LUND UNIVERSITY

A simplified non-linear model of a drum boiler - turbine unit

Åström, Karl Johan; Eklund, Karl

Link to publication

Total number of authors:
2

A simplified non-linear model of a drum boiler-turbine unit†

K. J. ÅSTRÖM and K. EKLUND

Lund Institute of Technology, Division of Automatic Control, Sweden

In the analysis of power systems it is highly desirable to have models of the different power generators. In this paper we present a model for a drum boiler whose purpose is to describe the gross behaviour of the boiler. The major control variables are fuel flow and control valve setting. The output variables are drum pressure and active output power. The model is verified by experiments on a 160 Mw boiler.

1. Introduction

This paper presents a simple non-linear model for a drum boiler-turbine unit having the form

$$\frac{dp}{dt} = \alpha[-f(p, u_2) + g(u_1, u_3)], \\ P_0 = f(p, u_2),$$

$$(1.1)$$

where the state variable p equals drum pressure. The control variables are fuel flow u_1, control valve setting u_2 and feedwater flow u_3. The output $P_0 = f$ is the output power and g is the input power.

It is shown that the model agrees well with measurements for the actual boiler in the range half power to full power. It is also shown that the model can be derived using physical arguments. This gives the form of f and g as

$$f(p, u_2) = \alpha_4[u_2 p^{5/8} - \alpha_5], \tag{1.2}$$

$$\alpha g(u_1, u_3) = \alpha_2 u_1 - \alpha_3 u_3, \quad \alpha = \alpha_1/\alpha_4. \tag{1.3}$$

The model is thus characterized by five parameters $\alpha_1, \ldots, \alpha_5$.

The structure of function f given by (1.2) is obtained by several approximations. Other structures are possible.

The model can be used in several different ways, e.g.

(i) As a boiler-turbine model in power systems studies.
(ii) As a tool to design experiments for delivery testing of boilers.
(iii) To understand how a boiler behaves under different operating conditions. In particular to define concepts like regulating power and storage capacitance.
(iv) To synthesize optimal trajectories for large load changes.

The paper is organized as follows. Section 2 describes the experiments that were used as a basis for the modelling. A derivation of a simplified model based on physical considerations is given in § 3. In particular it is shown what assumptions are required in order to arrive at the simplified model. Some alternative approaches are also given. A crude method of estimating the parameters of the simplified model is given in § 4. Section 5 contains a comparison of the model output with experimental data. It should be observed that the model parameters are *not* adjusted to fit each individual run. In § 6 are given some of the conclusions about the dynamics of a drum boiler that can be drawn from the model.
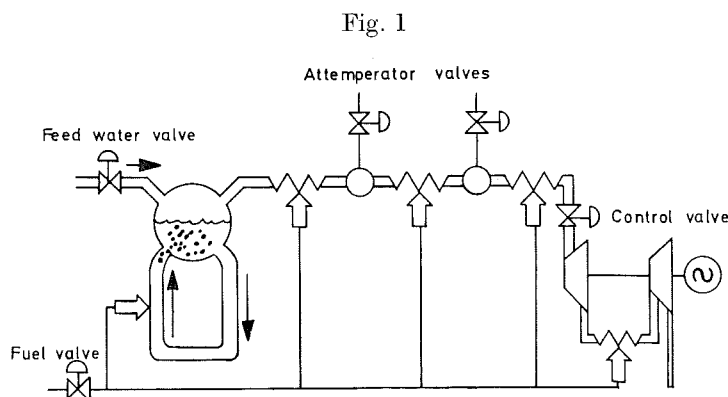
2. Experiments

The experiments were made on the boiler unit P16 and the turbine unit G16 at Öresundsverket of Sydsvenska Kraft AB in June 1969. The boiler is an oil-fired drum boiler designed for a maximum power of 160 Mw. The experiments were designed for two tasks :

(i) To obtain a qualitative understanding of the behaviour of the boiler during Transient loads, and

(ii) to obtain a detailed linear model for steady state operation.

None of the experiments were designed specifically for the purpose of developing non-linear mathematical models valid for a wide operating range. The possibility to obtain such models was not originally anticipated.

A schematic diagram of the boiler-turbine unit is shown in fig. 1. In the experiment the variables, fuel flow, feedwater flow, two attemperator flows and the control valve position were considered as inputs. The recorded outputs were drum pressure, generated electric power, drum level, temperatures and pressures in various parts of the system. In the experiments 24 variables were recorded on a datalogger. In the graphs given in this report only a selection of these variables will be given. A detailed account of the experiment is given by Eklund (1971).

Fig. 1

Schematic diagram of the boiler-turbine unit.

The major controllers used during normal operation of the boiler are controllers for air-fuel ratio, drum level, drum pressure, steam temperature and

Table 1.   A complete list of experiments discussed in the paper.

Experiment	Power level	Input	Remark
A	140 Mw	Fuel flow	Small perturbations. Regulators for steam temperature, drum level and drum pressure (power) disconnected.
B	130 Mw	Control valve	Same as A except that the control valve setting is changed.
C	70 Mw	Fuel flow	Same as A except that the experiments are done at half power.
D	70 Mw	Control valve	Same as C except that control valve setting is changed.
E	90–160 Mw	Fuel flow	Wide power range. Drum pressure controller disconnected.
F	75–90 Mw	Control valve	Drum pressure controller disconnected.

turbine power.   When these controllers are in operation they may change control variables which we would like to have at specific values during the experiment.   In many experiments the ordinary regulators were therefore disconnected.

A list of the experiments discussed in this paper is given in the table 1.   A detailed discussion of each experiment follows.
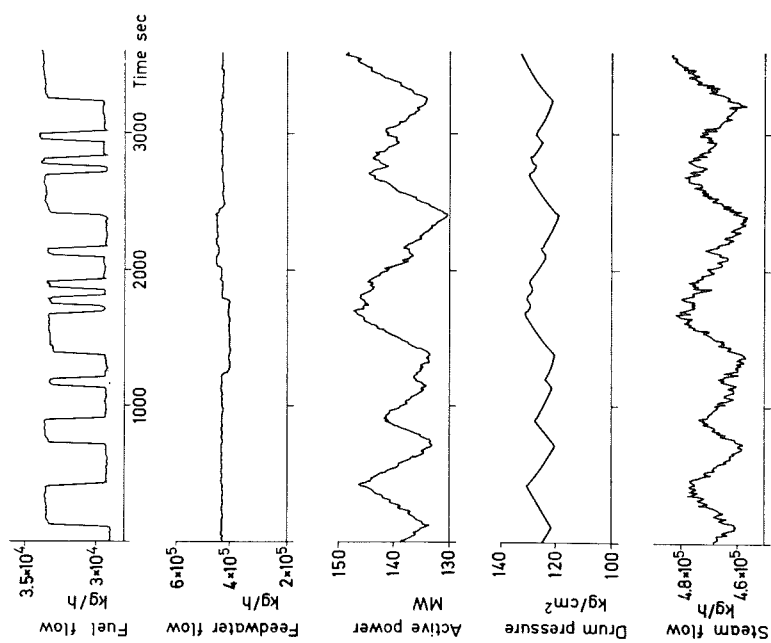
Experiments A and B

Both experiments were designed for the purpose of determining linear models for steady state operation.   The input, fuel flow in experiment A and control valve position in experiment B, was changed manually according to a prescribed programme.   The input signal was chosen on the basis of preliminary experiments and *à priori* estimates of system dynamics.   The sampling rate was 10 sec and the duration of the experiment was one hour.   The operating conditions were chosen so that the output power was slightly below the maximum power.   The feedwater flow was also changed manually in both experiments.   This was necessary in order to keep drum level within acceptable limits.

Figures 2 and 3 show the responses of active power, drum pressure and steam flow in the experiments.   It is clear from these figures that the drum pressure responds in more or less the same way to changes in fuel flow and control valve position.   The responses of active power and steam flow are, however, very different in experiment A and experiment B.   The smooth changes of active power and steam flow to a step change of fuel flow in fig. 2 are replaced by very fast initial changes to a step change in control valve setting in fig. 3.

In both fig. 2 and fig. 3 there is a good qualitative agreement between responses of active power and steam flow.

