



# **Volatility Based Sentiment Indicators for Timing the Markets**

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**Master Thesis of Finance**

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**Abstract:** VIX, published by the Chicago Board Options Exchange, is a well known implied volatility estimator. In this paper we assess its capability to be used as a sentiment indicator, and to give signals for a short term investment strategy. It will be proved and discussed how VIX-based strategies – also known as “Contrarian” strategies – can be effective as they lead to higher returns than the market. We also propose a purer sentiment indicator derived from VIX that gives more accurate market timing signals. We call this indicator “Net Emotional Volatility Index” (NEVI). It proves to have interesting properties, a highly significant statistical relationship with the market return, and a considerable power to time the market. The results of our back-testing for the period 2001-2002 and 2006-2007 using the two indicators are presented, compared and discussed. Possible explanations of the information that NEVI brings and why its signals work are provided.

*Key words: VIX, GARCH, market timing signals, sentiment, volatility*

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## 1. Introduction

Volatility is a major factor in the financial markets and volatility forecasts are used every day in the process of portfolio choice, risk management, hedging and pricing.

The goal of this paper is to assess the capability of volatility based indicators to measure investors' sentiment, to time the markets and to give signals for a short term investment strategy. First, we address this problem by studying the signaling quality of the famous "Contrarian" indicator VIX (Implied Volatility Index). Second, we propose a completely new volatility based indicator, NEVI (Net Emotional Volatility Index), defined as the difference between VIX and a GARCH (1,1) forecast over the same time horizon, which according to us has a higher signaling power in a certain market environment.

The market timing power of the two indicators is examined both with the use of regressions and with the help of out-of-sample investment strategies back-testing based on their signals. In the case of VIX, we utilize a "Contrarian" strategy, where we take positions contrary to the general market sentiment. We use NEVI, instead, in what we called a "Sentiment Momentum Strategy", where we follow the people's expectations. The usefulness of both indicators for predicting the market direction is tested in two historical periods. The first one is 2001-2002, when the market was comparatively more volatile, and the second one is 2006-2007, when the market was less volatile. This gives us a better idea of the behavior of the two indicators.

The work is organized as follows. In the first part we briefly present the "Contrarian" investment philosophy and introduce VIX, giving an overview of its characteristics and properties. In the second part we introduce and define the NEVI indicator, arguing why it might be a purer sentiment measure and a better market timing indicator. In the third part we present the data series that we work with, and conduct a regression analysis for both VIX and EVI, proving that one is a "Contrarian" indicator, while the other is a "Momentum" indicator. In the next part of the work we explain the investment strategies based on the two indicators and present the results. Finally we give two explanations why the Net Emotional Volatility Indicator NEVI works.

## **2. Contrarian Market Sentiment Indicator - Implied Volatility Index**

### **2.1. Contrarian Investing**

“Contrarian” investing is an investment philosophy that is closely related to the field of behavioral finance. It is followed by those investors who believe that the crowd behavior is typical for the market participants and that it often leads to the mispricing of the traded assets. Some “Contrarian” strategies have gained a considerable popularity among agents as they are widely believed to be able to time when the markets are going to change directions. The basis of any “Contrarian” strategy is to recognize when the majority of the market participants have become too extreme in their optimism or fear. At such occasion the “Contrarian” investors normally open positions against the general market trend, as they believe that market reversal will follow.

Taking the right side at the top or the bottom of the market leads to high returns but determining whether the market is excessively bullish or bearish is not an easy task. There exist a large number of market characteristics that are thought to be signs of overbought or oversold markets. For example the extremely low values of the Put-Call Ratio (indicator derived by dividing the number of the traded put options to the number of call options) can be interpreted as an overbought market, while the high values of the indicator are usually perceived as an alarm that the market is oversold. The fully invested portfolios of the mutual funds can also be used as an indication that the market might have surpassed its potential and that a drop in prices is to follow. In such cases the good news coming to the market do not have a big effect since all the money are already invested, while the bad news can cause a sale. It can turn out very profitable for every investor to be able to measure the market sentiment and use this as a weapon in his or her trading arsenal.

### **2.2. Implied Volatility Index (VIX)**

VIX is an index that measures the implied volatility conveyed by the option prices. When the option prices and all other option pricing factors, but volatility, are

known, the implied volatility is derived with a reverse calculation using a mathematical model. VIX was introduced in 1993 by the CBOE and originally it was based on the Black-Scholes option pricing model, considering only at-the-money put and call options on the S&P 100 index with a residual life of 30 days. Soon after its introduction in 1993, the VIX was widely accepted as a benchmark for stock market volatility.

In September 2003, the CBOE changed the formula it uses to calculate the VIX, in an attempt to come up with a more accurate gauge of the expected market volatility. The new VIX is not any longer based on the Black-Scholes OPM, but uses a newly developed formula<sup>1</sup>:

$$\sigma^2 = \frac{2}{T} \sum_t \frac{\Delta K_t}{K_t^2} e^{RT} Q(K_t) - \frac{1}{T} \left[ \frac{F}{K_o} - 1 \right]^2$$

Where,

$\sigma$  is  $\frac{VIX}{100}$ , meaning that  $VIX = \sigma \times 100$

**T** is Time to expiration. Precisely:

$$T = \{M_{\text{Current day}} + M_{\text{Settlement day}} + M_{\text{Other days}}\} / \text{Minutes in a year}$$

Where  $M_{\text{Current day}}$  = # of minutes remaining until midnight of the current day

$M_{\text{Settlement day}}$  = # of minutes from midnight until 8:30 a.m. on settlement day

$M_{\text{Other days}}$  = Total # of minutes in the days between current day and settlement day.

**F** is the Forward index level derived from at-the-money index option prices<sup>2</sup>

$K_i$  is the Strike price of  $i^{\text{th}}$  out-of-the-money option; a call if  $K_i > F$  and a put if

$K_i < F$

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<sup>1</sup> Details on the formula and relevant calculations are available at:

<http://www.cboe.com/micro/vix/vixwhite.pdf>

<sup>2</sup>  $F = \text{Strike Price} + e^{RT} * (\text{Call Price} - \text{Put Price})$ . The at-the-money Strike Price is the strike price at which the difference between the call and put prices is smallest.

$\Delta K_i$  is the Interval between strike prices – half the distance between the strike on either side of  $K_i$ :  $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$ . ( $\Delta K$  for the lowest strike is simply the difference between the lowest strike and the next higher strike, Likewise,  $\Delta K$  for the highest strike is the difference between the highest strike and the next lower strike.)

$K_0$  - First strike below the forward index level, F

$R$  – Risk-free interest rate of expiration

$Q(K_i)$  – The midpoint of the bid-ask spread for each option with strike  $K_i$ .

The new VIX incorporates data from prices of options with a wide variety of strikes, capturing the whole volatility skewness. It is based on S&P 500 index, instead of the S&P 100. The old VIX is now called VXO. It continues to be estimated only with the help of at-the-money options on the S&P 100 index. The new VIX has been re-calculated from 1993 in order to insure data homogeneity. The changes are not insignificant but the basic idea of the volatility index remains the same. It is a measure of the expected market volatility over a 30 day time horizon.

There are many papers focusing on volatility forecasting methods, where VIX is considered to be the main expression of what is defined as implicit volatility, and its capability to forecast realized volatility is compared to that of other methods such as historical based volatility and Model Based Forecast (especially GARCH). Results are different and often contradictory.

Blair, Poon, and Taylor (2001)<sup>3</sup> compare historical methods with VIX and find that not much incremental information is provided by the former in comparison with the

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<sup>3</sup> Blair, B.J., S. Poon, and S.J. Taylor, 2001, Forecasting S&P100 volatility: the incremental information content of implied volatilities and high-frequency index returns, *Journal of Econometrics* 105, 5–26.

latter. Martens and Zein (2002)<sup>4</sup> conclude that implied volatility provides superior volatility forecasts compared to GARCH models.

