SCHOOL OF ECONOMICS AND MANAGEMENT
Lund University

Master Thesis

15 ECTS credits

Pricing Skewness and Kurtosis Risk on the Swedish Stock Market

Andreas Johansson

Tutor: Birger Nilsson

Department of Economics, Lund University, Sweden

September 2005

Abstract

This paper investigates the role of higher moments on the Swedish stock market 1979-2004 using the asset pricing framework developed in Fang & Lai (1997). The models are estimated using a two-step ordinary least squares procedure and, in addition, an instrumental variables approach to account for the potential problem of errors in variables. Estimations have been made on the full period and in two sub-periods. The results show that the asset pricing performance improves when augmenting the standard capital asset pricing model with third (skewness) and fourth (kurtosis) moments. Further, we find that both skewness and kurtosis risk carries statistically significant risk premiums. Our results are in line with the results of Fang & Lai (1997) and other surveys covering the similiar area, like Kraus & Litzenberger (1976). The results presented in this survey can further be used by investors on the Swedish stock market, to make asset management even more effective by take into account the effect of skewness and kurtosis in asset return distribution.

Keys: Skewness, Kurtosis, CAPM, mean-varaince

I want to thank my tutor Birger Nilsson* for emminent supervision. I also want to thank Isak Ahlbom** for composition of the data.

*Nilsson is a PhD in Economics at Lund University, Lund School of Economics.

**Ahlbom is a MSc in Economics from Lund University, Lund School of Economics.

Contents

1 Introduction

In 1964, William Sharp brought the capital asset pricing model (CAPM) to the world. CAPM has since then been the absolutely most popular model for asset pricing, and is a cornerstone of financial economics. But, the CAPM theory has become more and more criticized for its rather strong assumptions about the market condition. The whole mean-variance theory used by the capital markets has after different stock market events, like the dot com crash, been more and more questioned.

In recent years a number of empirical models have been detected, which are not consistent with the standard asset pricing models such as the mean-variance CAPM. Among early published studies within this area, we have Banz (1981) who relates expected returns to firm size, while Fama and French (1992) later on link expected returns to the ratio of book to market value. Although the persistence of different anomalies over time is still subject to debate, the evidence suggests that the mean-variance CAPM is not a satisfactory description of complete market equilibrium. These pricing anomalies may be related to the possibility that irrelevant factors appear to be priced while other important ones are not. The third moment, skewness, of a distribution, has been investigated in several papers; but the forth moment, kurtosis, of a distribution has had a lack of attention.

Fang and Lai (1997) derive a four moment CAPM where, as well as variance, skewness and kurtosis contributes to the risk premium of an asset. They apply their model on all stocks continuoysly listed on the New York Stock Exchange over the period January 1969 through December 1988. They find positive risk premiums for conditional skewness and conditional kurtosis over the period. The purpose of this paper is to do a similar study on the Swedish stock market. If we find similar risk premiums on the Swedish stock market, these results can be used in investment processes, to perform more effective asset management.

The reminder of this paper is organized as follows. In Section 2 we discuss the literature and selected research papers of interest. In Section 3 we discuss the central theory behind capital asset pricing models and show the properties of skewness and kurtosis. Section 4 presents the econometric methods used. In Section 5 we discuss the properties of the data material. In section 6 we present the results from the econometric estimations made on the data material in section 5. We also comment and analyse the empirical results in this section. Section 7 is conclusions. Section 8 is references. Appendix A contains relevant graphs. Appendix B is the program made in Gauss.

2 Previous studies

In this section we make a review of selected papers in the field, including the article by Fang and Lai (1997). The purpose is to present a view of different approaches to the inclusion of skewness and kurtosis into capital asset pricing models.

2.1 Fang and Lai (1997)

Fang and Lai (1997) derive a capital asset pricing model which takes into account, not only the effect of systematic variance and systematic skewness, but also the effect of systematic kurtosis in the return generating process. They use all stocks that were continuously listed on the New York Stock Exchange from January 1969 through December 1988 to test the model. The rate of return on US Treasury bills is used as a proxy for the risk-free interest rate. The value-weighted NYSE composite index is used as the market portfolio. By doing ordinary least squares (OLS) and instumental variable estimation (IVE) regressions using a forth moment capital asset pricing model, risk premium for systematic variance, systematic skewness and systematic kurtosis is estimated. All stocks are arranged into portfolios ranked by their beta estimates. Fang and Lai use sub periods of five years to estimate the risk premiums. They conclude that investors have a preference for positive skewness in their portfolios and thus require a higher expected return for assets when the market portfolio is negatively skewed and vice verse. Further they conclude that investors are averse to variance and kurtosis in their portfolios and require higher excess rates of return for bearing higher systematic variance and systematic kurtosis.

2.2 Other relevant papers

Kraus and Litzenberger (1976) initiate the discussions about higher order moments of CAPM in the paper "Skewness Preference and The Valuation of Risk Assets". They extend the CAPM to include the effect of skewness on asset valuation. Kraus and Litzenberger argue that moments beyond the third is irrelevant since it is assumed that the investor's expected utility is only definable over the first three central moments of the probability distribution. They further conclude that investor's have a preference for positive skewness. By estimating betas and gammas (skewness) for the NYSE stocks from January 1926 to December 1935, which are further ranked and composed into portfolios, Kraus and Litzenberger achieve evidence of significant risk-premiums for skewness. Thus, a third moment CAPM provides a more effective asset pricing process than the basic two-moment mean-variance CAPM.

Kraus and Litzenberger (1983) later on derive a three moment consumption-oriented capital asset pricing model, which apart from mean and variance, also includes the effect of skewness on equilibrium expected rates of return. They show that quadratic characteristic lines are sufficient for a subset of assets to be priced according to the three moment CAPM, given an efficient capital market where individuals have homogeneous probability beliefs and non increasing absolute risk aversion. According to their empirical tests, the three moment CAPM shows a negative relationship between systematic skewness and asset returns. Thus, positive systematic skewness is preferable to negative.

Dittmar (2002) investigates non-linear pricing kernels where the risk factor is endogenously determined. A quadratic pricing kernel which relates the expected return of assets to co-variance with the return on aggregate wealth is tested. Dittmar argues that since the co-skewness of a random variable x with another random variable y can be represented as a function of $Cov(x, y)$ and $Cov(x, y^2)$, the quadratic pricing kernel is consistent with a three-moment CAPM. From this assumption he concludes that a cubic pricing kernel is, in the same way consistent with a fourth-moment CAPM, thus in which the agents have preferences over the fourth moment, kurtosis. Dittmar further concludes that moments beyond the fourth are difficult to interpret as they are not restricted by standard preference theory. However, the fourth moment, kurtosis, has according to Dittmar both a utility-based and intuitive rationale. Dittmar's theoretical assumptions are confirmed by his empirical results.

Zhou (1993) rejects the mean-variance efficiency given the normality assumption. However, Zhou concludes that the efficiency of the mean-variance model can not be rejected when using alternative distributions, thus the Normal is the most efficient one of the used distributions.

Brocket and Kahane (1992) explore the relationship between moment preferences and expected utility theory. They compare the decision process of individual projects in an economy allowing arbitrary distributions of returns, thus the distributions are not restricted to the Normal distribution. Brocket and Kahane conclude that a risk averse investor may, when choosing between two projects with positive expected returns, prefer the project which has lower mean with higher variance but with lower positive skewness.

Campbell and Siddique (2000) investigate if asset returns have systematic skewness, and how the reward for taking this skewness risk is settled. They use an asset pricing model that incorporates conditional skewness. By using different factors such as industry, size, book-to-market ratios etc., they create portfolios and determine the impact of skewness on the rate of returns. They conclude that systematic skewness is important and bring a risk premium, on average, of 3.60 percent per year. However, no effect of kurtosis has been taken in consideration.

3 Theoretical Asset Pricing Models

In this section we state the assumptions behind pricing models, and give a brief review for the properties of a distibution.

3.1 Mean-Variance

In 1952 Markowitz published a paper entitled "Portfolio Selection". He showed how to create a portfolio with the greatest possible expected rate of return, mean, given a certain level of risk, variance. Ever since, it has been a cornerstone within financial economics and used by the market to create efficient portfolios. An efficient portfolio has to be constructed, such as there exist no other portfolios with higher expected return given the same variance, or with lower variance given the same expected return. If this criterion is not fulfilled, the portfolio is said to be inefficient. The variance is build upon the Normal distribution, which is a symmetric measure. This means that high negative expectations, in the outer bound of the left tail of the Normal distribution, is given the same "value" in terms of risk as high positive expectations, in the outer bound of the right tail, of return. A rational investor always prefer more to less, but in the mean-variance framework, the investor is assumed to be indifferent between high probability of negative outcomes and high probability of positive outcomes according to the Normal distribution.

3.2 Capital Asset Pricing Model – CAPM

The general idea about CAPM is to determine the asset prices in market equilibrium. The classical CAPM was introduced by Sharp (1964), Lintner (1965), and Mossin (1996), and it is noted that the structure of capital market equilibrium is presented in a way, relating the equilibrium asset returns to the one and only risk factor, the market beta.

The first model of CAPM has through time lost some of its earlier appeal. In the paper "A critique of the asset pricing theory´s tests" Roll (1977) criticises the validity of CAPM. He argued that the only testable implication of the CAPM is the mean-variance efficiency of the market portfolio, and since the true market portfolio is non observable, it is not possible to test the true validity of CAPM.

For CAPM to hold there are some assumptions, which have to hold (Haugen 2001):

Assumption I: Investors can choose between portfolios on the basis of ex-

pected return and variance.

According to CAPM, the only variables sufficient, to make a portfolio choice is the expected returns and variances. CAPM assumes that the return of all asset's follow a Normal distribution. However, the return distributions of assets are not always Normal and seldom the same.

For a Normal distribution the only relevant parameters, are the expected value and the variance. According to CAPM, higher variance is associated with more risk. A higher expected value is preferable to a lower expected value, since the investor wants to maximize their risk-adjusted returns.

To illustrate the distribution of asset returns, consider figure *5 (page 18)*, which shows the distribution of AFGX for the last 25 years. According to the Jarque-Bera test, AFGX does not follow a Normal distribution.

Assumption II: All investors have the same expectation and investment horizon of security returns

CAPM assumes that all investors share the same belief about the market. This assumption is far from completely true. We would claim that the different expectations about the market are as many as there are investors on the market.

Assumption III: Perfect capital market

CAPM assumes that there are no transaction costs associated when buying or selling assets. We also assume that there are no taxes on capital gains, no restrictions on short selling and that we can borrow and lend at the risk free interest rate.