Fig. 3

Experiment B showing responses of active power, drum pressure and steam flow to changes in control valve position and feedwater flow.

Fig. 2

Experiment A showing responses of active power, drum pressure and steam flow to changes in fuel and feedwater flows.

Fig. 5

Experiment D showing responses of active power, drum pressure and steam flow to changes in control valve position and feedwater flow.

Fig. 4

Experiment C showing responses of active power, drum pressure and steam flow to changes in fuel and feedwater flows.

Experiments C and D

These experiments are performed at half-power but are otherwise analogous to experiments A and B. The results of these experiments are shown in figs. 4 and 5. The behaviour of the system at half-power is similar to that at full power.

Experiment E
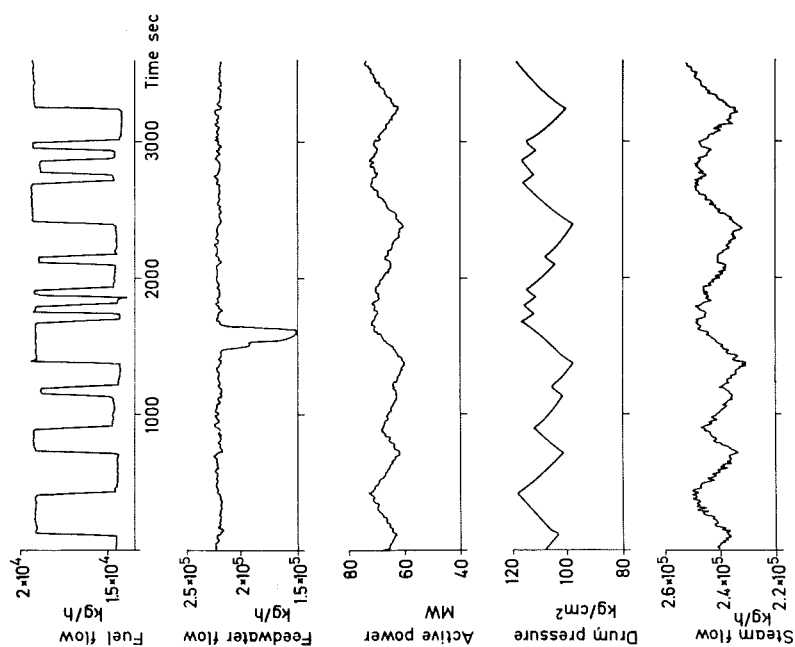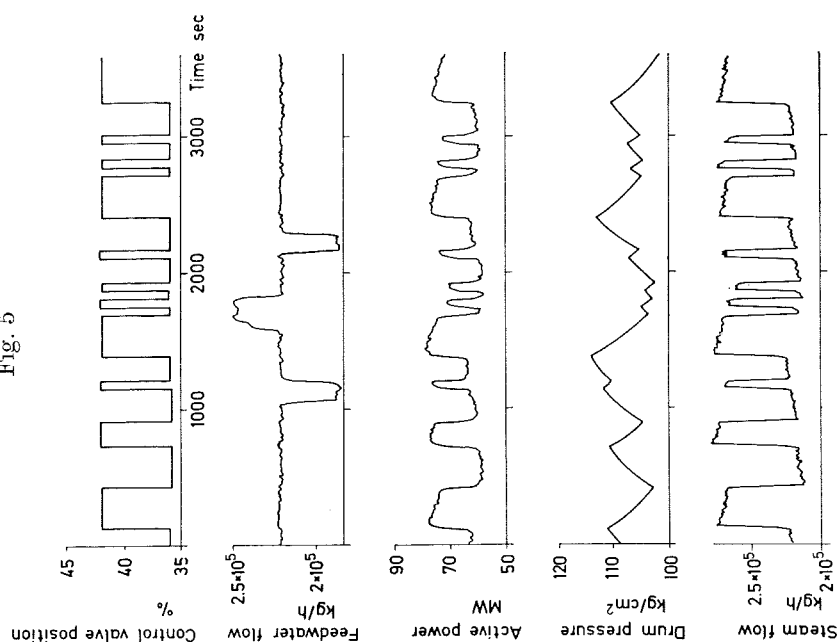
The purpose of this experiment was to study a fairly normal load change from 90 Mw to 160 Mw (full load). The control valve was fully opened. All regulators except the fuel flow regulator were in operation. The fuel flow was increased manually during the experiment.

Fig. 6
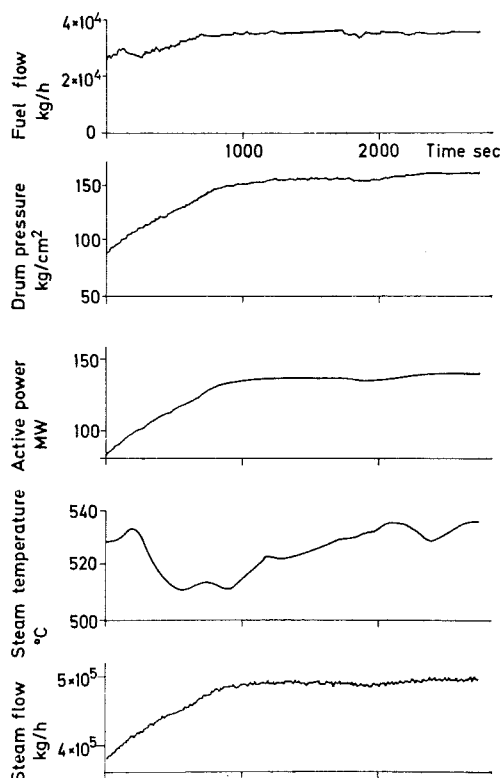
Experiment E showing responses of drum pressure, active power, steam temperature after reheater and steam flow to a changing fuel flow.

The result of the experiment is shown in fig. 6. The curves indicate that the changes in active power are proportional to the changes in steam flow and drum pressure.

Experiment F

In this experiment all controllers are in operation except the drum pressure controller which normally controls the fuel flow valve. The control valve is

Fig. 7
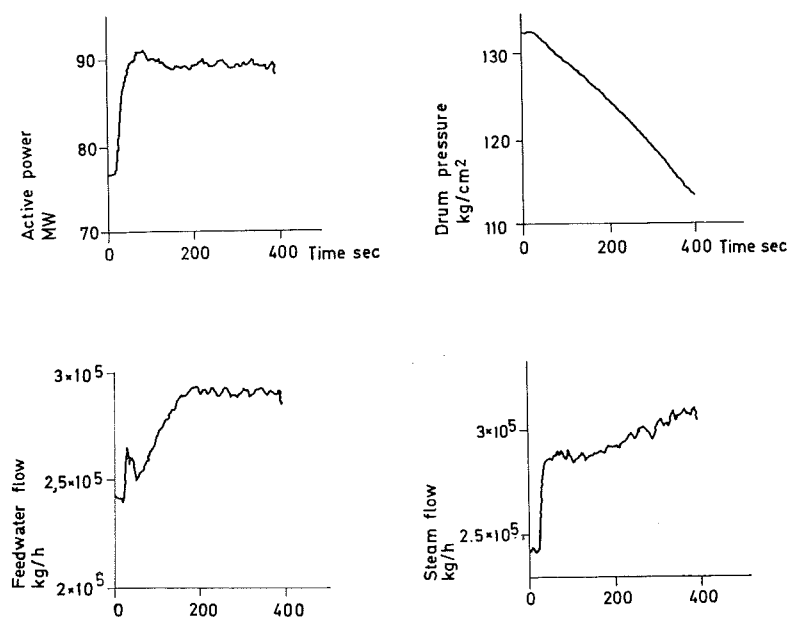
Experiment F showing responses of active power, drum pressure and steam flow to a change in control valve position (not shown) and feedwater flow.

roughly half closed which gives an initial drum pressure of $132 \, \text{kg/cm}^2$. The fuel flow is initially set to give 77 Mw active power. The control valve is manipulated manually.

The valve is first opened as quickly as possible to yield an output power of 90 Mw. This took about 15 sec. The valve is then manipulated manually in order to maintain a constant output power. The changes of major process variables during the experiment are shown in fig. 7. As seen from this diagram the pressure curve has almost a constant slope. The bend in the pressure at $t = 200$ sec is due to the changes in feedwater flow and steam flow.

