time. To this end let τ be the time since the last change of the heater on-off signal and let K be the inverse of T_r, i.e. $K = 1/T_r$. Notice that this is not the same K as in the previous section. The rules for the fuzzy scheme inspired by Scheme 2) in R91, p. 14 are

1) If TAUS then KN
2) If TAUL then KZ

where TAUS, KN, TAUL, and KZ means τ is small, K is nominal, τ is large, and K is zero respectively. Introduce the membership functions

$$\begin{cases} \mu_{\text{TAUS}}(\tau) = \exp(-\alpha\tau) \\ \mu_{\text{TAUL}}(\tau) = 1 - \exp(-\alpha\tau) \end{cases} \tag{32}$$

for TAUS and TAUL, where α is some positive constant. Further let

$$\begin{cases} \mu_{\text{KN}}(K) = \begin{cases} 1, & K = K^{nom} \\ 0, & K \neq K^{nom} \end{cases} \\ \mu_{\text{KZ}}(K) = \begin{cases} 1, & K = 0 \\ 0, & K \neq 0 \end{cases} \end{cases} \tag{33}$$

be the membership functions for KN and KZ, where K^{nom} is the nominal value of K, i.e. the value of K that results from the design of the averager when only considering the heater dynamics.

Then using the max-min-inference rule introduced in Z73, and taking as value of K the mean of the resulting membership function will give

$$K(\tau) = \frac{\mu_{\text{TAUS}}(\tau) \cdot K^{nom} + \mu_{\text{TAUL}}(\tau) \cdot 0}{\mu_{\text{TAUS}}(\tau) + \mu_{\text{TAUL}}(\tau)} = K^{nom} \exp(-\alpha\tau) \tag{34}$$

The scheme for T_r is thus given by

$$T_r(\tau) = T_r^{nom} \exp(\alpha\tau) \tag{35}$$

where T_r^{nom} is the nominal time constant.

Since T_r^{nom} is the time constant chosen in the design of the averager when the constraints are neglected, there is only one parameter, α, to be tuned for the fuzzy anti windup scheme. Further notice that the expression for $T_r(\tau)$ is given explicitly. Thus the implementation of the scheme will be computationally cheap. A discrete time implementation is given by

$$\begin{cases} T_r(k+1) = \exp(\alpha h)\,[1 + |u_3(k) - \eta(k)|]\, T_r(k) + |u_3(k) - \eta(k)| T_r^{nom} \\ \eta(k+1) = u_3(k) \end{cases} \tag{36}$$

where h is the sample interval.

Summary

The constraints imposed by the room dynamics and the hydraulic characteristics will cause windup of the heater water reference value. In this section a fuzzy anti windup scheme inspired by the so called conditional integration methods has been designed to prevent this windup.

The scheme has only one tuning-parameter. This will depend on the dynamics of the heated rooms as well as on the hydraulic characteristics. To find a good value of the parameter, simulations have to be performed. This will be done in Section 5.

Further the scheme is given explicitly as a function of the time since the heater on-off signal last changed its value. Thus the implementation of the scheme will be computationally cheap, i.e. no special purpose signal processor will be needed, which is often the case with fuzzy control.

5. Simulations

In this section the fuzzy anti windup scheme designed in Section 4 will be simulated. This will give a feeling for how to chose the parameter in the anti windup scheme. Further it will be seen that anti windup improves the performance.

In Section 5.1 the model is briefly described. It is a non-linear differential algebraic equation with 32 states. The simulation program used is Simulab, S91. In Section 5.2 the experimental conditions are described, and in Section 5.3 the results of the simulations are given. More than 120 hours of simulations have been performed on a Digital DEC-station 5000/200 and a Sun Sparc Station ELC. Finally, in Section 5.4 the results are summarized.

Model

Parts of the model, the heater and the averaging scheme, have already been described in the previous sections. Here the hydraulics and the room dynamics will be described briefly. For more detailed information see R92a, R92b, R92c, R92d, R92e.

