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Adding commodity futures to the Swedish stock portfolio

A good strategy for better diversification?

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ABSTRACT

By using data from Ecovision on the DJ AIG commodity index made up of commodity futures and the OMX index consisting of Swedish equities I have estimated the correlation between Swedish equities and US commodity futures. The correlation has been examined in a multivariate GARCH setting by using the BEKK model. The purpose has been to examine if commodity futures from a US exchange market can improve the *Sharpe ratio* of a portfolio made up of Swedish equities. The main interest is to see if commodity futures can keep their good properties when they are affected by FX markets. The *akaike criterion* shows that the BEKK model used is a good fit for the data.

The result shows a statistically significant low correlation between the OMX index and the DJ AIG during the test period. DJ AIG has also during the period had approximately the same rate of return as the OMX. Hence this paper shows that adding commodity futures will diversify the portfolio and therefore improve the *Sharpe ratio* thus lowering volatility at a given rate of return.

1 INTRODUCTION

1.1 Background

Historically commodity futures have not been a widely used asset class (G.Gorton & K.G. Rouwenhorst, 2006). Historically, direct commodity investment has been a small part of investors´ asset allocation decision (Robert J.Grier, 2000). It was not up until fifteen years ago that commodity futures was starting to be seen as a serious alternative to stocks for investment purpose. Since then the interest in using commodity futures as a alternative investment has grown.(Henry G.Jareki 2007). Commodity futures have in a recent study by G.Gorton & K.G. Rouwenhorst (2006) showed very good properties as a hedge against inflation and for diversifying the portfolio. Most studies have so far been done in America.

The basic idea of portfolio theory is that you want to have as many different assets as possible and you also would like them to be as uncorrelated as possible (Robert A Haugen, 2001). A combination of uncorrelated asset with expected positive return will have a lower volatility then each individual asset at a given return. Thus will a diversified portfolio give the same expected return but to a lower risk.

So why do not investors add commodity futures to there portfolios?

Historically commodity futures in US have shown about the same returns as equities and at the same time showing a low correlation with equities and bonds (G.Gorton & K.G. Rouwenhorst, 2006). Hence it serves as a good complement to a more classical portfolio made up of bonds and equities. The commodity futures have also historically shown a positive correlation to the US inflation when equities and bonds have shown a negative correlation (G.Gorton & K.G. Rouwenhorst, 2006). This means that commodity futures would present a good hedge against inflation in a portfolio of stocks and bonds, lowering expected portfolio volatility thus improving the *Sharpe ratio*.

Commodity futures have been out in the cold for a long time and are still in general being considered a risky investment (Henry G.Jareki 2007). Perhaps this is because they are harder to understand, needs to be rebalanced or people just have not been aware of the benefits they may bring to the portfolio. Historically commodity futures have been a good alternative investment in the US, but are they still? Would these good properties be kept intact if the investor is outside the US and is being affected by FX markets and different inflation rates?

More and more focus has been placed on commodities since the burst of the IT bubble in 2001 and increasingly so over the last couple of years. The low cost of capital, fast growing economies in china and India, the growing number of hedge funds are some of the factors being mentioned as reasons for the latest year's dramatic increase in asset prices. There have been sharp increases in asset value for stocks, real estate and commodities during the last couple of years. Since 2004 there has been an increase in commodity values. This has happened during a bull market for stocks in a period with a low inflation rate. Has any of this influenced the commodity futures market changing the fundamental market structure to become more speculative? Hedge funds for instance have been placing very large bets on commodity price fluctuations. Has this lead to any change in the correlation between stocks and commodity futures?

1.2 Research on commodity futures

The research on commodity futures performance and volatility has been scarce if you compare to equities. Is there any risk premium involved? Is the expected return positive? Have there been excess returns over t-bills? Recently there have been some papers shedding light on these issues for the American market. Probably the most comprehensive paper on commodity futures is *facts and fantasies about commodity futures* by Gary Gorton and K.Geert Rouwenhorst from 2006. This paper examines the return and variance on commodity futures between 1956 and 2004 and also looks at the correlation between commodity futures, stocks and inflation. To my knowledge there has been no paper on the correlation between OMX and a tracking index of commodity futures like DJ AIG that will be used in this paper.

1.3 Question

Will a well diversified portfolio consisting of Swedish equities (OMX index) be improved by adding the Dow Jones AIG commodity index to the portfolio?

1.4 Purpose

The purpose is to find out if the *Sharpe ratio* of a portfolio consisting of Swedish stocks can be improved by adding commodity futures from the US. To evaluate how FX markets and different inflation rates will effect the investment in commodity futures from a Swedish investors perspective and if it will change the correlation between equities and commodity futures compared to the correlation in the US.

1.5 Notations

The paper will be focused on commodity futures because of the complications that arise from using actual commodities in a portfolio. Commodity futures posses the important characteristics of commodities but lack many of the practical problems involved with investment in actual commodities. When investing in commodities there will be problems with storage, producing, quality control, logistics etc for the investor. Hence when evaluating commodities you need to be able to quantify and price all of these problems.

1.6 Limitations

There will be no regard taken to transactions cost because of the problem of correctly pricing them and removing them from the results. During this paper I will therefore assume similar transaction costs for commodity futures as stocks and bonds. The return from indexes being used will not be adjusted for inflation.

2 Theory

In this chapter the theoretical background of the thesis will be presented along with financial products and topics necessary for a satisfactory analysis of the subject.

2.1 Commodity

A basic good used in commerce interchangeable with all other commodities of exactly the same sort. Commodities are commonly used as inputs in the production of other goods like pork=food oil=energy etc. The quality of a given commodity that is traded has been standardized, and is essentially the same all over the world. When they are traded on an exchange market commodity must meet specified minimum standards, commonly often known as a basis grade (Investopedia).

2.2 Futures

A future is a financial obligation of a buyer to buy (or a seller to sell) a commodity or a financial asset in the future at a predetermined price so a holder of future contract is obligated to fulfil it. It is good to remember that futures are often fully collateralized to secure the transaction and that no money change owner upon agreement (G.Gorton & K.G. Rouwenhorst, 2006). Future contracts are standardized contracts with very specific details on quality, volume, size, etc traded on organised exchange markets (Robert A.Haugen, 2001). The underlying asset can be anything from pork bellies to oil or financial assets like bonds or equities.

2.2.1 Notation

In this paper commodities and commodity futures as an asset class are explored.This means that there is no need of going through the details of individual future contracts and specific commodity future contracts. But it could be worth nothing that there are a lot of different contract types on a lot of different commodities. Some of the largest trading places for commodities and commodity futures are NYBOT, CBOT and LME.