3.   A simple physical model

The simplified model was developed by a combination of data analysis and physical arguments. The work was significantly guided by the results of a concurrent study (Eklund 1971) aimed at developing a detailed linear model of the boiler. The results of the experiments described in the previous section can be summarized by the responses, shown in fig. 8.

The behaviour shown in fig. 8 can be explained by a first order model given by the eqns. (1.1) and (1.2), where f is linear in p. A closer analysis of the experimental data indicates, however, that a linear model is not sufficient. The possibility of using a first order model is also supported by the observation that the steam temperatures do not greatly affect the responses. We will thus attempt to derive a physical model of first order and find what approximations are needed for this purpose.

Fig. 8
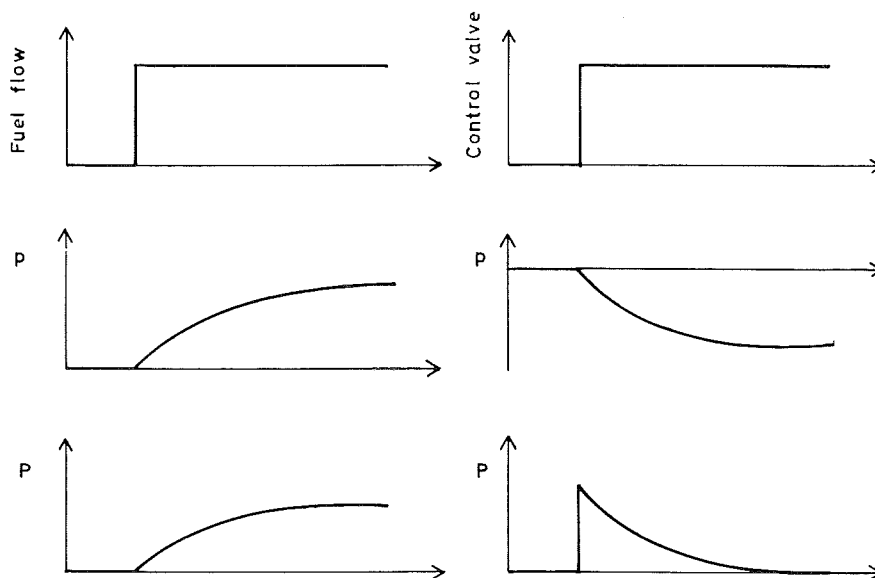
Shows qualitative responses of drum pressure (p) and output power (P) to step changes in fuel flow and control valve. Notice that the 'time constants' associated with all the variables are approximately the same.

Energy balance

The starting point in the modelling is to consider the boiler as a reservoir of energy, as illustrated by the block diagram of fig. 9.

Fig. 9
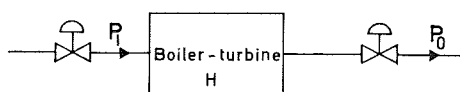
A schematic picture of the boiler as an energy reservoir.

Energy is fed to the reservoir by the fuel and the feedwater. The boiler-turbine unit delivers energy in the form of active power. The energy is stored in iron masses, water and steam. The amount of energy stored in each part is a complicated function of temperature and pressure. An energy balance gives

$$\frac{dH}{dt} = P_i - P_0, \tag{3.1}$$

where H is the total energy stored in the boiler, P_i the input power and P_0 the output power. To obtain a mathematical model it is now necessary to express H, P_i and P_0 in terms of significant variables of the boiler. Since the total energy is distributed over the boiler in a rather complicated way it is in general necessary to introduce temperatures and pressures all over the system

as state variables. However, the experiments described in Section 2 indicate that the gross features can be described by a first order dynamics. We will thus try to express the stored energy as a function of one variable, namely the drum pressure. This means in essence, that it is assumed that the distribution of the energy over the boiler does not greatly change over the operating range. The reason for choosing drum pressure rather than any other variable is that drum pressure is a significant measure of the state of the boiler. Furthermore, it is measured and the rate of change of drum pressure is an important constraint in the operation of the boiler.

Input power P_i is assumed to be a function of fuel flow and feedwater power. The input power associated with air flow and coolant flows to the attemperators are neglected. They are indeed small and their inclusion in the model gives only marginal effects.

The output power is a function of the control valve position and the steam temperature and pressure at the turbine.

There are two other output flows from the boiler which both represent power outputs, namely the flows of combustion gases and condensate. The flow of combustion gas is approximately proportional to the fuel flow. If this flow is neglected it only means a reduced effect of the fuel flow. Similarly if the enthalpy of the condensate is assumed constant, the condensate flow can be taken into account by changing the feed-water flow accordingly.

After these preliminaries we will now develop the model in detail by expressing the stored energy H, the input power P_i and the output power P_0 as functions of the control variables and the state variable.

Stored energy

The major part of the boiler energy H is stored in iron and water masses. The stored energy thus depends on material temperatures, which are essentially determined by steam temperatures, steam pressures and the load. The energy of steam masses is a minor part.

The critical simplifying assumption is that the distribution of energy stored in iron, water and steam masses do not change during transients. This implies that any energy dependent variable could be used as a measure of stored energy. Tube material temperatures are usually not measured and for good operation the boiler is constructed so as to keep turbine inlet temperatures constant over a wide load range. However, the drum pressure will change significantly according to changing load. For this reason and others stated previously drum pressure is chosen as the measure of energy storage.

Assuming that mass content of the drum is constant the energy storage of water and steam can be approximated by

$$H = H(p) = ap + b, \tag{3.2}$$

in the pressure range 50–150 bar, where p is the drum pressure and a and b constants. According to our energy distribution assumption this is believed to be a reasonable approximation of total energy storage of the boiler.

Input power

Input and output power will be computed with respect to the enthalpy level of the drum defined by the drum pressure.

Boiler input power depends on fuel and feedwater flow. The power of feedwater is the flow multiplied with the enthalpy difference between feedwater and saturation state in the drum. For simplicity the enthalpy difference is assumed to be constant. This is further discussed in Section 5. Input power is thus modelled as

$$P_i = a_1 u_1 - a_2 u_3, \qquad (3.3)$$

where u_1 is the fuel flow, u_3 the feedwater flow and a_1, a_2 constants. It is assumed that the efficiency of the boiler is constant. This also implies that the air flow during fast changes of the fuel flow always can be manipulated to meet the demand.

Output power

Neglecting the pressure drop across superheaters and turbine, active power can be modelled as steam flow multiplied by the enthalpy drop across the turbine, i.e.

$$P_0 = b_1 q \Delta h, \qquad (3.4)$$

where b_1 is a constant and

$$q = q(u_2, p) \quad \text{steam flow,}$$

$$\Delta h = \Delta h(p) \quad \text{enthalpy drop across turbine.}$$

The dependence of temperature is as stated before neglected. Equation (3.4) implies that active power is zero when drum pressure is zero. This is not true since the flow is not free from losses. This can be taken into account by adding a constant b_2 to eqn. (3.4). Output power is then modelled as

$$P_0 = b_1 q \Delta h + b_2. \qquad (3.5)$$

Assume that

$$q = b_3 u_2 \sqrt{p}, \qquad (3.6)$$

Fig. 10
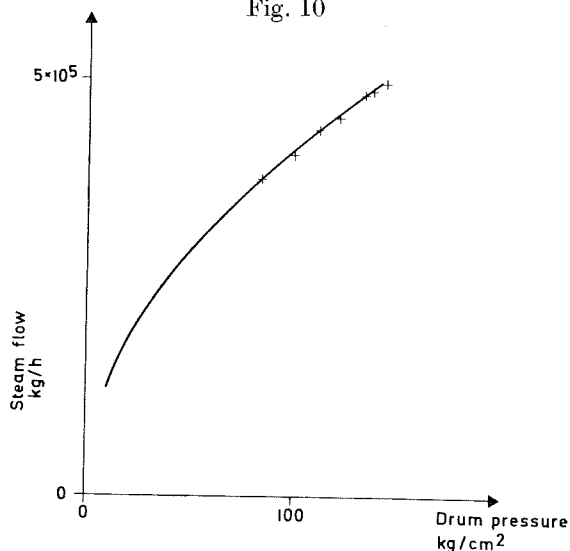
Steam flow in experiment E as a function of drum pressure for fully opened control valve. (+) is field data and the continuous curve is computed according to eqn. (3.6).

where b_3 is a constant and it is assumed that u_2 is proportional to the open valve area.   Using data from experiment E the coefficient b_3 is determined. In fig. 10 field data and computed values of steam flow, using $b_3 = 41 \cdot 5$, are shown.

