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Economic growth featuring costly extraction of low entropy natural resources

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Abstract

This thesis explores a model of economic growth in which natural resources are complements to labour and capital in production and the extraction of those resources requires labour and capital to be diverted from production. In other words, capital and labour have two complementary roles in the model and their allocation between the roles help determine total output as well as economic inequality. Environmental limits to the extraction of natural resources are also entered into the model. The end result is a model which is both practical for empiric purposes and familiar in structure to neoclassical models despite that it's assumptions are based on ecological economics.

Keywords: Economic growth, costly resource extraction, entropy, ecological economics

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1. Introduction

In growth economics today we tend to see the environment as something that can be added in an ad hoc manner to the fringes of our theories (for example as externalities or a non-renewable, substitutable resource input), otherwise centred on traditional economic concepts such as labour and capital. But this is a flawed approach since the economical and environmental systems are very similar as well as deeply interrelated and interdependent. In other words, the economy is an open thermodynamic subsystem of the environment (Daly, 1996, p49), a fundamental fact that should be the starting point of economic theory rather than an afterthought. One important aspect of this interrelation is the flow of natural resources through every vein of the economy; just as the sun powers the biosphere, resources from the biosphere power our economy in a very concrete and physical way (Georgescu-Roegen, 1971, p230). This insight leads me to believe that if we want to face and hopefully solve the economic challenges of a growing population, falling oil reserves and declining environmental health we can simply no longer afford to abstract from the immense importance of low entropy natural resources and the limits to their extraction. This realisation has taken root in our fellow discipline ecological economics (see for example Daly, 1999, p52) but has yet to translate into growth theory (a few notable exceptions are Georgescu-Roegen, 1971, England, 2000 and Kraev, 2002).

This thesis seeks to amend this by further developing the growth model in Hermansson (2008) which was built upon the insight that low entropy natural resources are complements to capital and labour in production. In that model, the extraction of these vital natural resources was crudely modelled as an automatic, exogenous flow. This is clearly unsatisfactory, to simply state that natural resources are hugely important to production without attempting to explain why some economies in time and space have access to more resources only brings the reader half way to answering the research question; Why are we so rich and they so poor? This thesis is an attempt to remedy this and complete the model by endogenising the input flow with the help of the concept costly resource extraction.

This has seldom been attempted, and never in a context where natural resources are assumed to be complements to capital and labour (Krutilla & Reuveny, 2004, p165). There is ample reason to believe that this new way of modelling the input flow may have significant implications for the results of the model. First of all, the new way of modelling resource extraction features several areas of choice for both individuals and governments which might help explain the different growth paths experienced by different economies. Second, growth and development economics have long realized that the primary and secondary sectors play different roles in economic growth. By modelling resource extraction as costly, a primary sector which is subject to other dynamics than production can effectively be created within the growth model. That economies are dominated by different sectors might in this thesis consequently explain their different growth paths. Finally, previous research has shown that how extraction of natural resources is modelled can have large impacts on the outcome of the model. In the context of the Ramsay-model, costly resource extraction has been shown to give rise to several possible steady states and balanced growth paths in the economy, which undermines the hypothesis of conditional convergence (ibid. p167). In addition, if the resource base features non-linearities, which is likely, that might also give rise to non-linearities in the growth path of the economy (for example Arrow et al., 2000).

The goal of this thesis is to further explore how a growth model built on ecological economics and costly resource extraction can help explain growth differences and economic inequalities. Concretely that entails providing the analytic solution to a mathematical model of a closed economy with certain assumptions about the extraction and importance of natural resources. The result will be the identification of sources of differences in both growth rates and output levels between economies in time and space. A division of labour and capital between the tasks of extracting natural resources and refining them into output will be the most prominent new feature of the model. The allocation of resources to extraction mimics a primary sector and will actually simplify the model leaving the final version very similar to the familiar Solow model, with the addition of an expression of the allocation which helps explain the Solow residual.

The thesis is structured into six chapters including this introduction. Chapter two briefly presents the theory so far, including the mathematical growth model. Chapter three sets the focus on natural resource extraction and presents and discusses the new assumptions which will shape the revised growth model. Chapter four takes these assumptions as given and show how the new version of the model can be solved for steady state level and growth rate of output, it also discusses transitional dynamics and static efficiency. Chapter five presents the results, namely the implications for growth and inequality that can be derived from a model with these types of assumptions. The last chapter discusses the strengths and weaknesses of the model and the individual assumptions, suggests future work and offers a few concluding remarks.

2. The model so far

The model briefly presented in this chapter was first presented in Hermansson (2008) (unless otherwise stated that is the source of all claims in this chapter) and constitutes an attempt to formalise some insights from ecological economics into growth theory. These insights are that the economy is best viewed as a subsystem of the environment and that low-entropy natural resources are complements to capital and labour in production. These insights are based on the laws of thermodynamics, some of the strongest scientific principles to date.

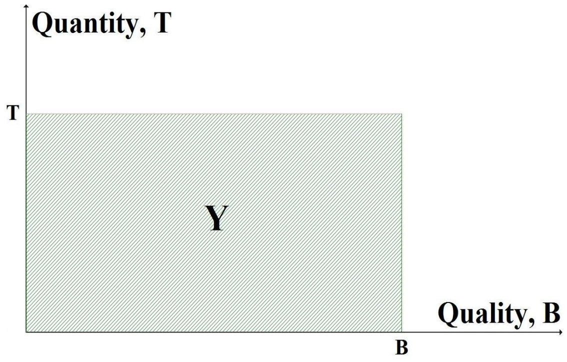
2.1. The economy and its environment

The first law of thermodynamics states that energy can neither be created nor destroyed. The second law of thermodynamics tells us that entropy, a measure of disorder, always increases within a closed system. Entropy is the fundamental difference between valuable inputs and waste (Daly, 1992, p22f). Consequently, the economy cannot be a closed system, because the quality of the energy stored in the economy would deteriorate with every physical process, leaving us less potential for further production and less consumption possibilities (Nationalencyklopedin, headword "entropilagen"). In short, everything would slowly turn to waste. This contrasts with the standard view within economics in which the economy is understood as a closed circular system of households and firms. From a thermodynamic viewpoint, a growing, circular, physical system is an impossibility (or rather an extreme quantum improbability).

Rather, the economy should be understood as an open subsystem of the environment (Daly, 1996, p49). The environment provides the economy with low-entropy matter-energy and *there is no other such significant source*, all wastes of the economy end up in the environment and the economy is wholly dependent on larger environmental processes such as atmospheric and climate regulation (Barbier, 1990, 10). Furthermore, it is exceedingly difficult to draw a line between the two systems, indicating that they are very closely linked. These are characteristics of the environment-economy relationship which have always held. But that does not mean that they will hold forever. Man-kind has begun to directly harvest solar energy with for example solar panels and some second generation biofuels, and it has begun to isolate it's wastes, indicating that the world economy might be growing past parts of it's historic relationship with the biosphere. However, these tendencies are so far insignificant in size compared to the scale of the two systems and can safely be ignored in this thesis. And ultimately, the economy can never outgrow it's dependence on the sun or similar entropy phenomena. For purpose of this thesis, the most important feature of the environment-economy relationship is that the environment is the economy's source of needed low-entropy matter-energy in the form of natural resources; fossil fuels, minerals, crops, animals, wind, rain and so on. The flow of these resources into the economy is in this version of the model seen as exogenous and automatic.

2.2. Economic processes and value

The purpose of production is to create output of some value to consumers. The model presented in this chapter relies on a certain interpretation of the nature of that value. I believe that output brings satisfaction to consumers through alteration of physical reality, for example the creation of goods, display of entertainment, transportation of the consumer herself or perhaps the change of binary values in a computer. In other words, output always has a physical dimension. But these physical changes are not random or arbitrary, they are purposeful and aim to create goods and services with some specific utility to consumers. This means that production and output has two dimensions, one material and one intangible which involves understanding the needs and wants of consumers. The total value of output can be described in a similar two-dimensional manner, Y=BT, where Y signifies output, T signifies the quantity of output and B signifies the quality of the output or how good that output is at satisfying the needs and wants of consumers.



Since this thesis is mostly concerned with the importance of natural resources and their extraction, focus is definitely on the material dimension of production. But it is important to say a few words here on the quality dimension. Quality is in this model the result of the design of the product or service, not of the production process itself. In other words, a firm cannot increase the quality of it's products by simply adding more capital, labour or natural resources to the production line, that will only increase the number of products. To produce a product of higher quality, the firm must instead change the nature of their product, not of their production. This is what Herman Daly terms qualitative development (Daly, 1999, p6). In reality, these changes often go hand in hand, but it is useful to separate them in theory. This qualitative dimension is very similar to how technology is usually envisioned in growth theory, an exogenous and possibly unlimited factor with constant returns to scale, and it will enter the production function in much the same way.

2.3. Funds and flows

This model is based on the groundbreaking work of Nicholas Georgescu-Roegen (1971) who among other things identified the roles of different factors of production. In every production process every input plays one of two (or both) roles, a fund or a flow. *Flows* are that which is transformed into products and waste, *funds* are the agents of transformation which are not themselves transformed (ibid., p230). Consider for example the making of a table, the carpenter and his tools (labour and capital, the funds) are largely unchanged by the making of the table whilst the wood which becomes a table (the flow) is obviously and irreversibly changed by the process, part of this change is the desired output. The flows are composed of indestructible matter-energy, which can only be transformed but neither created nor destroyed. In other words, the role of the traditional factors of production, capital and labour (the funds), in this context is not to "create" output but rather to make possible the transformation of natural resource input flows into valued output flows. The funds can be envisioned as a type of catalysts for the production process, they trigger and determine but do not take part in the changes. What constitutes a fund or a flow is a matter of the specific production process in question.