The house to be heated has two rooms. For each room there are 4 state variables modeling the temperatures of the air in the room, of the furniture in the room, and of the inside and outside of the wall respectively.

The radiators in each room are modeled with 8 state variables each. To avoid an oscillating behavior the parameter h_5 in R92a has been modified to be equal to x.

The heater, the house, and the radiators are connected via the hydraulics. This is modeled with one state each for the flow and return temperatures, and with one state for each temperature sensor. Further there are P-controllers acting on the valve positions in the rooms to control the room temperatures.

Experimental Conditions

Here the parameters and input signals that are constant for all simulations will be given. Only the ones which are not given in or have been altered as compared to R92a, R92b, R92c, R92d, R92e will be discussed.

For the heater in R92c the parameter p_1 has been set to 18000. Further p_{10} has been set to 100. The temperature u_4 outside the heater has been kept constant equal to 15. The initial values for the heater have been set to 20.

For the house in R92e the parameter p_4 has been set to 15, the parameter p_5 has been set to 20, the parameter p_{11} has been set to 1.0, the input signals $u_4(j)$ have been set to 1.0, and the out-door temperature u_1 to 10. Further the initial values for the house have been set to 20 for the furniture and the air temperatures and 15 and 10 for the wall temperatures.

For the radiators in R92a the parameter $p_6(j)$ have been set equal to $y_1(j)$, where y_1 is defined as in R92e. Further p_7 has been set to 70 and p_8 to 55. The initial values for the radiator states have been set to 20.

For the hydraulics in R92b the parameters p_1 and p_2 have been set to 70 and 55 respectively. Further p_9 has been set to 40 and $p_{10}(j)$ have been set equal to $y_1(j)$, where y_1 is defined as in R92e. The input signal u_1 has been set constant equal to 0.21. All states of the hydraulics have initial values equal to 20.

Results

The simulation experiments performed have been done to design a good anti windup scheme and to compare this with averaging schemes without anti windup. The experiments performed contain set point changes for the room temperatures and load disturbances. The simulation time is 48 hours.

First different constant values of the time constant in the averaging scheme have been used. The experiments performed are two set point changes for the temperature in room 1. After 24 hours the reference value is set to 22, and then after another 12 hours it is reset to 20. The results are seen in figures 4 and 3. It is seen that, for a time constant of 2 hours, the step responses are fast. However, the behavior is almost unstable. For the larger time constant, 6 hours, the behavior is stable but too slow.

In figures 15–17 the same experiments as described above have been performed. However, now the fuzzy anti windup scheme is used to modify the time constant from a nominal value of 2 hours to a higher one. The parameter α in the fuzzy scheme has been chosen to be $\alpha_1 = \ln(1.5)/3600$, $\alpha_2 = \ln(1.5)/1800$, and $\alpha_3 = \ln(1.5)/900$. Thus the time constant will have a 50 % higher value than the nominal one after 60 minutes, 30 minutes, and 15 minutes respectively. It is seen that the larger α is made the slower will the variations in the reference value be. In the sequel the value $\alpha = \alpha_3 = \ln(1.5)/900$ will be used.

In figures 18 and 19 step responses with different sizes of the steps are shown. The step sizes are 1 and 5. Further in figures 20 and 21 the behavior with respect to load disturbances of sizes 30 and 100 are shown. It is seen that the fuzzy windup scheme manages well both with respect to set point changes and with respect to load disturbances.

Summary

I this section the fuzzy anti windup scheme has been simulated and compared with different constant values of the time constant.

It has been seen that it is not possible to find a constant value of the time constant with sufficiently good performance. Although the value of 2 hours seemed to be reasonably good, this value will most certainly be bad for large set point changes.