The agreement is very good and eqn. (3.6) is accepted as the steam flow.

To compute the output power P_0 we still need an expression for the enthalpy drop across the turbine.   In a power plant the main part of the power is generated by the middle and low pressure parts of the turbine.   The output power is therefore estimated from the enthalpy drop in the pressure interval 1 to 35 bar. Assuming inlet temperature T_0 and condenser pressure p_0 constant the theoretical enthalpy drop for $T_0 = 530°C$ and $p_0 = 0 \cdot 04$ bar is shown in fig. 11.   The curve will pass close to origo and we assume that

$$\Delta h = b_4 p_i{}^r, \qquad (3.8)$$

where p_i is the turbine inlet pressure.

Fig. 11
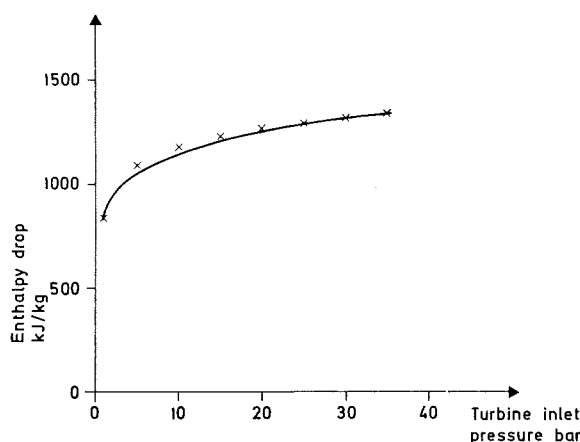
The theoretical enthalpy drop across a turbine.   Steam inlett data $T_0 = 530°C$, 1 bar $\leqslant$ p $\leqslant$ 35 bar and condenser pressure $p_0 = 0 \cdot 04$ bar.   ($\times$) is data from steam tables, and the continuous curve is computed according to eqn. (3.8), using $r = 1/8$.

Figure 11 shows the result, using $r = \frac{1}{8}$, and b_4 determined for good fit. There is a clear difference but the accuracy is believed to be satisfactory. Of course, there is the possibility to determine the value of r in this special case more accurate.   The value of r will change very little with changing inlet steam temperature and the pressure range is not less than 50 bar.

The drum pressure is taken to be proportional to the turbine inlet pressure p_i. Accepting these approximations the output power is

$$P_0 = \alpha_4 (u_2 p^{5/8} - \alpha_5), \qquad (3.9)$$

where α_4 and α_5 are constants.   The constants can be estimated from a diagram showing output power as a function of drum pressure.

Summary

Summing up the results we find that the boiler can be modelled by the eqns. (3.1), where the stored energy H is given by (3.2), the input power P_i by (3.3) and the output power by (3.9). Hence

$$a\frac{dp}{dt} = -\alpha_4(u_2 p^{5/8} - \alpha_5) + a_1 u_1 - a_2 u_3.$$

Introducing the normalized coefficients

$$\alpha_1 = \alpha_4/a, \quad \alpha_2 = a_1/a, \quad \alpha_3 = a_2/a, \tag{3.10}$$

we get

$$\begin{cases} \dfrac{dp}{dt} = -\alpha_1(u_2 p^{5/8} - \alpha_5) + \alpha_2 u_1 - \alpha_3 u_3 \\[2mm] P_0 = \alpha_4(u_2 p^{5/8} - \alpha_5), \end{cases} \tag{3.11}$$

which can be written as (1.1), (1.2) and (1.3) with $\alpha = \alpha_1/\alpha_4$.

The model (3.11) is a first order nonlinear differential equation. If the input signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are continuous functions and $p(t) > 0$, then there exists a unique solution to eqn. (3.11). Assuming that u_1, u_2 and u_3 are constants the stationary solution is given by

$$p = \left(\frac{\alpha_2 u_1 - \alpha_3 u_3 + \alpha_5 \alpha_1}{\alpha_1 u_2}\right)^{8/5}. \tag{3.12}$$

The steady state output is given by

$$P_0 = \frac{\alpha_4}{\alpha_1}(\alpha_2 u_1 - \alpha_3 u_3). \tag{3.13}$$

This implies that output power equals input power in the steady state.

Linearization of eqn. (3.11) gives

$$\frac{dp}{dt} = -\frac{5\alpha_1 \bar{u}_2}{8\bar{p}^{3/8}}\Delta p - \alpha_1 \bar{p}^{5/8}\Delta u_2 + \alpha_2 \Delta u_1 - \alpha_3 \Delta u_3, \tag{3.14}$$

where Δv is the deviation of the variable v from its steady state value $\bar{v}$. The time constant T is

$$T = \frac{8\bar{p}^{3/8}}{5\alpha_1 \bar{u}_2} = \frac{8a\bar{p}}{5a\alpha_1 \bar{u}_2 \bar{p}^{5/8}} = \frac{8}{5} \cdot \frac{H(\bar{p}) - H(0)}{P_0(\bar{p}) - P_0(0)}, \tag{3.15}$$

where the last equality follows from (3.2), (3.9) and (3.10).

Since the drum pressure p is assumed always to be greater than zero the linearized eqn. (3.14) is asymptotically stable. The time constant is inversely proportional to the control valve position $\bar{u}_2$. For constant drum pressure this means that a reduction of $\bar{u}_2$ to half its previous value will double the time constant T. Recalling the picture of the boiler as an energy reservoir we find that the time constant of the derived model can be interpreted in the usual manner as stored energy divided by heat flow.

Defining the efficiency of the boiler-turbine unit as

$$\eta = \gamma \cdot \frac{P_0}{u_1}, \tag{3.16}$$

where P_0 is the output power in Mw and u_1 is the fuel flow in kg/h. The constant γ is a factor converting the oil flow into an energy flow. Using eqn. (3.13) we get

$$\eta = \gamma \left[\frac{\alpha_4 \alpha_2}{\alpha_1} - \frac{\alpha_4 \alpha_3}{\alpha_1} \cdot \frac{u_3}{u_1} \right]. \tag{3.17}$$

The boiler model efficiency is thus constant if the ratio between feedwater and fuel flow are constant. During short time periods the efficiency can be temporarily increased using the storage capacity of the drum by decreasing the feedwater flow.

Another important property of the model is that output power will respond directly to a change in control valve position. This is approximately the physical behaviour shown in experiments B and D. The smooth responses in experiments are due to the large reheater of this particular boiler.

Finally it should be emphasized that the structure of the function f, that is the output power is not unique. It not only depends on the set of approximations used but also of the approach taken when defining the reference level for input and output power.

4. Crude parameter estimates

The non-linear boiler-turbine model developed contains five unknown parameters $\alpha_1, \ldots, \alpha_5$. In this section we will estimate the numerical values of these parameters for the boiler used in the experiments. A schematic diagram of this boiler as well as the experimental data, which will be used here, were shown in Section 2. The parameters will be determined from at least two different experiments in order to test the invariance. Small differences should indicate a suitable model structure. The type of experiments to be used is discussed and a suitable set recommended.

*Parameters α_4 and α_5*

It follows from the physical arguments of Section 3 that the output power is related to drum pressure by the equation

$$P_0 = f(p, u_2) = \alpha_4 (u_2 p^{5/8} - \alpha_5). \tag{4.1}$$

Notice that this equation also holds in the transient stage. To determine the parameters α_4 and α_5 (and also to determine if the physical arguments leading to (4.1) are reasonable) it is thus necessary to have an experiment where p and P_0 vary over a reasonable range. Data from experiment E are shown in fig. 12 together with curves, computed from (4.1).