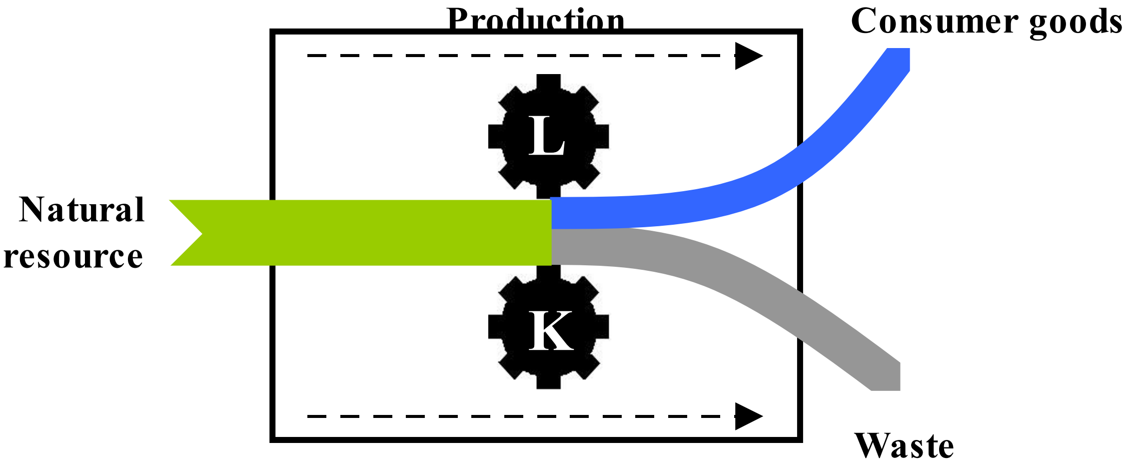


Figure 2.2: Aggregate production involving capital (K), labour (L) (the funds) and natural resources which are turned into consumer goods and waste (the flows).

To fully understand the interplay between funds and flows in production we must return to the concept of entropy and how all processes are subject to entropic limitations. Entropy is a physical concept which is inversely related to the potential to do physical work stored in a system, for example as the potential in a battery. I.e. the less entropy in a given system, the greater the potential for physical work which can be used for production. Low entropy can be said to be the ultimate resource, the common denominator of all natural resources (Daly, 1992, p16). The total potential, i.e. the total quantity of this ultimate resource, is the ultimate limit to the quantity of output that can be created by a given economy. Since the funds are defined as remaining unchanged by the production process, the total potential for work can be found in the flows. This is why the economy needs natural resources, stored in them is the economy's potential to do work.

Low entropy can be created, for example through industrial processes where energy is concentrated or matter given form, but only at the expense of a greater entropy increase in another part of the system (Nationalencyclopedin, headword: entropilagen). Hence, the economy will always need an external source of low entropy, it is an open subsystem. Let us consider as an example one of the most important natural resources of our age, oil. It is important because there are practical techniques for using it to create useful products and services. Homogeneous and concentrated oil is conveniently stored in large underground reservoirs which can be accessed with machinery. This oil has predictable qualities which allow us to design engines to run on it and indeed run our entire economy on it. If oil could only be found dispersed and diluted, and if burning it was an unpredictable business, oil would never have become a cornerstone of our economy. This usefulness can be equated with the low entropy embodied by the oil.

The role of the funds in this entropy setting is to catalyst desirable change, the potential stored in the natural resources must so to speak be put to good use. Oil in itself is no good for transporting a container, you also need a truck to make sure that the potential in the oil is used to transport the container where it is needed. In other words, capital and labour can be used to guide the potential into the economic processes that create useful output. The economy needs both physical potential, which is stored in the flows, and something to guide and catalyse that potential, namely the funds. Therefore the funds and flows are fundamentally complements in the production process.

So, there is physical potential stored in the flows, and funds harness that potential for useful purposes. In an ideal economy, all of the potential goes into the output and all of the low entropy can now be found in the products. But in all real economic processes, low entropy is lost in at least one of two ways. First, the unwanted by-products still contain some production potential (low entropy) when they are disposed of and leave the production process. Second, the combined entropy of the products and wastes is higher than the entropy previously embodied by the inputs. This is a result of the irreversibility of all economic processes, we cannot turn wastes and outputs into inputs again (Gillett, 2006, p60). The amount of production potential that is embodied by the output, i.e. the quantity of output, can consequently be described as a quota of the production potential embodied by the inputs (inputs less losses). This quota can be understood as the entropic efficiency of the production process.

This efficiency can reasonably be said to depend on two things, the technological level in the economy and the amount of capital and labour per unit of natural resources. The intuition for the first statement is straightforward, a higher technological level indicates better designed production processes which utilizes productive potential to a higher degree. For example, less than one percent of the fuel use of a standard car is used to actually move the driver and passengers (Lovins, http://www.ted.com/index.php/talks/amory_lovins_on_winning_the_oil_endgame.html), with better designed cars we could transport more goods with less oil, reflecting a higher technological level. The key to avoiding entropy losses is to not convert energy into low grade heat which dissipates into the environment (Gillet, 2006, p60). A lesson not taken to heart in our present economy. The maximum technological level is unity, such processes would turn inputs into output with zero losses. That A cannot grow larger than one is simply a testament that no technology, no matter how advanced, can break the laws of thermodynamics (Daly, 1992, p24).

The intuition behind the importance of capital and labour is fairly easy as well. Our current economy is a so called throughput economy, most resources take a fairly linear route through the economy, only being used in the production of one good. In contrast, the economy could mimic natural ecosystems in which each waste product is used as an input in another process, minimizing entropy losses in the system. Such an economy could be called a roundput economy. What is needed for the economy to begin using natural resources and wastes in this efficient manner? Simply put it needs capital and labour which have been designed or trained for the purpose. And since the funds (capital and labour) and the flows (natural resources or “wastes”) are complements in production, that will be the result if the economy is saturated with funds whilst the flows are scarce. In other words, efficiency will rise with the fund to flow ratio.

The maximum efficiency of production is one (a hundred percent). The “first” percent of efficiency is probably much easier to attain (by applying funds) than the “last” percent. It is reasonable to assume that the best resources are used first in every stage of the production process. The not so good resources, and indeed the wastes, probably require more capital and labour to be turned into valued output, leading to decreasing returns to the funds with respect to efficiency increases. A simple way to graphically demonstrate these insights is by the following diagram.

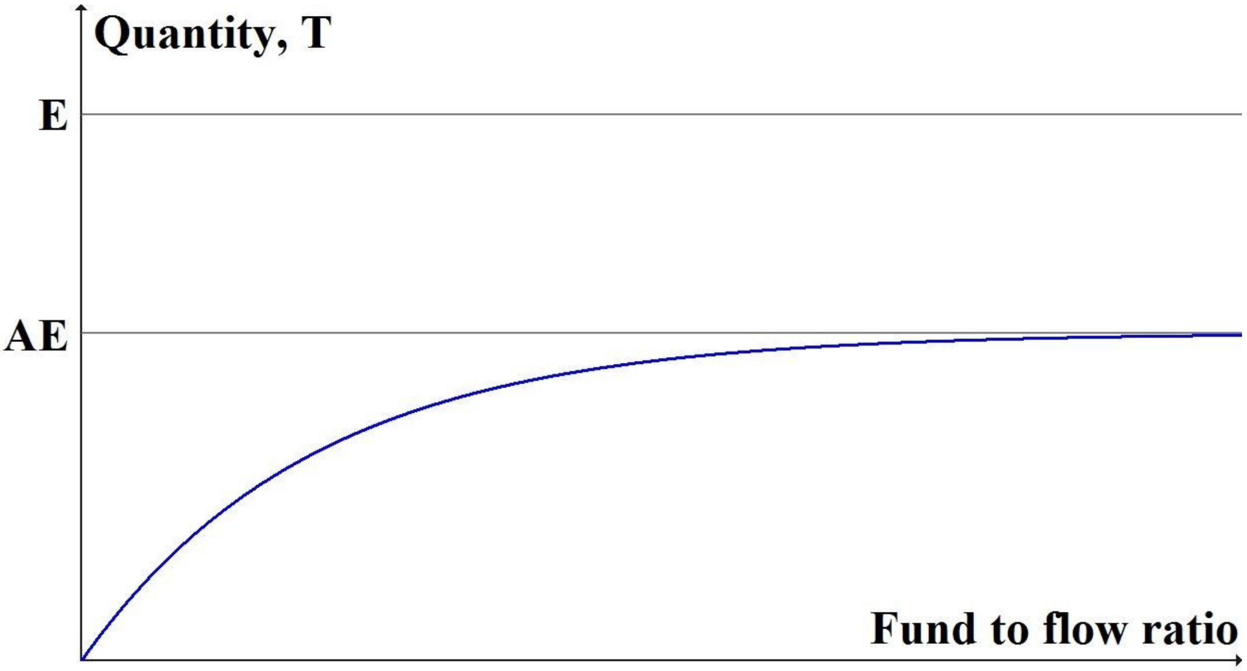


Figure 2.3: Output quantity as a function of the amount of capital and labour to input flows, converging on the maximum limit set by technological limits and the availability of input flows.

To summarize, the role of fund factors of production is to transform input flows into valued output. More funds per unit flow clearly does this more efficiently, minimizing entropy losses. But no amount of capital and labour can overcome the technological limitations of the production process in question. So technology poses another limit to production and is ultimately limited itself by the laws of nature, prominently the laws of thermodynamics. The next section will attempt to bring this chapter together in a mathematical model.

2.4. The mathematical model

The production function which captures the insights in this chapter looks as follows,

$$Y = BAE\left(1 - e^{-K^{\alpha} L^{1-\alpha} E^{-1}}\right) \quad , \qquad \text{Equation 2.1}$$

in which Y is the total value of output, B represents the qualitative dimension, A is a technology variable between zero and one, E is the total inflow of production potential (low entropy natural resources), K is capital and finally L is labour. In short it represents the qualitative dimension times the quantitative dimension. The quantitative dimension approaches its maximum E when the fund to flow ratio approaches infinity and technology approaches unity. Capital and labour are assumed to be complements. A graphical representation is shown below.