Giot (2002)<sup>5</sup> strongly concludes in favor of implied volatility as represented by VIX: *“In the out-of-sample study and while (almost) no volatility forecast is both unbiased and efficient, we find that VIX/VXN based volatility forecasts are the closest to being unbiased, have the highest information content (with a statistical fit that increases with the time horizon)”... “ Moreover, the in-sample study shows that GARCH-type forecasts have little incremental information over that contained in the implied volatility indexes”... “Moreover, they compete very favorably with volatility forecasts based on historical returns and are simple to use, both in a volatility forecasting and market risk evaluation framework.”*

Other authors have also highlighted the predictive power of VIX but also point out that other information can improve forecasting results. Becker, Clements, White (2005)<sup>6</sup> conclude *“It is shown, as in previous research, that there is a significant positive correlation between the VIX index and future volatility. In contrast to much of the previous literature, it has been demonstrated that the VIX is not an efficient volatility forecast in the sense that other available information can improve upon the VIX as a volatility forecast. In previous work, it has been shown that implied volatility often dominates other, model-based forecasts of volatility. The results presented here do not contradict such a finding; they merely suggest that an improved forecasting model could potentially be found”*.

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<sup>4</sup> Martens, M., and J. Zein, 2002, Predicting financial volatility: high-frequency time-series forecasts vis-a-vis implied volatility, Mimeo, Erasmus University Rotterdam.

<sup>5</sup> P. Giot, December 13, 2002, The information content of implied volatility indexes for forecasting volatility and market risk. Department of Business Administration & CEREFIM at University of Namur, Belgium

<sup>6</sup> R. Becker, A.E. Clements, S.I. White - On the informational efficiency of S&P500 implied volatility – The North American Journal of Economics and Finance - 1<sup>st</sup> December 2005

All these papers are focused on the predictive power of the indicator and are concerned with the efficiency and unbiasedness of the volatility estimator. For our work we will change the focus towards the research of a reliable indicator for timing the market and for implementing a short term portfolio investment strategy.

Many investors tend to view VIX just as a tool for measuring the market fear. During times of financial distress and general market insecurity, when stock prices usually decline, VIX is normally expected to rise. The greater the market risks, the higher the index would rise. Such behavior of VIX is clearly due to the riskier conditions in the market, since volatility is the most common measure of risk. It is amplified by the hedging actions of the market participants, which include the use of options. Such demand for options leads to an increase of their premiums and so VIX rises.

Many “Contrarian” investors believe that the bottoms of the VIX value translate into tops in the market, and the tops of the VIX index coincide with the bottoms in the market. It is widely believed among them that when the implied volatility is extremely high the market is shaking in fear and it is a good moment to buy, while when the implied volatility is too low the agents are too optimistic and it is a good moment to sell.

When the investors’ reaction to a particular bad event is too strong due to excess fear, the market will drop more than reasonable. However, with the time, market participants will assess the market conditions more carefully and the mispricing will be corrected. On the other hand, if the majority of investors are overoptimistic and do not react to a really bad event, or overprice the effect of good news, later they will realize their mistake and the market will go down as a result.

### **3.0. NEVI (Net Emotional Volatility Index)**

The investors that blindly follow the VIX signals are likely to make mistakes from time to time. By taking long positions when VIX has extremely high values for example, they take it for granted that the majority of the investors overreact and that the market will have a correction soon. However the only reason to believe that markets are overreacting is that VIX climbs high above/falls low below the average values of the



indicator. In our opinion extreme values of the implied volatility can be a result of both the behavior of the investors or the properties of the volatility itself, so that not all VIX signals might be accurate sentiment alarms, while at the same time VIX might miss some good market opportunities. We can neither be sure when the investors are overreacting and if their fears are unreasonable or rational, nor be sure how long it will take them to realize and correct their irrational behavior. However we can try to estimate how big their fear or optimism actually is.

We argue that high VIX does not automatically translate into (unreasonably) high fear and for that reason we propose a new and more reliable sentiment market timing indicator based on volatility forecasts. We created an indicator that we called Net Emotional Volatility Index (NEVI). This indicator is a proxy for the sentiment of the investors. It is calculated as the difference between the market based implied volatility VIX and the model-based forecast over the same period ((GARCH (1,1) forecast)). We argue that this indicator is a purer sentiment gauge than VIX alone is.

GARCH (1,1) (Generalized Autoregressive Conditionally Heteroscedastic) model is an econometric tool for modeling time-varying conditional variances. It is acknowledged for being able to account for volatility clustering and excess kurtosis, and is widely recognized for providing accurate forecast of return volatility. Usually, it is found that a GARCH (1,1) model is sufficient to capture the volatility dynamics<sup>7</sup>, so that only one lagged squared error and one lagged variance is needed:

$$\sigma_t^2 = \omega + \alpha\eta_{t-1}^2 + \beta\sigma_{t-1}^2$$

Considering the widely recognized properties of GARCH (1,1), and after obtaining a positive result with the test for ARCH/GARCH effect on the S&P 500 sample, we decided to use this model in our research.

As good as GARCH (1,1) is, it suffers from several limitations. Since the GARCH models are conditional and autoregressive, the forecasts are dependent on the observations of the immediate past, and past observations are incorporated into the present. As any econometric model, the GARCH forecast follows a mathematical pattern.

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<sup>7</sup> See "Empirical Finance Lecture Notes Part 1", Fall 2007, by Hossein Asgharian, Associate Professor at Lund University, Sweden.

The parametric characteristic of the model leads to a better performance when the market conditions are comparatively stable. To forecast volatility for the next 30 days, for example, GARCH models reiterate the calculation and tomorrow's forecast becomes the last observation in the time series to forecast volatility for the day after tomorrow, and so on. Since the forecasts are smoothed by the past observations, the model needs a certain time to account for the unanticipated volatility jumps and when a shock is registered it takes a certain number of observations for the model to capture the change.

Implied volatility on the other hand includes information known by the market agents about the historical behavior of the volatility but it includes also the sentiment of the option traders with regards to the development of the volatility in the near future. Therefore the VIX value incorporates the volatility forecast based on the historical volatility pattern, as well as the volatility expectations of the investors, resulting from their emotional reaction to the current market environment. In our opinion the GARCH (1,1) forecast can be considered as a proxy for the volatility that the option traders expect based on the historical behavior of the market. The implied volatility is obtained by adding the emotional reactions and the expectations of the investors, based on the current market events and phenomena, to the model based forecast. Our theory finds support in the recent work of Becker, Clements and McClelland (BCM) (2008)<sup>8</sup>, who argue that as a market determined forecast, VIX can reflect information that cannot be captured by a model based forecast. In addition they also show that VIX have incremental information in comparison to a Model Based Forecast (MBF), when it comes to explaining future jump activity. This is the part of volatility that we call emotional. Our intuition is that a good proxy for the emotional volatility is the difference between the market determined volatility forecast VIX and a model based volatility reiterated forecast as presented by a GARCH (1,1) model.

Daily VIX fixing takes into account the historical volatility trend, the volatility clustering effect, and the everyday news and events that can affect the emotions and expectations of the option traders as well. Conversely a model based forecast, like

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<sup>8</sup> Ralf Becker, Adam Clements and Andrew McClelland; "The Jump Component of S&P 500 volatility and the VIX index", March 2008

GARCH (1,1), can capture the first two components but not the latter (at least not immediately). Corporate news and economic indicators, political changes and technological innovations, natural disasters and terrorist attacks instantly affect the attitude of the investors (optimism, pessimism) and as a result the implied volatility (VIX) moves accordingly. Market events that are expected to happen or are happening suddenly at the moment can change the perception of the investors about the recent or more distant price changes, as well as their expectations about the future. On the contrary a model based forecast, is much more dependent and smoothed by the past, and as a result the forecast reacts relatively slower to expected jumps in the market. The reason for that is that the coefficients of the model are estimated on the basis of many years of observations. To obtain a forecast for tomorrow these coefficients are applied to today's volatility and to today's difference between the market actual return and the return expressed by a mathematical model. Therefore, given a shock that is able to affect the S&P 500, the change in the S&P volatility forecast for tomorrow will be transmitted through today's change in the return, as error with respect to a mathematical model, and through today's registered volatility. Both components will be weighted by  $\beta$ ,  $\alpha$ , and will also stand the effect of the coefficient  $\omega$ . We could say that shock impacts are "filtered" and such filtering becomes stronger when we use GARCH models with reiteration of forecasts (when we use tomorrow's forecast as a base for the forecast for the day after tomorrow and so on, like what we do in the following section)). The final output of a 30 days GARCH (1,1) reiterated forecast is surely less affected by today's shocks, news, and events and by the relevant emotional reactions of the market agents, than what could be the volatility implicitly assumed by the options traders. The latter, and so VIX, is certainly more directly responding to the above mentioned factors since shocks' effects are not "filtered" to obtain the forecast. Given that the other variables involved in the option pricing model are not changed we just plug the new market price in the formula and through a reverse calculation we get the implicit volatility. This insures a much faster reactions of the implied volatility to the market shocks.