2.3 Commodity futures market

A commodity futures market is a public market place where commodities are contracted for purchase or sale at an agreed price for delivery at a specified date. These purchases and sales, which must be made from through a broker who is member of an organized exchange, are made under the conditions of a standardized futures contract. (R.L.Lerner, 2000)

2.4 Pricing of Commodity futures

Commodities are a physical investment you can literally walk up and touch. Commodities many times need to be stored sometimes they also needs feeding, refrigeration, or regulated secure storage. This means that investing in commodities has extra cost above the spot price they will also be quite cumbersome to include in a normal investment portfolio not having access to storage facilities. The result being that normal investors will not have live cattle in there back yard, a basement full of copper, and a tank truck of oil on there drive way even if all of the above will give higher expected return and lower volatility. Different commodities have very different properties and problems associated which are illustrated below with a table from an article by Jim Finnegan (2004). Finnegan (2004) compared Gold and heating oil in 7 different categories: *Spot market pricing, Seasonality, Risk of supply interruption, Transportation/storage and insurance costs, Consumption levels relative to inventory, Risk of spoilage/loss, and it's collateral value for borrowing.*

Table 1

CHARACTERISTIC	Gold	Heating Oil
Spot Market Pricing	Global	Regional
Seasonality of Production and Consumption	None	Highly Seasonal
Risk of Supply Interruption	Low	High
Transportation/Storage and Insurance Costs	Low	Moderate
Consumption Levels Relative to Inventory	Low	High
Risk of Spoilage/Loss	None	Moderate
Collateral Value for Borrowing	High	Low

Here you can see that gold very much can be treated like a financial asset like stocks or bonds and the formula that is taught in basic economics all over the world is applicable with the assumptions of an efficient market and no free lunches (no risk free arbitrage possibilities). This means that there will have to be two different ways of pricing commodity futures. The most commonly used is *Catango*, the other *Normal backwardation*.

Catango is defined as follows below.

$F_{0-t} = S_0 e^{(r+w)T}$ where $F_{0-T} \geq S_0$ most hold.

Where F is the future price, S the spot price, r is the interest rate, w is the cost of storage, and T is the time period. This is called a cantango moment where the "price" of the future increases as T increases. This will be suitable for commodities that have similar properties to gold.

But applying the same formula to the price of heating oil would require quite a few assumptions about the future. For instance the seasonality and risk of interruption is not taking in to consideration at all. There are also high transaction costs and high costs of storage. This means that the formula would not present an accurate value of heating oil futures because it oversimplifies of the real dynamics. It will become complicated for market participants to lock in an arbitrage possibility in differences between spot and futures prices according to the formula above. Hence another type of model will have to be used for explaining the future prices. The model applicable would be normal backwardation.

2.5 Normal backwardation

One of economies founding fathers presented a solution to this problem called normal backwardation where you ad a risk factor to the price of the future much like a stock price here the future price will fall with *T*.

Here $F_{0-T} \leq S_0$ is the situation so the longer the time period the higher the risk premium.

In the 1930 Keynes and Hicks come up with the theory of *normal backwardation* a theory says that on average the risk premium goes to the buyer. Hence the producer is buying an insurance against a price decline, a *hedge.* To supply this hedge the buyer will want a positive expected return. So the future contract is bought slightly under expected spot price on expiration date. This means that on average the buyer of futures will gain from the transaction thus the transaction involves a risk premium for the buyer. This means that the buyer of futures will on average gain even if the spot price is expected to decline.

There are a lot of incentives for hedging and they are presented in the chapter hedging (should be noted that there are a lot of reasons why not to hedge.).

The important factor is that producers of commodities are selling the futures resulting in the buyers of this futures are taking the entire risk of commodity price fluctuation.

To get the market back to equilibrium we will have to ad a convenience yield to the equation. This will get the market back into equilibrium from a mathematicians point of view. Defined as below:

$F_{0-t} = S_0 e^{(r+y+w)T}$ Where *y* is the convenient yield. The convenient yield can be defined as the risk premium the buyer demands for taking on the risk of volatility in the market.

2.6 Reasons for Hedging

2.6.1 Theories of motivations behind risk management.

The theory of shareholder value maximisation states the point that risk management can increase firm value by reducing costs associated with financial distress, fixing levels of taxable income and managerial risk aversion according to *Smith and Stulz* (1985). Hedging will also reduce the cost of financial distress and in smaller producers of commodities it also helps to offset the risk taken by poor diversification since a large amount of the personal wealth is tied to one asset class and many times to one asset in this asset. This means that the primary objective of hedging for a firm is to reduce volatility of future cash flows. Lowering volatility means that the firm will be able to have higher debt capacity and less underinvestment problems according to *Shapiro and Titman* (1986) the firm will also have less deadweight costs from not honouring long term commitment to buyers and suppliers.

2.6.2 Taxes

Taxes are one of the biggest incentives for hedging in the commodity futures market. Since taking on debt creates a tax shield that lowers the cost of capital making debt favourable over equity. This means that when hedging you can increase the debt without increasing the cost of financial distress and through that increase the size of the tax shield increasing firm value (Ekholm, Nguyen, 2006)

2.6.3 Cost of external financing

Hedging can also improve firm value by increasing the access to internal funding since external funding often is more costly because of imperfections in financial markets. Management has an information advantage over creditors making creditors undervaluing the value of projects. This information asymmetry creates a situation where some firms even haft withdraw from undergoing profitable projects since creditors are undervaluing future cash flows. By hedging the firm can secure a higher proportion if internal funding and decreasing problems of under investments.

2.7 Previous work on commodity prices and commodity futures

Already in 1965 Fama found that prices on commodities were characterized by volatile periods being followed by more tranquil periods. Moreover the variance was changing over time also the unconditional distributions of the price changes was leptokurtic rather then normal distributed. This shows the importance of using GARCH models when working with commodity futures even if the GARCH model was not invented in 1965. H.G. Jareki (2007) says that there has been many misunderstanding about investing in commodities mainly that because many commodity investors has gone bankrupt people have started to believe that commodities are very volatile.

In fact commodity price fluctuations are not more volatile than stocks and this misconception probably comes from that commodity futures can be bought on such a small margin as 5%. This creates a situation where small changes in price can result in very large changes in profits and loss (Jarecki, 2007). G.Gorton and K.G.Rouwenhourst (2006) looked at commodity futures as an asset class during the time period 1959 through 2004. This is probably the most comprehensive investigation of commodity futures historical earnings and volatility. The result clearly shows that fully collateralized commodity futures have been showing they same high returns as stocks and higher than bonds. The volatility has been just below the volatility of stocks essentially the risk premium for commodity futures and stocks has been the same. The have by this shown that Sharpe ratio between stock and commodity futures are pretty much the same.

But during this 45 year period they have shown negative correlation with stocks and bonds. The negative correlation mostly comes from that commodity futures behaves differently over the business cycle. Commodity futures often have higher returns in the end of a business cycle where stocks have high returns in the beginning. Commodity futures are also positively correlated with inflation when stocks are negatively correlated (Gorton ,Rouwenhourst, 2006).

2.8 Required Finacial Theory

2.8.1 Financial Risk.

Risk will here be expressed in volatility, since it is the most widely accepted measurement of risk in financial markets. A high volatility in the return distribution is a riskier investment then one with a low volatility for simplicity the volatility will be expressed in standard deviation. When we use volatility as the main measure of financial risk we are talking about the risk for the entire portfolio and not single assets. "The volatility of the entire portfolio return depends on the variances and covariance between the risk factors of the portfolio, and the sensitivities of individual assets to these risk factors" (*Carol Alexander, 1998*). Standard deviation is the square root of variance defined as $\sigma^2 = \text{var}(x) = E\left[(X-\mu)^2\right]$ and $\mu = E(X)$. One problem when using volatility as a measure of risk is that you can miss the impact of events with a very large impact in a single event. The only way to lower the financial risk is to diversify. As you include more and more assets in a portfolio the variance of the single assets will start to lose importance and as you go towards infinite only the correlation between the assets will be of importance. This means that the correlation is of importance when composing a portfolio hence including a new asset that has low correlation will improve the portfolio risk profile much more than including assets with low volatility.