The control valve position is normalized to the interval $[0, 1]$. The parameters α_4 and α_5 are simply estimated using two points from experimental data. We get

$$\alpha_4 = 11 \cdot 45, \quad \alpha_5 = 8 \cdot 2. \tag{4.2}$$
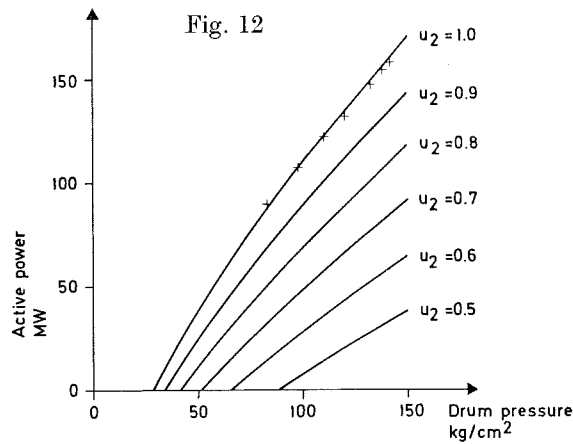
Fig. 12

Active power as a function of drum pressure.   Field data (+) is taken from experiment
      E.   The continuous curves are computed using eqn. (4.1) and $\alpha_4 = 11 \cdot 45$ and
      $\alpha_5 = 8 \cdot 2$.

The family of curves in fig. 12, corresponding to different values of u_2,
gives for every drum pressure the generated active power.   The diagram can
be used to calculate the instantaneous power response to a step change of
control valve position.   Figure 12 will also be used to compute the normalized
values of control valve position used in the simulations.

Parameter α_1

The parameter α_1 will be estimated from a dynamical experiment.   From
the model eqn. (3.11) it is clear that if u_1 and u_3 are constant the change
$\Delta dp/dt$ of dp/dt at time t_1 due to a change Δu_2 of u_2 is

$$\Delta \frac{dp}{dt} (t_1) = -\alpha_1 p^{5/8}(t_1) \Delta u_2(t_1). \tag{4.3}$$

An estimate of α_1 is easily obtained using data from experiment B, where
the boiler was excited by u_2, using a square wave type signal.   In fig. 13 the
first part of the drum pressure response is shown.   We chose to determine
$\Delta dp/dt$ at $t_1 = 400$ sec.   Using the construction in fig. 13 we get

$$\Delta \frac{dp}{dt} (t_1) = \frac{20}{400} \text{ kg/cm}^2\text{s}. \tag{4.4}$$

Fig. 13
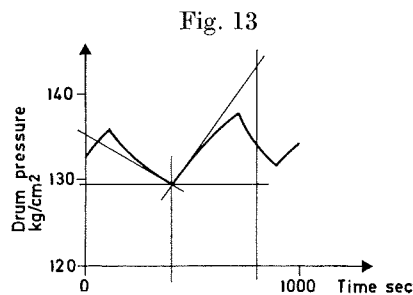
Construction to determine $\Delta dp/dt$ of drum pressure response from experiment B.

The change $\Delta u_2(t_1)$ in normalized units is read in fig. 12 knowing the drum pressure at $t_1 = 400$ sec and the active power before and after this time. The values of the active power are determined from the construction in fig. 14. Note that the value of active power after the change is extrapolated using the continuation of the response. This is necessary since the model assumes a momentarily response, but the real process shows a more smooth response.

Fig. 14
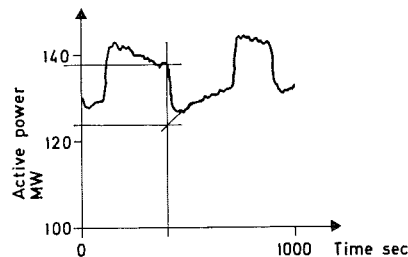
Construction to determine the values of active power before and after the change of u_2 at $t_1 = 400$.

From fig. 12 the change $\Delta u_2(t_1)$ is found to be

$$\Delta u_2(t_1) = -0 \cdot 064. \tag{4.5}$$

All variables required to determine α_1 are now known. The same method can be applied to experiment D. The resulting estimates of α_1 are shown in table 2.

An experiment of the type, used in experiment F, is especially attractive when estimating α_1. If the fuel flow and feedwater flow are kept constant and the control valve position is manipulated to give a step shaped response in active power it is seen from the model equation that this should result in a constant derivative of drum pressure. This is verified by experiment F, but here the feedwater flow has also been changed. The influence of the change appears as a bending of the drum pressure response. The estimate of parameter α_1, using experiment F, is also given in table 2.

Table 2. Estimated values of parameter α_1, using three different experiments.

Experiment	Estimate of α_1
B	$0 \cdot 038$
D	$0 \cdot 033$
F	$0 \cdot 036$

The variations of the estimate are not alarmingly large. Model sensitivity to parameter variations will be further commented on in Section 5.

Parameter α_2

Using the same type of arguments as when deriving eqn. (4.3) a formula to estimate parameter α_2 is

$$\alpha_2 = \frac{\Delta \dfrac{dp}{dt}(t_1)}{\Delta u_1(t_1)}. \tag{4.6}$$

The different values obtained at $t_1 = 400$ sec from two experiments are given in table 3. The change $\Delta u_1(t_1)$ is measured in ton/hour.

Table 3. Estimated values of parameter α_2, using two different experiments.

Experiment	Estimate of α_2
A	0·018
C	0·022

Parameter α_3

The remaining parameter is α_3. To assure an acceptable accuracy in steady state values of drum pressure this parameter is determined from steady state values. From several estimates we chose a reasonable mean value

$$\alpha_3 = 4\cdot4 \ . \ 10^{-4}. \tag{4.7}$$

Recommended experiments

Summarizing the experience, obtained by applying the estimation procedure discussed in this section, the following recommendations can be made :

(i) Make one experiment, where the steam valve is suddenly changed and then manipulated in order to give a constant output power. This experiment will give a drum pressure curve with a constant slope (like in experiment F) from which the parameter α_1 can be determined. Notice that it is important to keep other inputs constant during the experiment.

(ii) The parameter α_2 can be determined from perturbation experiments when small changes in the fuel flow are made (similar to experiments A and C). It is recommended to make the experiments at different power levels.

(iii) The parameter α_3 can be determined from steady state values or from a perturbation experiment when the feedwater flow is changed.

(iv) Make one experiment where the control valve is fully open and the fuel flow is increased from low power to full power. This experiment makes it possible to determine the function (4.1) relating output power to drum pressure and to determine the parameters α_4 and α_5. (Experiment E was of this type.) If possible the experiment should also be repeated for different settings of the control valve.

These experiments will give rough estimates of the model parameters directly. Using more sophisticated data analysis it is also possible to get more accurate estimates of the parameters if the experimental data is analyses using nonlinear identification techniques.

5. Comparison with experiments

The comparison is based on approximate mean values of numerical parameter values developed in the previous section. The model for Öresundsverket power station unit P16–G16 then is

$$\frac{dp}{dt} = -0{\cdot}035(u_2 p^{5/8} - 8{\cdot}2) + 0{\cdot}02u_1 - 4{\cdot}4 \, . \, 10^{-4}u_3$$

$$P_0 = 11{\cdot}45(u_2 p^{5/8} - 8{\cdot}2).$$

(5.1)

In the sequel we will discuss efficiency, steady state values and time constants. The model responses to the input signals used in the experiments are also shown.

Efficiency

From a set measurements including experiments A, B, C, D, E and F we obtain

$$\eta = 0{\cdot}395\text{--}0{\cdot}405 \quad \text{full load range}$$
$$\eta = 0{\cdot}340\text{--}0{\cdot}360 \quad \text{half load range.}$$

The efficiency of the boiler-turbine unit is thus decreased when load is decreased. The theoretical values of model efficiency derived from eqn. (3.17) inserting steady state values from experiments A, B, C and D are within the interval $\eta = 0{\cdot}395 - 0{\cdot}415$. The efficiency of the *boiler model* is thus roughly constant over a wide load range while the efficiency of the *plant* decreases with decreasing load. In the simulations this discrepancy is taken care of by adjusting the fuel flow so that an efficiency of 0·405 is obtained in all cases.