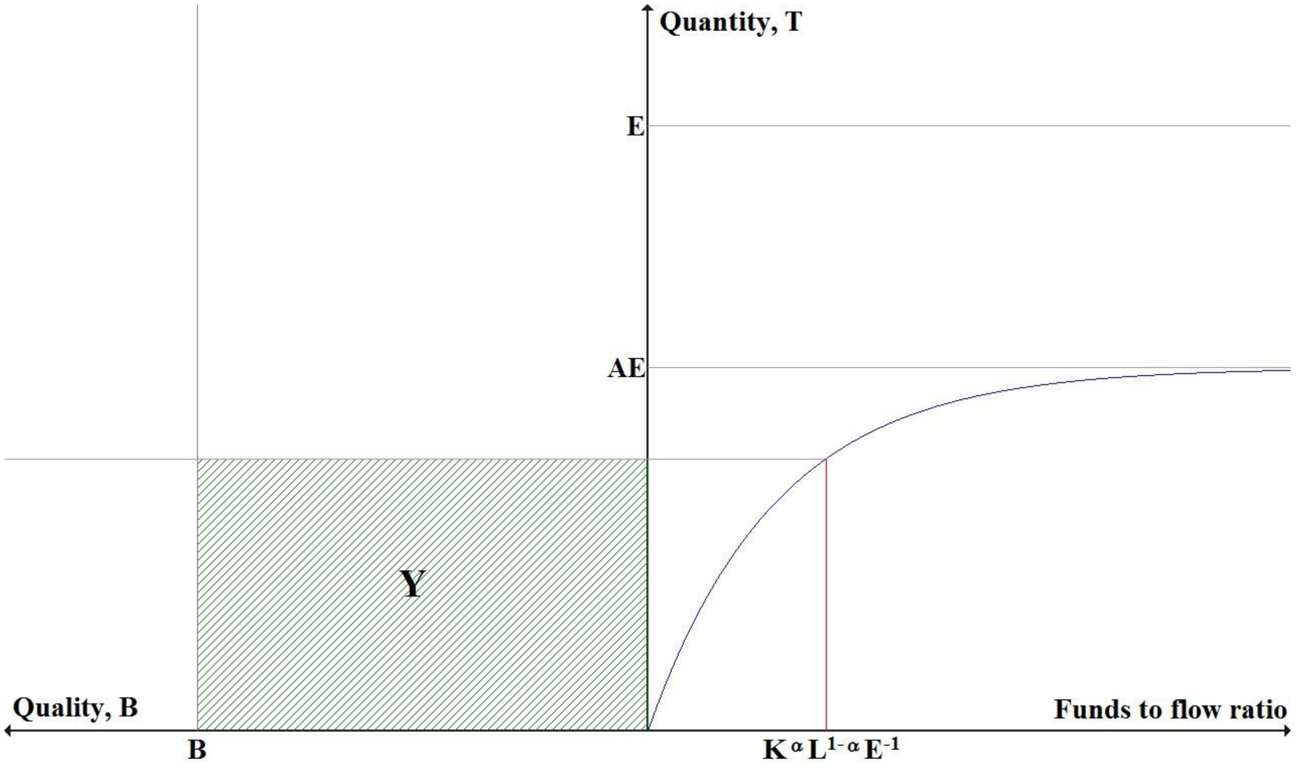


Figure 2.3: Total output (the area Y) as a function of total low entropy inputs, E, technology A, the fund to flow ratio, $K^{\alpha}L^{1-\alpha}E^{-1}$, and the qualitative dimension, B. In other words, how much input, how well we use it and what we use it for.

The major feature of this production function is that capital, labour and natural resource inputs all experience diminishing returns to scale, with substitution possibilities at the margin although funds and flows are complements. The quantitative dimension is bounded by the finitude of production potential embodied by the inputs whilst the quality dimension is possibly unlimited. It is exceedingly simply to create a per capita version of this production function. As B, A and $1-e^{-K^{\alpha}L^{1-\alpha}E^{-1}}$ all reflect characteristics of the economy as a whole, they are not affected by the transformation. Only the input of low entropy must be corrected to per capita form.

$$\frac{E}{L} \equiv \varepsilon \quad , \qquad \text{Eq 2.2}$$

$$y = B \varepsilon A (1-e^{-K^{\alpha}L^{1-\alpha}E^{-1}}) \quad . \qquad \text{Eq 2.3}$$

The growth and level of per capita output is hence dependent on the qualitative dimension, the amount of input flow per capita and technological level as well as capital and labour. Capital accumulation is assumed to be a result of investment (which equal savings) less depreciation. The savings rate is assumed constant.

$$\dot{K}=s_K Y-dK \quad . \qquad \text{Eq 2.4}$$

Per capita capital accumulation can be described as follows;

$$\dot{k}=s_K Y-(n+d)k \quad . \qquad \text{Eq 2.5}$$

The growth and level of B, A, L and E are assumed exogenous. If g_X indicates the growth rate of the variable X, the growth of y can be described as;

$$g_y=g_B+g_A+g_E-g_L+(\alpha g_K+(1-\alpha)g_L-g_E)\frac{(K^{\alpha}L^{1-\alpha}E^{-1})e^{K^{\alpha}L^{1-\alpha}E^{-1}}}{1-e^{K^{\alpha}L^{1-\alpha}E^{-1}}} \quad . \qquad \text{Eq 2.6}$$

2.5. Recapitulation of the conclusions

The above model allowed for a few conclusions on historic and future growth. In the context of the model, there are four types of growth: (1) qualitative steady state growth that raise absolute and per capita incomes, (2) transition growth of input flows with decreasing but positive returns to absolute and per capita output (because the economy is a subsystem of the environment, inputs cannot grow forever (Daly, 1992, pxiii)), (3) transition labour growth with positive effects on absolute output but decreasing and potentially negative returns to per capita output and finally (4) transition growth of capital or technology with positive returns to both absolute and per capita output.

Historic growth in per capita output has most probably not been limited to the qualitative growth associated with the steady state. Transition phenomena like population growth, capital accumulation and increasing inputs from the environment, as well as technological progress, have driven development from one steady state to another, more prosperous one. Furthermore, this type of growth cannot continue forever within the boundary set by limited natural resources. Only qualitative steady state growth is possibly boundless, and without risk of falling per capita incomes.

It may however be warranted to deviate from the "safe" steady state growth path if benefits exceeds costs. But we are receiving clear signals that environmental costs are mounting fast (for example in the form of climate change) and that they may threaten long term stability. There is also the threat of depleting terrestrial sources of low entropy such as oil, indicating that E in the long run will have to converge on the sustainable yield from renewable resources. But there are also signs that the regenerative limit has been reached, if not overstepped, for many resources, as exemplified by overfishing of the seas. And the real limit may be environmental degradation rather than depletion of a specific resource stock which might be more unpredictable and occur before depletion (Barbier, 1990, p8). Quantitative growth in inputs is consequently becoming an increasingly non-viable deviation from the steady state. If inputs are to be held stable, population growth can quickly become a downward drag on per capita incomes. This leaves two fruitful avenues for future economic growth, increasing the size and efficiency of the capital stock (raising K and A) and qualitative development (in the form of raising B).

3. Costly resource extraction

In the model above the inflow of low entropy natural resources was assumed to be automatic and exogenously determined. This is obviously a flawed assumption, in reality extraction of natural resources is a conscious effort by individuals and firms. But the extent of resource extraction is not wholly endogenous, it must also depend on the environment from which the resources are extracted, government decisions and so on. This chapter attempts to revise the model to capture these and other insights.

Extraction is a hugely diverse process and agriculture might for example seem to have little to do with hydroelectric power plants. Consequently, the challenge in the task to model extraction is to create a single, mathematically simple description of this diverse process without losing to many important insights. Due to time and other limitations as well as the focus of the thesis, extraction is here modelled as one primary sector with homogeneous inputs, outputs and internal dynamics, i.e. an aggregate process which reflects the average conditions of all part processes. In other words, the assumptions made in this chapter are sometimes overly simple, but in those cases more realistic assumptions are briefly presented and in some cases discussed in the last chapter as venues for future work.

3.1. A flow from a stock

In this model, the environment contributes to production in the form of a flow that brings low entropy into the economy. But a flow cannot originate from nothing, it flows from something and the characteristics of its source limits the size and quality of the flow. It is therefore important to describe and analyse the source to understand the flow. The source of the low entropy flow into the economy has always been the biosphere of our planet, it provides crops, animals, fossil fuels, atmospheric patterns and all the other phenomena from which the economy draws low entropy (Barbier, 1990, p10). The biosphere in turn is sustained by a flow of energy from another source, the sun, which might eventually become the economy's primary (direct) source of low entropy.

It is useful to describe the flow and source as being composed of the same building blocks, similar to how a river and the lake it runs from are both made of water. So in this thesis the biosphere is described as a stock of low entropy, even though it in reality is a much more complex phenomena, of which a portion flow into and sustain the economy. In other words, the flow and stock are considered homogeneous, every unit of flow similar to every other unit of flow and stock and so on. In one way this is realistic, low entropy, the ultimate resource, is a homogeneous physical state which can be measured in any thermodynamic system. In another way this assumption of homogeneity disguises the fact that all stocks and flows are different in the sense that they can be more or less hard to come by or utilize. For example, within natural resource economics resource deposits are often differentiated depending on the amount of energy required to extract the resource (Hanley, 2007, p244). This perspective is lost with the current homogeneity assumption but could be incorporated in a technology variable discussed later in the chapter. In addition, different resources are most often used for different specific purposes. The famous and dramatic example in Lozada (2006, p78) illustrates this; a man dying in a desert with ample hydrogen and oxygen but no appropriate method to make them react into water. Although Lozada meant this to prove that the entropy concept has limited usefulness for economics, I believe this instead illustrates that conversion between between different forms of low entropy (the low entropy of a tomato versus the electric potential in a battery) is difficult and wasteful, i.e. primarily a result of our limited technology and not of the difference between low entropy stocks. Besides, this thesis deals with aggregated processes, the entire economy's capacity to do work, and thereby avoids such extremes that can be found in specific examples involving men in deserts. When low entropy is in focus (i.e. as the main function of the flows) natural resources are in a sense very homogeneous, making this assumption reasonable.

It is clear from the water analogy that the flow cannot become larger than the stock, once the lake is empty, the river dries up. It is also clear that if the outbound flow is larger than the flow which replenishes the stock, the stock and thereby the outbound flow, will be reduced to zero over time. In other words, for the economy to be sustainable in any real sense, the flow must be smaller than the biosphere's capacity to regenerate low entropy, this is sometimes called the maximum sustainable yield (Harris, 2002, p278). For non-renewable resources, this flow is at or near zero, making these resources less important for the long run since their depletion is only a matter of time. This maximum sustainable yield is the long-term limit to extraction on this earth, when all non-renewable resources are gone.

But as history has shown us, there is no guarantee that the economy or it's individual actors will adhere to the long-term optimal level of extraction. Overfishing, overforesting and overuse of marginal lands are all examples of this. Luckily, society has created other, political, ethical and economical, limits to extraction to temper short-term overextractive behaviour. These can take the form of quotas, environmental standards, property rights or even natural parks. One of the most important such limitations of today is the cap on oil production and sales agreed on by the OPEC countries. These limits are called human-made in this thesis.