2.8.2 Sharpe ratio

The *Sharp ratio* measures the expected return per unit of risk the definition is as follows.

$$sr_i = \frac{E[R_i] - R_f}{\sigma_i}$$

The *Sharpe ratio* is a measure of the risk adjusted return on the portfolio. By using the Sharpe ratio we can measure if excess return is due to excess risk or a better portfolio diversification. A higher *Sharpe ratio* means a better performance after risk adjusting. The *Sharpe ratio* will here be used to determine if portfolio has improved or just added extra risk.

2.8.3 Minimum variance portfolio

The *minimum variance portfolio* (MVP) is the combination of assets that gives the portfolio with the lowest possible variance. If you are a risk adverse investor who wants to achieve an expected return above T-bills the MVP is the route to go. The MVP will be located on the *efficient set* meaning it will dominate all other portfolios with same expected return. Mathematical the MVP means you optimize the equation of the variance with regard to the weights of the assets.

2.8.4 Asset classes

An asset class will be defined as an investment that will improve the MV- portfolio and the efficient set. The four most common asset classes are equities, bonds, real estate, and commodities. Some will argue that hedge funds and Private equity could be regard as asset classes but this will be disregard from since this argument have not been supported by any paper that they actually improve the efficient set. Intuitively this is easy to understand since hedge funds and private equity are based upon the other asset classes. For pension funds and institutions a portfolio where you not only ad commodity futures but also real estate could be of interest. In the case of private investors most of them already have a high exposure to the real estate market and it's quite cumbersome to include. Therefore this paper will not try to improve a portfolio further by including real estate even if it's a tempting idea from a theoretical point of view.

2.8.5 Covariance Matrix

The covariance matrix is a matrix of the covariance between the elements of a vector it is defined as follows: $V = \begin{matrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{i1} & \cdots & \cdots & \sigma_{in} \end{matrix}$

Where $\sigma_{11} = \text{var}(x) = E\left[(X-\mu)^2\right]$ and $\mu = E(X)$.

In the GARCH frame work this will be revaluated for every new observation.

2.9 Correlation

Correlation is how two different stocks or in this case two different asset classes correlate with each other over time. The correlation is measured from -1 to +1 where +1 is perfectly correlated assets.

The definition of correlation is as follows: $Corr\left(X,Y\right)=\dfrac{Cov\left(X,Y\right)}{\sqrt{Var\left(X\right)Var\left(Y\right)}}$

When you combine different assets you want them to be as uncorrelated as possible. In reality it's very hard to find both fully correlated asset and fully uncorrelated. Here X and Y are two different return series. For two series to be unconditionally correlated they need to be jointly stationary. If they are not jointly stationary the correlation will be very unstable and tend to jump between states over time. The time series in this paper are not jointly stationary.

Because of the risk of extreme events creating ghost features that will linger on in series for longer time then necessary an *exponentially weighted moving averages* (EWMA) could be used instead of historical values, this will give a higher weight to events closer in time and allow the impact from spikes to decay faster resulting in a smoother series. This will be done by using a smoothing parameter Risk metrics that constructed the model is using $\lambda = 0{,}94$.

The definition of *EWMA* is as follows: $\dfrac{x_{t-1}+\lambda_{t-2}x_{t-2}+\lambda^{2}x_{t-3}+\ldots\lambda^{n-1}x_{t-n}}{1+\lambda+\lambda^{2}\ldots\lambda^{n-1}}$

If it will go towards infinity it will be defined as follow: $\left(1-\lambda\right)\sum_{i=1}^{\infty}\lambda^{i-1}x_{t-i-1}$.

2.10 Effect of inflation, foreign exchange markets and interest rates on commodity futures

Exchange rates, inflation rates, and interest rates will all affect the results from investments done abroad. Since Sweden is such a small country the commodity futures market is too small and illiquid to present a true complement to stocks. A Swedish investor will therefore have to look abroad if he would like to introduce an exposure to the commodities market. I have chosen to work with the commodity futures index *DJ AIG* traded on the *New york stock exchange* intuitively it should not be as good for hedging the Swedish inflation as a commodity index in Sweden especially if the majority of the portfolio is in Swedish equities and bonds. However many economist today do argue that financial markets over the world are becoming more and more integrated. This is an argument made among others by Iraj J fooladi , john Rumsey (2006) and Cristansen (2003). Cristansen (2003) for instance also finds strong evidence for volatility spill over effects from US and Europe into individual markets. If the world's financial markets to some extent are integrated with each other perhaps it will be not such a bad idea to look for asset diversification abroad.

Which are the major macro factors that can offsets the gain shown when adding commodity futures to a Swedish equity portfolio instead of an American portfolio?
The two major macro factors in this case would be exchange rates and inflation rates. There is also of interest if higher inflation rates in one of the countries to some extent can be offset by the exchange rates. According to the purchasing power parity PPP "*movements in internal price levels corresponds to movements in the exchange rate*."(L.oxelheim and C,Wihlborg, 2005).

The PPP is defined as follows:

$\frac{S_t}{S_{t-1}} = \frac{P_t^{DC} / P_{t-1}^{DC}}{P_t^{FC} / P_{t-1}^{FC}}$ where P^{DC} = price level domestically and P^{FC} = price level foreign and

S= spot rate (DC unit per FC) formula from (L.Oxelheim and C.Wihlborg, 2005). The PPP holds in the long run according to L.Oxelheim and C.Wihlborg and in the short run if commodity arbitrage is perfect. Many economist argue that there are deviations from the PPP in the short run and there can be deviations being sustained over periods of a couple of years. Many believe that the PPP is a mean reverting process where deviations are unstable states that eventually will revert back to PPP. There can also be some commodities that are more sensitive then others and some that are less sensitive.

There is one major macro economic factor that will be disregarded that could have effect on a commodity futures portfolio and that is the changes in consumer behaviour. Mad cow disease was such a situation where consumer preferences where changed almost over a night. A situation like this is also likely to affect more than one commodity and most probably in different directions. Since these effects will be very hard to adjust the portfolio for and it is likely that many of them will cancel each other out they will here be disregarded from.

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If it will go towards infinity it will be defined as follow: $\left(1-\lambda\right)\sum_{i=1}^{\infty}\lambda^{i-1}x_{t-i-1}$.

3 Data

This chapter will present the data material the thesis is based upon and from where it has been collected.