For our purposes, this argument - that could be a drawback of GARCH models in other researches - could actually become a useful advantage. The difference between the

two volatility values can be a good proxy of the reactions of the market agents to today's events, news, and shocks. This is what we call emotional volatility.

According to our arguments above, VIX can be decomposed into two parts – rational market volatility and emotional volatility. The emotional volatility derived from VIX is a purer sentiment indicator than VIX itself is, and it can probably be used more successfully in an investment strategy. To our best knowledge, no one has ever discussed in the literature before the indicator that we created and named Net Emotional Volatility Index (NEVI).

#### 4. Data Set and Regression Analysis

We have gathered daily closing values of the VIX for the period January 1997 – December 2007. We set two estimation windows, 1997-2000 and 1997-2005, to measure the statistic properties of the indicator, and two event windows, 2001-2002 and 2006-2007 respectively, which are used in the following sections for a back-testing analysis of the portfolio strategies based on VIX.

The resulting statistical properties for VIX in the two estimation windows are presented below in Table 1.

Table 1. Statistical Properties of VIX for the period 1997-2000 and 1997-2005

| Deciles |       | Period 1997-2000   |        | Deciles |       | Period 1997-2005   |        |
|---------|-------|--------------------|--------|---------|-------|--------------------|--------|
| 0.1     | 19.23 | Mean               | 23.91  | 0.1     | 13.57 | Mean               | 22.099 |
| 0.2     | 20.1  | Median             | 22.94  | 0.2     | 16.54 | Median             | 21.5   |
| 0.3     | 21.04 | Standard Deviation | 4.692  | 0.3     | 19.02 | Standard Deviation | 6.463  |
| 0.4     | 21.9  | Sample Variance    | 22.015 | 0.4     | 20.22 | Sample Variance    | 41.779 |
| 0.5     | 22.91 | Kurtosis           | 3.389  | 0.5     | 21.5  | Kurtosis           | 0.500  |
| 0.6     | 24.03 | Skewness           | 1.564  | 0.6     | 22.85 | Skewness           | 0.629  |
| 0.7     | 25.08 | Range              | 29.51  | 0.7     | 24.52 | Range              | 35.51  |
| 0.8     | 27.01 | Minimum            | 16.23  | 0.8     | 27.11 | Minimum            | 10.23  |
| 0.9     | 29.48 | Maximum            | 45.74  | 0.9     | 30.78 | Maximum            | 45.74  |

Afterwards, we addressed the empirical estimation of the above described NEVI. We proceeded subtracting the average of GARCH (1,1) forecasts for the next 30 days from each value of the VIX times series. Precisely, we built a GARCH (1,1) model starting from the observations of S&P 500 in two estimation windows, 1997-2007 and 1997-2002. Then we used these models to forecast volatility for the periods 2001-2002 and 2006-2007. It is worth noting that just for the GARCH coefficients we used enlarged estimation windows and therefore we output an “in sample” forecast. We believe that this is the best solution for our back-testing purposes, since an “out of sample” forecasting, that normally sounds more rigorous, in this case would cause a bigger approximation, making simulations more distant from reality. In reality, in fact, every agent can daily recalculate and update his/her GARCH Model. An “out of sample” forecasting, instead, would create a simulation in which the agents cannot update their GARCH models. The coefficients are presented in Table 2 below.

Table 2: GARCH (1,1) Coefficients

| Garch Coefficients | $\omega$  | $\alpha$ | $\beta$  |
|--------------------|-----------|----------|----------|
| 1997-2007          | 0,0000012 | 0,070164 | 0,922884 |
| 1997-2002          | 0,0000093 | 0,098557 | 0,851944 |

Once the GARCH coefficients are obtained, for each day we forecast the volatility for the next 30 days, reiterating the forecast, which means that every forecast becomes the last observation for the next forecast. Finally we take the average of these values. We call this Average GARCH Forecast (AGF).

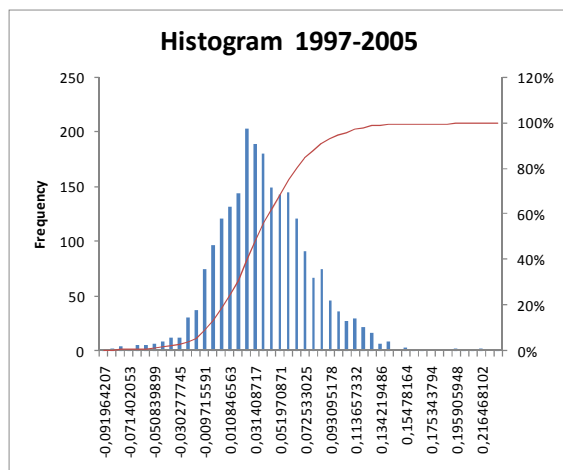
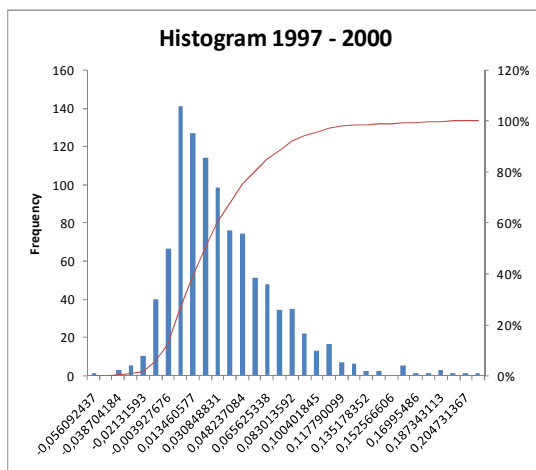
The difference between VIX and AGF is what we already called Net Emotional Volatility Index (NEVI). It is normally positive, with a positive mean and median value. Below in table 3 are presented the Histogram, the Deciles distribution and other statistical properties for NEVI in two estimation windows 1997-2000 and 1997-2005 <sup>9</sup>:

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<sup>9</sup> Estimation windows now are 2 years shorter than the ones used to estimate GARCH coefficients, therefore exactly equal to the ones used for VIX. Deciles and other NEVI statistical properties measured in such estimation windows will be used in the following sections for back-testing analysis of NEVI based portfolio strategies in the event windows, 2001-2002 and 2006-2007.

Table 3. Statistical Data for NEVI

| Deciles |        | Period 1997-2005   |        | Deciles |        | Period 1997-2000   |        |
|---------|--------|--------------------|--------|---------|--------|--------------------|--------|
| 0.1     | -0,76% | Mean               | 0.0361 | 0.1     | -0,64% | Mean               | 0.0297 |
| 0.2     | 0,55%  | Median             | 0.0333 | 0.2     | 0,08%  | Median             | 0.0216 |
| 0.3     | 1,71%  | Standard Deviation | 0.0376 | 0.3     | 0,75%  | Standard Deviation | 0.0362 |
| 0.4     | 2,48%  | Sample Variance    | 0.0014 | 0.4     | 1,43%  | Sample Variance    | 0.0013 |
| 0.5     | 3,33%  | Kurtosis           | 1.3161 | 0.5     | 2,16%  | Kurtosis           | 2.5768 |
| 0.6     | 4,21%  | Skewness           | 0.4047 | 0.6     | 3,06%  | Skewness           | 1.2633 |
| 0.7     | 5,29%  | Range              | 0.3221 | 0.7     | 4,20%  | Range              | 0.2695 |
| 0.8     | 6,51%  | Minimum            | -0.092 | 0.8     | 5,56%  | Minimum            | -0.056 |
| 0.9     | 8,37%  | Maximum            | 0.2302 | 0.9     | 7,87%  | Maximum            | 0.213  |



The fact that NEVI is normally positive and that the mean and the median of its distribution are also positive could be considered as a proof in favor of the literature that considers VIX as a provider of incremental information in comparison to other forecasting method.