The general CAPM assumes in equilibrium the relationship in equation *1* where β_i is calculated according to equation *2*.

$$E(r_i) - r_f = \beta_i \left[E(r_m) - r_f\right] \tag{1}$$

$$\beta_i = \frac{Cov(r_m, r_i)}{Var(r_m)} \tag{2}$$

where r_i is the return of stock i and r_m is the return of the market portfolio.

3.3 The moments of a distribution

3.3.1 Mean - μ

The mean is sometimes referred to as the first moment of a distribution The mean is also equal to the expected value of the distribution and is defined as equation *3* implies (Verbeek, 2004).

$$\mu = E[r_t] \tag{3}$$

where r_t is the return of an asset at time t.

3.3.2 Variance - σ^2

The dispersion of the distribution is referred to as the variance, which is the square of the standard deviation. The variance can be defined according to equation *4* (Verbeek, 2004).

$$\sigma^2 = V\{r_t\} = E\{(r_t - \mu)^2\} \tag{4}$$

where r_t is the return of an asset at time t and μ is the mean return stated in equation *3*.

3.3.3 Skewness -S

The third moment is known as skewness. Skewness measures the extent to which a distribution is non symmetric about its mean. A Normal distribution is not skewed, thus is has a skewness of zero. The skewness of a distribution is defined by equation *5* (Verbeek, 2004). A Normal distribution is illustrated in figure *1*, with a skewness of zero, whether figure *2* shows a negatively skewed distribution.

$$S = \frac{E\{(r_t - \mu)^3\}}{\sigma^3} \tag{5}$$

where r_t is the return of an asset at time t and μ is the mean return stated in equation *3* and σ is the standard deviation.

An asset is defined as having "positive co-skewness" with the market when the residuals from the regression of its returns, and the market returns are positively correlated with squared market returns. Therefore, an asset with positive (negative) co-skewness

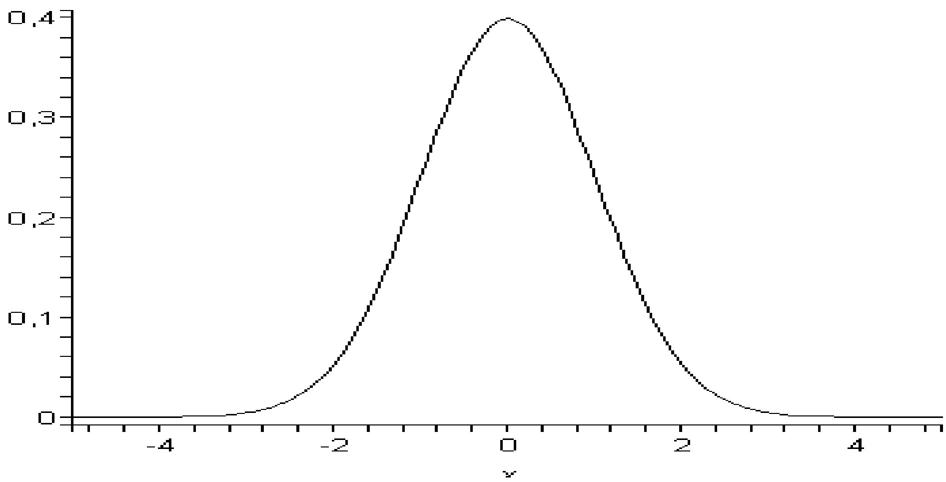

Figure 1: Normal Distribution

reduces (increases) the risk of the portfolio to large absolute market returns, and should yield a lower (higher) expected return in equilibrium (Verbeek, 2003).

3.3.4 Kurtosis - K

The fourth moment of a distribution, kurtosis, measures the peakedness and fat-tailedness of the distribution. A Normal distribution has a kurtosis of three and a distribution with a higher kurtosis than three is said to have excess kurtosis. So kurtosis above three means a more peaked distribution with fatter tails than the Normal one. Consider figure *3*, the thin line illustrates a Normal distribution, whereas the thick line is a distribution with excess kurtosis, thus it has a higher peak than the Normal distribution. Kurtosis for a distribution can be defined as equation *6* (Verbeek, 2004) implies.

$$K = \frac{E\{(r_t - \mu)^4\}}{\sigma^4} \tag{6}$$

where r_t is the return of an asset at time t, μ is the mean return stated in equation *3* and σ is the standard deviation.

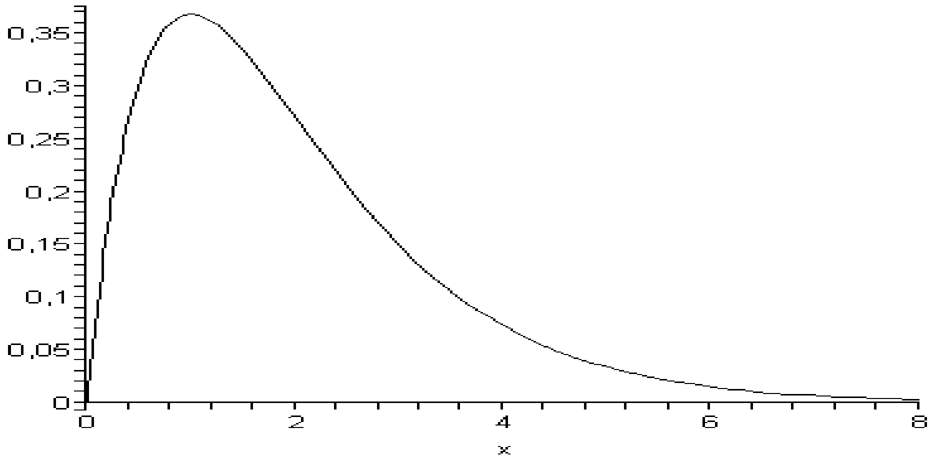

Figure 2: Negatively Skewed Distribution

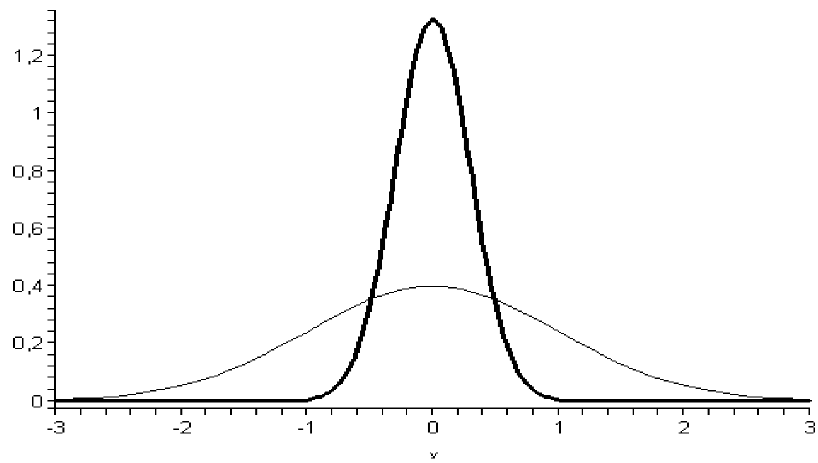

Figure 3: Normal distribution (thin line) and distribution with excess kurtosis (thick line)

4 Econometric Estimation

This section is a reproduction of the methodological descriptions in Fang and Lai (1997). The purpose is to produce an expression to maximise an investor's expected utility subject to not only mean and variance, but also skewness and kurtosis. We also explain the assumptions behind the instrumental variables approach.

4.1 Econometric Survey

Fang and Lai (1997), assume that there are n risky assets and one risk-free asset. All assets are assumed to have limited liability and their returns are accrued only in the form of capital gains. The capital market is also assumed to be perfect and competitive with no taxes or transaction. All investors have the same beliefs about the return of the security and seek to maximize their expected utility. The utility can be represented by the mean, variance, skewness, and kurtosis of terminal wealth, subject to the budget constraint.

Suppose an investor invests x_i of his wealth in the i-th risky asset, and $1-\sum x_i$ in the risk-free asset. The mean, variance, skewness and kurtosis of that investor's portfolio are $X'(\overline{R}-R_f)$, $X'VX$, $E[X'(R-\overline{R})/(X'VX)^{1/2}]^3$, and $E[X'(R-\overline{R})/(X'VX)^{1/2}]^4$, where $X'=(x_1,x_2,...,x_n)$ is a $(nx1)$ vector of the holdings in risky assets.

Fang and Lai argue that because of the relative percentage invested in different assets, the portfolio can be rescaled. The investor's preference, which is a function of the mean, variance, skewness, and kurtosis of the terminal wealth, can thus be defined over the mean, skewness, and kurtosis, subject to a unit variance. The increase of the mean and skewness of terminal wealth is assumed to increase the investor's utility. On the other hand, an increase of the kurtosis of terminal wealth increases the probability of extreme outcome of terminal wealth which will result in either a gain or a lost to the investor. Therefore, the marginal utility of kurtosis is assumed to have a negative impact on the utility. To maximize the investor's expected utility of terminal wealth, Fang and Lai sets up the following Langrangian expressions, subject to the budget and unit variance constraints:

$$MaxU\{X'(\overline{R}-R_f),E[X'(R-\overline{R})]^3,E[X'(R-\overline{R})]^4\}-\lambda(X'VX-1) \tag{7}$$

where λ represents the Lagrangian multiplier of unit variance constants. To solve for the investor's portfolio equilibrium condition, we take the first-order condition of equation *7*. By doing that we get equation *8*

$$\overline{R} - R_f = \varphi_1 VX + \varphi_2 Cov[(X'(R - \overline{R}))^2, R] + \varphi_3 Cov[(X'(R - \overline{R}))^3, R] \tag{8}$$

where $Cov[(X'(R-\overline{R}))^i, R]$ is the covariance vector of asset return R with the portfolio return $X'(R - \overline{R})^i$ for $i = 1, 2, 3$.