3.1 Sources

Because of the large amount of data and major interest is on the asset class itself the focus will be on an index of commodity futures. This will be extrapolated with the help of *Ecovision.* The index being used for the estimation is the *DJ AIG index.* The stock price index used is the OMX index for the Swedish market. The CPI index for the inflation in the US and in Sweden is also recieved from Ecovision. For the exchange rates between US dollars and the Swedish krona the daily spot price will be used for simplicity. The CPI index will be used to take away effects from inflation and deflate the index if this will prove to be necessary. The exchange rate index of spot prices is used to compare the American commodity index to the Swedish stock market. This is also done to see if the PPP will interfere when using foreign commodity futures. For the commodity index and the stock market index daily closing prices will be used, the CPI will consist of monthly observations. The exchange rates that will be used are daily closing prices on the spot market. When looking at monthly data the closing price of the last day of trade will be used. The US commodity futures index will be denominated in the Swedish krona using the same day's closing price on the spot market and the stock exchange no account is taken for the different closing times and openings times of the Swedish stock exchange and the American stock exchange.

4 Methodology

This chapter goes through research methods and also presents how regression and regression analysis has been done.

4.1 Choice of method

As has already been stated the purpose of the thesis is to estimate the correlation between commodity futures in US and equities in Sweden to be able to draw conclusions of the *sharpe ratio*. The estimation will be done by looking at daily and monthly returns after exchange rate changes from DJ AIG and the Swedish OMX index over the Swedish stock market. The estimation will be done in a Multivariate GARCH frame work and the covariance from the MGARCH estimation will be used to estimate the correlation between the equity index and commodity futures. The correlation between the Swedish stock exchange and the American commodity futures index are the main interest since the inflation and macro variables will affect commodities and exchange rates at the same time so by looking at the correlation we can draw conclusions of the possible effects commodity futures will have the portfolios *sharpe ratio*.

This correlation coefficient is then compared to the historical coefficient from the paper by Garton and Reawenhaurst to see if there has been any significant change over the last five years and if the is any major difference between the correlations if the stock index is from Sweden and the commodity index is from US. Because the indexes are not deflated there is possible for the correlation to be higher then Garton and Reawenhurst (2006).

5 GARCH FRAME WORK

This chapter goes through the GARCH frame work and the model used for estimating the correlation.

Since the object of this paper is to compare correlation over time a EWMA could have been enough to capture the time dependent coefficients however with heteroscedasticity a *Generalized autoregressive conditional heteroscedasticity* (GARCH) model will instead be used for constructing the covariance variance matrices. It is very common for financial time series to have autoregressive conditional heteroscedasticity. The covariance matrix is a matrix of the covariance between the elements of a vector it is defined as follows:

$$V = \begin{matrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{i1} & \cdots & \cdots & \sigma_{in} \end{matrix}$$

Where $\sigma_{11} = \mathrm{var}(x) = E\left[(X-\mu)^2\right]$ and $\mu = E(X)$.
In the GARCH frame work this will be revaluated for every new observation.

The GARCH model framework will allow the variance and covariance to be dependent of previous lags. It will also be able to capture dependency on previous lags between different time series. The GARCH framework is less likely to over fit the sample as EWMA or ARCH. It is also less likely to break non-negative constraints. Using time dependent variance and covariance will enable the model to capture clustering effects in the data. It has been shown that variance in financial markets is high during certain periods and low during other. (Brooks**,** 2004**)** When the variances change over time it means that the time series has heteroscedasticity, it has a changing volatility. Using GARCH will also allow mean reverting i.e. if there is a long term means periods of high volatility mean reverting will decrease volatility over time and periods with low volatility will increase over time. The series is auto regressive it regress on *itself* (Chris brooks, 2005) .

"There is also evidence that GARCH models generate more realistic long-term forecast than exponentially weighted moving averages. This is because the GARCH volatility and correlation term structure forecasts will converge to the long-term average level, which may be imposed on the model, whereas the exponentially weighted moving average model forecast average volatility to be the same for all risk horizons"(Carol Alexander, 2000). Andersen, Bollerslev Cristoffersen, Diebold from 2005 tells us why using the GARCH framework presents a more accurate picture of reality then RW (Random walk). "As is well known (e.g Nerlov and Wage, 1964, Theil and Wage, 1964) exponential smoothing is optimal if and only if squared returns follow a "random walk plus noise" model (a "local level model in the terminology of Harvey 1989) in which case the minimum MSE forecast at any horizon is simply the current smoothed value. The historical records of volatilities of numerous assets (not to mention the fact volatilities are bounded below by zero) suggest , however, that volatilities are unlikely to follow random walks, and hence that the flat forecast function associated with exponential smoothing is unrealistic and undesirable for volatility forecasting purposes." (Andersen, Bollerslev Cristoffersen, Diebold 2005)

Since the GARCH model is dependent on its previous legs in its simplest form

It is defined as follows

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-t}^2$$

σ_t^2 is known as the conditional variance one-period ahead estimate for the variance calculated based on any past information thought relevant. (Chris brooks, 2005)

To see this consider that the squared return at time t relative to the conditional variance is given by

$$\varepsilon_t = u_t^2 - \sigma_t^2$$

Or

$$\sigma_t^2 = u_t^2 - \varepsilon_t$$

Using the latter expression to substitute in for conditional variance in

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-t}^2 + \beta u_{t-1}^2 - \left(u_{t-1}^2 - \varepsilon_{t-1}\right)$$

Rearranging

$$u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta u_{t-1}^2 - \beta\varepsilon_{t-1} + \varepsilon_t$$

So that

$$u_t^2 = \alpha_0 + (\alpha_1 + \beta)u_{t-1}^2 - \beta\varepsilon_{t-1} + \varepsilon_t$$

The final expression is an ARMA (1,1) process for the squared errors(Brooks, 2005).

Since the GARCH model uses the maximum likelihood function to estimate the parameters I will also briefly explain it here.

5.1 Maximum likelihood

Maximum likelihood picks the mean and variance that makes the most probable distribution of the sample. Under the assumption of normal distribution it does this by maximising the following equation

$$L = -\frac{T}{2}\log(2\pi)\sum_{t=1}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{(y_t - u - \phi y_{t-1})}{\sigma_t^2}$$

Most often maximum likelihood equations are solved numerically the process of maximising the function is done by minimising the error variance. Therefore should yield the same result as with OLS, if OLS would have been available.

5.2 Bivariate/Multivariate GARCH model

The multivariate GARCH model has many similarities to the univariate GARCH model but it will also include equations for time dependent covariance i.e. how the covariance between to or more assets differ over time. The first one and the one with the least amount of restrictions imposed on itself is The original Vech proposed by Engle and Wooldrige (1988) defined as follows:

$$VECH(H_t) = C + AVECH(\Xi_{t-1}\Xi'_{t-1}) + BVECH(H_{t-1})$$
$$\Xi_t \mid \psi_{t-1} \sim N(0, H_t),$$

Even in its simplest form with only two assets the unrestricted VECH model there is 21 parameters that has to be estimated as the number of assets included increase there will be an extremely large amount of parameters to be estimated.

The biggest problem with the VECH model is that it’s not guaranteed to return a covariance matrix that is positive semi definitive. This means that the variance covariance matrix will have positive numbers on the leading diagonal and will also be symmetrical around this about this leading diagonal. These properties are intuitively appealing as well as important from a mathematical point of view, for variances can never be negative, and the covariance between two series is the same irrespective of which of the two series is taken first. (Chris Brooks, introductory econometrics for finance, 2002)

Therefore I have chosen to use the BEKK model that only has 9 parameters to be estimated in the bivariate case.

5.3 BEKK

In the BEKK model A and B are assumed to be diagonal with 3 elements each.