In the Appendix (Pictures 5 and 6) we plotted NEVI time series together with S&P 500 returns. A brief look at those graphs supports the idea that a relationship between NEVI and S&P 500 returns is likely to exist, and that this relationship should be generally positive in the long run (High NEVI is followed by S&P in the long term).

Can we actually consider the NEVI index as a proxy for market fear/confidence? We deepen our argument with an analysis of the correlation coefficients that we find in Table 4.

Table 4. Correlation table (sample 1/1997-12/2007)

|                    | VIX         | Log ret<br>SP 1<br>day | Log<br>Ret SP<br>5 days | Log<br>Ret<br>SP 10<br>days | Log<br>Ret SP<br>20<br>days | Log<br>Ret SP<br>30<br>days | NEVI |
|--------------------|-------------|------------------------|-------------------------|-----------------------------|-----------------------------|-----------------------------|------|
| VIX                | 1           |                        |                         |                             |                             |                             |      |
| Log ret SP 1 day   | 0,04        | 1                      |                         |                             |                             |                             |      |
| Log Ret SP 5 days  | <b>0,08</b> | -0,06                  | 1                       |                             |                             |                             |      |
| Log Ret SP 10 days | <b>0,09</b> | 0,31                   | 0,70                    | 1                           |                             |                             |      |
| Log Ret SP 20 days | <b>0,12</b> | 0,23                   | 0,50                    | 0,68                        | 1                           |                             |      |
| Log Ret SP 30 days | <b>0,14</b> | 0,19                   | 0,39                    | 0,53                        | 0,80                        | 1                           |      |
| NEVI               | 0,47        | <b>-0,20</b>           | <b>-0,10</b>            | <b>-0,14</b>                | <b>-0,05</b>                | <b>0,03</b>                 | 1    |

From the analysis of the table we see that the sign of the correlation coefficients between S&P 500 returns and VIX are different from the signs of the correlations coefficients between S&P 500 returns and NEVI. It is interesting to note that in the latter case, the sign of the coefficient changes if we move from the 1, 5,10, 20 days return to the 30 days return. So it looks like for high value of NEVI the market should be expected to decrease in the short run while in a little longer term it appears to recover. If these relationships could be considered reliable, in a short term strategy – within a month time horizon - high NEVI values should trigger the opening of a short position while low NEVI values should trigger the opening of a long position. This shows that agents tend to overreact, but it takes some time before a correction follows.

We can check the reliability of these hypothesized relationships by running some regressions between the S&P 1, 5, 20 and 30 days returns using NEVI and VIX as independent variables.

Table 5:Regression Coefficients Table (1/1997- 12/2007)

| Regression coefficients table |                       |        |          |         |
|-------------------------------|-----------------------|--------|----------|---------|
| Dependent Variables           | Independent Variables |        |          |         |
|                               | NEVI                  | cost.  | VIX      | cost    |
| S&P 1 day Return              | -0,0587               | 0,002  | 7,27E-05 | -0,0013 |
| Prob                          | 0,0000                | 0,0000 | 0,0234   | 0,0612  |
| R squared                     | 0,0388                |        | 0,0018   |         |
| S&P 5 days Return             | -0,0636               | 0,0003 | 0,0002   | -0,0047 |
| Prob                          | 0,0000                | 0,0000 | 0,0015   | 0,0000  |
| R squared                     | 0,0095                |        | 0,0058   |         |
| S&P 10 days Return            | -0,1178               | 0,0055 | 0,0004   | -0,0072 |
| Prob                          | 0,0000                | 0,0000 | 0,0000   | 0,0001  |
| R squared                     | 0,0202                |        | 0,0089   |         |
| S&P 20 days Return            | -0,0646               | 0,0056 | 0,0007   | -0,0126 |
| Prob                          | 0,0038                | 0,0000 | 0,0000   | 0,0000  |
| R squared                     | 0,0029                |        | 0,0142   |         |
| S&P 30 days Return            | 0,0372                | 0,0044 | 0,0010   | -0,0172 |
| Prob                          | 0,1701                | 0,0009 | 0,0000   | 0,0000  |
| R squared                     | 0,0006                |        | 0,0191   |         |

We can immediately note that all NEVI regressions are very significant (at 1% level or more), with an exception only in the case of S&P 30 days return, where we have a significance threshold at 17%. R squared levels are at a relatively low level, but in line with those found by similar regression on S&P returns in the available literature. What is very important to highlight is that the table above confirms the existence of a relationship between the S&P 500 returns and NEVI that changes sign when we go from the very short term (1, 5, 20 days) to the 30 days time horizon. Conversely such a change in the sign of the relationship is not observable when we use VIX as a regressor. In that case in fact, the sign is always positive and the size of the coefficient is very small. The significance although remains very high. This is consistent with the wide market belief that VIX is “Contrarian” indicator and that it is good to buy when VIX is high, and sell



when it is low. NEVI on the other hand is confirmed to be a “Momentum” indicator in the short term (up to 30 days), so high NEVI values should trigger the opening of short positions while low NEVI values should trigger the opening of long positions.

In general, the regressions confirm that NEVI conveys different information than VIX does, and that this information could be exploited for setting a successful portfolio strategy and for realizing effective short term market timing.

## **5.0. Comparing VIX and NEVI Investment Strategies**

### **5.1. Contrarian Strategy Based on VIX**

The back-testing of a “Contrarian” strategy that follows the signals of VIX turned out to give returns above the returns of a simple Buy-and-Hold the S&P 500 index strategy. The rules of our strategies are very simple. We only trade the S&P 500 index. We take long positions when the indicator climbs high and we open short positions when it reaches low values. To set the thresholds that trigger buying and selling we use the deciles registered during the estimation windows, 1997-2000 and 1997-2005. The back-testing is conducted for the periods 2001-2002 and 2006-2007. In the first strategy the positions are always closed within a fixed time horizon of 30 days and there is a restriction by which we can open only one short position and one long position at a time. In the second strategy we change the constrain. Instead of “one position at a time”, the signals to open a position become effective only if in the prior 5 days there was no other signal. This way we can avoid multiple signals due to the same market shock. In the third strategy, instead of a fixed holding period, we have an exit signal when the indicator moves back and gets closer to its median value. Again in this strategy we follow the constrain of “one position at a time”.

The first strategy respects the same rules that are set by Simon and Wiggins<sup>10</sup> in their study and the results obtained are similar. In establish the rules for the second and the third strategy, instead, we do not follow any prior study and simply try to improve the first one.

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<sup>10</sup> David Simon, Roy Wiggins, “S&P Futures Returns and Contrary Sentiment Indicators”, 2001

All signals (both for opening and closing) are registered at the market close and buying and selling orders are placed to be executed when the market session opens on the following day. Transaction costs are set to be equal to 0.1% both for buying and selling. In addition to that, we do not use our own money to trade. We assume that we start with no cash but just with a credit line. When we have long position we borrow and we pay 4% annual interest rate for the number of days we hold that position. When we open a short position instead, we get 2% interest for the holding period. To be more accurate in measuring the performance we assume that the above mentioned interest rates are always paid or gained on the cash position balance (so, for example, if we do not have opened positions, we carry the profit or the loss of the previous position and we pay or gain interest on that). At the end of the two year back testing, we measure the strategy performance considering the ratio between the final cash position, after deducting net interests, and the used capital (borrowing from credit line), weighted by the relevant number of days.

Following the above mentioned rules, we trade on the VIX signals and we open long positions when the indicator goes above the 8<sup>th</sup> deciles, and open short positions when it falls below the 2<sup>nd</sup> deciles. The results of the investment strategy are presented in Table number 6 below.