The behavior of the anti windup scheme seems to be robust with respect to the tuning parameter α. Thus it is easy to find a good value of this parameter. Further it has been seen that the anti windup scheme is robust with respect to different sizes of set point changes and load disturbances. Thus the overall performance of the anti windup scheme seems to be good.

In all simulations there have been stationary control errors for the room temperatures. This may be due to the characteristic curve not being correctly adjusted. Perhaps it would be possible to use this control error to manually adapt the curve. This would be most practical, since the curve must vary with the house characteristics. This variation is not only present when the heating is installed, i.e. startup variation due to different houses, but also when e.g. a guest room is taken into use, i.e. is heated after not having been used for a longer period of time. This is due to the fact that one of the assumptions when deriving the characteristic curve in R89 is that all rooms must be equipped with controllers acting on the valves. If a room is not heated at all, then there is no control, and the assumption is not fulfilled.

6. Conclusions

In this report an observer for the reference value of the flow temperature of a heating installation has been designed. Possible speeds of the observer have been investigated. First a nominal fast design has been made utilizing the heater dynamics. Then an anti windup scheme has been designed by means of fuzzy logic to slow it down when large disturbances interfere with the nominal design.

The observer is composed of two parts: an averager with a time constant to be chosen, and a characteristic curve relating the stationary flow temperature demand to the stationary power demand. It has been seen that the time constant of the averager should be made dependent of the user adjustment of the characteristic curve. Further a lower bound on the time constant has been obtained. By means of the method of harmonic balance the amplitude and frequency of the control error signal has been related to the averager time constant. Thus the nominal design is easily done.

In an example it has been found that a reasonable value of the time constant is 2 hours. It must be stressed that the value of the time constant is dependent on the heater characteristics. It is easily seen that the higher the power and the smaller the water volume and mass of the heater is made, the smaller may the time constant of the averager be made. Further by introducing hysteresis the period of oscillation may be increased in order to find a practical value.

It has been seen in simulations that it is not possible to find a constant value of the time constant with sufficiently good performance. Thus anti windup has to be considered. A fuzzy anti windup scheme inspired by the so called conditional integration methods has been designed to prevent this windup. The scheme has only one tuning-parameter. This will depend on the dynamics of the heated rooms as well as the hydraulic characteristics. Further the scheme is given explicitly as a function of the time since the heater on-off signal last changed its value. Thus the implementation of the scheme will be computationally cheap, i.e. no special purpose signal processor will be needed, which is often the case with fuzzy control. The behavior of the anti windup scheme seems to be robust with respect to the tuning parameter. Thus it is easy to find a good value of this parameter. Further it has been seen that the anti windup scheme is robust with respect to different sizes of set point changes and load disturbances. Thus the overall performance of the anti windup scheme seems to be good. The scheme has with success been incorporated in the Sigmagyr RVP110 controller which has been on the market for about a year.

Further Research

As was mentioned above, the idea in this report is to design a steady state observer for the reference value of the flow temperature. If more measurements had been to disposal, it would most certainly have been possible to design a more sophisticated observer giving higher performance of the overall control. This would have been more expensive due to the need of e.g. a sensor for the return temperature. However, it would for comparison be interesting to know how far from optimality the proposed scheme is. Further it may perhaps also be possible to build a better observer for the reference value of the flow temperature if another state space representation is considered without extra measurements.

In all simulations there have been stationary control errors. This may be due to the characteristic curve not being correctly adjusted. Perhaps it would be possible to use this control error to manually adapt the curve. This would be most practical, since the curve must vary with the house characteristics. This variation is not only present when the heating is installed, i.e. startup variation due to different houses, but also when e.g. a guest room is taken into use, i.e. is heated after not having been used for a longer period of time. This is due to the fact that one of the assumptions when deriving the characteristic curve in R89 is that all rooms must be equipped with controllers acting on the valves. If a room is not heated at all, then there is no control, and the assumption is not fulfilled.