In order to move from individual equilibrium for investors to market equilibrium, a separation theorem which assumes all investors hold the same probability beliefs and have identical wealth coefficients is assumed. By assuming that the portfolio held by investors must be the market portfolio in order to fully clear the market. Equation *9*, which is the asset pricing model with skewness and kurtosis, is derived from *8* by letting R_m be the market portfolio with $R_m = X'_m(R - R_f)$ and $X'_m C X_m = 1$,

$$\overline{R} - R_f = \varphi_1 Cov(R_m, R) + \varphi_2 Cov(R_m^2, R) + \varphi_3 Cov(R_m^3, R) \tag{9}$$

Fang and Lai (1997) rewrite equation *9* to create the linear empirical version of the four-moment CAPM as equation *10.*

$$\overline{R_i} - R_f = b_1\beta_i + b_2\gamma_i + b_3\delta_i, \quad i = 1, ..., n, \tag{10}$$

where $\overline{R_i}$ is the expected rate of return on the i-th risky asset; $\beta_i = Cov(R_i, R_m)/var(R_m)$, is defined as the systematic variance of asset i; $\gamma_i = Cov(R_i, R_m^2)/E[(R_m - E(R_m))^3]$, is the systematic skewness of asset i; $\delta_i = Cov(R_i, R_m^3)/E[(R_m - E(R_m))^4]$ and is defined as the systematic kurtosis of asset i. The parameters b_1, b_2, b_3 are the market premiums for respective risks.

Fang and Lai (1997) derive a cubic market model, equation *11*, which is consistent with the four-moment CAPM:

$$R_{it} = \alpha_i + \beta_i R_{mt} - \gamma_i R_{mt}^2 + \delta_i R_{mt}^3 + \varepsilon_{it}, \quad i = 1, ...n; \quad t = 1, ...T, \tag{11}$$

where β_i,γ_i, and δ_i are multiple regression coefficients identical to the parameters in equation *10.*

According to utility theory, the market premiums for the respective risks, b_1, b_2, b_3, are suppose to have the following signs; $b_1 > 0$ since higher variance is connected with a higher probability for uncertain outcomes, b_2 has the opposite sign of market skewness since $\frac{d\overline{W}}{dm_w}$ is negative under non-increasing absolute risk aversion, and $b_3 > 0$, since positive kurtosis increases the probability of an negative outcome according to our four-moment CAPM. The market distribution properties are presented in figure *5* for the full period 1979-2004, and in figure *7* and *8* in Appendix A, for the sub-periods.

In our survey we will use two different procedures to estimate the parameters, namely ordinary least square (OLS) and instrumental variables (IV). The IV estimation will be further described in detail in section 4.2.

We will use Gauss statistical package for the data processing. A program (see Appendix B) is created which read our dataset of over 600 series, market portfolio (AFGX) and risk-free rate. After running the program, Gauss will provide us with all statistical information necessary to make an analysis.

4.2 Instrumental Variables (IV)

In the OLS framework there are certain assumptions that must be fulfilled in order to get an efficient OLS estimator. Assumption one says that the expected value, the mean, of the error term is zero. Assumption two implies that the independent variable(s) and the residuals are independent, which also is the requirement for consistency of the OLS estimator. Assumption three assumes homoskedasticity, thus that the error terms have a constant variance over time. Assumption four implies no autocorrelation, thus zero correlation between different error terms (Verbeek, 2004). If assumption two, $E\{x_i\varepsilon_i\} = 0$, does not hold, the OLS is said to be unbiased and inconsistent. However, to achieve a consistent estimator, it is necessary to make sure that the model is statistically identified. To do this we need to impose further conditions. Assume we have two independent variables. From assumption two we can set up moment conditions as: $E\{(y_i - x'_{1i}\beta_1 - x'_{2i}\beta_2)x_{1i}\} = 0$ and $E\{(y_i - x'_{1i}\beta_1 - x'_{2i}\beta_2)x_{2i}\} = 0$ which will hold for OLS. But since assumption two does not hold we use an instrumental variable z, which is uncorrelated with the error term. If such an instrument can be found, the second moment condition can be replaced as $E\{(y_i - x'_{1i}\beta_1 - x'_{2i}\beta_2)z_{2i}\} = 0$. The instrumental variables estimator can then be solved trough the following restrictions:

$\frac{1}{N}\sum_{i=1}^{N}(y_i - x'_{1i}\overset{\wedge}{\beta}_{1,IV} - x'_{2i}\overset{\wedge}{\beta}_{2,IV})x_{1i} = 0$

$\frac{1}{N}\sum_{i=1}^{N}(y_i - x'_{1i}\overset{\wedge}{\beta}_{1,IV} - x'_{2i}\overset{\wedge}{\beta}_{2,IV})z_{2i} = 0$

The solution to this system leads to equation *12*, which is the estimator for β.

$$\beta_{IV} = (Z'X)^{-1}Z'y \quad \text{(Greene, 1997)} \tag{12}$$

Fang and Lai (1997) used instruments suggested by Durbin (1954), and are argued to be the most efficient estimator of this type:

$$\mathbf{Z'}= \begin{pmatrix} 1, & 1, & 1, & .. & 1 \\ r_{11} & r_{12} & r_{13} & .. & r_{1n} \\ r_{21} & r_{22} & r_{23} & .. & r_{2n} \\ r_{31} & r_{32} & r_{33} & .. & r_{3n} \end{pmatrix}$$

where r_{1i} is the rank order of portfolio i´s beta among all the stock i´s betas; r_{2i} is the rank order of stock i´s co-skewness; and r_{3i} is the rank order of stock i´s co-kurtosis among all the stocks co-kurtosis, all estimated from equation *12*

To estimate the significance of the IV-parameters we use equation *13* to calculate the asymptotic variance of the IV-parameters.

$$\widehat{\sigma}^2(Z'X)^{-1}(Z'Z)(X'Z)^{-1} \text{ (Green, 1997)} \tag{13}$$

As our IV-estimator is asymptotic Normal distributed, we use the two tailed Normal probability distribution to calculate the p-values for the estimated parameters.

5 Data

The data being used in this survey consists of 629 monthly time series of all Swedish stocks being traded between 1979-12-30 and 2004-12-30. The data material is brought from Thomson DataStream. The time series early in the material have been put together using old order books, provided by Stockholm Stock Exchange. However, stocks from the smallest lists such as Nya Marknaden and Göteborgslistan will not be included due to lack of observations. Stock returns are presented in return index form (RI), and has been corrected for dividends, emissions and splits. The data, which is a unique material in Sweden has been provided and put together by I. Ahlbom (2005).

Since the true market portfolio is non observable, the market portfolio used in this study is Affärsvärldens generalindex (AFGX), which serves as a good proxy for the market portfolio. AFGX is a value weighted index, with base value 100, 1995-12-29.

As risk free rate of return, we have used Swedish monthly 3-month Treasury bill rate provided by EcoVision. The 1-month and 3-month rate are the most liquid rates, and will therefore serve as a good proxy for risk free rate. As is seen in figure *4*, the difference between the 3-month rate and 1-month rate is very small. We therefore conclude that the choice between the two rates has a little impact in our model. The monthly return was calculated using equation *14*.

$$R_{monthly} = \frac{\text{Yearly interest rate}}{12} \tag{14}$$

All stock series consists of between 3 and 303 observations each, and was arranged to have a perfect one day match to AFGX and the risk free rate.

To avoid small-sample problems, time series with 30 observations or less are excluded from our estimations.

AFGX does not have a kurtosis of three, and according to the Jarque-Bera test we do not have Normal distributed returns. The fact that the index does not follow a Normal distribution is a major drawback for the mean-variance theory, which relies heavily on the Normal distribution assumption. Out of the 538 series with more than 30 observations, 95.5 % shows significant excess kurtosis. This is illustrated in figure *6*

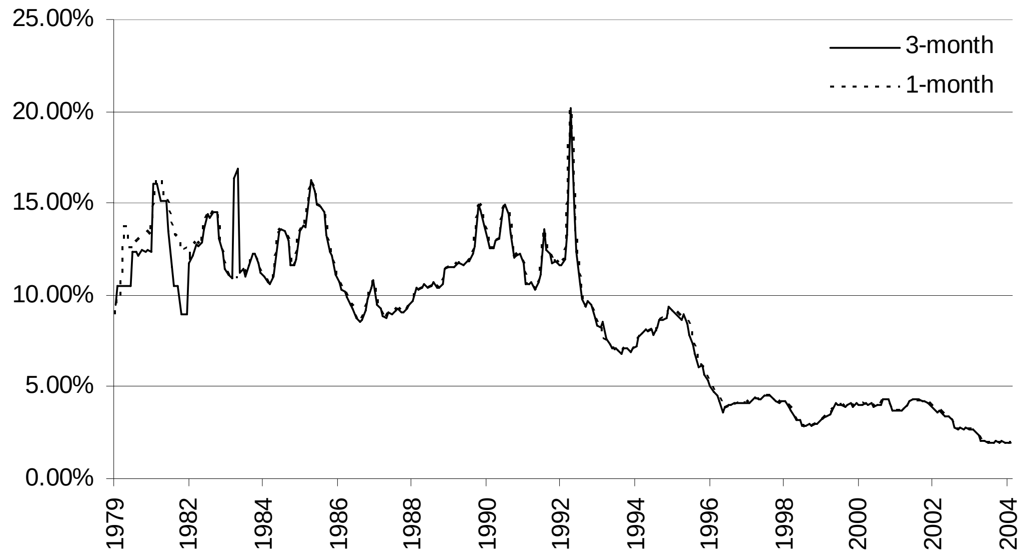

Figure 4: The graphs illustrate the Swedish 1-month and 3-months treasury bill rate.

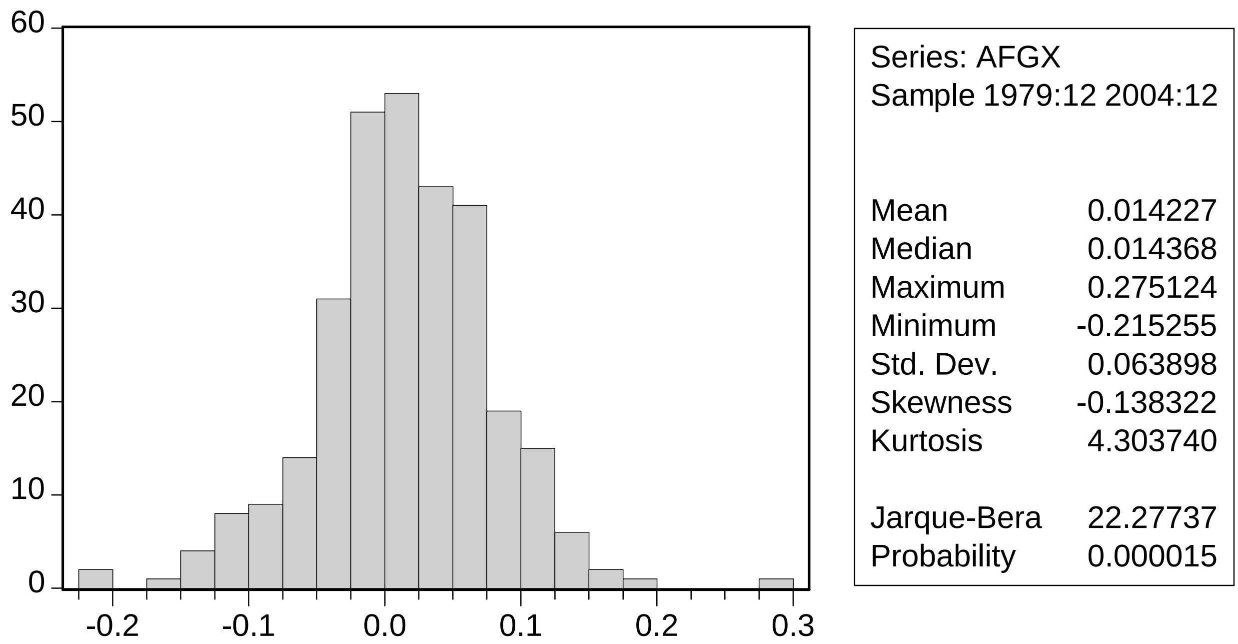

Figure 5: Decribes the return distribution of AFGX 1979-12-30 to 2004-12-30.