**Table 6. VIX Contrarian Strategies Results**

| Strategy | Period                  | Rules  | Number win/total long positions | Number win/total short positions | Annualized Excess return | Quasi Sharpe Ratio |
|----------|-------------------------|--|---------------------------------|----------------------------------|--------------------------|--------------------|
| 1        | 12/30/2005 – 12/31/2007 | Positions opened on the following day, 1 position at a time, 30 days holding period                  | 2/2; 100%                       | 4/11; 36%                        | -12,08%                  | -2,48              |
| 1        | 01/02/2001 – 12/30/2002 | Positions opened on the following day, 1 position at a time, 30 days holding period                  | 6/10; 60%                       | 3/3; 100%                        | 17,04%                   | 1,26               |
| 2        | 12/30/2005 – 12/31/2007 | Positions opened on the following day, not more than 1 position every 5 days, 30 days holding period | 3/4; 75%                        | 2/5; 40%                         | 13,97%                   | 3,21               |

|   |                            |  |           |           |        |      |
|---|----------------------------|--|-----------|-----------|--------|------|
| 2 | 01/02/2001 –<br>12/30/2002 | Positions opened on the following day, not more than 1 position every 5 days, 30 days holding period | 3/6; 50%  | 3/3; 100% | 5,16%  | 0,47 |
| 3 | 12/30/2005 –<br>12/31/2007 | Positions opened on the following day, signal to close when VIX moves back to the median             | 3/3; 100% | 1/3; 33%  | 21,40% | 3,79 |
| 3 | 01/02/2001 –<br>12/30/2002 | Positions opened on the following day, signal to close when VIX moves back to the median             | 3/3; 100% | 3/3; 100% | 23,42% | 4,43 |

In the first and second columns are specified the strategy and period for which it is tested. The third column contains the strategy rules, while the fourth and the fifth column present the number of profitable long and short positions, compared to the total number of long and short trades. The annualized excess return (the return above the interest rate that we set at 4% level) is given in the sixth column, and the seventh column displays a Quasi Sharpe ratio for the investment strategy.

To make a comparison, the simple strategy of holding the S&P 500 index in the period 2001-2002 produced an average annualized return - in excess of the 4% interest rate - negative and equal to - 21.26%. For the period 2006-2007, the same simple strategy resulted in an average annualized excess return positive and equal to 3.30%

In the first strategy – where we do not use any signal to close the positions and only hold them for 30 days, respecting the constrain of opening one position at the time – for the period 12/30/2005 – 12/31/2007 we open 13 positions, getting a profit from 6 of them. Only 4 of the 11 signals for short selling are correct. As a result, the annualized return for the 2 years is negative -12.08%. For the period 01/02/2001 – 12/30/2002 however, the excess return is 17.04%, and 9 of 13 market timing signals are accurate.

In order to refine the strategy we introduce a constrain by which a signal to open a position becomes effective only if in the prior 5 days there was no other signal. This way we can avoid that the indicator passes the threshold multiple times due to the same

market news, triggering multiple signals. Open positions are still kept for 30 days. For the period 12/30/2005 – 12/31/2007 the change brings an improvement to the result. Out of 9 signals 5 are correct. The annualized excess return is considerably higher, 13.97%. Conversely, for the period 01/02/2001 – 12/30/2002 the strategy performs worse, and the final result is an excess return of 5.16%.

Addressing the third strategy, the major change is that we define a signal for closing the positions, instead of simply holding them for 30 days. The closing signal we chose is quite consistent with the opening signal. It is triggered when VIX comes back closer to its median value. To avoid duplication of signals we use also the “one position at a time constrain”. For the period 12/30/2005 – 12/31/2007 this strategy leads to an annual excess return of 21.40%. Out of the total 6 signals, 4 are correct. For the period 01/02/2001 – 12/30/2002, the annual excess return is 23.42%, and all 6 signals are accurate.

According to the results of these 3 strategies, VIX is truly a “Contrarian” indicator. Simon and Wiggis reached the same conclusion in their already mentioned study. All the strategies lead to better results in terms of percentage return, than simply the strategy buying and holding the S&P 500 index. In order to check if the greater profitability is not simply due to the higher risk involved, we have estimated a Quasi-Sharpe Ratio for each strategy. The Quasi-Sharpe Ratio is the excess return of each strategy (as above defined) divided by the annualized standard deviation involved by the strategy:

$$\text{Quasi Sharpe Ratio} = \frac{\text{Return} - \text{Interest Rate}}{\text{St.Dev. of Returns}}$$

This ratio represents therefore a “risk reward” measurement. The Quasi Sharpe ratio for the strategy of buying and holding the S&P 500 in the period 2006-2007 is equal to 0.25, and for the period 2001-2002 it is -0.89. All VIX based strategies except one express a Quasi Sharpe Ratio higher than the one registered by simply holding the S&P 500 index for the same period. Therefore, not only we get higher returns, but also a higher compensation per unit of risk. This shows that the above described VIX strategies

are not simply based on “buying volatility”, or accepting higher risks, but are really effective in managing portfolio.

The best strategy out of the three, both in terms of returns and risk (as measured by the Quasi Sharpe Ratio) is the one in which we introduced a closing signal. Another thing to notice is that overall the strategies performed better for the period 01/02/2001 – 12/30/2002, with the little exception for the second strategy. The ratio of the correct signals to the total number of signals is higher for that period. The annualized standard deviation of the S&P 500 returns for the 2001-2002 period is 23.96%, while for the 2006-2007 period it is 13,38%. Obviously, in the times of higher market turbulence the VIX signals better, which is intuitive and consistent with the belief of the Contrarians. Following the VIX signals during the Dot.com breakdown and the NYC terrorist attack resulted in a higher portfolio returns and less unprofitable trades, compared to the returns in the 2006-2007 period, which was characterized with lower volatility. VIX works better in a more dynamic environment, which is usually characterized with more emotions involved. According to Bruce Schneier<sup>11</sup>, people exaggerate spectacular but rare risk and downplay common risks, and the results of our VIX strategy are consistent with that theory.

It is worth noting that all such strategies are highly *path dependent*: It is not trivial if the market takes an upward or downward direction at the beginning of the strategy, despite that the final arrival point can be the same. The strategies will output different results in different conditions.

Pictures 1, 2, 3 and 4 in the Appendix show graphically that VIX may have a predictive power over the returns of the market. It is possible to see a pattern in which when VIX hikes to high levels it is usually followed by an increase in the returns of S&P 500, while when it drops low, the market falls as well.

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<sup>11</sup> The Psychology of Security, January 21, 2008. Bruce Schneier is a world renown security technologist and author. Among his books are “Applied Cryptography”, describing how secret codes work, “Secrets and Lies” on computer and network security, and “Beyond Fear”, studying matters such as personal safety, crime, corporate security, etc.

## 5.2. Momentum Strategy Based on NEVI

In order to test the quality of the newly created indicator NEVI, we replicate the above described three strategies, this time using NEVI instead of VIX as an indicator that triggers the buy and sell signals. We use NEVI as a “Momentum” indicator, opening long positions when the indicator goes below the 2nd deciles and short positions when it goes above the 8<sup>th</sup> deciles. In other words, instead of going against the trend, like in the “Contrarian” strategy, this time we follow it. We have a “Momentum” strategy based on the emotional expectations of the investors. The Correlations and the regression coefficients mentioned previously, in fact, show that the relationship between NEVI and the S&P returns within the 1,5,10, and 20 days horizon has a negative sign, while VIX’s relationship has a positive sign. The two indicators, therefore, have to be used in opposite ways. The results of the NEVI based strategies are presented in Table 7 below.