Yet another limit to extraction is the technological level of the economy. The simplest way to incorporate this limit is to think of technology as a binary value which determines if a resource is accessible or not. For example, all resources at the bottom of the ocean can be labelled as inaccessible with our technology, making the effective stock considerably smaller. These types of resources are often labelled subeconomic and/or undiscovered (Harris, 2002, p232f). Few resources are truly inaccessible though, it is merely a matter of the cost of accessing them, but this feature will be discussed further below, in the section devoted to technology in extraction. The effective stock can consequently be redefined as the stock of natural capital that can be extracted within technological and political limits from a biosphere of limited size. The technological limit has and will become less restrictive over time, an effect which might balance the depletion of non-renewable resources.

Together, environmental, technological and human-made limits to extraction define what is in this thesis called the effective stock of low entropy, i.e. the amount of low entropy available for extraction. One way to mathematically express the importance of the effective stock is the following inequality;

$$E \leqslant N \quad , \qquad \text{Eq 3.1}$$

where E signifies the flow of low entropy and N signifies the effective stock of low entropy. The flow must always be smaller than or equal to the stock and the economy's extraction of low entropy is thus limited. Note that the only strict short-term environmental limit is the size of the biosphere (the total amount of low entropy stored in the biosphere) and that the long-term limit is the regenerative capacity of the biosphere, the definition of the effective stock is in other words dependent on the time perspective. The long term perspective might seem the most prudent in the eyes of a benevolent social planner, but only the short term limits are likely to affect a decentralised economy directly. If the stock of natural capital is defined in the short term this indicates that N is large and that $E<N$ will be valid in almost all cases leaving $E=N$ a rare phenomena. The next section will discuss how extraction is carried out within these limits.

3.2. Diversion of resources for extraction

The main thrust of this thesis is that extraction of natural resources is not automatic or free. But how exactly should the cost be incorporated in growth theory? What type of resources are diverted to extraction? This thesis follows the approach taken by Krutilla and Reuveny (2004, p168) which is based on the idea that capital and labour is needed for extraction. This is a quite realistic assumption, indeed the bulk of labour and capital in our history have been devoted to agriculture or other types of extraction. It is also easy to use in empirical studies as there is often data available on the use of labour and capital in different economic sectors. In other words, this type of approach mimics how economies are differentiated into primary and secondary sectors. This can be used to introduce in growth theory the different conditions faced by for example agriculture compared to industrial production. And as shall be shown later in the thesis the approach can result in a simple and manageable model as well. Due to the advantages of the approach, in this thesis extraction is understood as an active effort by firms and individuals using capital and labour to supply the economy with low entropy natural resources.

A convenient mathematical formulation of this diversion of resources is a quota of the total production funds, labour and capital;

$$s_E K^{\alpha} L^{1-\alpha} \quad , \qquad \text{Eq 3.2}$$

where s_E is a number between zero and one. The funds now available for production are;

$$(1-s_E) K^{\alpha} L^{1-\alpha} \quad . \qquad \text{Eq 3.3}$$

This formulation is based on four critical assumptions. First, that the extractive or primary sector is self-sufficient in terms of low entropy, i.e. that it does not need low entropy to create low entropy. This is in a way a blunt simplification. It has been said that we no longer eat potatoes made of potatoes, we eat potatoes made of oil and synthetic fertilizer. And of course, the oil which fuels our cars is made partly from the potatoes eaten by oil workers. The assumption is also problematic if the amount of low entropy needed to produce low entropy changes over time or between economies. But it is a simplification which is both useful and which has merit. Since the net effect of extraction is always positive, low entropy flow into production, a model in which the primary sector provides the primary sector with low entropy can be collapsed into a model where the primary sector does not need low entropy without any important insight for growth being lost. At least not as long as the model does not differentiate between different types of natural resources or attempts to describe open economies. The weaknesses of this assumption could be compensated by the technological variable discussed later in the chapter.

The second assumption behind the formulation above is that capital and labour are both diverted to extraction in the same extent. In mathematical terms, that s_E is the same for both labour and capital. This is likely a very inaccurate assumption and somewhat problematic for the results. Not only is capital predominantly used in industrial production rather than agriculture, but this is also something that varies between economies, perhaps explaining differences in growth rates and wealth. However, this weakness does not contradict or affect the main results, rather it is something that could be investigated separately in future studies.

The third assumption is that alpha is homogeneous across sectors. This seems reasonable, the relationship between labour and capital, how they are substitutes to each other, is probably not very dependent on the task they are to perform. In any event, if there are large differences in this relationship depending on the task, this is a problem even without costly resource extraction and this thesis merely reproduces a neoclassic error.

The quota s_E is seen as exogenous in this thesis, mostly because the choice of individuals and firms to enter the primary or secondary sectors is very complex. In many cases it involves relocating and overcoming strong social and legal boundaries. This probably also makes changes in the quota a slow process. However, the historic experience of urbanisation during the last decades indicates that wage differences between sectors (which is a result in the model) is a powerful driver of change which might be used to approximate the expected changes and values in the quota.

These diverted resources are assumed to be the only thing needed to extract a flow of low entropy. In other words, the size of the input flow is a function of the amount of capital and labour used to create it;

$$E = F\left(s_E K^{\alpha} L^{1-\alpha}\right) \quad . \qquad \text{Eq 3.4}$$

The next section discusses the role of technology and the efficiency of capital and labour in extracting the low entropy flow.

3.3. Technology in extraction

In the course of this chapter it has been suggested that technology plays many important roles in extraction and the mathematical formulation of the process. These roles are:

1. A limit to which resources that can be extracted or accessed or alternatively the relative cost of accessing different resources.
2. A determinant of which resources that subsequently can be used in production and therefore worthy of extraction.

3. A factor which determines the productivity of capital and labour in producing a flow of low entropy.

The first role was easily incorporated in the effective stock of natural capital as yet another limit to the amount of resources that can be extracted. The second role is really a feature of production but worth mentioning here because it ties into the assumption made in this chapter that the effective stock and flows of low entropy are homogeneous. Modern day economies are nowhere near to utilizing the total amount of low entropy in some of it's inputs, we have for example no method for extracting the potential in nuclear bindings except for a fraction of the potential of a few radioactive isotopes. Our technological efficiency for certain types of low entropy is in other words very low. But the homogeneity assumption transforms this inefficiency of certain processes to an inefficiency in the entire economy, both production and extraction, through the technology variable A presented in the last chapter. In reality it makes certain resources not worthy of extraction.

The productivity of labour and capital in extraction, the third role of technology, must be defined in terms of both how much low entropy they can extract and the productivity of labour and capital in production. This makes it by far the most difficult role of technology to enter into the model. It affects how much labour and capital that should be diverted to extraction, the size of the input flow as well as total and per capita output. It should be entered in the model as a Harrod-neutral factor in extraction determining the output flow;

$$E = F(C s_E K^{\alpha} L^{1-\alpha}) \ , \qquad \text{Eq 3.5}$$

in which C signifies the productivity augmenting technology. However, the inclusion of this technology factor makes the model much more mathematically complicated, especially if the variable is to grow over time. Due to the limited scope of the thesis, this variable C has therefore been normalized to unity with respects to both the productivity of capital and labour in production and the amount of low entropy capital and labour can extract. This means that for every unit of capital and labour (together), one unit of low entropy flow into production. It also means that the entire difference in marginal productivity between labour and capital in the primary versus secondary sectors can be fully explained by the allocation of resources. Finally, since C is always equal to one, it does not change with the other conditions in the extraction process. What this means will become clearer after the next chapter and will be brought up again in relation to the results and yet again in the discussion. The next section will provide the final expression for the extraction of natural resources.

3.4. The size of the input flow

The sections above presented three conditions for the extraction of low entropy;

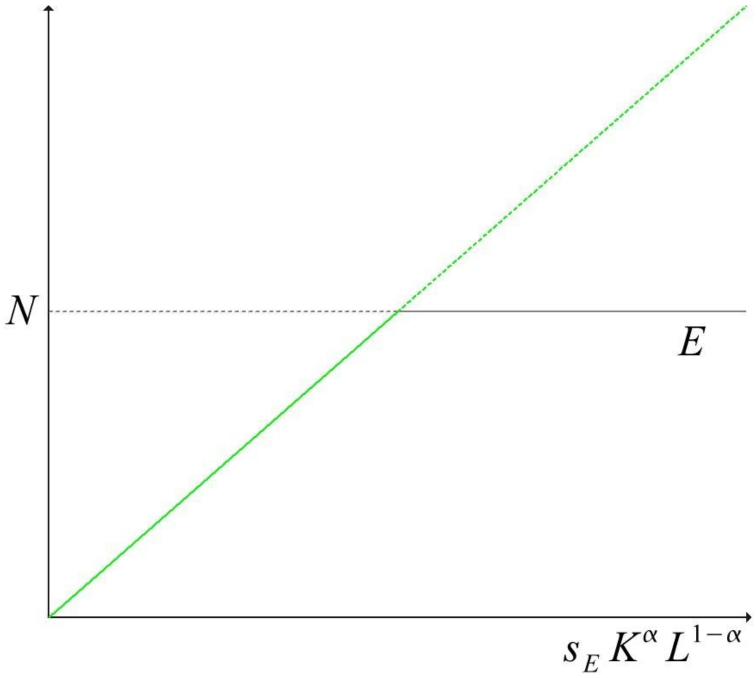
$$E \leqslant N \ , \qquad \text{Eq. 3.6}$$

$$E = F\left(C s_E K^{\alpha} L^{1-\alpha}\right) \ , \qquad \text{Eq. 3.7}$$

and that C is equal to one, constant and thus possible to ignore. These three conditions can easily be combined;

$$E = MIN\left(s_E K^{\alpha} L^{1-\alpha} \middle| N\right) \ . \qquad \text{Eq. 3.8}$$

In words, the diverted funds extract a flow of entropy within the environmental, technological and human-made limits. This expression can be illustrated with figure 3.1. In other words, the efficiency of capital and labour to extract a natural resource flow does not change as the economy approaches the limit to extraction. In a sense this seems odd, the first unit of natural resources should be simpler to extract than the very last. But this assumption seems more reasonable if human-made rather than technological or environmental limits are in focus. If society limits extraction based on other considerations than the economical viability of individual stocks, the ease of extraction might not reflect proximity to limits. After the limit has been reached, no amount of capital or labour will increase the size of the input flow.