The BEKK model ensures positive semi definitines and reduces the parameters that have to be estimated to 9 it does this without imposing any large restrictions.

If there is n-series that has to be estimated it will be defined as follows.

$$H_t = W'W + A'H_{t-t}A + B'\Xi_{t-1}\Xi'^{\prime\prime}_{t-1}B$$

Where W is an $n \times n$ triangular matrix. B and A are $n \times n$ matrices. (Brooks, 2005)

In the model used in this paper the matrices are as follows:

C = 2 x 2 low triangular
A = 2 x 2 diagonal
B = 2 x 2 diagonal

The maximum likelihood function in the BEKK model is defined as follows below.

$$\ell(\theta) = -\frac{TN}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\left(\log\left|H_t\right| + \Xi_t' H_t^{-1}\Xi_t\right) \qquad \text{(Brooks, 2005)}$$

The BEKK model will be estimated by using the statistical package from Eviews.

The correlation will be derived by dividing the time dependent covariance with the product of the time dependent standard deviations at every point in time. The result will be a time series of time dependent GARCH estimated correlations.

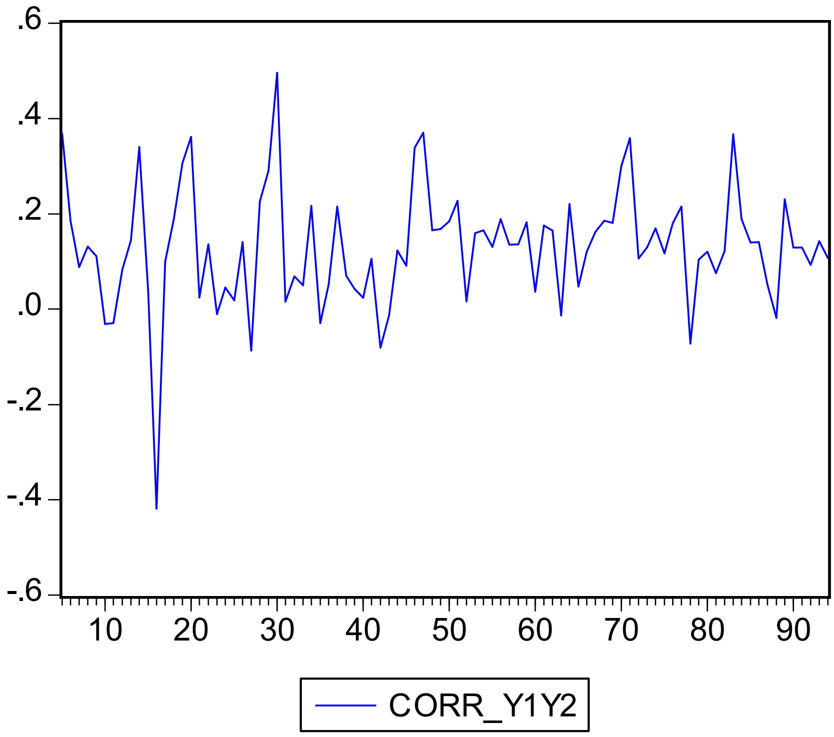
$$Corr\left(X, Y\right) = \frac{Cov\left(X, Y\right)}{\sqrt{Var\left(X\right)Var\left(Y\right)}} \quad \text{(Vesterlund, 2005)}$$

6 Empirical findings

Here the results are presented.

Since the time period is only just over 7 years the frequency of the observations are monthly and daily this means that results over a longer time period could differ, because of the fact that correlations have a tendency to be stronger over time.

Figure 1: Monthly correlations between OMX and DJ AIG, 2000-2007 MGARCH BEKK



The average monthly correlation is 0,24 between 2000 and the first quarter of 2007. During this time the monthly correlation varies between -0,4 and 0,5. -0,4 and 0,5 are extreme values and for the last four years the variation have been between -0,1 and 0,4.
During this time the average monthly return has been 0,446% for commodity index in denominated in Swedish kronor and 0,359% for the Swedish OMX index. During this time the historical variance is around 0.15 higher then G.Gorton and K.G.Rouwenhourst average on American stocks.

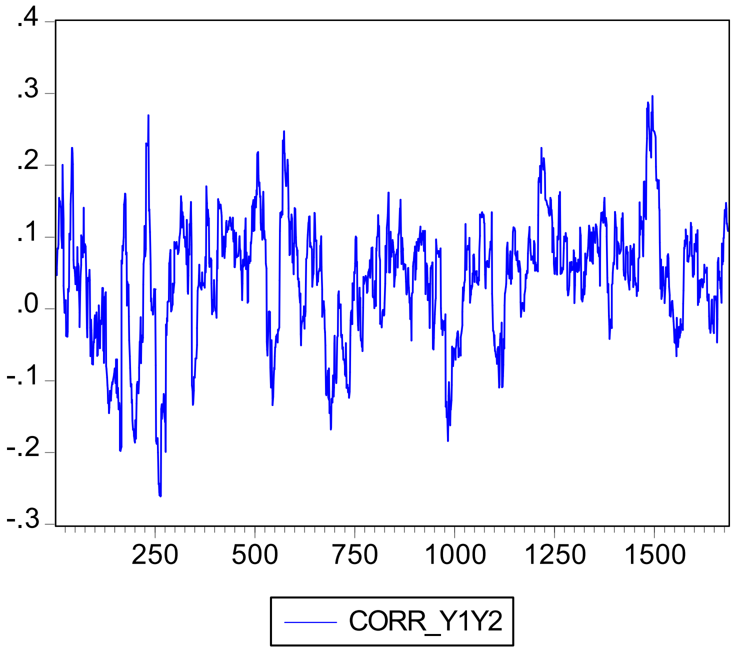
Table 2: parameters estimate

W(1)	0.053928
W(2)	0.004918
W(3)	0.017912
A(1)	0.149975
A(2)	0.824656
B(1)	0.350751
B(2)	0.465664

Table 2 above is the parameters estimate of the BEKK model defined as below

$$H_t = W'W + A'H_{t-t}A + B'\Xi_{t-1}\Xi_{t-1}^{\prime\prime}B$$

Figure 2: Daily correlations between OMX and DJ AIG, 2000-2007 MGARCH BEKK



Daily observations gives support for the trend in monthly observations but are more volatile. The correlation do stay within 0,3 and -0,3 range commodity futures can therefore be considered to have a low correlation to OMX.