**Table 7. NEVI Strategies Results**

| Strategy | Period                  | Rules  | Number win/total long positions | Number win/total short positions | Annualized Excess return | Quasi Sharpe ratio |
|----------|-------------------------|--|---------------------------------|----------------------------------|--------------------------|--------------------|
| 1        | 12/30/2005 – 12/31/2007 | Positions opened on the following day, 1 position at a time, 30 days holding period                  | 9/15; 60%                       | 4/4; 100%                        | 12,65%                   | 1,46               |
| 1        | 01/02/2001 – 12/30/2002 | Positions opened on the following day, 1 position at a time, 30 days holding period                  | 3/7; 43%                        | 8/10; 80%                        | 25,35%                   | 1,93               |
| 2        | 12/30/2005 – 12/31/2007 | Positions opened on the following day, not more than 1 position every 5 days, 30 days holding period | 9/15; 60%                       | 7/8; 87.5%                       | 17,51%                   | 3,15               |
| 2        | 01/02/2001 – 12/30/2002 | Positions opened on the following day, not more than 1 position every 5 days, 30 days holding period | 4/13; 31%                       | 8/13; 62%                        | -10,66%                  | -0,84              |

|   |                          |   |  |               |            |        |      |
|---|--------------------------|---|--|---------------|------------|--------|------|
| 3 | 12/30/2005<br>12/31/2007 | – | Positions opened on the following day, signal to close when VIX moves back to the median | 10/11;<br>91% | 7/8; 87.5% | 47,52% | 4,20 |
| 3 | 01/02/2001<br>12/30/2002 | – | Positions opened on the following day, signal to close when VIX moves back to the median | 3/6; 50%      | 5/8; 62.5% | 3,87%  | 0,43 |

The back-testing of the NEVI investment strategy shows excellent results in comparison with simply holding the S&P 500 index (which yields an average annualized excess return of  $-21.26\%$  in 2001-2002 and of  $3.30\%$  in 2006-2007). It performs well even in comparison with the VIX based strategies. For the period 2006-2007 NEVI has a stronger signaling power than VIX in every version of the strategies ( $12.65\%$  vs  $-12.08\%$ ,  $17.51\%$  vs  $13.97\%$  and  $47.52\%$  vs  $21.40\%$ ). During 2001-2002 NEVI performs much better than the S&P 500 index but outperforms VIX only in the first strategy where the holding period is simply 30 days ( $25.35\%$  vs  $17.04\%$ ). As we already mentioned, the earlier period was more dynamic, with the results of the Dot.com fall and the terrorist attack shaking the confidence of the investors. VIX did better in this period probably because the market volatility spiked higher and more often, allowing the followers of the signals to profit from the mean reverting properties of volatility and the fear of the investors. On the other hand NEVI does better during 2006-2007, when the market is comparatively calm. With the more stable volatility in 2006-2007 the signals of VIX were less accurate, while our sentiment indicator, who relies solely on sentiment and not mean reversion, was able to give better signals. In 2001-2002 the performance is worse, and one explanation for that could be that GARCH 1,1 is not the best model for highly volatile markets. Finding a better mathematical based forecast during volatile periods to fit in our indicator could be a subject to further research.

NEVI based strategies prove to be efficient also when we look at the Quasi Sharpe ratios. They are all greater (also in the case when it is negative) than the equivalent ratio calculated for the strategy of simply holding the S&P 500 index. The higher profitability is not due to the higher risk.

## **6. Interpretation of NEVI Indicator**

Correlation and regression coefficients highlight with a strong level of significance that NEVI conveys information about the S&P 500 return in the very short term. Our back testing confirmed the validity of such an indicator. Although the strategies we implemented present drawbacks in terms of path dependency and lack of stop loss barriers, they are a proof that in order to maximize the returns in the short run it is useful to consider the information embedded in NEVI.

What remains open to be fully understood is the nature of the relationship from a financial point of view. We propose two different explanations why NEVI works and what information it conveys.

### **6.1. NEVI is a pure sentiment indicator**

In our work we defend the argument that VIX is not a pure sentiment indicator since it contains two major parts – the autoregressive volatility forecast, and the expectations and the emotional reactions of the option traders. Thus it is not very reasonable to consider VIX as an accurate sentiment indicator, since it does not tell us how big the fear or the emotions are. Spikes in the indicator can be due to the volatility properties itself, or due to the emotional reactions to the market, but we cannot determine which one is the reason in any given case. The strategy to buy when implied volatility is high and to sell when it is low actually exploits both the mean reversion properties of the volatility and the investors' sentiment. When the implied volatility is high and there is fear in the market, Contrarians bet that volatility will revert back to its mean (meaning that the market will calm down), and so they buy.

On the other hand NEVI is a purer sentiment indicator, since it removes the autoregressive volatility forecast from VIX, together with the other volatility effects. Its value represents only that part of the implied volatility that is not based on the volatility pattern. We call this part “emotional volatility”. Thus NEVI is an attempt to remove the effects of the volatility properties on VIX and to represent solely the sentiment of the



investors. With the help of the regressions we could find a relationship, according to which it will take approximately a month for the investors to realize that their reaction was extreme. In the proposed NEVI strategy we exploit that observations and follow the sentiment, instead of going against it.

## **6.2. Institutional investors lead the market**

NEVI could simply be seen as a proxy for the sentiment of professional investors and for that reason it is more convenient to use it like a “Momentum” indicator rather than a “Contrarian”. NEVI, in fact, measures the difference between the volatility expectations of the option traders and the volatility forecast based on a GARCH (1,1) Model. Let’s assume that the latter can be seen as a proxy for the reaction expected by all of the market (all the agents trading on NYSE with the stocks composing the S&P 500), while VIX represents the expectations of the option traders only. In the latter group usually professional and institutional investors, such as bulge brackets investment banks, pension funds, other funds managers and hedge funds, play an important role. They strongly affect the market due to their significant trading activity and reputation. Their forecasts and fears often become reality as they are followed by the rest of the market.

A “Contrarian” strategy can be successful only if the majority of the market is wrong. The more the indicator at the base of our strategy reflects the behavior of the mass of the market, the higher is the probability to find such mistakes and exploit them in order to get a higher portfolio return. Conversely, if NEVI reflects the behavior of agents whose expertise and knowledge is above the average, it is likely more convenient to implement a “Momentum” strategy with the indicator, since such agents are better than the rest of the market in predicting patterns.

## 7.0. Conclusion

Our research finds that volatility based indicators can be successfully used for the purpose of measuring the investors' sentiment, timing the market and implementing a short term investment strategy.

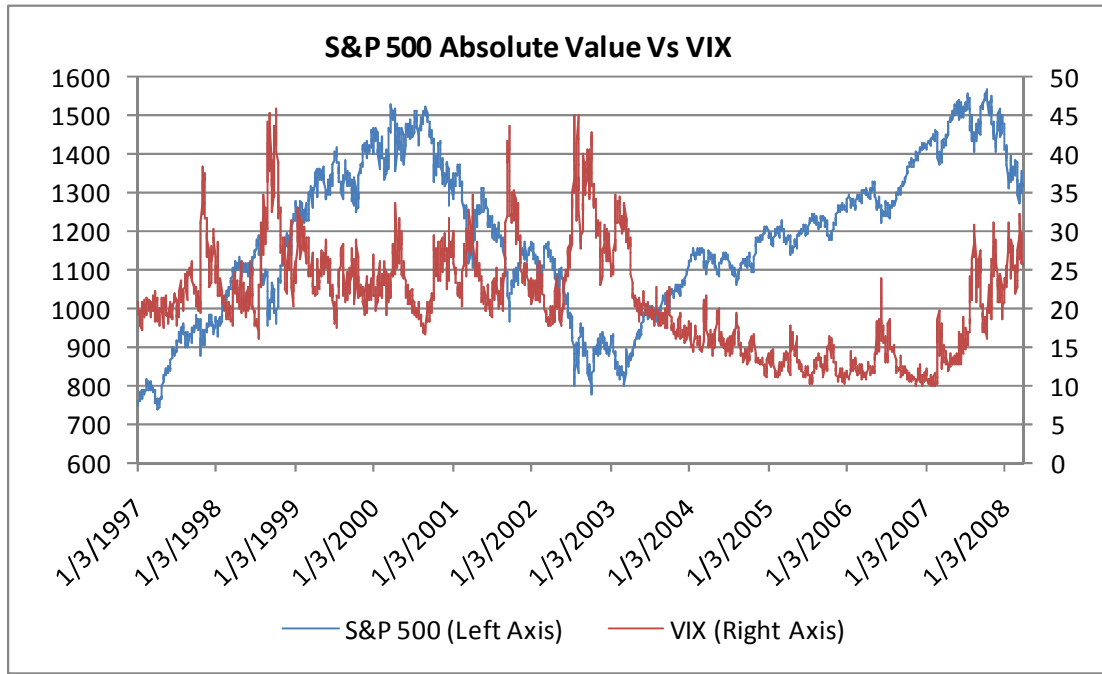
The implied volatility index VIX, calculated by the CBOE, is truly a useful indicator that is worth to have in one's trading arsenal. The positive relationship between S&P 500 return and VIX verifies the widespread belief that VIX is a "Contrarian" indicator. Following a strategy to buy when VIX is high (fear in the market is high), and to sell when it is low (investors feel secure) beats the market.