3.5. The returns to extraction

The thrust of this chapter has been that labourers and owners of capital devote their time and resources to extraction. But the question still remains, how are they compensated for their efforts? Just as in neoclassical theory labour and capital are assumed to be reimbursed their respective marginal products. The only difference is that the marginal product of a specific unit of capital or labour is dependent on the sector in which it is employed. In other words, the factors of production that work in the primary sector share the returns to the natural capital flow that they have created (the share taken by landowners and holders of property/extraction rights are ignored or assumed part of the share claimed by capital). They share these returns according to their relative importance in extraction, i.e. capital receives α times the total returns to extraction and labour receives $(1-\alpha)$ times the total returns to extraction. These wages and factor rents are not necessarily, or even probably, equal to those enjoyed by labour and capital in the production sector. The implications of this will be further discussed in chapter five. The next chapter however sets up the mathematical model and solves it analytically.

4. Growth rates and steady states

In the last chapter resource extraction was explored and some important assumptions formed. This chapter attempts to solve the model under these new assumptions. The main difference between the previous version of the model and this revised model is that the production funds in the economy, capital and labour, are now divided between production and extraction of resources. Under these assumptions of division of resources, the flows can now be described in terms of the effective stock of natural capital and the funds,

$$E = MIN\left(s_E K^{\alpha} L^{1-\alpha} \middle| N\right) \quad . \qquad \text{Eq. 4.1}$$

The model has different implications for wealth and inequality depending on the relative scarcity of natural capital and capital and labour, each case is explored separately in this chapter.

4.1. When natural capital is abundant

Let us for now assume that $s_E K^{\alpha} L^{1-\alpha} \leq N$ so that $E = s_E K^{\alpha} L^{1-\alpha}$. As was argued in chapter three this is most probably the normal scenario. In this case the production function looks as follows;

$$Y = BAE\left(1 - e^{-(1-s_E)(K^{\alpha} L^{1-\alpha})E^{-1}}\right) \quad , \qquad \text{Eq. 4.2}$$

where $(1-s_E)$ represents the share of the production funds used in production. Substituting for E gives;

$$Y = BAs_E K^{\alpha} L^{1-\alpha}\left(1 - e^{\frac{-(1-s_E)(K^{\alpha} L^{1-\alpha})}{s_E K^{\alpha} L^{1-\alpha}}}\right) \quad . \qquad \text{Eq. 4.3}$$

As the production funds feature in both numerator and denominator of the right-most term, they cancel out, leaving;

$$Y = BAK^{\alpha} L^{1-\alpha} s_E\left(1 - e^{\frac{s_E - 1}{s_E}}\right) \quad . \qquad \text{Eq. 4.4}$$

Output is now determined by the value per unit output, technology, the amounts of production funds and how they are distributed. Per capita output can easily be arrived at;

$$y = BAk^{\alpha} s_E\left(1 - \frac{e^{s_E - 1}}{s_E}\right) \quad . \qquad \text{Eq. 4.5}$$

To simplify the analysis of the steady state, it is useful to rewrite the equation above as;

$$\hat{y} = \hat{k}^{\alpha} s_E\left(1 - \frac{e^{s_E - 1}}{s_E}\right) \quad , \qquad \text{Eq. 4.6}$$

where $\hat{y} \equiv \frac{y}{BA}$ and $\hat{k} \equiv \frac{k}{BA}$ signify output and capital per capita and quality and technology unit. Just as in the previous version of the model, capital accumulation is determined by;

$$\dot{K} = s_K Y - dK \qquad \text{Eq. 4.7}$$

$$\Leftrightarrow \dot{\hat{k}} = s_K \hat{y} - (d+n+g_B+g_A)\hat{k} \quad , \qquad \text{Eq. 4.8}$$

in which g_X indicates the growth rate of variable X and labour growth is assumed exogenous. These equations are enough to allow us to model the steady state. This is fairly simple as the model is now very similar to the familiar Solow model (as presented in for example Jones, 2002, p18ff) since $s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)$ is a constant, because s_E is an exogenous constant parameter in the model. Also, since we have assumed that natural capital is abundant, many of the conditions on the steady state that were necessary in the previous version of the model can now be dropped. I.e. labour growth can be positive and the input flows are only limited by the size of labour and capital. Since the model is so similar to the Solow model with technology, a good place to start exploring the steady state is by stating that $\bar{\hat{k}}$ is constant, where the bar sign indicates variables in a steady state. This gives the following;

$$\dot{\hat{k}} = 0 \Leftrightarrow s_K \bar{\hat{y}} = (d+n+g_B+g_A)\bar{\hat{k}} \quad . \qquad \text{Eq. 4.9}$$

Remembering the equality in equation 4.6 allows $\hat{y}$ to be substituted for;

$$\Rightarrow s_K \bar{\hat{k}}^{\alpha} s_E\left(1-e^{\frac{s_E-1}{s_E}}\right) = (d+n+g_B+g_A)\bar{\hat{k}} \qquad \text{Eq. 4.10}$$

$$\Leftrightarrow \bar{\hat{k}}^{1-\alpha} = \frac{s_K s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)}{d+n+g_B+g_A} \qquad \text{Eq. 4.11}$$

$$\Leftrightarrow \bar{\hat{k}} = \left(\frac{s_K s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)}{d+n+g_B+g_A}\right)^{\frac{1}{1-\alpha}} \quad . \qquad \text{Eq. 4.12}$$

This equation gives the capital per capita, technology unit and quality unit in the steady state, given a certain savings quota, allocation of resources to extraction, depreciation, population growth, qualitative growth and technological progress a well as certain factor shares. To arrive at the steady state output per capita, technology and quality unit, simply insert equation 4.12 in equation 4.6;

$$\bar{\hat{y}} = \left(\frac{s_K s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)}{d+n+g_B+g_A}\right)^{\frac{\alpha}{1-\alpha}} s_E\left(1-e^{\frac{s_E-1}{s_E}}\right) \quad . \qquad \text{Eq. 4.13}$$

The growth rate of $\bar{\hat{y}}$ in a steady state is zero because $\dot{\bar{\hat{y}}} = \dot{\bar{\hat{k}}} = 0$, a result of the definition of the steady state. The growth rate of output per capita may however be positive, if technology or the qualitative dimension improves;

$$g_y = g_{\hat{y}} + g_B + g_A \quad . \qquad \text{Eq. 4.14}$$

This can be arrived at by “taking logs and derivatives” of the definition of $\hat{y} = \frac{y}{BA}$. In other words, the steady state growth rate of per capita output is equal to technological and qualitative growth. Worth remembering though is that technological progress cannot continue forever if we have an understanding of technology which is compatible with the laws of thermodynamics. The steady state can be graphically demonstrated with the following diagram.

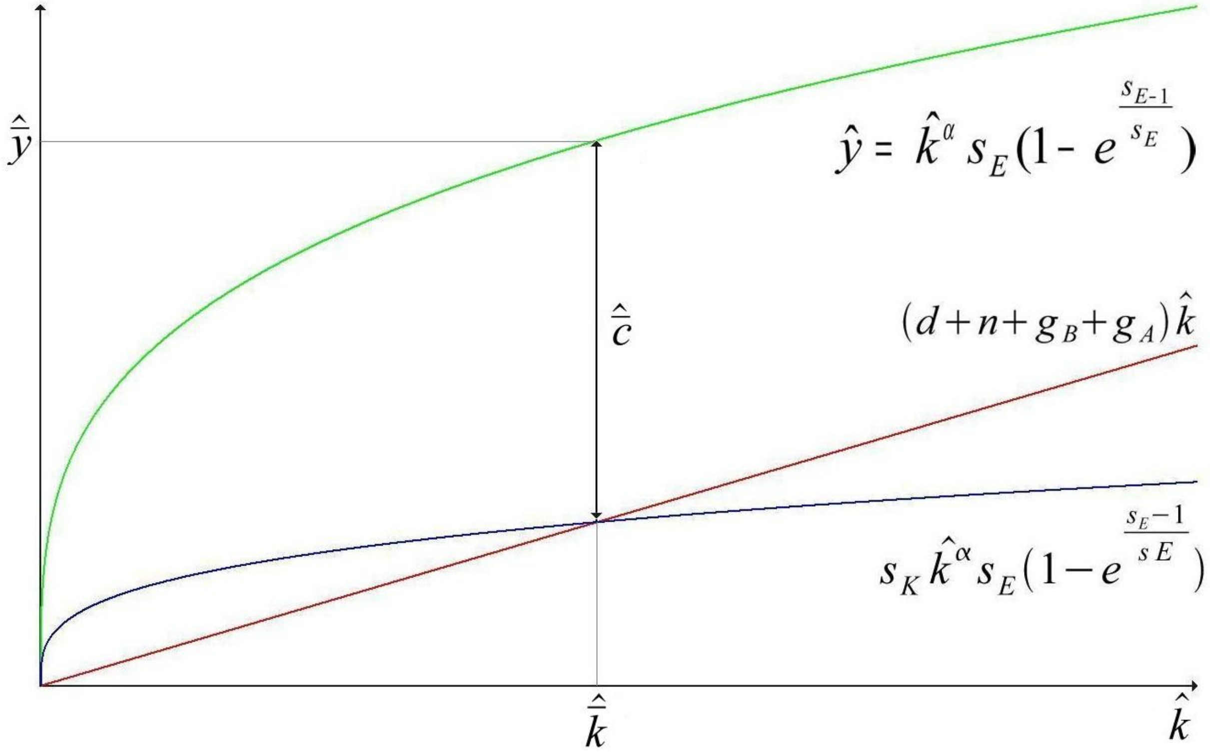


Figure 4.1: The steady state featuring a balance between investment and depreciation (and pressures due to population growth etc.) given parameter values. Also featured, steady state consumption.

As was established above, $\hat{y}$ is constant in the steady state. But real economies are probably seldom in their respective steady states. It is therefore useful to know something about how economies move towards their steady states, their transitional dynamics which arise when the variables in equation 4.12 are subject to exogenous changes. In this model there are two types of transitional dynamics, those traditionally related to capital accumulation and those related to changes in s_E. Per capita, quality and technology unit capital accumulation is determined by;

$$\dot{\hat{k}} = s_K \hat{y} - (d + n + g_B + g_A)\hat{k} \quad . \qquad \text{Eq. 4.15}$$

If the savings quota rises, or one of the far right variables decreases, investment will exceed depreciation (and population growth etc.) leading to capital accumulation at a positive but decreasing speed until $\dot{\hat{k}}=0$ and the economy is once again in a steady state. If depreciation or one of the other far right parameters increases, or if the savings quota falls, depreciation will exceed investment leading to capital accumulation at a negative but rising speed until $\dot{\hat{k}}=0$ and the economy finds itself in a steady state once more. These patterns are similar to those found in the Solow model.