Figure 3: Monthly Covariance and variance between OMX and DJ AIG, 2000-2007MGARCH BEKK

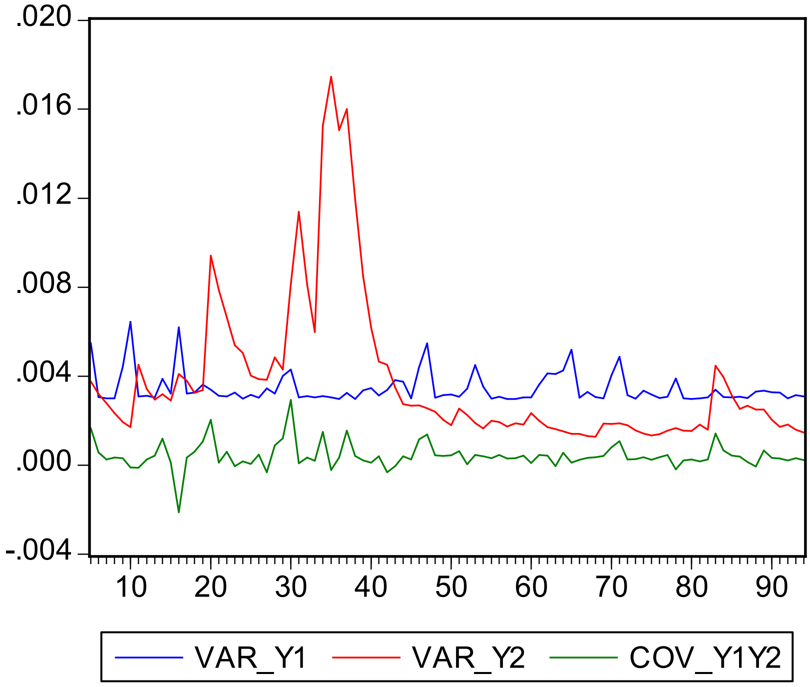
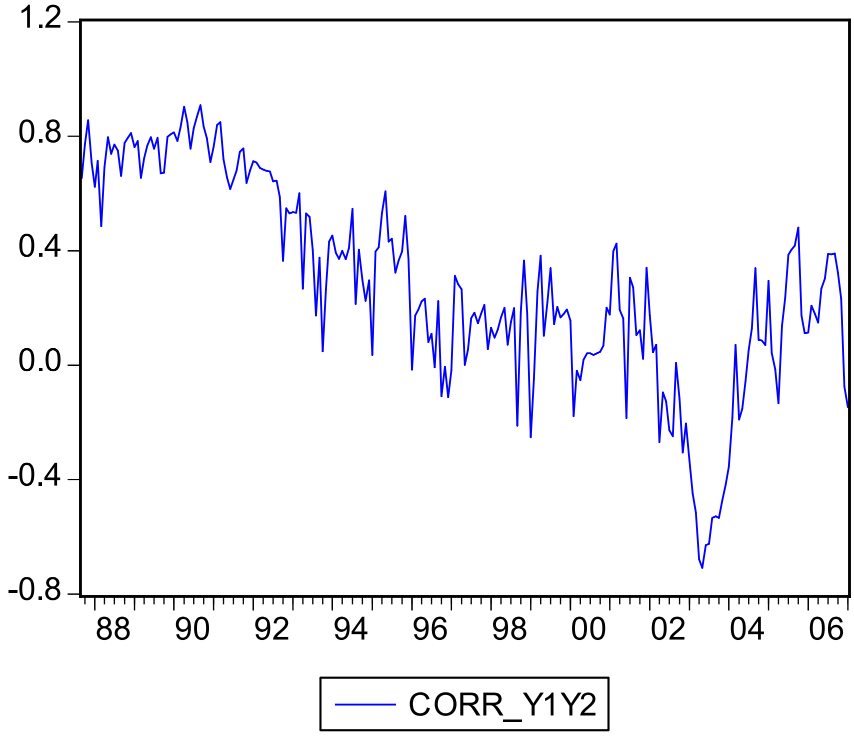


Figure 3 shows the variance for the *OMX*(Y2) and *DJ AIG*(Y1) index and their covariance. figure 3 clearly shows how the *OMX* index variation shoots through the rough during the years of 2002-2004 after that the *DJ AIG* have had a higher variation. So how tightly are then Swedish and American inflation tied together?

Well the ties have been weakened ever since the beginning of the nineties. This means that for twenty years ago the correlation between Swedish and American inflation was very high. Since 1991 when the Swedish economy had one gigantic crisis the ties has been declining. That crisis resulted in the Swedish government being forced to let the krona float against other currencies. Since then there has been a slowly decreasing correlation of the inflation in Sweden and America. The commodity index has already been proved to have a high positive correlation with the American inflation. Because of lesser ties between the different inflations the negative correlation between commodities and stocks in America will not hold over the ocean. They still present an interesting alternative to stocks but will not be able to perfectly match the results from the American market.

Figure 4: Monthly correlation between Swedish and US inflation.



The figure above shows the correlation between Swedish and American inflation. Where we clearly can see how the correlation has steadily been decreasing since the nineties.

7 Analysis

Here the analysis of the results will be presented.

G.Gorton and K.G.Rouwenhourst article from 2006 showed very small correlation between monthly returns and negative correlation for longer time periods. The hope was to find similar results for the Swedish market the results wasn't as strong as for the US market, where there was a very high variation in the correlation. It ranged from -0,4 up to around 0,5 with a majority of the observations shoving a positive correlation. This is far better then most stocks but quiet a long way from being the perfect hedge. There is no significant trend to be observed and the correlation could almost best be described with a random walk.

7.1 Question: Do the FX market have a large impact on the correlation?

What the results clearly shows is also that the situation will get more complicated when you ad the presence of the FX-market. The correlation has a high variation higher than in the G.Gorton and K.G.Rouwenhourst article on the US market. This could be due to the effect that the value of the dollar affects the demand for many commodities. Since the commodities are bought in dollar a decrease in dollar would make for an increase for commodities denoted in dollar. This would at the same time decrease the value of a portfolio denoted in Swedish kronor but bought in dollar. This could perhaps to some extent be avoided with the help of FX futures. Using FX futures would probably either increase the volatility or it would mean lowering expected rate of return. Choosing the path of using FX futures as a hedge against movements in the dollar the expected profit from the commodities futures will to some extent be offset by loses on FX futures. This is because if we assume normal backwardation for one type of futures the same most apply to all type of futures bought under the conditions. But are FX futures really bought under same conditions as commodity futures? Cash is the most liquid asset of all meaning that you could have both parties of a contract looking for a hedge but in opposite markets. Any arbitrage possibility can be exploited in seconds differing quite a lot from the pork bellies market or the oil market.

7.2 Question: Is a foreign commodity futures portfolio a good hedge for the Swedish investor?

Still, despite all of this, the correlation is significantly lower than one. During the same time period the historical rate of return on a monthly basis has been slightly higher for commodity futures than for the OMX index. Hence the DJ AIG has a positive expected rate of return at the same level as the OMX but a low correlation making it an asset class that would improve the *Sharpe ratio* of the MVP portfolio. Also during the same period the variance has practically been the same but and even slightly better on the commodity futures index. By using the *Sharpe ratio* we can logically see how this improves the *MVP-portfolio* and obviously also improves the efficient frontier. A Swedish investor will not fully benefit from all the properties the American investors enjoy. However the Swedish investor will none the less have improved the portfolio both in regards of risk and expected return. The outcome from both the GARCH and the historical model shows statistically significant results that a portfolio consisting of weighted Swedish stocks will be improved by adding a portfolio of US commodity futures.

The DJ AIG is well diversified and traded on the New York stock exchange it is therefore much more liquid then anything denoted in the Swedish krona. Since it is relatively liquid and has a low correlation with OMX and also a return equal to OMX it could be considered as a hedge for the OMX and would definitely be a good strategy for diversifying a portfolio.