While acknowledging the value of VIX, we argue that it cannot be considered as a pure sentiment indicator. Therefore we created and proposed a new volatility based indicator, NEVI, which we believe is a purer sentiment indicator. It is defined as the difference between VIX and a GARCH (1,1) forecast. The GARCH (1,1) forecast measures the volatility forecast based on the market pattern, while the difference (NEVI) is a measure of the sentiment of the option traders. Changes of NEVI are not influenced by the volatility properties as their effect is removed when we subtract the GARCH (1,1) from VIX. There is a significant short term negative relationship between NEVI and the S&P 500 return (less than one month) which makes NEVI a suitable market timing indicator for a short term "Momentum" strategy.

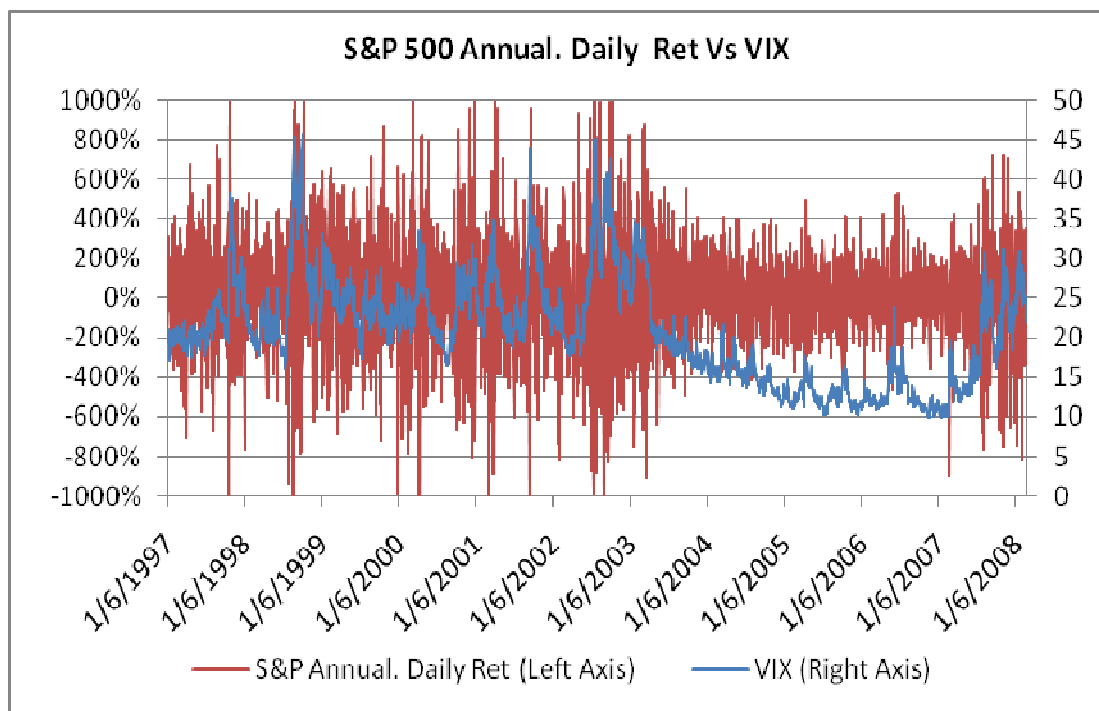
NEVI based strategies beat the VIX based strategies in 4 of the 6 studied cases. The results show that the VIX has a better signaling power when the market is more volatile (2001-2002), while NEVI performs much better when the market is relatively calm (2006-2007). We believe this is likely due to the fact that the GARCH model does not perform at its best in capturing volatility patterns when the markets are very unstable and/or to the fact that the "qualified" expectations expressed by NEVI are likely to be fulfilled in a less volatile market rather than in an unstable one. Finding other financial meanings for NEVI indicator could be a subject of further research.

## Appendix

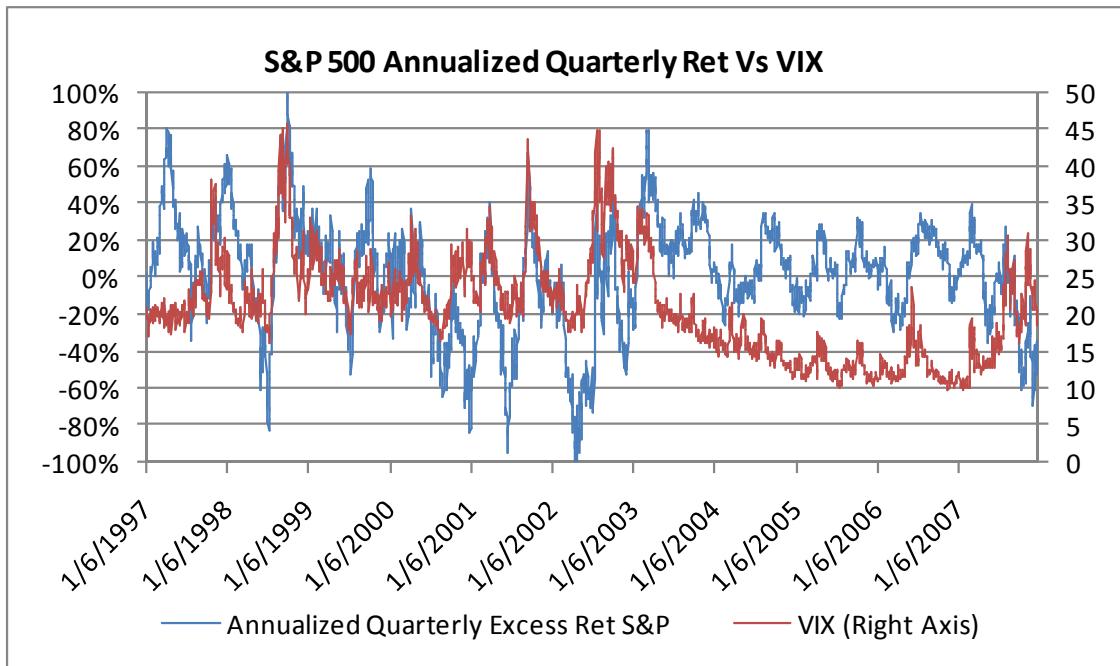
Picture 1: S&P 500 Absolute Value vs. VIX



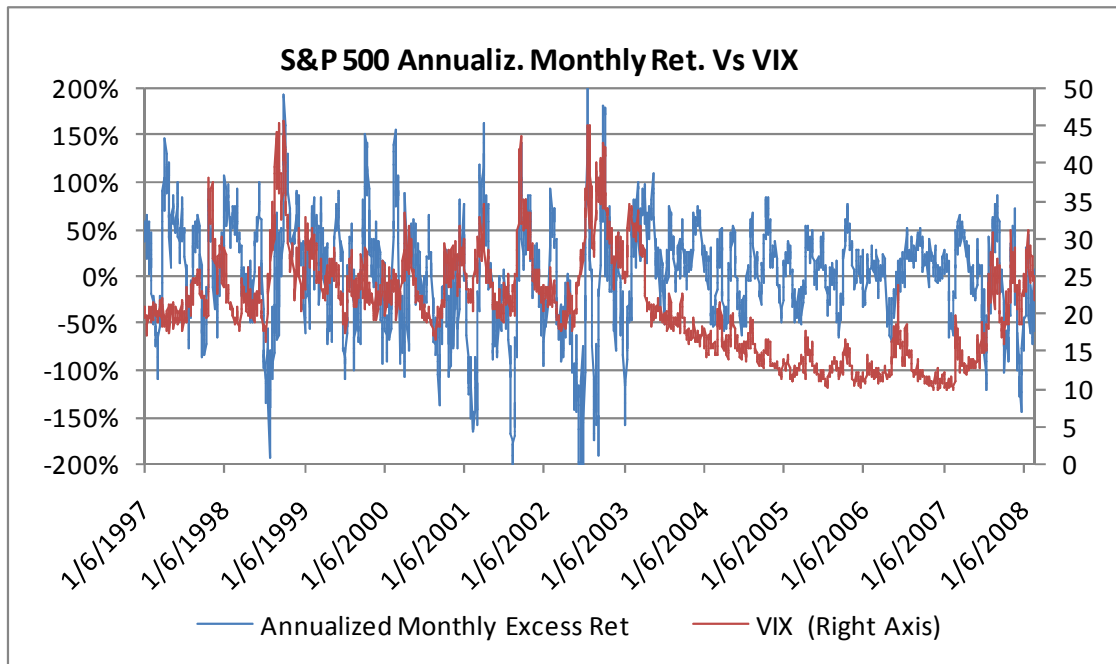
Picture 2: S&P 500 Annualized Daily Returns vs. VIX