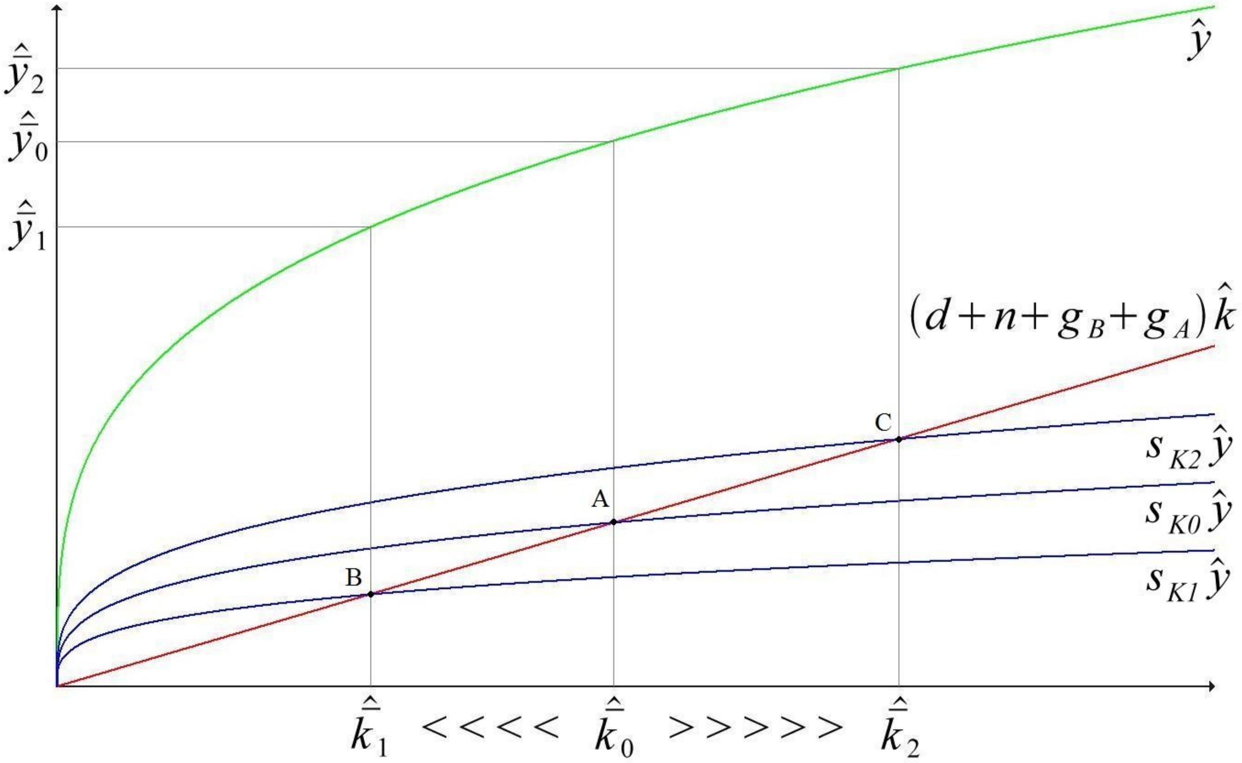


Figure 4.2: A rise in the savings quota from for example s_{E0} to s_{E2} causes $\dot{\hat{k}}$ to exceed 0, $\dot{\hat{k}}$ then falls as the economy approaches a new steady state at point C. A fall in the savings quota from for example s_{E0} to s_{E1} causes $\dot{\hat{k}}$ to fall short of 0, $\dot{\hat{k}}$ then rises as the economy approaches a new steady state at point B.

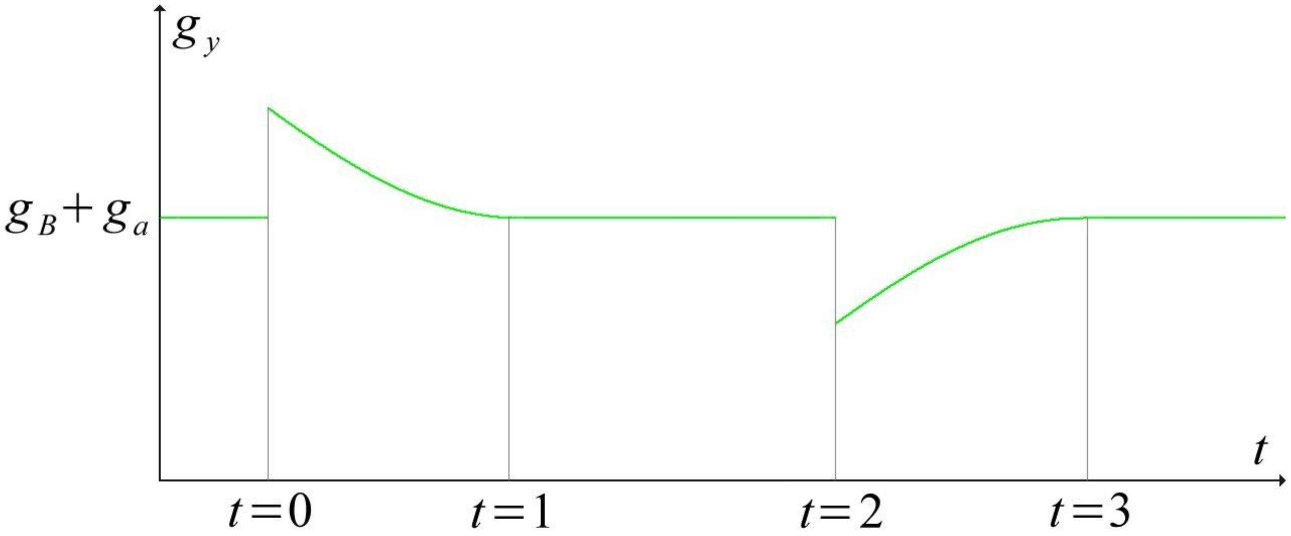


Figure 4.3: At time zero the savings quota experiences a permanent rise which causes the growth rate of y to exceed gB+gA, at time one investment once more equals depreciation. At time two the savings quota experiences a permanent fall which causes the growth rate of y to fall short of gB+gA, at time three investment once more equals depreciation.

The other type of transitional dynamics in this model arise when s_E changes. The next chapter will deal with static efficiency in this variable, but it must at this point be mentioned that output is maximised at a certain value of s_E between zero and one. In other words, there can be both too much and too little labour and capital diverted to extraction. If s_E moves towards its optimal value (from either direction) there will be two positive effects in terms of increased output. The first one can be derived from the equation describing output in the steady state;

$$\hat{\hat{y}}=\left(\frac{s_K s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)}{d+n+g_B+g_A}\right)^{\frac{\alpha}{1-\alpha}} s_E\left(1-e^{\frac{s_E-1}{s_E}}\right) \quad . \qquad \text{Eq. 4.16}$$

The rightmost term, $s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)$, indicates that $\hat{\hat{y}}$ is directly dependent on the allocation of capital and labour. An improvement in that allocation will therefore result in an immediate positive effect on output.

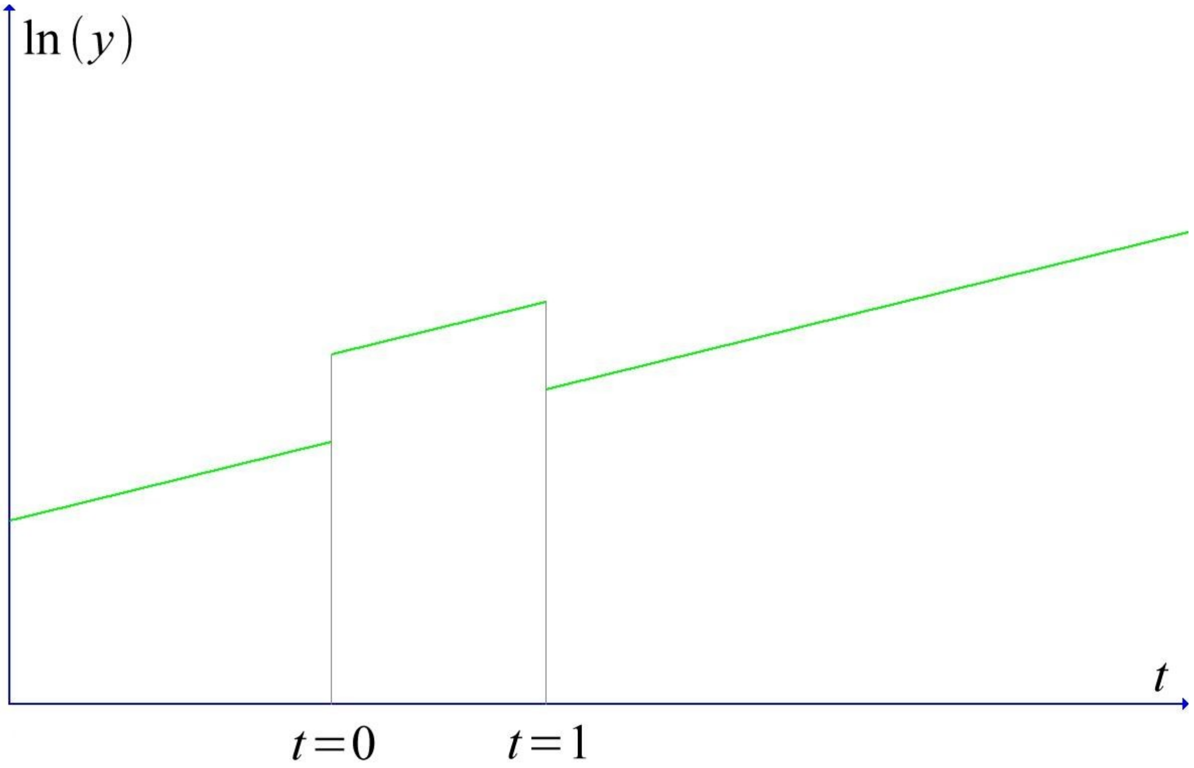


Figure 4.4: At time zero, s_E experiences a permanent move towards it's optimal value leading to an immediate rise in output per capita. At time one, s_E experiences a permanent move away from it's optimum leading to an immediate fall in output per capita.