If the averaging scheme has a constant time constant, then it can be shown that there is no bias in the estimate of the average power. This may not be the case for the averaging scheme with anti windup. However, if the value of

α is chosen small, the bias may be made sufficiently small. It would of course be interesting to investigate how large the bias is.

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Appendix—Figures

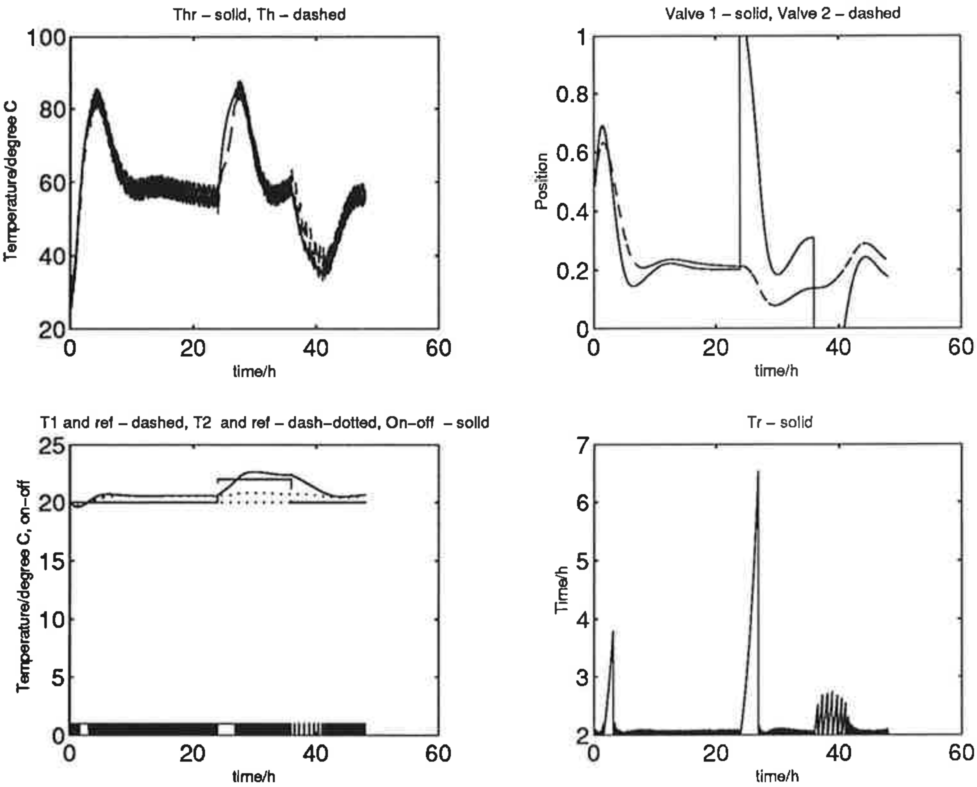

Figure 15. Simulation with a nominal value of the time constant $T_r^{nom} = 2$ h and a value $\alpha = \ln(1.5)/3600$ of the fuzzy parameter.