7.3 __Summary__

Commodity futures will be affected by the FX markets and the different inflation rates as concluded before but will nonetheless improve the *Sharpe ratio* for a portfolio made up of mainly Swedish stocks. Because of the low correlation between Swedish and US inflation this results could probably be analogues to the rest of the world meaning that the results shows that probably a foreign commodity futures is a good strategy for diversifying any portfolio mainly made up of equities. This is only intuitive and is not proven mathematically but still shows an interesting point.

This paper points in the direction that many times we could probably be looking in the wrong direction when we are trying to diversify among equities when we instead should be looking for including so many asset classes as possible instead.

8 Conclusions

This paper examines the correlation between a American commodity futures index (DJ AIG) and a Swedish stock index. The purpose is to find out if a Swedish investor can improve the *Sharpe ratio* of his portfolio by adding commodity futures from a foreign stock exchange.

By using the BEKK model which is a multivariate GARCH model time dependent variances and covariance's for the two time series are estimated. These estimates are then used for constructing a time dependent correlation between the two time series.

The results from these estimations show a low correlation between the OMX and the DJ AIG. However the correlation is higher then in a previous paper by Gorton and Rouwenhourst (2006) that looked at correlation over a longer time period for the American market using a historical model. Hence the results are that the correlation will be affected by the different inflations, the FX market and Macro variables. Commodity futures have during the period showed approximately both the same return and variance as the OMX. Hence the low correlation means that commodity futures will improve the *Sharpe ratio* of a portfolio consisting of the OMX index. Despite the fact that commodity futures are affected by the factors named earlier they will still be a god investment for a Swedish investor mainly invested in Swedish stocks.

8.1 Fit of the model

The GARCH model is an appropriate model for looking at data that is expected to differ over time. It can fully capture the variation over time both in variance and covariance. The *akaike criterion* shows that the model is a good fit for the model during the time period tested. This shows that the results are stable enough for answering the question asked in the introduction.

8.2 Future research in the subject

It would be of great interest to test a strategy where the FX market risk is hedge away. This would probably be done by using FX derivates. It would also be interesting to examine the possibility of extracting the result from this and previous papers to a more general setting to see if the very good result in both America and Sweden could be applied to more markets.

9 References

Articles

Orthogonal Methods for Generating Large Positive Semi-definitive Covariance Matrices: Carol Alexander, 2000

G.Gorton and K.G.Rouwenhourst, 2006, “Facts and Fantasies about commodity futures”, *Financial Analyst Journal,* Vol 62, No 2 pp 47-69

Robert F.Engle and Kenneth F.Kroner, 1995, Multivariate simultaneous GARCH model, *Econometric Theory*, Vol. 11, No. 1 (Mar., 1995), pp. 122-150

Henry G Jarecki, 2007, Commodities create the right mix, *Futures magazine,* Vol 36, No 3, pp 66-70.

Torben G. Andersen & Tim Bollerslev & Peter F. Christoffersen & Francis X. Diebold, 2005, "Practical Volatility and Correlation Modeling for Financial Market Risk Management," *CFS Working Paper Series Center for Financial Studies.*

Iraj J. Fooladi and John Rumsey, 2006, “Globalization and portfolio risk over time, The role of exchange rate”, *Review of Financial Economics*, vol 15, issue 3, pages 223-236

Christianse,C. (2003), ‘Volatility-Spillover Effects in European Bond Markets’, Working paper, Aarhus School of Business.

Smith Jr, C, and R Stulz, 1985,” The determinants of firms' hedging policies”, *Journal of Financial and Quantitative Analysis 20*, pages 391–405

Shapiro, A.C, Titman, S, 1986.” An Integrated approach to corporate risk manegemant”. In: Joel M.stern, Donald, H. Chew, Jr, *The revolution in Corporate finance*. Blackwell, Oxford,UK, pp. 215-229.

Hanna Ekholm, Chi Nguyen, 2006, “Hedging core and Non-core Risks: Evidence from the forestry and Paper Industry”, Master thesis, Lund University Department of Business Administration.

Bibliography

Joakim Westerlund, 2005, *Introduktion till ekonometri,* Lund, Studentlitteratur.

Carol Alexander, *Risk management and Analysis: Measuring and Modelling Financial Risk*, John Wiley & Sons, 1st edition, 1998.

L.Oxelheim and C.Wihlborg, 2005, *Corporate performance and the exposure to macroeconomic fluctuations,* Stockholm, Norsteds Akademiska förlag.

R.L.Lerner, 2000, http://www.turtletrader.com/beginners_report.pdf

Haugen, Robert A. 2001.*Modern investment theory*. 5 ed. New Jersey: Prentice Hall International Inc.

Chris Brooks, 2005, *Introductory econometrics for finance.* 6 ed. Cambridge: Cambridge University press

Electronic sources

http://www.investopedia.com/university/futures/

http://www.turtletrader.com/beginners_report.pdf; R.L.Lerner, 2000,

“The Natur of commodity index returns”, Robert J.Greer,2000, *Journal of alternative investments*, http://www.iinews.com/site/pdfs/JAI_Summer_2000_Greer.pdf

Jim Finegan jan 2004 finical engineering news http://www.fenews.com/fen35/index.html

Ecovison

Ewievs, ver:5

10 Appendix

For the inflation

LogL: BVGARCH
Method: Maximum Likelihood (Marquardt)
Date: 07/02/07 Time: 12:14
Sample: 1987M09 2007M01
Included observations: 233
Evaluation order: By observation
Estimation settings: tol= 1.0e-06, derivs=accurate numeric
Initial Values: MU(1)=0.00218, MU(2)=0.00118, OMEGA(1)=0.00036, BETA(1)=0.80342, ALPHA(1)=0.46781, OMEGA(3)=0.00043, OMEGA(2)=0.00000, BETA(2)=0.77456, ALPHA(2)=0.38836
Convergence not achieved after 4 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
MU(1)	0.001989	0.000109	18.16516	0.0000
MU(2)	0.001127	7.43E-05	15.17034	0.0000
OMEGA(1)	0.000268	7.45E-05	3.590779	0.0003
BETA(1)	0.900599	0.032786	27.46937	0.0000
ALPHA(1)	0.358004	0.078267	4.574157	0.0000
OMEGA(3)	0.000247	5.57E-05	4.440595	0.0000
OMEGA(2)	4.02E-05	6.74E-05	0.595926	0.5512
BETA(2)	0.795604	0.024829	32.04369	0.0000
ALPHA(2)	0.580659	0.100835	5.758525	0.0000
Log likelihood	2463.807	Akaike info criterion		-21.07131
Avg. log likelihood	10.57428	Schwarz criterion		-20.93800
Number of Coefs.	9	Hannan-Quinn criter.		-21.01755