Steady state values

It is important that the model predicts steady state values accurately since the model should cover at least the upper half of the load range. The steady state value of drum pressure does also influence the increment in active power when e.g. control valve position is altered. In table 4 computed and measured steady state values of drum pressure are given. The maximum difference is roughly 3 kg/cm² which means about 3 Mw active power when control valve is fully opened. The covered load range is from 70 Mw to 160 Mw active power. The percental error then range from 1·8% to 4·3%.

Table 4. Steady state values of drum pressure. Measured and computed from the model (5.1).

Experiment	Drum pressure	
	Measured	Model
A	125	125·2
B	132·5	135·6
C	108	106·7
D	109	110·0

Time constants

An expression for the time constant of the linearized model equation is given by eqn. (3.15). Using the initial steady state values from the four experiments A, B, C and D the time constant can then be evaluated from eqn. (3.15). The result is given in table 5.

Table 5.   Time constant T for four different experiments.   Computed from the model (5.1).

Experiment	T sec.
A	280
B	307
C	353
D	350

The maximum likelihood identification method has also been applied to determine linear models relating the input variables, fuel flow and control valve position to the output variables, drum pressure and active power. The dominant time constant found was within the interval 200–800 sec. The time constant increased when decreasing the load. This indicates that the model responses will appear a little too fast but the load dependence of the time constant is correct.

Simulations

The computed model responses compared to the measured ones for the five experiments A through F are given in figs. 15–19. Since the control valve position was not recorded automatically in the experiments no calibrating curve referring real values to normalized can be computed with an acceptable degree of accuracy. In the simulations mean value and deviations for u_2 was computed in one point and then used throughout the experiment. All other input sequences used in the simulations are the measured sequences. Note that the steady state value of model responses is altered to agree with the measured ones. The real error is previously given in table 4.

Inspecting the discrepancies of responses there are deviations which require some comments.

In fig. 16 changes of feedwater flow at $t \sim 1000$ sec cause knees in the drum pressure response of the model which are not present in the measured curve. On the other hand, in figs. 17 and 18 the effect of feedwater is in accordance with measurements.

The discrepancies of active power responses at the time of changes in figs. 16 and 18 can be explained by two arguments. The control valve position exciting the model is computed and not the real input signal. This is clearly causing the disagreement at time $t = 3200$ in fig. 16. The smooth shape of measured active power curves is caused by the reheater of the boiler. The reheater is not included in the model which partly explains the differences. On this point the model behaviour can be improved by introducing a differential equation for the reheater. This is easily done but at the expense of increased model complexity.

Fig. 15
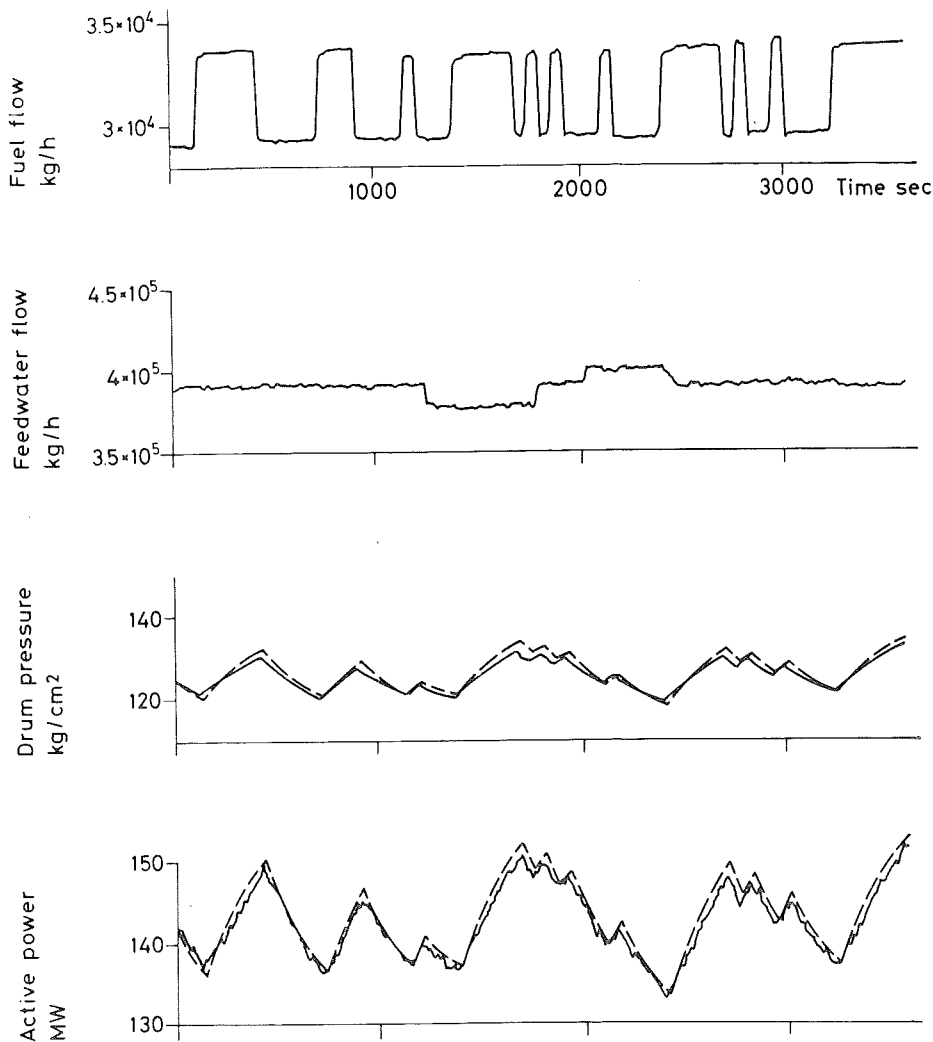
Comparison of measured boiler-turbine (solid) and model (dashed) responses of experiment A.