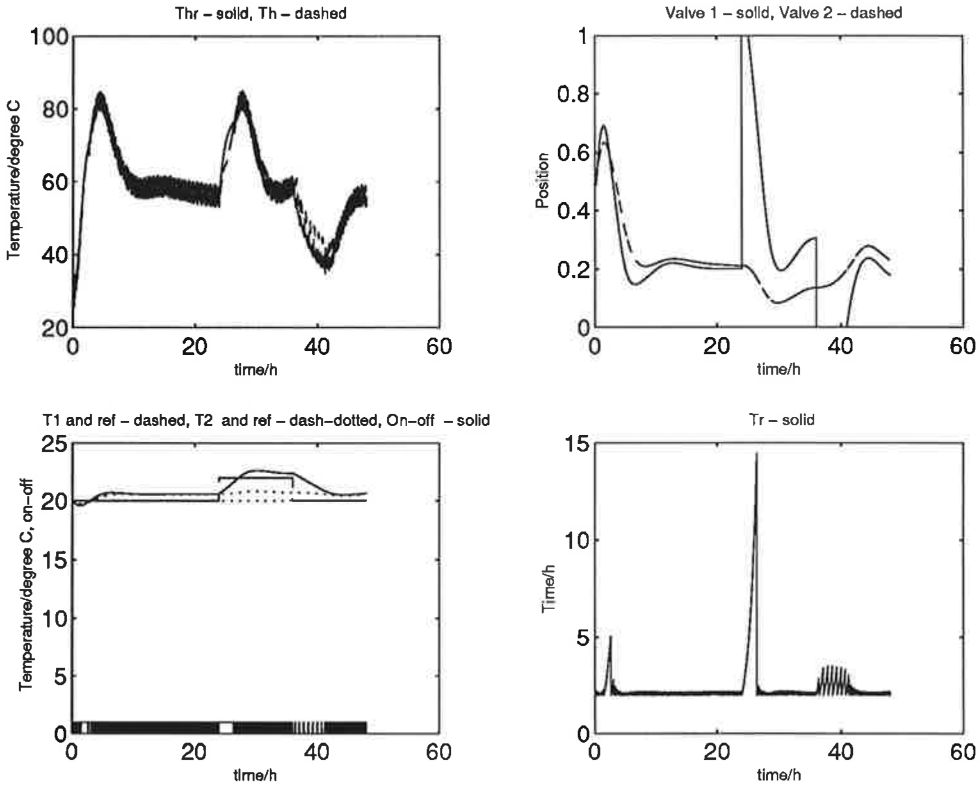

Figure 16. Simulation with a nominal value of the time constant $T_r^{nom} = 2$ h and a value $\alpha = \ln(1.5)/1800$ of the fuzzy parameter.