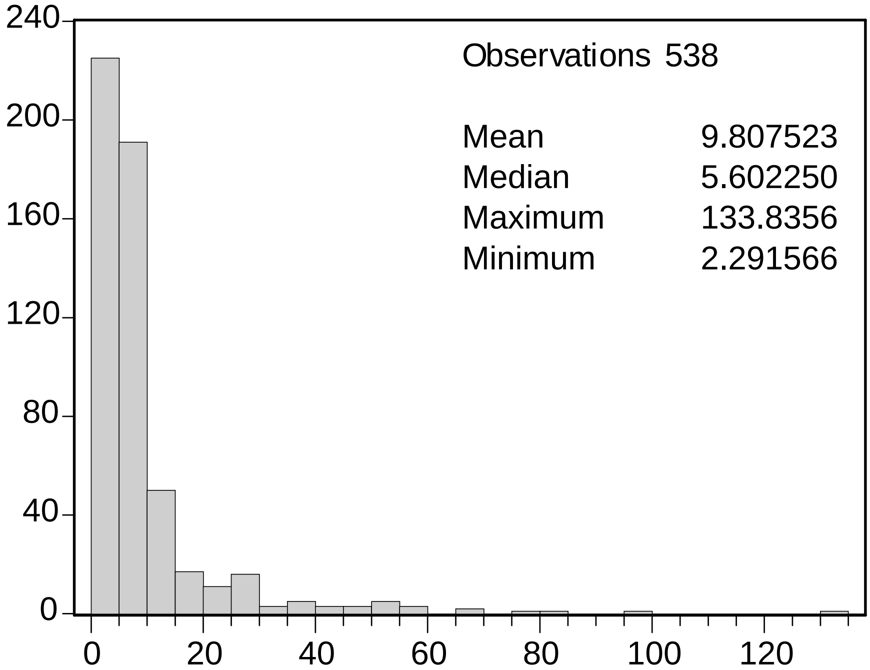

Figure 6: The kurtosis distribution of the stocks included in our study.

6 Empirical Results

In this section we will present the results obtained according to the estimations made. A short introduction of the properties in Table 1, as well as an analytical discussion will be presented.

6.1 Empirical Results - Risk Premiums

Table 1 reports estimated risk premiums for beta (b_1), co-skewness (b_2) and co-kurtosis (b_3). We will for comparison sake present four different pricing models; A. Four-Moment CAPM, B. Three-Moment CAPM, C. Two-Moment CAPM, and D. CAPM with beta and co-kurtosis. The ordinary least square (OLS) and the instrumental variable estimation (IVE) are reported for the full period 1979-2004 and for two non-overlapping sub-periods, 1979-1991 and 1992-2004. The adjusted R-square has been calculated using equation *15.*

$$R^2 = 1 - \frac{\frac{SSR}{n-k-1}}{\frac{SST}{n-1}} \text{(Woolridge, 6.21)} \tag{15}$$

where SSR is the sum of squared residuals, SST is the sum of squared deviations from the mean of the dependent variable, n is the number of observations and k is the number of explanatory variables.

R_i in Table 1 denotes the average monthly deflated excess return on stock i; β_i the estimated beta, γ_i estimated co-skewness and δ_i the estimated co-kurtosis. b_1, b_2, b_3 denote the market risk premiums for the respective risks. All rates in the table are measured per month.

The first sub-period, 1979-1991, is characterized as a period with relatively high interest rates (consider figure *14*), whereas the second sub-period has lower interest rates. The interest rate peak in early 1992 was the result of a changed monetary decision, when the Swedish Krona lost the fixed rate properties to other currencies.

AFGX K denotes the kurtosis of AFGX defined in equation *6* and AFGX S is the skewness of AFGX defined as equation *5.*

Table 1 - Estimated Risk Premiums

	1979-2004		1979-1991		1992-2004	
# series	538		237		431	
AFGX K	4.275191		4.715563		4.001397	
AFGX S	-0.137633		-0.550233		0.184692	
Coefficent	OLS	IVE	OLS	IVE	OLS	IVE
A. Four-Moment CAPM: $R_i = b_0 + b_1\beta_i + b_2\gamma_i + b_3\delta_i$						
b_0	-0.03719*	-0.01574	-0.00711*	0.00015	-0.02906*	-0.00913
b_1	0.04968*	0.01301*	0.00770*	-0.00050*	0.04755*	0.01765*
b_2	0.00621*	0.00248*	0.00173*	0.00022*	-0.00620*	-0.00230*
b_3	0.00067*	0.00015*	0.00028*	-0.00002*	0.00058*	0.00017*
Adj. R^2	0.618	0.347	0.072	0.003	0.647	0.462
B. Three-Moment CAPM: $R_i = b_0 + b_1\beta_i + b_2\gamma_i$						
b_0	-0.03421*	-0.01343	0.00669*	0.00002	-0.02990*	-0.00851
b_1	0.04781*	0.01503*	0.00755*	0.00078	0.04530*	0.01791*
b_2	0.00649*	0.00201*	0.00171*	0.00031	-0.00672*	-0.00284*
Adj. R^2	0.705	0.377	0.051	0.014	0.718	0.470
C. Two-Moment CAPM: $R_i = b_0 + b_1\beta_i$						
b_0	-0.03447*	-0.01127	-0.00762*	0.00157	-0.03310*	-0.01040
b_1	0.04902*	0.01418*	0.00919*	0.001	0.04976*	0.01706*
Adj. R^2	0.318	0.157	0.032	0.002	0.310	0.177
D. CAPM with beta and co-kurtosis: $R_i = b_0 + b_1\beta_i + b_3\delta_i$						
b_0	-0.02403*	-0.00642	-0.00612*	0.00072	-0.02252*	-0.00266
b_1	0.03732*	0.01285*	0.00708*	0.00024	0.03840*	0.01614*
b_3	0.00028*	0.00010	0.00030*	0.00013	0.00029*	0.00014*
Adj. R^2	0.406	0.229	0.051	0.024	0.403	0.256

* denotes significance at the 5%-level

6.2 Analysis

As is seen in Table 1, the Three-Moment CAPM has the highest adjusted R-square when focusing on the full period, 1979-2004. The adjusted R-square is 0.705 in the OLS case and 0.377 in the IVE case. Due to the properties of the different estimation methods used, it is irrelevant to compare the individual adjusted R-squared for OLS and IVE as the IV-estimates is based on other assumptions than OLS. However, it is still highly relevant to compare the adjusted R-squared for every estimation method used. Note that b_0 for the IV-estimates are not significant in any of the cases. This implies that the parameters of the explanatory variables are highly representative for the IV estimations.

The traditional Two-Moment CAPM (C) has an adjusted R-square of 0.318 in the OLS case and 0.157 in the IVE case. Thus, the explanatory power when using only the first and second moment (C), mean and variance, is less than 50 percent compared with usage of the third moment (B), skewness, as well. This is strong evidence for an efficiency improvement by including skewness in the asset pricing process. The main Four-Moment CAPM (A) yields an adjusted R-square of 0.618 in the OLS case and 0.347 in the IVE case, which both are less than the R-square for the Three-Moment CAPM (B).

Consider the Two-Moment CAPM (C) and CAPM with beta and co-kurtosis (D). As stated earlier we have an adjusted R-square of 0.318 in the OLS case and 0.157 for the IVE. When incorporating the effect of co-kurtosis, the adjusted R-square increases to 0.406 (D) in the OLS case and to 0.229 in the IVE case. This is strong evidence for the importance of kurtosis in the asset pricing process.

For the sub-period 1979-1991 we have very poor R-square power at all moments and for all of the models. This can be the result of extreme condition on the market, for example the big movements at late 80th and early 90th, consider the volatility of AFGX, figure *9* in appendix A. The adjusted R-square is highest for the four-moment CAPM (A) in the OLS case with a R-square of 0.072. The two-moment CAPM (C) once again provide us with the worst R-square for our OLS estimates and as for the full period, CAPM with beta and co-kurtosis (D) increases the R-square compared to the Two-Moment CAPM (C). For the IV-estimations, we only have significant parameters for the four-moment CAPM (A).

For the second sub-period, 1992-2004, we have significant values for all risk premiums and the highest adjusted R-square for the three-moment CAPM (B) in the OLS case, thus imposing skewness has the best positive effect on explaining the model. Our Four-Moment CAPM (A) provides us with the second best explanation power for both the OLS as well as the IVE case. When comparing Two-Moment CAPM (C) and CAPM with beta and co-kurtosis (D), it is obvious that co-kurtosis contributes to an increased explanation power. All risk premiums are positive for both the OLS and IVE estimations.

As stated earlier the ordinary mean-variance model claims that the best way to create an efficient portfolio is to use mean and variance. According to our four-moment CAPM we have significant risk premiums for both skewness, b_3, and kurtosis, b_4, when looking at the whole period from 1979 to 2004, but we also have significant risk premiums for the sub-periods, except for model B, C, D in the IV-cases for the first sub-period. This result is consistent for both our OLS estimation as well as in our IV estimation. If we compare the OLS and IVE risk premiums, the premiums are not exactly the same. This can be the result from the errors in variable problem when estimating OLS. The IVE estimates are in general lower than the OLS estimates.