For stocks and commodity futures monthly

LogL: BVGARCH
Method: Maximum Likelihood (Marquardt)
Date: 07/02/07 Time: 11:48
Sample: 5 94
Included observations: 90
Evaluation order: By observation
Estimation settings: tol= 1.0e-06, derivs=accurate numeric
Initial Values: MU(1)=0.00212, MU(2)=0.01233, OMEGA(1)=0.04699,
BETA(1)=0.53090, ALPHA(1)=0.28034, OMEGA(3)=0.02146,
OMEGA(2)=0.00000, BETA(2)=0.82495, ALPHA(2)=0.38376
Convergence not achieved after 4 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
MU(1)	0.001976	0.006935	0.285002	0.7756
MU(2)	0.009708	0.006441	1.507363	0.1317
OMEGA(1)	0.053928	0.011368	4.743936	0.0000
BETA(1)	0.149975	1.064975	0.140825	0.8880
ALPHA(1)	0.350751	0.148192	2.366864	0.0179
OMEGA(3)	0.017912	0.008150	2.197776	0.0280
OMEGA(2)	0.004918	0.009396	0.523352	0.6007
BETA(2)	0.824656	0.090359	9.126491	0.0000
ALPHA(2)	0.465664	0.122254	3.808986	0.0001

Log likelihood	262.4966	Akaike info criterion	-5.633258
Avg. log likelihood	2.916629	Schwarz criterion	-5.383277
Number of Coefs.	9	Hannan-Quinn criter.	-5.532451

For stocks and commodity futures Daily

LogL: BVGARCH
Method: Maximum Likelihood (Marquardt)
Date: 08/23/07 Time: 13:16
Sample: 4 1684
Included observations: 1681
Evaluation order: By observation
Estimation settings: tol= 1.0e-05, derivs=accurate numeric
Initial Values: MU(1)=7.5e-05, MU(2)=0.00075, OMEGA(1)=0.00148,
BETA(1)=0.97944, ALPHA(1)=0.16991, OMEGA(3)=0.00155,
OMEGA(2)=0.00000, BETA(2)=0.95317, ALPHA(2)=0.28464
Convergence achieved after 18 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
MU(1)	0.000104	0.000323	0.320777	0.7484
MU(2)	0.000737	0.000286	2.572623	0.0101
OMEGA(1)	0.000792	0.000228	3.471921	0.0005
BETA(1)	0.993401	0.002104	472.2431	0.0000
ALPHA(1)	0.097066	0.014934	6.499863	0.0000
OMEGA(3)	0.001620	0.000185	8.739592	0.0000

OMEGA(2)	0.000292	0.000263	1.112951	0.2657
BETA(2)	0.951433	0.006260	151.9817	0.0000
ALPHA(2)	0.289125	0.018688	15.47076	0.0000

Log likelihood	9743.847	Akaike info criterion	-11.58221
Avg. log likelihood	5.796459	Schwarz criterion	-11.55315
Number of Coefs.	9	Hannan-Quinn criter.	-11.57145

Program used for the MGARCH BEKK model in Eviews

(Here an ‘ before the text means the program should be disregard as information)

Definitions

$H_t = W'W + A'H_{t-t}A + B'\Xi_{t-1}\Xi'_{t-1}B$ = H = omega*omega' + beta H(-1) beta' + alpha res(-1) res(-1)' alpha'

W=omega

A= beta

B=alpha

example program for EViews LogL object

```
'
' restricted version of
' bi-variate BEKK of Engle and Kroner (1995):
'
' y = mu + res
' res ~ N(0,H)
'
' H = omega*omega' + beta H(-1) beta' + alpha res(-1) res(-1)' alpha'
'
' where
'
' y = 2 x 1
' mu = 2 x 1
' H = 2 x 2 (symmetric)
' H(1,1) = variance of y1 (saved as var_y1)
' H(1,2) = cov of y1 and y2 (saved as var_y2)
' H(2,2) = variance of y2 (saved as cov_y1y2)
' omega = 2 x 2 low triangular
' beta = 2 x 2 diagonal
' alpha = 2 x 2 diagonal
'
' sample file execution time ~1/2 min on Pentium II 333
'
'change path to program path
' load workfile
load djaigdj10.wf1
```

```
' dependent variables of both series must be continues
smpl @all
series y1 = dlog(aig)

series y2 = dlog(dj10)

' set sample
' first observation of s1 need to be one or two periods before
' the first observation of s0
sample s0 2 93

sample s1 3 93

' initialization of parameters and starting values
' change below only to change the specification of model
smpl s0

'get starting values from univariate GARCH
equation eq1.arch(m=3,c=1e-6) y1 c

equation eq2.arch(m=3,c=1e-6) y2 c

' declare coef vectors to use in bi-variate GARCH model
' see above for details
coef(2) mu

mu(1) = eq1.c(1)

mu(2)= eq2.c(1)

coef(3) omega

omega(1)=(eq1.c(2))^.5

omega(2)=0

omega(3)=eq2.c(2)^.5

coef(2) alpha

alpha(1) = (eq1.c(3))^.5

alpha(2) = (eq2.c(3))^.5

coef(2) beta
```

```
beta(1)= (eq1.c(4))^.5

beta(2)= (eq2.c(4))^.5


' constant adjustment for log likelihood
!mlog2pi = 2*log(2*@acos(-1))


' use var-cov of sample in "s1" as starting value of variance-covariance matrix
series cov_y1y2 = @cov(y1-mu(1), y2-mu(2))

series var_y1 = @var(y1)

series var_y2 = @var(y2)


series sqres1 = (y1-mu(1))^2

series sqres2 = (y2-mu(2))^2

series res1res2 = (y1-mu(1))*(y2-mu(2))




' ..........................................................
' LOG LIKELIHOOD
' set up the likelihood
' 1) open a new blank likelihood object (L.O.) name bvgarch
' 2) specify the log likelihood model by append
' ..........................................................
logl bvgarch

bvgarch.append @logl logl

bvgarch.append sqres1 = (y1-mu(1))^2

bvgarch.append sqres2 = (y2-mu(2))^2

bvgarch.append res1res2 = (y1-mu(1))*(y2-mu(2))


' calculate the variance and covariance series
bvgarch.append var_y1 = omega(1)^2 + beta(1)^2*var_y1(-1) + alpha(1)^2*sqres1(-1)

bvgarch.append var_y2 = omega(3)^2+omega(2)^2 + beta(2)^2*var_y2(-1) + alpha(2)^2*sqres2(-1)

bvgarch.append cov_y1y2 = omega(1)*omega(2) + beta(2)*beta(1)*cov_y1y2(-1) +
alpha(2)*alpha(1)*res1res2(-1)


' determinant of the variance-covariance matrix
bvgarch.append deth = var_y1*var_y2 - cov_y1y2^2


' inverse elements of the variance-covariance matrix
bvgarch.append invh1 = var_y2/deth
```

```
bvgarch.append invh3 = var_y1/deth

bvgarch.append invh2 = -cov_y1y2/deth


' log-likelihood series
bvgarch.append logl =-0.5*(!mlog2pi + (invh1*sqres1+2*invh2*res1res2+invh3*sqres2) + log(deth))


' remove some of the intermediary series
' bvgarch.append @temp invh1 invh2 invh3 sqres1 sqres2 res1res2 deth

' estimate the model
smpl s1

bvgarch.ml(showopts, m=3, c=1e-7)


' change below to display different output
show bvgarch.output


graph varcov.line var_y1 var_y2 cov_y1y2

show varcov


' LR statistic for univariate versus bivariate model
scalar lr = -2*( eq1.@logl + eq2.@logl - bvgarch.@logl )

scalar lr_pval = 1 - @cchisq(lr,1)

series corr_y1y2 = cov_y1y2/@sqrt(var_y1*var_y2)
```