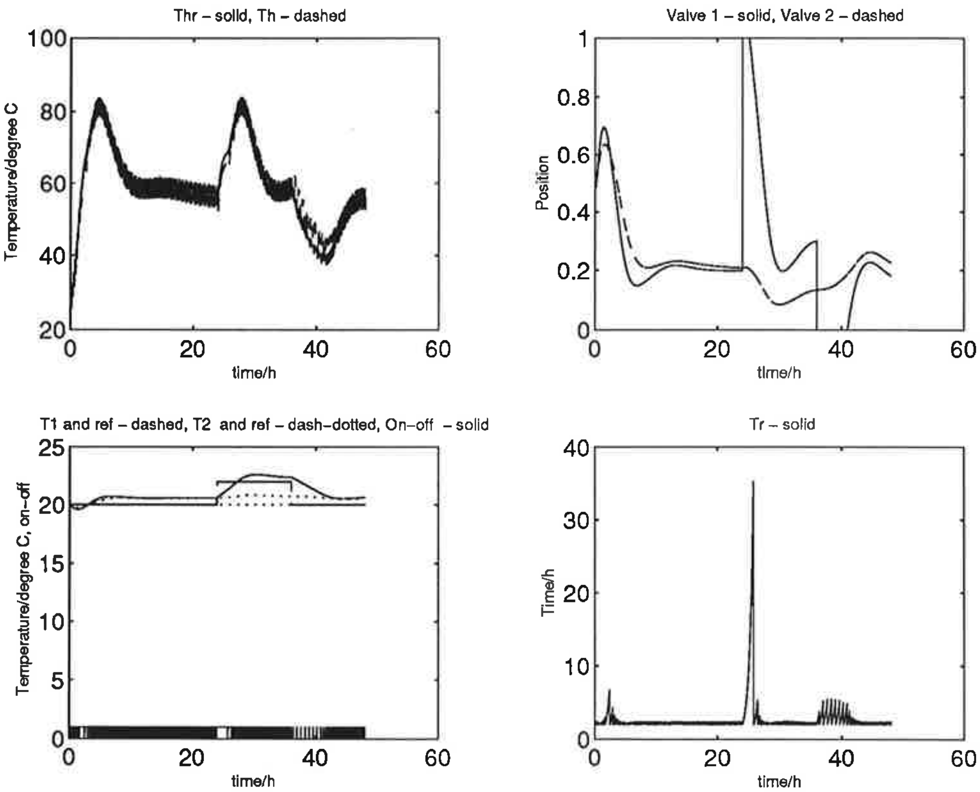

Figure 17. Simulation with a nominal value of the time constant $T_r^{nom} = 2$ h and a value $\alpha = \ln(1.5)/900$ of the fuzzy parameter.

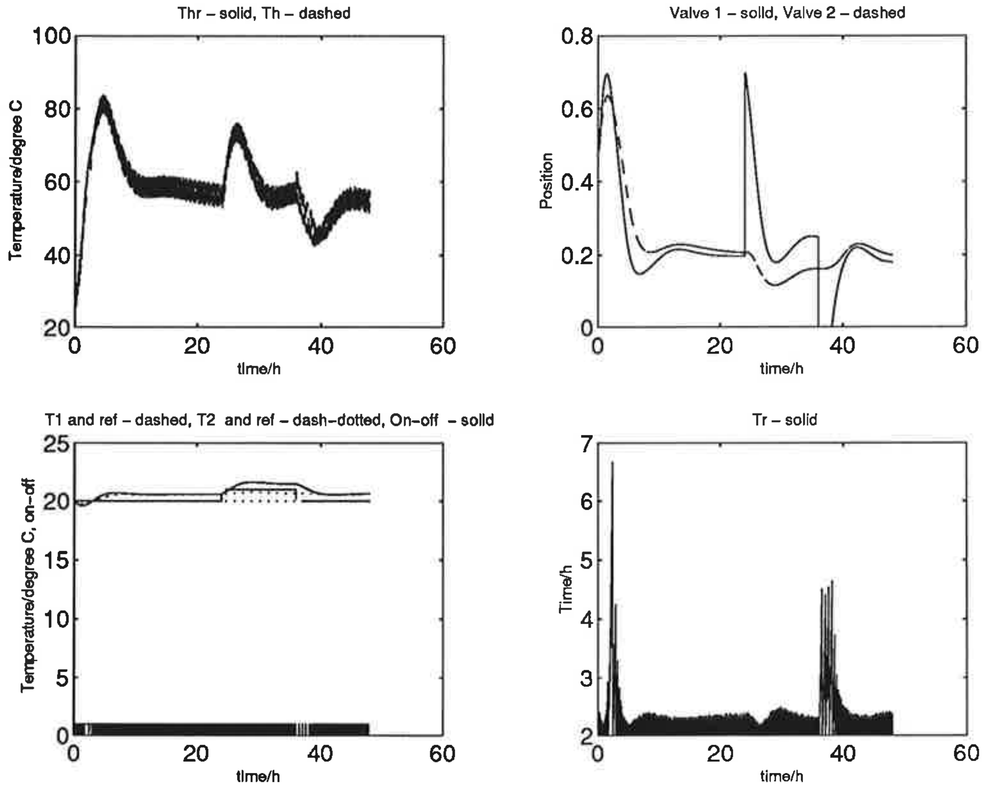

Figure 18. Simulation of the fuzzy scheme with step responses of hight 1 for the temperature in room 1.