For the full period, 1979-2004, we have a significant positive risk premium for conditional kurtosis, b_3 in the Four-Moment CAPM (A) and for CAPM with beta and co-kurtosis (D). It implies that the investor require higher expected rate of return when the kurtosis of an asset increases. The risk premium for conditional kurtosis is 0.067 percent per month in the OLS case and 0.015 percent per month in the IV case for the Four-Moment CAPM (A). Fang and Lai presented risk premiums for the IV estimation are in general lower than the premiums obtaind from the OLS regression. The risk premium for conditional kurtosis has a rather little impact on our model, compared to variance, but is still valid and different from zero. Also the conditional skewness, b_2, is positive and significant. As mentioned in section 3, the sign is supposed to have the opposite sign of the market skewness. As is seen in figure *5*, the market skewness is negative, thus the positive risk premium is consistent with theory discussed earlier. The risk premiums for the first sub-period, 1979-1991, are hard to interpret, taken in consideration the low explanation power and the insignificance in the IVE cases. Remarkable in this case is the negative value of the conditional kurtosis, b_3, for the IV estimation. This implies that high kurtosis is preferred to less. However, the risk premium for conditional skewness, b_2, is consistent with theory, as is seen in figure *7* in Appendix A, as the AFGX skewness for this period is negative.

The risk premiums for variance, b_1, and kurtosis, b_3, are positive and significant for the second sub-period, 1992-2004. The risk premium for skewness is negative. This is consistent with our earlier hypothesis since the skewness of the market portfolio is positive for this period, see figure *8* in Appendix A, and b_2 was suppose to have the opposite sign of market skewness.

Our results are similar to the results presented by Fang and Lai (1997) which as stated earlier cover the US stock market. The risk premiums presented by Fang and Lai are in general lower than our premiums. This suggests that an investor on the US Stock Market is given a lower compensation for taking more skewness and kurtosis risk than an investor on the Swedish stock market.

Both Kraus and Litzenberger (1976) and Campbell and Siddique (2000) argue that kurtosis is a superfluous measure to include in asset pricing whereas Dittmar (2002) and Fang and Lai (1997) propagate for the efficiency of kurtosis. However, they all argue for the importance of skewness in the asset pricing process. As Table 1 presents, the risk premium for skewness plays a much larger role than the risk premium for kurtosis, even though kurtosis alone contributes to an increased explanation power.

7 Conclusion

A capital asset pricing model which takes into account the effect of skewness and kurtosis has been used to show that in the presence of skewness and kurtosis in asset return distribution, the expected rate of return is not only related to the variance but also to the skewness and kurtosis. Investors can, by using these results create portfolios more efficient than ordinary mean-variance portfolios, by incorporating the third and fourth moments, skewness and kurtosis, of return distribution. It is shown that the distribution of stock returns seldom follow the normality assumption of a kurtosis of three.

There are some major differences between our study and studies made on other markets. First of all the liquidity of the Swedish stock market is relatively small compared to other major markets. However, the prices on the market are still settled by supply and demand. Another issue that separated our survey from many others is the number of stocks included. In our data material, we had almost 600 different stocks, whereas on for example the US stock market, it is possible to include over 10 000 different stocks. Although we had a smaller sample than other studies presented within the field, our results are still valid for the Swedish market.

Our results regarding the effect of skewness are in line with many other papers, for example Kraus and Litzenberger (1976) and Campbell and Siddique (2000). The effect of kurtosis on asset pricing presented in this paper are stronger than the results presented by Fang and Lai (1997) and Dittmar (2002), with a risk premium of 0.804 percent per year (0.067 % per month) according to the OLS estimation, and 0.18 percent per year (0.015 % per month) according to the IV estimation, for the full period. The effect of skewness has, as stated earlier, stronger effect than kurtosis, with a risk premium of 7.45 percent per year (0.621 % per month) according to the OLS estimation and 2.976 percent per year (0.248 % per month) according to the IV estimation for the full period. These premiums act as evidence for the importance of including higher moments. We further conclude that variance as a measure of risk is a relatively good measure with a big impact on the model, but not a completely covering measurement. The basic capital asset pricing model serves as a relatively good asset pricing model. But, whether you are an investment manager with a fund portfolio worth billions, or a small investor saving for your pension, a central rule of thumb is to never give anything away, to use as much available information as possible. Therefore, it is highly recommendable to use the results presented in this paper when making investment decisions to maximize the expected rate of return on investments.

8 References

Affärsvärlden, www.affarsvarlden.se, 2005

Banz, R.W. (1981), "The Relationship between Return and Market Value of Common Stocks", Journal of Financial Ecnonomics, 9, p. 3–18

Bodie, Z. Merton, R. (2000), Finance, Prentive Hall, New Jersey, USA

Brockett, Patrick L. and Kahane, Yehuda (1992), "Risk, Return, Skewness and Preference", Management Science, Vol 6, June 1992

Campbell, Lo, MacKinlay (1997), The Econometrics of Financial Markets, second printing, Princeton University Press, Monticello, USA

Campbell, R. Harvey & Siddiue, Akhtar (2000), "Conditional Skewness in Asset Pricing Tests", The Journal of Finance, Vol LV, NO 3

Dittmar, Robert F. (2002), "Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns", The Journal of Finance, Vol LVII

Durbin, James (1954), "Errors in Variables", Review of the International Statistical Insitute 22, p. 23-32

Fama, E., and French, K. R. (1995), "The Cross-Section of Expected Stock Returns", Journal of Finance, Vol. 47, No. 2, p.427-465

Fang, Hsing & Lai, Tsong-Yue (1997), "Co-Kurtosis and Capital Asset Pricing", The Financial Review, Vol 32 No. 2, May 1997, p. 293-307

Green, William H. (1997), Econometric Analysis, 3$^{\text{th}}$ edition, Prentice Hall

Haugen, Robert A. (2001), Modern Investment Theory, 5$^{\text{th}}$ edition, Prentice Hall, New Jersey, USA,

Hull, John C. (2003), Options, Futures, and other Derivatives, 5$^{\text{th}}$ edition, Prentice Hall, New Jersey, USA

Kane, Alex (1982), "Skewness Preference and Portfolio choice", The Journal of Financial and Quantitative Analysis, Vol. 17, p. 15-25

Kraus, Alan and Litzenberger, Robert (1976), "Skewness Preference and the Valuation Of Risk Assets", The Journal of Finance, Vol. XXXI, No. 4, September 1976

Kraus, Alan and Litzenberger, Robert (1983), "On the Distributional Conditions for a Consumption-Oriented Three Moment CAPM", The Hournal of Finance, Vol. 38, No. 5, p. 1381-1391

Richadson, Matthew & Smith, Tom (1993), "A Test for Multivariate Normality in Stock Returns", The Journal of Business, Vol. 66, p. 295-321

Roll, R.(1977), "A critique of the asset pricing theory's tests", Journal of Financial Economics 4, p. 129-176

Verbeek, M. (2004), A Guide to Modern Econometrics, 2^{th} edition, John Wiley & Sons, West Sussex, England,

Wooldridge, Jeffrey M. (2002), Introductory Econometrics, A Modern Approach, 2^{th} edition, Thomson South-Western, Mason, Ohio, USA

Zhou, Guofu (1993), "Asset-Pricing Tests Under Alternative Distributions", The Journal of Finance, Vol 48, No. 5