In the real world, this transition will reasonably not be as smooth as in the model. Capital and labour will experience many different types of obstacles in moving from for example agriculture to industrial production. For example, labour may have to move and learn new skills, capital has to be transported and probably rebuilt for a new purpose. The reasons for the easy and immediate change in the model are that capital and labour are assumed homogeneous and freely moveable between sectors in the economy. If human capital was introduced to replace raw labour, that framework might help model difficulties in moving labour. Until such improvements to the model can be made, this change is, unfortunately, unrealistically immediate.

Since output rises sharply and immediately when s_E goes towards optimality, investment rises sharply and immediately in the model. The result will be capital accumulation similar to that shown in graph 4.3. This is the second positive effect of a more optimal allocation of production resources. The effect is represented in equation 4.12 by the first instance of $s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)$, within the large brackets. The combined effects of an improved or worsened allocation is seen in the graph below.

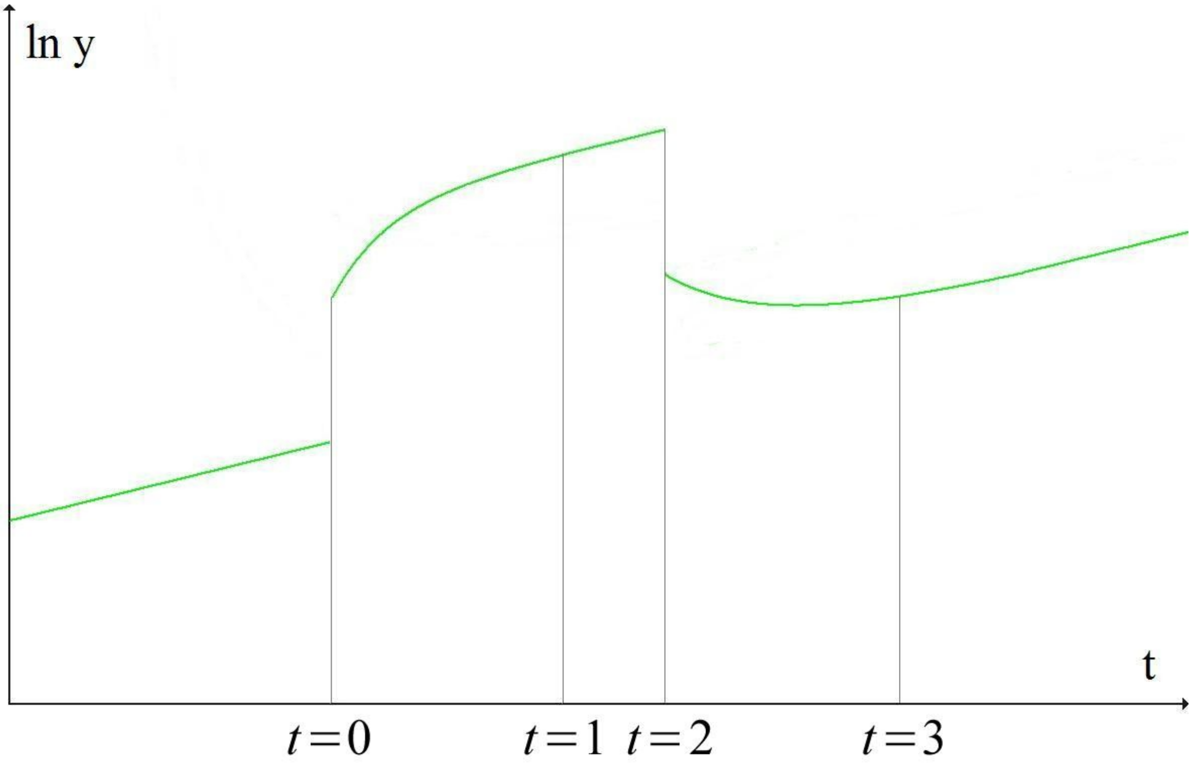


Figure 4.5: At time zero, sE experiences a permanent move towards it's optimal value leading to an immediate rise in output per capita and positive per capita accumulation of capital. At time one, the economy is in it's new steady state. At time two, sE experiences a permanent move away from it's optimum leading to an immediate fall in output per capita and negative per capita accumulation of capital. At time three, the economy is once again in it's steady state.

4.2. When natural capital is scarce

Section 4.1 explored growth rates and steady states when natural capital is abundant, what can be called empty-world-economics. This section looks at situations where the economy has no abundant source of low entropy and natural capital is scarce, full-world economics. Scarcity is of course a relative concept, here it is taken to mean that the extractive capacity of the economy is larger than the amount of low entropy to be extracted. In mathematical terms;

$$E = N < s_E K^{\alpha} L^{1-\alpha} \quad . \qquad \text{Eq. 4.17}$$

There are two main implications of this scarcity of natural capital. First, the economy can only achieve a steady state under a certain combination of growth rates. Second, static efficiency in s_E is now dependent on the quota between capital, labour and natural capital. Let us first investigate the steady state. Under the assumption of scarce natural capital, the production function looks as follows;

$$Y = BAN\left(1 - e^{-(1-s_E)K^{\alpha}L^{1-\alpha}N^{-1}}\right) \quad . \qquad \text{Eq. 4.18}$$

It is very similar to the first version of the model but the flows have been replaced by the effective stock of natural capital, the only other addition is the term (1-s_E) which indicates that resources are being diverted between production and extraction. Rewrite the equation in terms of $\hat{y}$ to analyse the steady state;

$$\hat{y} = \frac{N}{L}\left(1 - e^{-(1-s_E)K^{\alpha}L^{1-\alpha}N^{-1}}\right) \quad . \qquad \text{Eq. 4.19}$$

A steady state is defined as a state in which all variables grow at constant rates (possibly zero). The next equation describes the growth rate of $\hat{y}$ in terms of the growth rates of the other variables (part 1) and the current state of the economy (part 2).

$$\underbrace{g_{\hat{y}} = g_N - g_L + (\alpha g_K + (1-\alpha)g_L - g_N)}_{part\,1}\underbrace{\frac{K^{\alpha}L^{1-\alpha}N^{-1}e^{-K^{\alpha}L^{1-\alpha}N^{-1}}}{1-e^{K^{\alpha}L^{1-\alpha}N^{-1}}}}_{part\,2} \quad . \qquad \text{Eq. 4.20}$$

This expression was arrived at by taking the derivative of equation 4.19 with respects to time. For the growth rate of $\hat{y}$ to be constant, and consequently for the economy to be in a steady state, the expression $(\alpha g_K + (1-\alpha)g_L - g_N)$ must be zero. If it is not zero, the expression $\frac{K^{\alpha}L^{1-\alpha}N^{-1}e^{-K^{\alpha}L^{1-\alpha}N^{-1}}}{1-e^{K^{\alpha}L^{1-\alpha}N^{-1}}}$ will change in a way that results in a varying (monotonically falling or rising) growth rate for $\hat{y}$. This is not as abstract as it may sound, $(\alpha g_K + (1-\alpha)g_L - g_N)\frac{K^{\alpha}L^{1-\alpha}N^{-1}e^{-K^{\alpha}L^{1-\alpha}N^{-1}}}{1-e^{K^{\alpha}L^{1-\alpha}N^{-1}}}$ represents the growth rate of $\left(1 - e^{-(1-s_E)K^{\alpha}L^{1-\alpha}N^{-1}}\right)$, which is the entropy efficiency in the economy. This efficiency can be portrayed with the graph below.

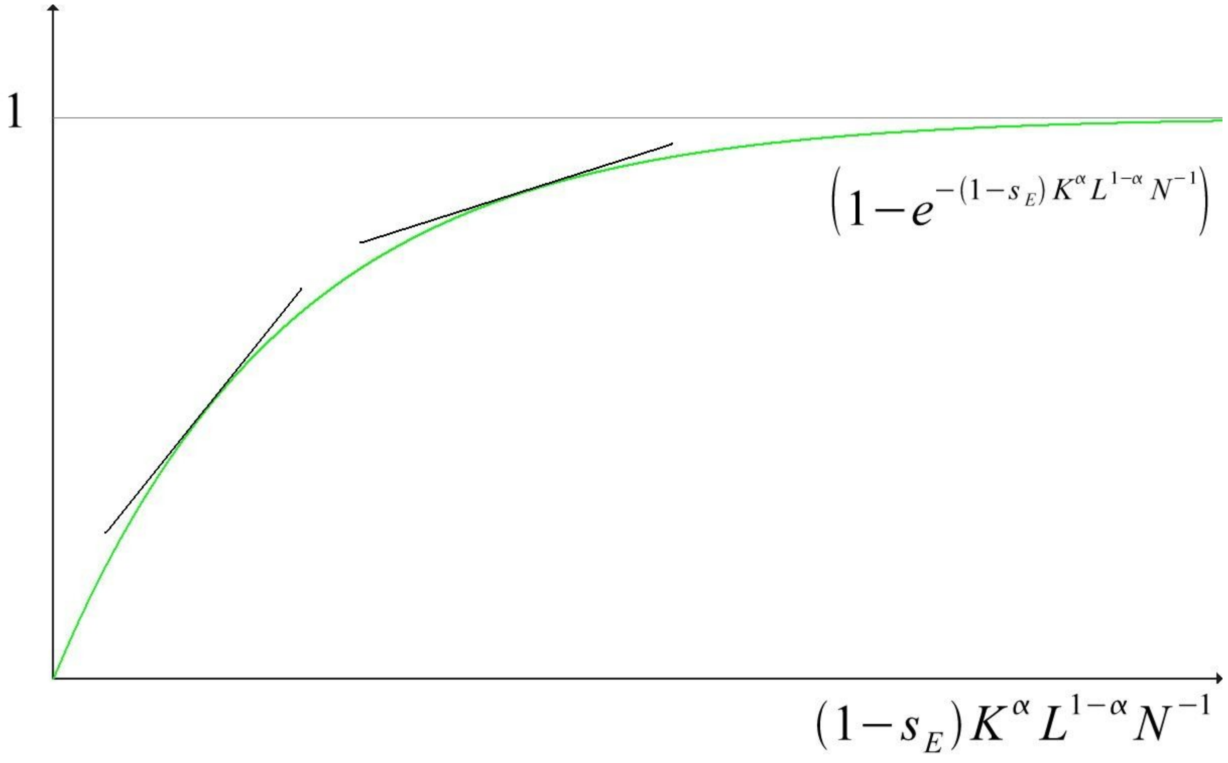


Figure 4.6: The entropy efficiency of the economy has a varying first derivative.