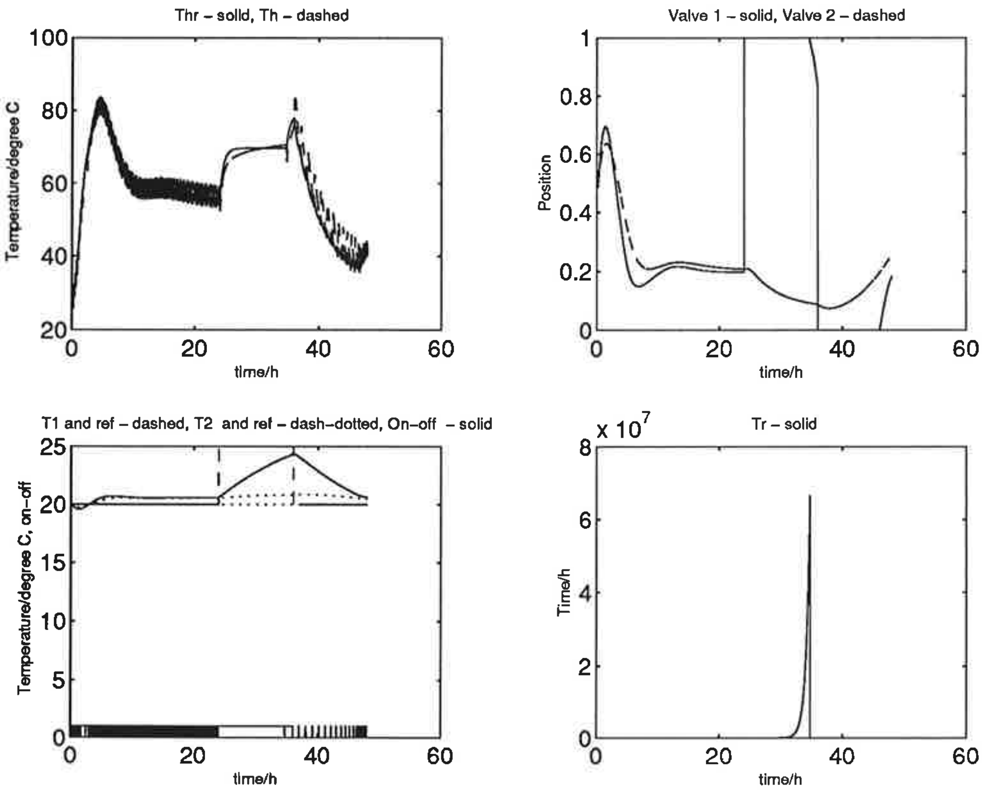

Figure 19. Simulation of the fuzzy scheme with step responses of hight 5 for the temperature in room 1.