In experiment E, shown in fig. 19, the feedwater temperature is also given. The large discrepancy in drum pressure and active power in the later half of the experiment is due to the decreased feedwater temperature. The enthalpy difference between feedwater and drum saturation enthalpy was assumed to be constant. The modelling of this difference requires a model of the economizer which means (at least) one additional state variable for feedwater temperature. If model complexity is allowed to grow it seems more appropriate to include this phenomenon than the effects, caused by the reheater.
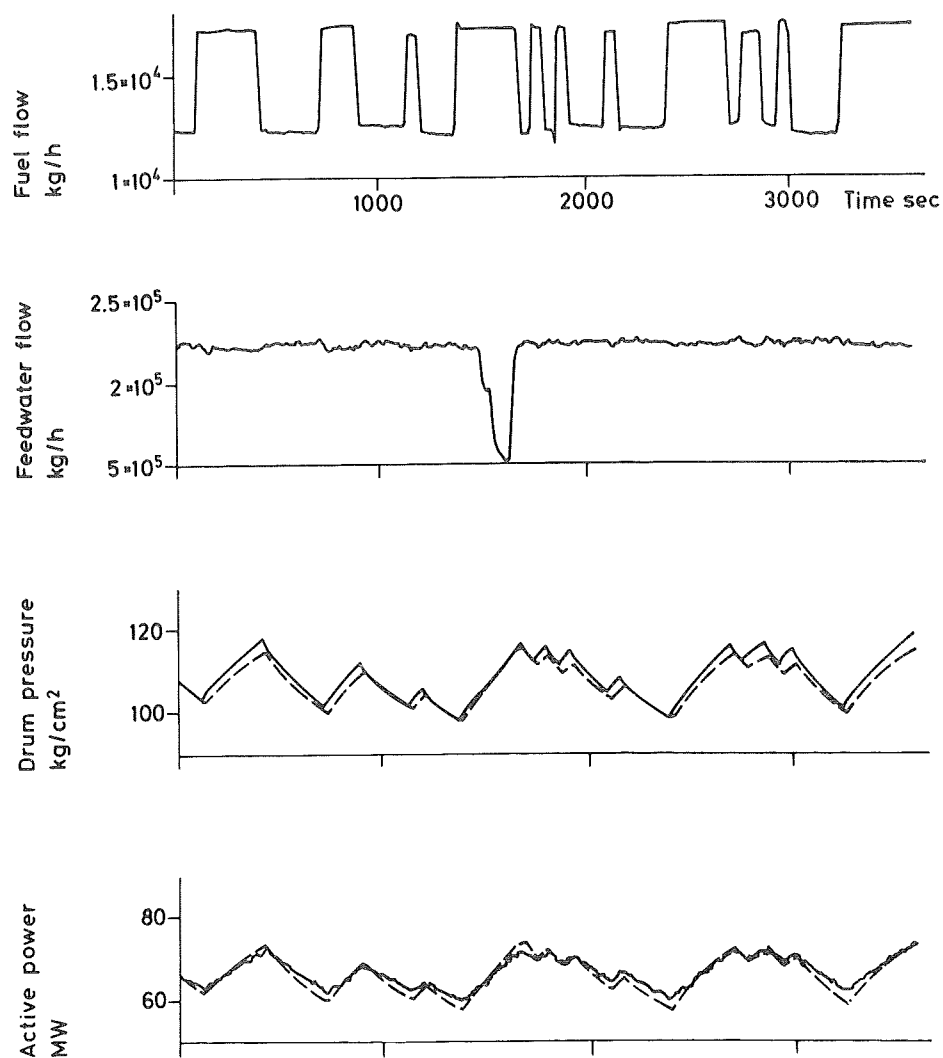
Any systematic investigation of the influence of parameter variations on model behaviour has not been made. Simple simulations, however, show that model properties not critically depend on the choice of model parameters.
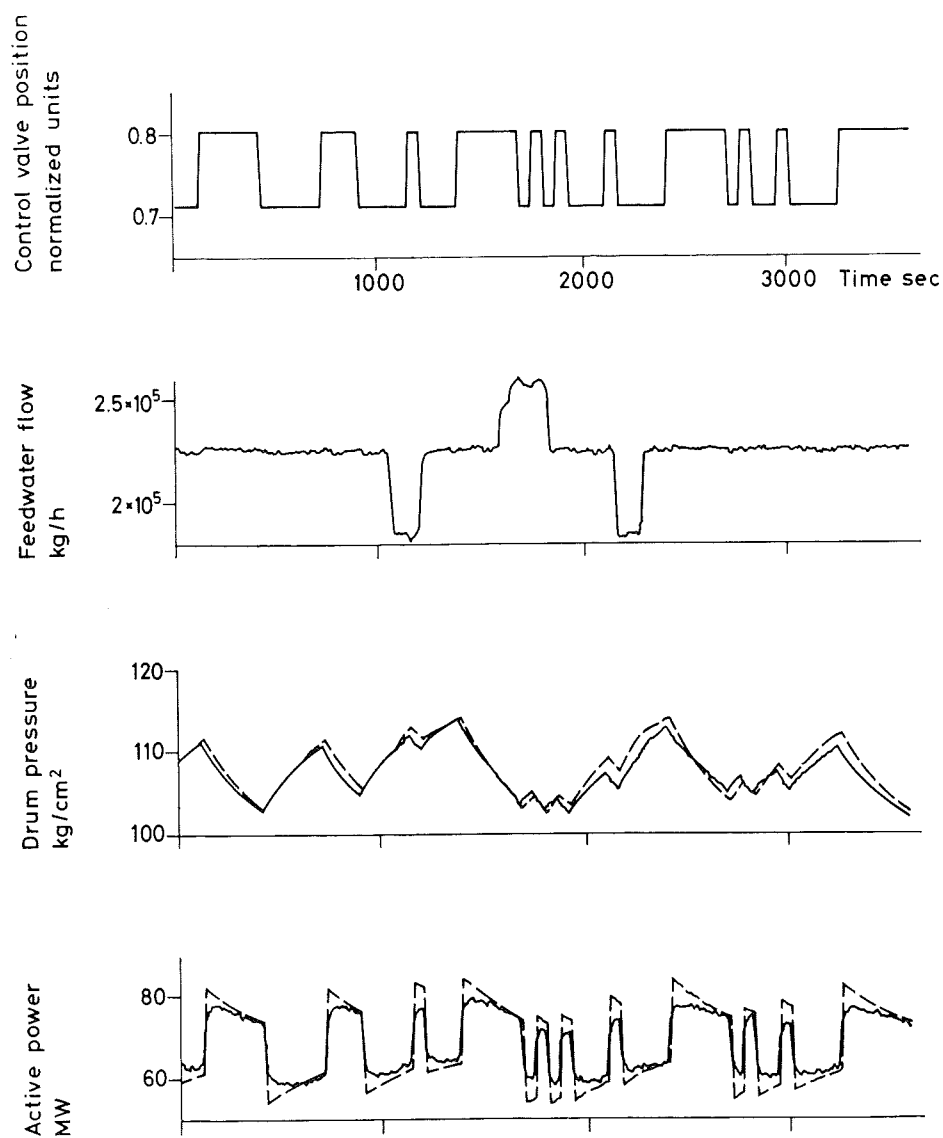
Fig. 16
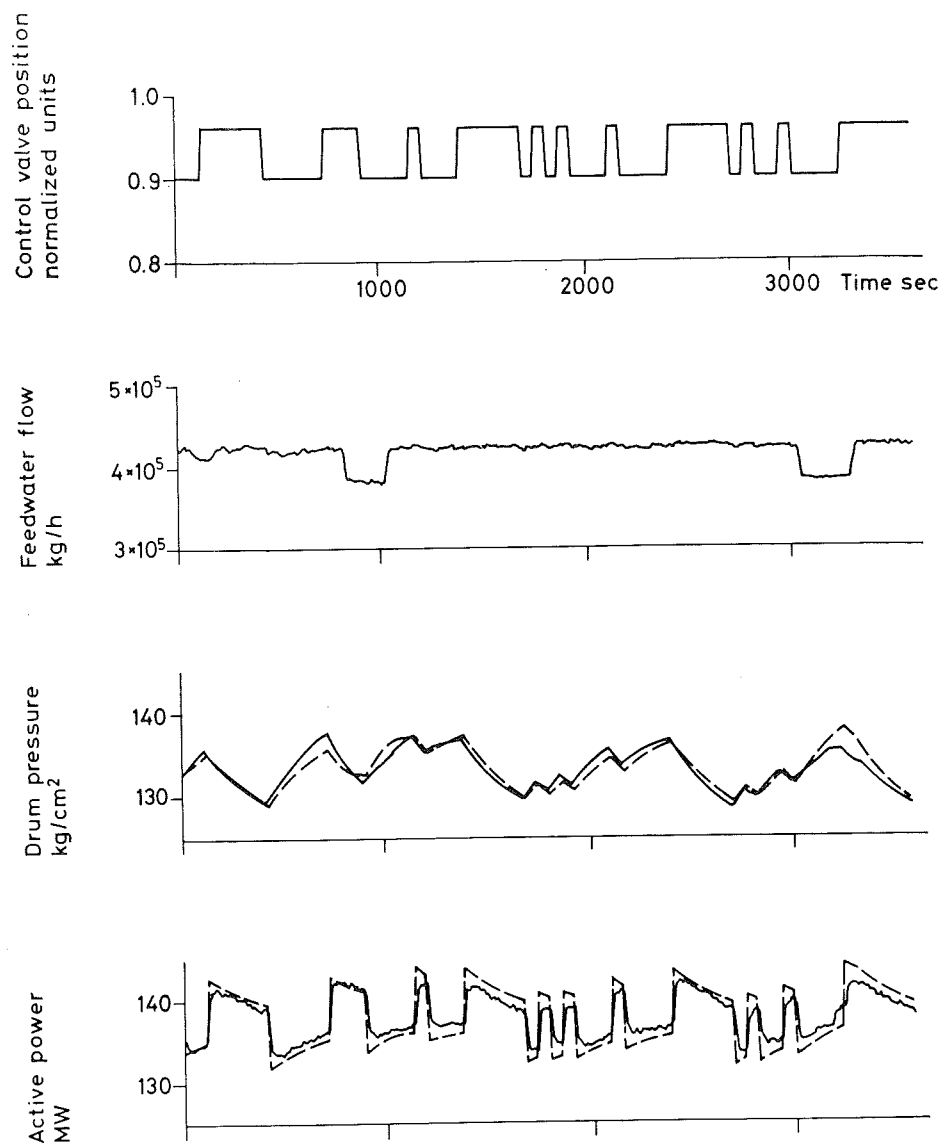
Comparison of measured boiler-turbine (solid) and model (dashed) responses of experiment B.
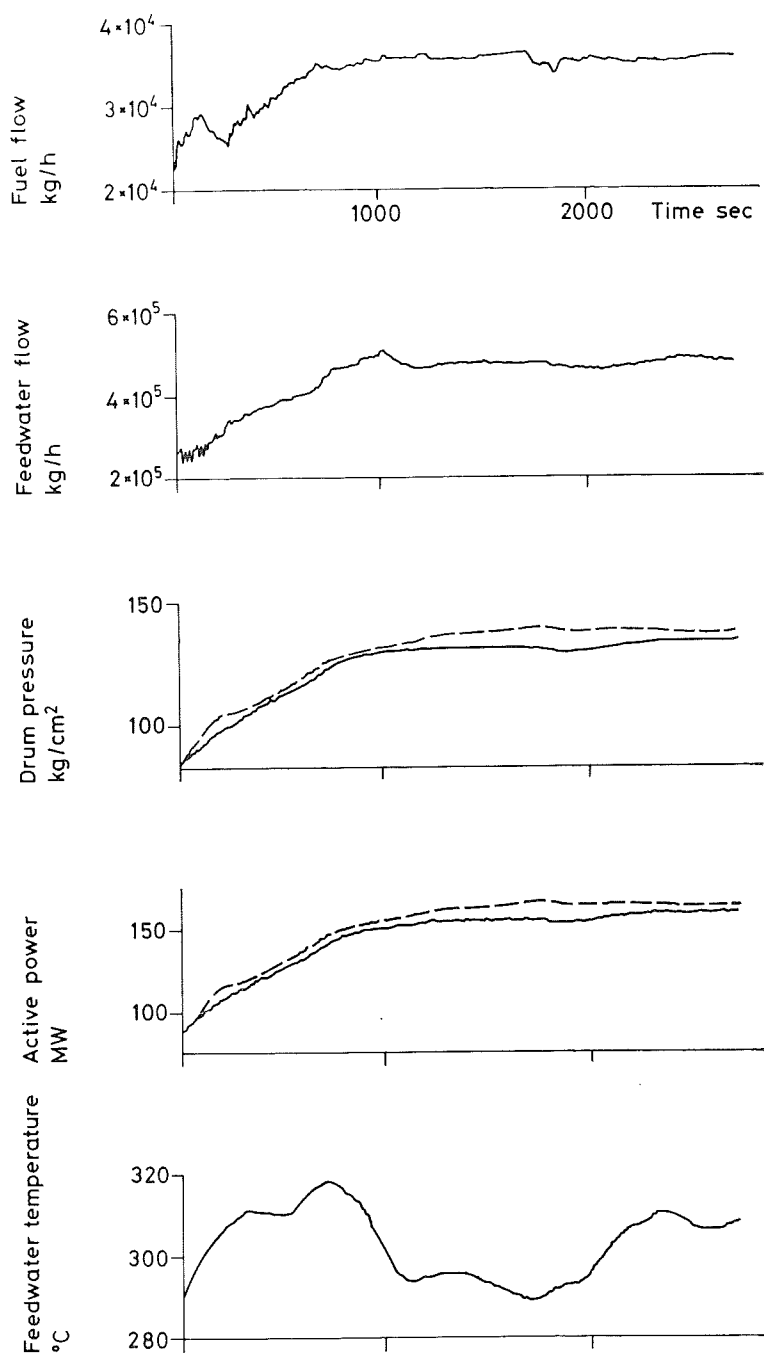
Fig. 17