Appendix A - Relevant graphs

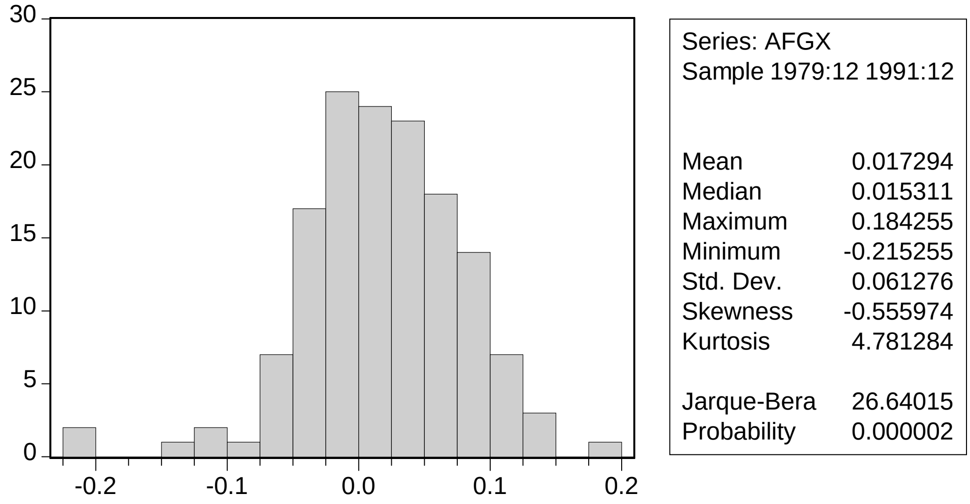

Figure 7: Market distribution 1979-1991

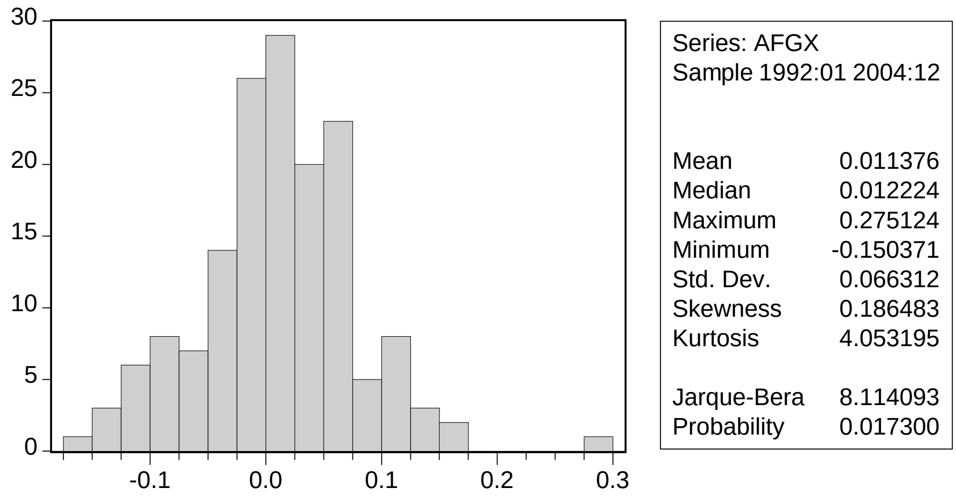

Figure 8: Market distribution 1992-2004

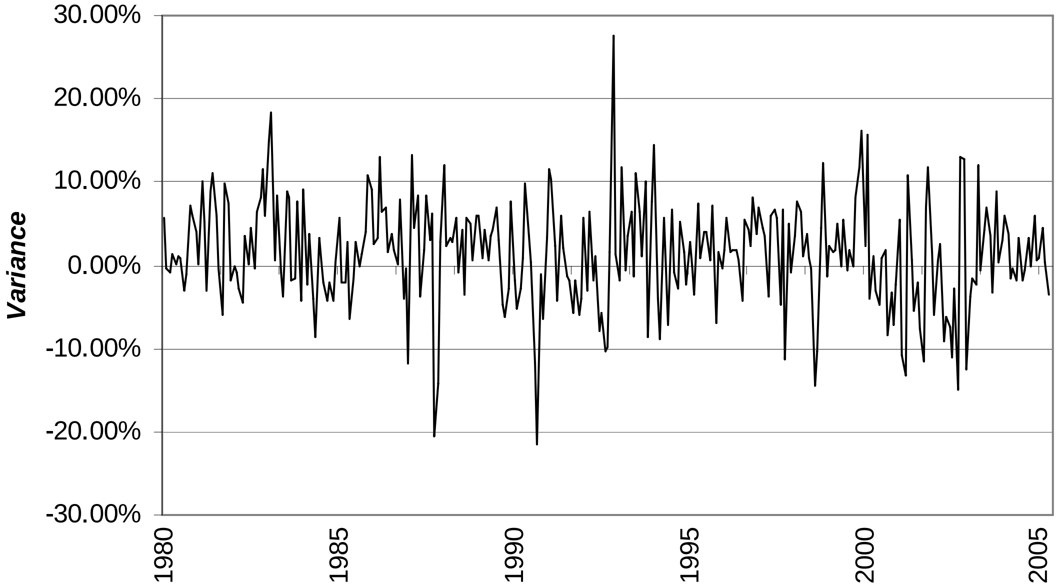

Figure 9: Market portfolio (AFGX) Variance 1979 - 2005

Appendix B - Program in Gauss

```
cls;
    /*Define data files*/
    data_return = "C:\\gauss\\datafil.xls";
    afgx_return = "C:\\gauss\\afgx.xls";
    rf_return = "C:\\gauss\\rf.xls";
    /*Enter data ranges and restrictive parameters*/
    time_start="151"; /*Default value=6*/
    time_stop="306"; /*Default value=306*/
    data_range = "B6:GY306";
    afgx_range = "C"$+time_start$+":C"$+time_stop;
    rf_range = "C"$+time_start$+":C"$+time_stop;
    observation = 30; /*The lowest number of observations required for a serie to be
included*/
    /*Import data*/
    dataset1 = spreadsheetreadm(data_return,"B"$+time_start$+":CY"$+time_stop,1);
    dataset2 = spreadsheetreadm(data_return,"B"$+time_start$+":DA"$+time_stop,2);
    dataset3 = spreadsheetreadm(data_return,"B"$+time_start$+":DJ"$+time_stop,3);
    dataset4 = spreadsheetreadm(data_return,"B"$+time_start$+":DQ"$+time_stop,4);
    dataset5 = spreadsheetreadm(data_return,"B"$+time_start$+":V"$+time_stop,5);
    dataset6 = spreadsheetreadm(data_return,"B"$+time_start$+":DM"$+time_stop,6);
    dataset7 = spreadsheetreadm(data_return,"B"$+time_start$+":BB"$+time_stop,7);
    dataset=dataset1~dataset2~dataset3~dataset4~dataset5~dataset6~dataset7;
    dataset_old_cols = cols(dataset);
    afgx = spreadsheetreadm(afgx_return,afgx_range,1);
    rf = spreadsheetreadm(rf_return,rf_range,1);
    dataset_cols = cols(dataset);
    dataset_rows = rows(dataset);
    i=1;
    j=1;
```

```
k=1;
counter=0;
counter1=1;
exclusion_counter=0;
/*Determine how many series are excluded*/
do while k <= dataset_cols;
if rows(packr(dataset[.,k])) <= observation;
exclusion_counter = exclusion_counter + 1;
endif;
k=k+1;
endo;
/*Delete those series from the dataset*/
dataset_1=zeros(dataset_rows,(dataset_cols-exclusion_counter));
do while j <=dataset_cols;
if rows(packr(dataset[.,j])) > observation;
dataset_1[.,counter1]=dataset[.,j];
counter1 = counter1 + 1;
endif;
j=j+1;
endo;
/*Define the new dataset without small series*/
dataset=dataset_1;
dataset_cols=cols(dataset);
/*Define matrices*/
alfa=zeros(dataset_cols,1);
beta=zeros(dataset_cols,1);
kurtosis=zeros(dataset_cols,1);
skewness=zeros(dataset_cols,1);
indep_capm=zeros(dataset_cols,3);
capm=zeros(dataset_cols,1);
capm_matrix=zeros(dataset_cols,3);
```

```
test_statistic=zeros(4,1);
kurtosis_real=zeros(dataset_cols,1);
skewness_real=zeros(dataset_cols,1);
/*screen off;*/
do while i <= dataset_cols;
/* —- 4th moment CAPM estimation —-*/
/*Arange the returns and risk free to match eachother*/
reg = dataset[.,i]~afgx[.,1]~-afgx[.,1]^2~afgx[.,1]^3~rf[.,1];
reg_packr = packr(reg);
if rows(reg_packr) <= observation;
counter = counter + 1;
goto label_end;
endif;
/*Define dependent and independent variables*/
dependent=reg_packr[.,1];
indep=reg_packr[.,2]~reg_packr[.,3]~reg_packr[.,4];
rf_packr=reg_packr[.,5];
/*Run Regression*/
print "——You look at the " ftos(i,"%*.*lf",1,0) "th regression——";
__con=1;
call ols("",dependent,indep);
{d1,d2,d3,d4,d5,d6,d7,d8,d9,d10,d11}=ols("",dependent,indep);
/*Use equation 4 to estimate our four-moment CAPM*/
R_expected=d3[1]+d3[2]*indep[.,1]+d3[3]*indep[.,2]+d3[4]*indep[.,3];
/*R_expected=d3[1]*indep[.,1]+d3[2]*indep[.,2]+d3[3]*indep[.,3];*/
CAPM[i,.]=meanc(R_expected)-meanc(rf_packr);
/*CAPM[i,.]=meanc(reg_packr[.,1])-meanc(rf_packr);*/
indep_capm[i,.]=d3[2]~d3[3]~d3[4];
/*indep_capm[i,.]=d3[1]~d3[2]~d3[3];*/
/*Calculate skewness and kurtosis*/
kurtosis_real[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^4)
```

```
)/(stdc(dependent[.,1])^4);
skewness_real[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^
3))/(stdc(dependent[.,1])^3);
alfa[i,1]=d3[1];
beta[i,1]=d3[2];
skewness[i,1]=d3[3];
kurtosis[i,1]=d3[4];
/*beta[i,1]=d3[1];
skewness[i,1]=d3[2];
kurtosis[i,1]=d3[3];*/
print " ";
/*print "——You look at the " ftos(i,"%*.*lf",1,0) "th regression——";
print " Constant is " d3[1];
print " X1 is " d3[2];
print " X2 is " d3[3];
print " X3 is " d3[4];*/
print " Kurtosis is: " kurtosis[i,1];
print " Skewness is: " skewness[i,1];
print " R^2 is: " d9;
print "—————————————————— ";
print " ";
label_end:
i=i+1;
endo;
/*Estimate values using IV*/
/*Create a rank matrix*/
kurtosis_rank=rankindx(kurtosis,1);
skewness_rank=rankindx(skewness,1);
beta_rank=rankindx(beta,1);
instrument_matrix=beta_rank~skewness_rank~kurtosis_rank;
instrument_matrix=ones(rows(instrument_matrix),1)~instrument_matrix;
```

```
iv_estimate=inv(instrument_matrix'*(/*alfa~*/beta~skewness~kurtosis))
*instrument_matrix'*capm;
/*Calculate significance for IV-parameters*/
/*Calculate residual variance*/
parameter_variance=(1/rows(beta))*sumc((capm-beta*iv_estimate[1]-sk
ewness*
iv_estimate[2]-kurtosis*iv_estimate[3])^2);
asy_var=parameter_variance*(inv(instrument_matrix'*(alfa~beta~skewn
ess~kurtosis))*
(instrument_matrix'*instrument_matrix)*inv((alfa~beta~skewness~kurtosis)
'*instrument_matrix));
test_statistic[1]=iv_estimate[1]/sqrt(asy_var[1,1]);
test_statistic[2]=iv_estimate[2]/sqrt(asy_var[2,2]);
test_statistic[3]=iv_estimate[3]/sqrt(asy_var[3,3]);
test_statistic[4]=iv_estimate[4]/sqrt(asy_var[4,4]);
p_value = 2*cdfnc(abs(test_statistic));
R_square_iv_4th=1-(sumc((capm-alfa-beta*iv_estimate[1]-skewness*iv
_estimate[2]-kurtosis*
iv_estimate[3])^2)/sumc((capm-meanc(capm))^2));
R_square_iv_4th_adjusted = 1-(1-R_square_iv_4th)*((rows(beta)-1)/(r
ows(beta)-3-1));
screen on;
print "————————————————";
print "——————4th moment CAPM—————-";
__con=1;
call ols("",capm,indep_capm);
{e1,e2,e3,e4,e5,e6,e7,e8,e9,e10,e11}=ols("",capm,indep_capm);
print "————————————————";
print " OLS estimates";
print "b0(constant) = " ftos(e3[1],"%*.*lf",1,6);
print "b1 = " ftos(e3[2],"%*.*lf",1,6);
```

```
print "b2 = " ftos(e3[3],"%*.*lf",1,6);
print "b3 = " ftos(e3[4],"%*.*lf",1,6);
print " ";
print "R^2 = " ftos(e9,"%*.*lf",1,6);
print "Adjusted R^2 = " ftos(1-(1-e9)*((rows(beta)-1)/(rows(beta)-3-1)),"
%*.*lf",1,6);
print "——————————————————";
print " IVE estimates";
/*print "bo(constant) = " ftos(iv_estimate[1],"%*.*lf",1,6);*/
print "b0 = " ftos(iv_estimate[1],"%*.*lf",1,6) " P-value = " ftos(p_value[
1],"%*.*lf",1,6);
print "b1 = " ftos(iv_estimate[2],"%*.*lf",1,6) " P-value = " ftos(p_value[2
],"%*.*lf",1,6);
print "b2 = " ftos(iv_estimate[3],"%*.*lf",1,6) " P-value = " ftos(p_value
[3],"%*.*lf",1,6);
print "b3 = " ftos(iv_estimate[4],"%*.*lf",1,6) " P-value = " ftos(p_value[
4],"%*.*lf",1,6);
print " ";
print "R^2 = " ftos(R_square_iv_4th,"%*.*lf",1,6);
print "Adjusted R^2 = " ftos(R_square_iv_4th,"%*.*lf",1,6);
print "——————————————————";
print "——————————————————";
print "Skipped " ftos(exclusion_counter,"%*.*lf",1,0) " out of " ftos(dat
aset_old_cols,"%*.*lf",1,0)
" series, because of a small number of observations in series.";
print "So, totally " ftos((dataset_old_cols-exclusion_counter),"%*.*lf",1
,0) " series are included";
print " ";
print "Kurtosis for AFGX for chosen period is: " ftos((meanc((afgx[.,1]-
meanc(afgx[.,1]))^4)/
stdc(afgx[.,1])^4),"%*.*lf",1,6);
```