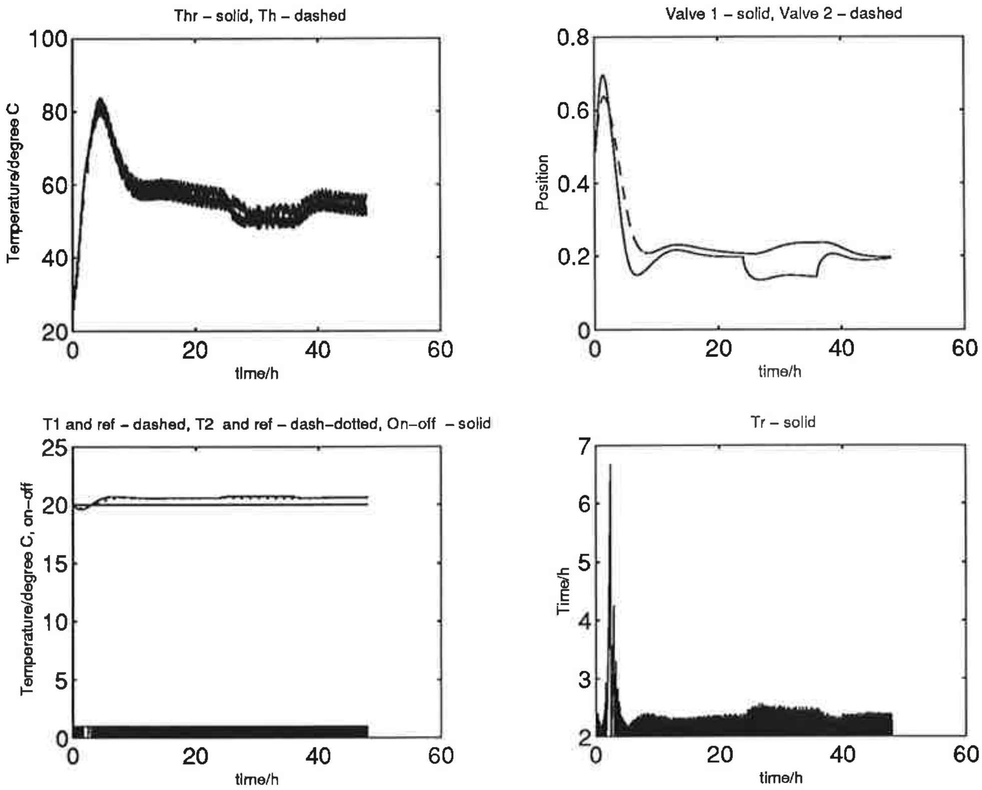

Figure 20. Simulation of the fuzzy scheme with load disturbance of size 30 W/m^2 in room 1.

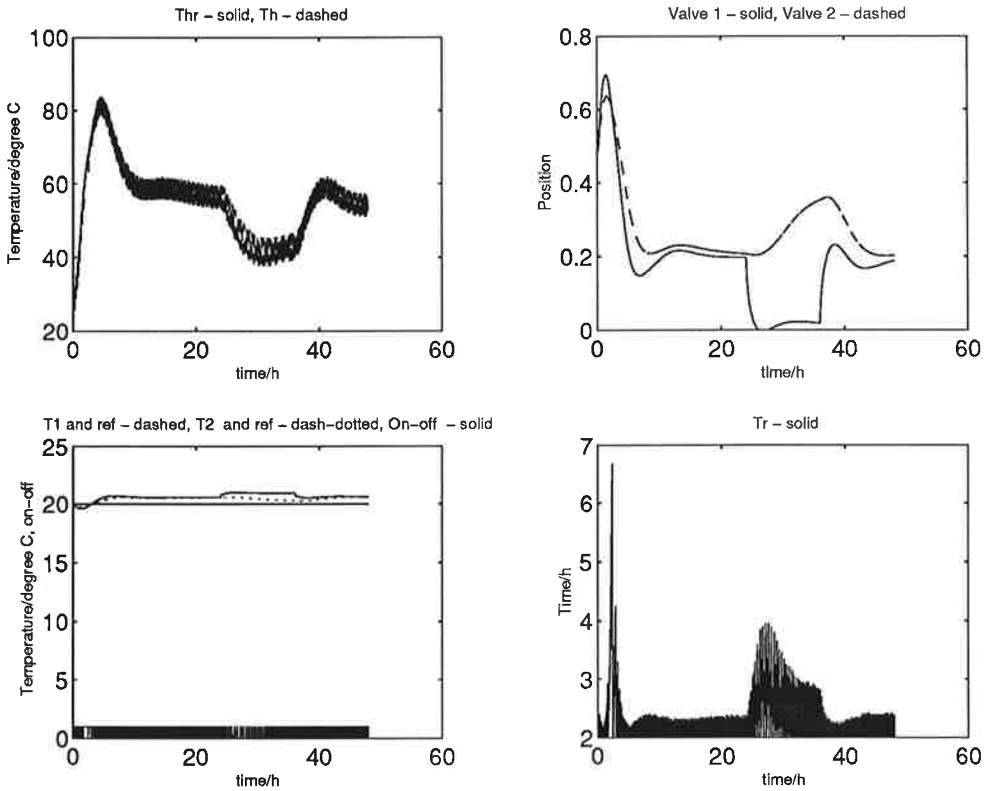

Figure 21. Simulation of the fuzzy scheme with load disturbance of size 100 W/m^2 in room 1.