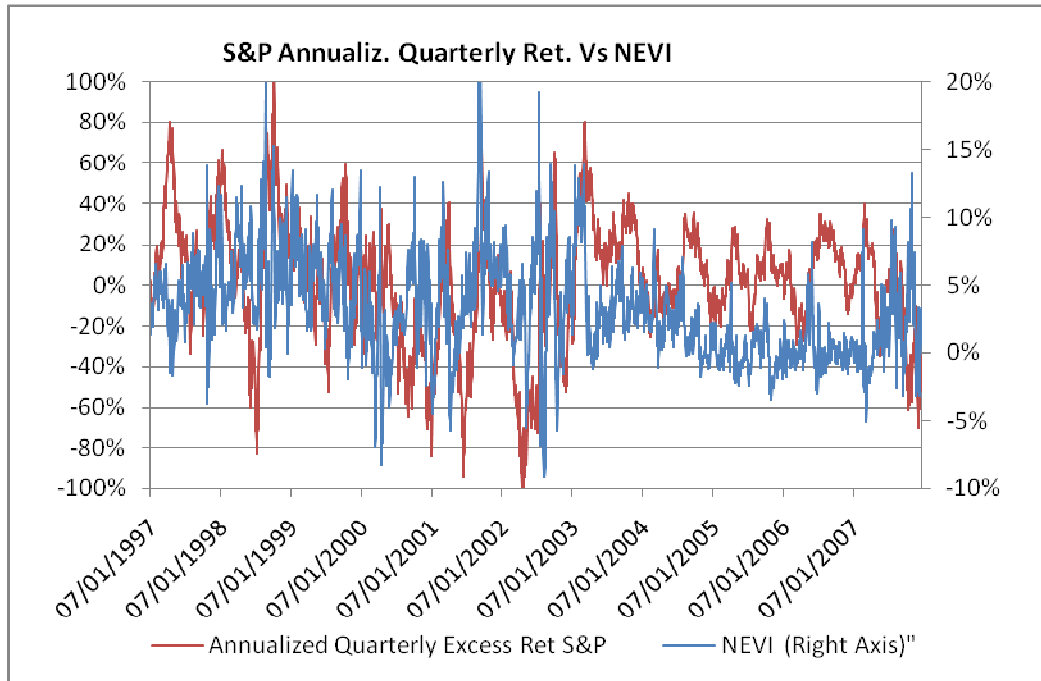
Picture 3: S&P 500 Annualized Quarterly Return vs. VIX



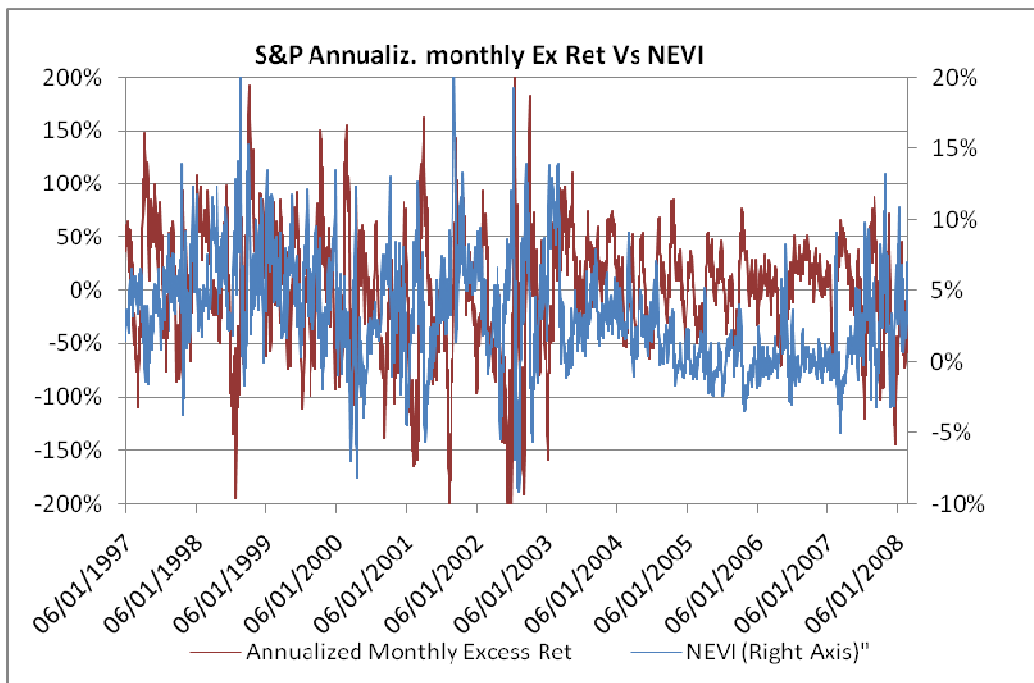
Picture 4: S&P 500 Annualized Monthly Return vs. VIX



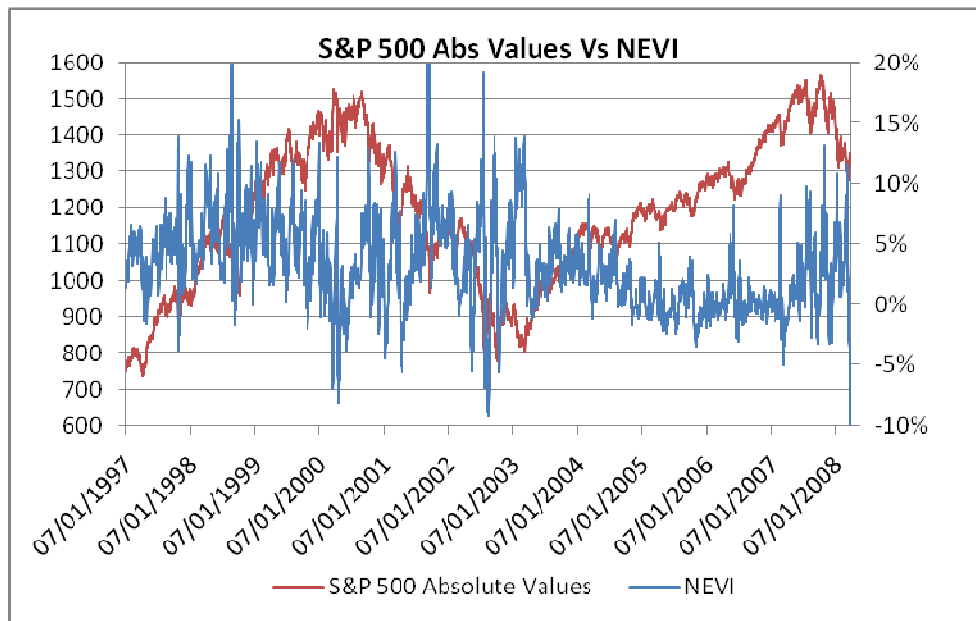
Picture 5: S&P 500 Annualized Quarterly Returns vs. NEVI



Picture 6: S&P 500 Annualized monthly Return vs. NEVI



Picture 7: S&P 500 absolute values vs. NEVI



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