Comparison of measured boiler-turbine (solid) and model (dashed) responses of experiment C.

Fig. 18

Comparison of measured boiler-turbine (solid) and model (dashed) responses of experiment D.

Fig. 19

Comparison of measured boiler-turbine (solid) and model (dashed) responses of experiment E.

## 6. Qualitative properties of drum boilers

The simplified model can be used in many different ways. As an illustration we will here show how it can be used to obtain a qualitative understanding of the behaviour of a drum boiler.

Assuming steady state we get from eqn. (1.1)

$$P_0 = f(p, u_2) = g(u_1, u_3),$$

i.e. the output power f equals input power g. In fig. 20 we show a graph of the function f for different values of u_2. Assume that the plant is operating in steady state with a given drum pressure and a given steam valve position $u_2 < 1$ (point A of the diagram). Now if the input power is increased the drum pressure and the output power will slowly increase until a new steady state condition is achieved (point B of the diagram). However, if the control valve is suddenly opened the output power will immediately increase to a new value indicated by point C in the diagram.

Fig. 20
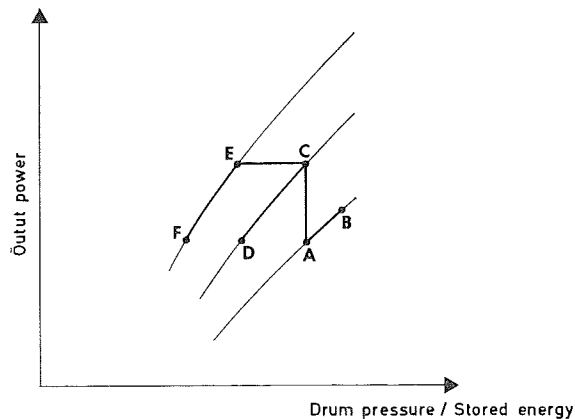
Relationship between output power and drum pressure or stored energy during load changes.

Keeping control valve position constant the pressure and output power will then slowly decrease until point D is reached. Also if control valve instead is manipulated (valve area increased) to keep output power constant point E of the diagram will be reached. The valve is now fully opened and the pressure and the output power decrease until a new steady state condition is assumed in point F. This is the operating procedure of experiment F.

The axis of abscissa can also be graded in energy, stored in the boiler, using eqn. (3.2), e.g.

$$H(p) = ap + b,$$

where

$$a = \frac{\alpha_4}{\alpha_1}.$$

This means that the decrease in energy storage for the operating schemes ACD and ACEF can be read from the diagram. The energy decrease is commonly referred to as storage capacitance.

The storage capacitance S can be expressed in several ways. For both schemes ACD and ACEF S can be given as total energy reduction

$$S_{ACD} = H_A - H_D \text{ Mwh},\qquad (6.1)$$

$$S_{ACEF} = H_A - H_F \text{ Mwh},\qquad (6.2)$$

where H_I is energy stored at point I of the diagram.

In the interval CE the rate of change of drum pressure is constant (u_1 and u_3 are kept constant). Then in this interval storage capacitance can be given as

$$S_{ACE} = \frac{H_A - H_E}{p_A - p_E} \text{ Mwh/bar}.\qquad (6.3)$$

Using the model equation still another measure can be given as

$$S'_{ACE} = \frac{H_A - H_E}{p_A - p_E} \cdot \frac{dp}{dt} \text{ Mw},\qquad (6.4)$$

where dp/dt is constant. The measures (6.3) and (6.4) are exact in the interval CE. Approximately they can be extended to the entire interval ACEF. It is clear from this discussion that considerable insight into the properties of the drum boiler is provided by the simple diagram. Also notice that the function f can be determined experimentally by observing the steady state relation between drum pressure and output power.

REFERENCES

A number of drum boiler models based on basic physical laws are available in literature. Model order is high ranging from 10 to 100 and the equations usually linearized.

ANDERSON, J. H., and KWAN, H. W., 1970, *Int. J. Control*, 1970, **12,** 977.

CHIEN, K. L., ERGIN, E. I., LING, C., and LEE, A., 1958, *Am. Soc. mech. Engrs*, **80,** 1809.

EKLUND, K., 1971, *Linear Drum Boiler-turbine Models*, Report 7117, Lund Inst. of Techn., Div. of Aut. Control, Lund.

PROFOS, P. 1962, Die Regelund von Dampfanlagen (Berlin : Springer-Verlag).

WILLIAMS, T. J., *et al.* : 1st–6th quarterly reports 1966–1967, Digital Control of a Steam Propulsion System of a Naval Vessel, Purdue Laboratory for Applied Industrial Control, Purdue Univ., Lafayette, Indiana.