```
print "Skewness for AFGX for chosen period is: " ftos((meanc((afgx[.
,1]-meanc(afgx[.,1]))^3)/
stdc(afgx[.,1])^3),"%*.*lf",1,6);
print "Mean kurtosis for series for chosen period is: " ftos((meanc(kurto
sis)),"%*.*lf",1,6);
print "Mean skewness for series for chosen period is: " ftos((meanc(ske
wness)),"%*.*lf",1,6);
/*goto label_avslut;*/
screen off;
/* ————3th moment CAPM———— */
/*Reset matrices and counter variable*/
i=1;
j=1;
k=1;
counter=0;
counter1=1;
alfa=zeros(dataset_cols,1);
beta=zeros(dataset_cols,1);
kurtosis=zeros(dataset_cols,1);
skewness=zeros(dataset_cols,1);
indep_capm=zeros(dataset_cols,2);
capm=zeros(dataset_cols,1);
capm_matrix=zeros(dataset_cols,2);
test_statistic=zeros(3,1);
/* RESET */
do while i <= dataset_cols;
/* —- 3th moment CAPM estimation —-*/
/*Arange the returns and risk free to match eachother*/
reg = dataset[.,i]~afgx[.,1]~-afgx[.,1]^2~rf[.,1];
reg_packr = packr(reg);
if rows(reg_packr) <= observation;
```

```
counter = counter + 1;
goto label_end2;
endif;
/*Define dependent and independent variables*/
dependent=reg_packr[.,1];
indep=reg_packr[.,2]~reg_packr[.,3];
rf_packr=reg_packr[.,4];
/*Run Regression*/
print "——You look at the " ftos(i,"%*.*lf",1,0) "th regression——";
__con=1;
call ols("",dependent,indep);
{d1,d2,d3,d4,d5,d6,d7,d8,d9,d10,d11}=ols("",dependent,indep);
/*Use equation 4 to estimate our third-moment CAPM*/
R_expected=d3[1]+d3[2]*indep[.,1]+d3[3]*indep[.,2];
/*R_expected=d3[1]*indep[.,1]+d3[2]*indep[.,2]+d3[3]*indep[.,3];*/
CAPM[i,.]=meanc(R_expected)-meanc(rf_packr);
indep_capm[i,.]=d3[2]~d3[3];
/*indep_capm[i,.]=d3[1]~d3[2]~d3[3];*/
/*Calculate skewness and kurtosis*/
/*kurtosis[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^4))/(
stdc(dependent[.,1])^4);
skewness[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^3))/(
stdc(dependent[.,1])^3);*/
alfa[i,1]=d3[1];
beta[i,1]=d3[2];
skewness[i,1]=d3[3];
/*beta[i,1]=d3[1];
skewness[i,1]=d3[2];
kurtosis[i,1]=d3[3];*/
print " ";
/*print "——You look at the " ftos(i,"%*.*lf",1,0) "th regression——";
```

```
print " Constant is " d3[1];
print " X1 is " d3[2];
print " X2 is " d3[3];
print " X3 is " d3[4];*/
print " Kurtosis is: " kurtosis[i,1];
print " Skewness is: " skewness[i,1];
print " R^2 is: " d9;
print "———————————————— ";
print " ";
label_end2:
i=i+1;
endo;
/*Estimate values using IV*/
/*Create a rank matrix*/
skewness_rank=rankindx(skewness,1);
beta_rank=rankindx(beta,1);
instrument_matrix=beta_rank~skewness_rank;
instrument_matrix=ones(rows(instrument_matrix),1)~instrument_matrix;
iv_estimate=inv(instrument_matrix'*(alfa~beta~skewness))*instrument
_matrix'*capm;
/*Calculate significance for IV-parameters*/
/*Calculate residual variance*/
parameter_variance=(1/rows(beta))*sumc((capm-alfa-beta*iv_estim
ate[1]-skewness*iv_estimate[2])^2);
asy_var=parameter_variance*(inv(instrument_matrix'*(alfa~beta~sk
ewness))*
(instrument_matrix'*instrument_matrix)*inv((alfa~beta~skewness)'*
instrument_matrix));
test_statistic[1]=iv_estimate[1]/sqrt(asy_var[1,1]);
test_statistic[2]=iv_estimate[2]/sqrt(asy_var[2,2]);
test_statistic[3]=iv_estimate[3]/sqrt(asy_var[3,3]);
```

```
p_value = 2*cdfnc(abs(test_statistic));
R_square_iv_3th=1-sumc((capm-beta*iv_estimate[1]-skewness*iv_
estimate[2])^2)/
sumc((capm-meanc(capm))^2);
R_square_iv_3th_adjusted = 1-(1-R_square_iv_3th)*((rows(beta)-1
)/(rows(beta)-2-1));
screen on;
print "——————————————————————";
print "—————-3th moment CAPM——————————";
__con=1;
call ols("",capm,indep_capm);
{e1,e2,e3,e4,e5,e6,e7,e8,e9,e10,e11}=ols("",capm,indep_capm);
print "——————————————————————";
print " OLS estimates";
print "b0(constant) = " ftos(e3[1],"%*.*lf",1,6);
print "b1 = " ftos(e3[2],"%*.*lf",1,6);
print "b2 = " ftos(e3[3],"%*.*lf",1,6);
print " ";
print "R^2 = " ftos(e9,"%*.*lf",1,6);
print "Adjusted R^2 = " ftos(1-(1-e9)*((rows(beta)-1)/(rows(beta)-3-
1)),"%*.*lf",1,6);
print "——————————————————————";
print " IVE estimates";
print "bo(constant) = " ftos(iv_estimate[1],"%*.*lf",1,6) " P-value = "
ftos(p_value[1],"%*.*lf",1,6);
print "b1 = " ftos(iv_estimate[2],"%*.*lf",1,6) " P-value = " ftos(p_va
lue[2],"%*.*lf",1,6);
print "b2 = " ftos(iv_estimate[3],"%*.*lf",1,6) " P-value = " ftos(p_va
lue[3],"%*.*lf",1,6);
print " ";
print "R^2 = " ftos(R_square_iv_3th,"%*.*lf",1,6);
```

```
print "Adjusted R2 = " ftos(R_square_iv_3th,"%*.*lf",1,6);
print "——————————————————————";
print "——————————————————————";
print "Skipped " ftos(exclusion_counter,"%*.*lf",1,0) " out of " ftos(d
ataset_old_cols,"%*.*lf",1,0)
" series, because of a small number of observations in series.";
print "So, totally " ftos((dataset_old_cols-exclusion_counter),"%*.*lf
",1,0) " series are included";
print " ";
print "Kurtosis for AFGX for chosen period is: " ftos((meanc((afgx[.,
1]-meanc(afgx[.,1]))^4)/s
tdc(afgx[.,1])^4),"%*.*lf",1,6);
print "Skewness for AFGX for chosen period is: " ftos((meanc((afgx[
.,1]-meanc(afgx[.,1]))^3)
/stdc(afgx[.,1])^3),"%*.*lf",1,6);
print "Mean kurtosis for series for chosen period is: " ftos((meanc(kur
tosis)),"%*.*lf",1,6);
print "Mean skewness for series for chosen period is: " ftos((meanc(s
kewness)),"%*.*lf",1,6);
screen off;
/* ——————2th moment CAPM—————— */
/*Reset matrices and counter variable*/
i=1;
j=1;
k=1;
counter=0;
counter1=1;
alfa=zeros(dataset_cols,1);
beta=zeros(dataset_cols,1);
kurtosis=zeros(dataset_cols,1);
skewness=zeros(dataset_cols,1);
```

```
indep_capm=zeros(dataset_cols,1);
capm=zeros(dataset_cols,1);
capm_matrix=zeros(dataset_cols,1);
test_statistic=zeros(2,1);
/* RESET */
do while i <= dataset_cols;
/* --- 2th moment CAPM estimation ---*/
/*Arange the returns and risk free to match eachother*/
reg = dataset[.,i]~afgx[.,1]~rf[.,1];
reg_packr = packr(reg);
if rows(reg_packr) <= observation;
counter = counter + 1;
goto label_end3;
endif;
/*Define dependent and independent variables*/
dependent=reg_packr[.,1];
indep=reg_packr[.,2];
rf_packr=reg_packr[.,3];
/*Run Regression*/
print "----You look at the " ftos(i,"%*.*lf",1,0) "th regression----";
__con=1;
call ols("",dependent,indep);
{d1,d2,d3,d4,d5,d6,d7,d8,d9,d10,d11}=ols("",dependent,indep);
/*Use equation 4 to estimate our second-moment CAPM*/
R_expected=d3[1]+d3[2]*indep[.,1];
/*R_expected=d3[1]*indep[.,1]+d3[2]*indep[.,2]+d3[3]*indep[.,3];*/
CAPM[i,.]=meanc(R_expected)-meanc(rf_packr);
indep_capm[i,.]=d3[2];
/*indep_capm[i,.]=d3[1]~d3[2]~d3[3];*/
/*Calculate skewness and kurtosis*/
/*kurtosis[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^4))/
```

```
(stdc(dependent[.,1])^4);
skewness[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^3))/(
stdc(dependent[.,1])^3);*/
alfa[i,1]=d3[1];
beta[i,1]=d3[2];
/*beta[i,1]=d3[1];
skewness[i,1]=d3[2];
kurtosis[i,1]=d3[3];*/
print " ";
/*print "——You look at the " ftos(i,"%*.*lf",1,0) "th regression——";
print " Constant is " d3[1];
print " X1 is " d3[2];
print " X2 is " d3[3];
print " X3 is " d3[4];*/
print " Kurtosis is: " kurtosis[i,1];
print " Skewness is: " skewness[i,1];
print " R^2 is: " d9;
print "———————————————— ";
print " ";
label_end3:
i=i+1;
endo;
/*Estimate values using IV*/
/*Create a rank matrix*/
beta_rank=rankindx(beta,1);
instrument_matrix=beta_rank;
instrument_matrix=ones(rows(instrument_matrix),1)~instrument_matrix;
iv_estimate=inv(instrument_matrix'*(/*alfa~*/beta))*instrument_matr
ix'*capm;
/*Calculate significance for IV-parameters*/
/*Calculate residual variance*/
```

```
parameter_variance=(1/rows(beta))*sumc((capm-beta*iv_estimate[1])^2);
asy_var=parameter_variance*(inv(instrument_matrix'*(alfa~beta))*(inst
rument_matrix'*
instrument_matrix)*inv((alfa~beta)'*instrument_matrix));
test_statistic[1]=iv_estimate[1]/sqrt(asy_var[1,1]);
test_statistic[2]=iv_estimate[2]/sqrt(asy_var[2,2]);
p_value = 2*cdfnc(abs(test_statistic));
R_square_iv_2th=1-sumc((capm-beta*iv_estimate[1])^2)/sumc((capm
-meanc(capm))^2);
R_square_iv_2th_adjusted = 1-(1-R_square_iv_2th)*((rows(beta)-1)/
(rows(beta)-1-1));
screen on;
print "--------------------------------------------------";
print "------------2th moment CAPM--------------------";
__con=1;
call ols("",capm,indep_capm);
{e1,e2,e3,e4,e5,e6,e7,e8,e9,e10,e11}=ols("",capm,indep_capm);
print "--------------------------------------------------";
print " OLS estimates";
print "b0(constant) = " ftos(e3[1],"%*.*lf",1,6);
print "b1 = " ftos(e3[2],"%*.*lf",1,6);
print " ";
print "R^2 = " ftos(e9,"%*.*lf",1,6);
print "Adjusted R^2 = " ftos(1-(1-e9)*((rows(beta)-1)/(rows(beta)-1-1)
),"%*.*lf",1,6);
print "--------------------------------------------------";
print " IVE estimates";
print "bo(constant) = " ftos(iv_estimate[1],"%*.*lf",1,6) " P-value = " fto
s(p_value[2],"%*.*lf",1,6);
print "b1 = " ftos(iv_estimate[1],"%*.*lf",1,6) " P-value = " ftos(p_value
[2],"%*.*lf",1,6);
```

```
print " ";
print "R^2 = " ftos(R_square_iv_2th,"%*.*lf",1,6);
print "Adjusted R^2 = " ftos(R_square_iv_2th,"%*.*lf",1,6);
print "————————————————————————";
print "————————————————————————";
print "Skipped " ftos(exclusion_counter,"%*.*lf",1,0) " out of " ftos(datase
t_old_cols,"%*.*lf",1,0) "
series, because of a small number of observations in series.";
print "So, totally " ftos((dataset_old_cols-exclusion_counter),"%*.*lf",1,0)
" series are included";
print " ";
print "Kurtosis for AFGX for chosen period is: " ftos((meanc((afgx[.,1]-m
eanc(afgx[.,1]))^4)
/stdc(afgx[.,1])^4),"%*.*lf",1,6);
print "Skewness for AFGX for chosen period is: " ftos((meanc((afgx[.,1]-m
eanc(afgx[.,1]))^3)/
stdc(afgx[.,1])^3),"%*.*lf",1,6);
print "Mean kurtosis for series for chosen period is: " ftos((meanc(kurtosis))
,"%*.*lf",1,6);
print "Mean skewness for series for chosen period is: " ftos((meanc(skewne
ss)),"%*.*lf",1,6);
screen off;
/* ——————CAPM with beta and co-kurtosis—————— */
/*Reset matrices and counter variable*/
i=1;
j=1;
k=1;
counter=0;
counter1=1;
alfa=zeros(dataset_cols,1);
beta=zeros(dataset_cols,1);
```

```
kurtosis=zeros(dataset_cols,1);
skewness=zeros(dataset_cols,1);
indep_capm=zeros(dataset_cols,2);
capm=zeros(dataset_cols,1);
capm_matrix=zeros(dataset_cols,2);
test_statistic=zeros(3,1);
/* RESET */
do while i <= dataset_cols;
/* —- CAPM with beta and co-kurtosis estimation —-*/
/*Arange the returns and risk free to match eachother*/
reg = dataset[.,i]~afgx[.,1]~afgx[.,1]^3~rf[.,1];
reg_packr = packr(reg);
if rows(reg_packr) <= observation;
counter = counter + 1;
goto label_end4;
endif;
/*Define dependent and independent variables*/
dependent=reg_packr[.,1];
indep=reg_packr[.,2]~reg_packr[.,3];
rf_packr=reg_packr[.,4];
/*Run Regression*/
print "——You look at the " ftos(i,"%*.*lf",1,0) "th regression——";
__con=1;
call ols("",dependent,indep);
{d1,d2,d3,d4,d5,d6,d7,d8,d9,d10,d11}=ols("",dependent,indep);
/*Use equation 4 to estimate our third-moment CAPM*/
R_expected=d3[1]+d3[2]*indep[.,1]+d3[3]*indep[.,2];
/*R_expected=d3[1]*indep[.,1]+d3[2]*indep[.,2]+d3[3]*indep[.,3];*/
CAPM[i,.]=meanc(R_expected)-meanc(rf_packr);
indep_capm[i,.]=d3[2]~d3[3];
/*indep_capm[i,.]=d3[1]~d3[2]~d3[3];*/
```

```
/*Calculate skewness and kurtosis*/
/*kurtosis[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^4))/(stdc(
dependent[.,1])^4);
skewness[i,1]=(meanc((dependent[.,1]-meanc(dependent[.,1]))^3))/(stdc(
dependent[.,1])^3);*/
alfa[i,1]=d3[1];
beta[i,1]=d3[2];
kurtosis[i,1]=d3[3];
/*beta[i,1]=d3[1];
skewness[i,1]=d3[2];
kurtosis[i,1]=d3[3];*/
print " ";
/*print "——You look at the " ftos(i,"%*.*lf",1,0) "th regression——";
print " Constant is " d3[1];
print " X1 is " d3[2];
print " X2 is " d3[3];
print " X3 is " d3[4];*/
print " Kurtosis is: " kurtosis[i,1];
print " Skewness is: " skewness[i,1];
print " R^2 is: " d9;
print "———————————————— ";
print " ";
label_end4:
i=i+1;
endo;
/*Estimate values using IV*/
/*Create a rank matrix*/
beta_rank=rankindx(beta,1);
kurtosis_rank=rankindx(kurtosis,1);
instrument_matrix=beta_rank~kurtosis_rank;
instrument_matrix=ones(rows(instrument_matrix),1)~instrument_matrix;
```

```
iv_estimate=inv(instrument_matrix'*(alfa~beta~kurtosis))*instrument_matrix
'*capm;
/*Calculate significance for IV-parameters*/
/*Calculate residual variance*/
parameter_variance=(1/rows(beta))*sumc((capm-alfa-beta*iv_estimate[1]-
kurtosis*iv_estimate[2])^2);
asy_var=parameter_variance*(inv(instrument_matrix'*(alfa~beta~kurtosis))
*(instrument_matrix'*
instrument_matrix)*inv((alfa~beta~kurtosis)'*instrument_matrix));
test_statistic[1]=iv_estimate[1]/sqrt(asy_var[1,1]);
test_statistic[2]=iv_estimate[2]/sqrt(asy_var[2,2]);
test_statistic[3]=iv_estimate[2]/sqrt(asy_var[3,3]);
p_value = 2*cdfnc(abs(test_statistic));
R_square_iv_beta_co_curtosis=1-sumc((capm-alfa-beta*iv_estimate[1]-
kurtosis*iv_estimate[2])^2)/
sumc((capm-meanc(capm))^2);
R_square_adjusted = 1-(1-R_square_iv_beta_co_curtosis)*((rows(beta)-
1)/(rows(beta)-2-1));
screen on;
print "————————————————";
print "——————-CAPM with beta and co-kurtosis——————————";
__con=1;
call ols("",capm,indep_capm);
{e1,e2,e3,e4,e5,e6,e7,e8,e9,e10,e11}=ols("",capm,indep_capm);
print "————————————————";
print " OLS estimates";
print "b0(constant) = " ftos(e3[1],"%*.*lf",1,6);
print "b1 = " ftos(e3[2],"%*.*lf",1,6);
print "b3 = " ftos(e3[3],"%*.*lf",1,6);
print " ";
print "R2 = " ftos(e9,"%*.*lf",1,6);
```

```
print "Adjusted R² = " ftos(1-(1-e9)*((rows(beta)-1)/(rows(beta)-2-1)),"%
*.*lf",1,6);
print "————————————————————";
print " IVE estimates";
print "bo(constant) = " ftos(iv_estimate[1],"%*.*lf",1,6) " P-value = " ftos(p
_value[1],"%*.*lf",1,6);
print "b1 = " ftos(iv_estimate[2],"%*.*lf",1,6) " P-value = " ftos(p_value[2],
"%*.*lf",1,6);
print "b3 = " ftos(iv_estimate[3],"%*.*lf",1,6) " P-value = " ftos(p_value[3]
,"%*.*lf",1,6);
print " ";
print "R² = " ftos(R_square_iv_beta_co_curtosis,"%*.*lf",1,6);
print "Adjusted R² = " ftos(R_square_adjusted,"%*.*lf",1,6);
print "————————————————————";
print "————————————————————";
print "Skipped " ftos(exclusion_counter,"%*.*lf",1,0) " out of " ftos(dataset_o
ld_cols,"%*.*lf",1,0)
" series, because of a small number of observations in series.";
print "So, totally " ftos((dataset_old_cols-exclusion_counter),"%*.*lf",1,0) " se
ries are included";
print " ";
print "Kurtosis for AFGX for chosen period is: " ftos((meanc((afgx[.,1]
-meanc(afgx[.,1]))^4)/stdc(afgx[.,1])^4),"%*.*lf",1,6);
print "Skewness for AFGX for chosen period is: " ftos((meanc((afgx[.,1]
-meanc(afgx[.,1]))^3)/stdc(afgx[.,1])^3),"%*.*lf",1,6);
print "Mean kurtosis for series for chosen period is: " ftos((meanc(kurtosis)),"%
*.*lf",1,6);
print "Mean skewness for series for chosen period is: " ftos((meanc(skewness)
),"%*.*lf",1,6);
```