As can be seen in the graph, the first derivative of the efficiency varies, indicating that the only stable (non-variant) growth rate for $(\alpha g_K+(1-\alpha)g_L-g_N)$ is zero. For $(\alpha g_K+(1-\alpha)g_L-g_N)$ to be zero, g_N must be equal to $\alpha g_K+(1-\alpha)g_L$. Out of these growth rates only g_K is endogenous and there is nothing to indicate that it would converge on a value which would satisfy the above conditions. In other words, when natural capital is scarce, only chance will bring the economy into a steady state. This is of course not inherently bad, it could mean that per capita output is growing due to large improvements in technology or product design but at a varying rate. But it does imply that population growth in this situation is a larger drag on per capita output than usual. Since the limited stock of natural capital now determines the flow of low entropy into the economy, labour has lost the opportunity to produce additional output through extraction, lowering the returns to labour in general whilst population growth of course still entails more mouths too feed. This is an effect which will have to be outweighed by for example technological progress in times of scarcity.

5. Implications for growth and inequality

This chapter brings into focus those features of the model which result in differences in growth rates and levels of output, i.e. those feature which help answer the research question, why are we so rich and they so poor? Several of these sources of inequality are old news, in the sense that they have been well explored in the body of growth economics, and will only be briefly handled here. But there are two results which are more novel and which will receive a more thorough investigation in separate sections. Those previously well explored sources of differences between economies are differing savings rates and differences in technological levels, which in this case includes differences in the design of products and production processes.

The savings rate affects the accumulation of capital, particularly in closed-economy models, since it determines investment. The value of s_K does not affect the steady state growth rate of output or consumption in the model, but it does affect which steady state the economy converges towards. There is in other words an optimal value which maximises consumption at every point in time. This optimal value for s_K is often called the golden rule of savings and is a familiar feature of Solow type models. Per capita consumption in these kinds of models is equal to output less investment, i.e. $c=(1-s_K)y$. Steady state consumption;

$$\hat{c}=(1-s_K)\left(\frac{s_K s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)}{d+n+g_B+g_A}\right)^{\frac{\alpha}{1-\alpha}} s_E\left(1-e^{\frac{s_E-1}{s_E}}\right) , \qquad \text{Eq. 5.1}$$

can be maximised by taking the first derivative of this expression with respect to s_K and setting it equal to zero. Just as in the Solow model, the maximum is found when the savings quota equals alpha, the factor share of capital. Economies with savings rates close to the golden rule will have more consumption possibilities at each point in time, and economies with rising savings rates will experience faster-than-steady-state output growth as a result. Savings rates are therefore powerful determinants of the historic growth paths experienced by economies.

Differences in technological levels are in this model represented by two variables, B and A. Although the importance of technology in production have been separated into one part subject to the laws of thermodynamics (the design of production processes) and one part which only reflects the usefulness of output (the design of products) it is no surprise that it has a major impact on output and growth. Economies which utilize more efficient production processes and create better products will quite reasonably reap larger returns to all factors of production.

5.1. Allocation of resources

The addition of costly resource extraction to a model where natural resources and capital and labour are complements has resulted in a new source of differences between economies. Since the extraction and production processes presents two vital tasks for labour and capital, the allocation of the resources between the two activities is crucial to the creation of output. If too much capital and labour is diverted to extraction, there will not be enough funds to refine the natural resources, and vice versa. The optimal allocation of labour and capital, s_E, can be found by finding the maximum value of the expression $s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)$, since s_E always affects output and consumption in the form of this expression which is positively related to output. This maximum can be found by taking the first derivative of the expression with respect to s_E and setting it equal to zero. Under the current assumptions, output is maximised when approximatively forty-seven percent of capital and labour is employed in the primary sector extracting low entropy ($s_E\left(1-e^{\frac{s_E-1}{s_E}}\right)$ is then approximatively zero point three).

Since this result is dependent on both the logarithmic nature of the expression and the units in which capital, labour and natural capital are measured, it should perhaps be interpreted with a pinch of salt. It is probably enough to say that output is maximised when roughly half of all capital and labour is employed in each activity. This result is also dependent on the normalization of the technology variable to one. Another reason to doubt this number is the assumption that all natural resources are used as inputs in production and not sold as final products. If this assumption was to be relaxed the optimal allocation would shift in the favour of the primary sector.

If applied to our historical context these insights indicate that the phenomenal growth experiences of industrialised countries not only represents the importance of capital, but also a shift from extraction to production in the form of a monumental shift away from agricultural to industrial production. This feature of the model might also help explain the current divide between countries dominated by primary and secondary sectors, respectively.

The allocation of resources does not only affect total output and growth however, it also affects inequality within societies (or within a global economy). Since labour and capital have two different tasks in the model (extraction and production) wages and factor rents no longer need to be homogeneous within an economy even though labour and capital are. If s_E is above it's optimal value, too much resources are employed in the primary sector indicating lower returns these factors than to factors in the secondary sector. If s_E is below it's optimum, the opposite holds.

5.2. Natural resource endowments

In a setting with closed economies, the stock of natural capital is equal to the domestic stock of natural capital. This means that countries with rich natural resource endowments (compared to their populations) can experience higher per capita incomes (given a certain technological level etc.) before they move into the phase where there are environmental limits to extraction. There are also time advantages, an economy which has an unproblematic relationship with it's environment can focus on improving it's technology etcetera, and it will probably be more resilient when scarcity finally does occur. In reality, scarcity might not be all that bad, it is in fact a driver of innovation, as argued by for example Ayres (1998).

In Krutilla and Reuveny (2004) and Arrow et al. (2000) it was argued that the economy's resilience on natural resources might give rise to non-linearities in the economy's growth path. These non-linearities in this thesis takes the form of the scarcity phase of growth explored in section 4.2. When the economy moves into this phase, when the economy's extractive capacity surpasses the effective stock, new and non-balanced growth paths arise. This does not happen once and for all however, since the effective stock changes both due to depletion and technological progress. This means that the economy might move between phases over time, creating a crooked and quite unpredictable growth path, which could be illustrated with simulations in which the effective stock changes over time.

6. Discussion

The two sections of this chapter are devoted to discussing the model from the perspective of it's major strengths and weaknesses and pointing to rewarding avenues for future work, respectively.

6.1. Strengths and weaknesses

6.1.1. Convenient and familiar intuition

The first version of the model, in Hermansson (2008), had the disadvantage of having the traditional factors of production, labour and capital, enter the production function in the index of a logarithmic expression. Although there are good reasons to believe that this reflects important insights, it does not fit the neoclassic intuition of most growth models. In the version explored in this thesis capital and labour enter the production function as factors directly related to output (at least when natural capital is abundant). Indeed the model is almost identical to the Solow model when a few preconditions are met. This allows the intuition that is the size and action of the traditional factors of production which power the economy, but with the addition that this is only possible when the economy is an open subsystem of a larger system from which to draw entropy. If these two models are roughly interchangeable in most situations it implies that the basic empirical fit of the Solow model will also benefit the model in this thesis, immediately giving it backing in the data even before any empirical studies have been conducted. It also implies that the Solow model (and models based on it) is a fairly reasonable approximation of an ecological economics perspective on growth.

6.1.2. Explaining the Solow residual

But the model presented here also introduces the two new sources of differences in growth rates and output levels that were explored in the last chapter. Hopefully these will help explain the so called Solow residual, the unexplained variation in the data. It seems likely that both the move from extraction to production indicated by the rampant urbanism of the last decades and environmental limits should have a strong effect on growth and wealth, since that has been suggested in many previous studies. The question of course remains whether the assumptions in this model represents the best approximation or the most correct way for these important insights to enter growth theory. In many previous studies the importance of natural resources have been modelled by including natural resources as a factor of production. These types of assumptions have generally not received strong support in the data (see for example Jones, 2002, p169ff). In this thesis natural resources enter the model in a less direct way, as a division of capital and labour and as the condition that the flow must be smaller than the stock. This allows for greater flexibility for the impact of natural resources, even though they are vital for production their scarcity is not always felt throughout every economy. Hopefully this flexibility will suit the data better.

6.1.3. Need for calibration

One of the major difficulties in the model is how to compare quantities of capital, labour and stocks of natural resources on the same scale. The expression $E=MIN(N|s_E K^{\alpha} L^{1-\alpha})$ and the logarithmic production function are examples of features of the model that require this type of comparison and calibration. These difficulties led to the exclusion of the technology variable C, there is no point in including it unless there is a way to calibrate it with appropriate units. To be correctly compared, capital, labour and natural resources in the extraction process must be remeasured in entropy terms instead of for example the number of workers or the energy content of a barrel of oil. An alternative is to directly measure how much capital and labour that is currently needed to produce for example one barrel of oil, repeat the process with the most common forms of low entropy resources and thereby approximate (for the given technological level) the relationship between quantities of funds and flows. Until these calibrations are made the model is incomplete in the sense that the optimum allocation of resources cannot be pinned down (but it is still possible to determine that there is an optimum).

6.1.4. Empirical ease of use

When natural capital is abundant, the probably most common scenario, the only additional data (compared to the standard Solow model) needed for simulation or statistical processing of the model is the relative sizes of the primary and secondary sectors (with the possible addition of the calibration mentioned in the previous section). And this data is readily available for most economies. This way of entering natural resources is much easier than measuring the precise amounts of natural resources available to each economy for each year of study.

6.2. Future work

There are four main avenues for future work that I wish to point out in conclusion to this thesis. The first of these is quite naturally to use the model in empirical studies, to test and calibrate it against data as well as making predictions about future growth. As argued above, the model is now suited for quite simple empirical investigation. The remaining areas for future work is about relaxing the simplifying assumptions of this thesis.

The first unrealistic and slightly distorting assumption was made already in Hermansson (2008). In this model all natural resources are assumed to be inputs to the production process and nothing else. In reality many natural resources, mostly foods, are sold directly to consumers without any significant production or refinement. One way to incorporate this insight in the theory is to create two output goods, one that has been directly extracted from the environment and one industrial good. The analytical solution to such a model would also include an equilibrium between the two goods determined by both supply (the focus in this thesis) and demand. The relative prices of the two goods would determine both how much labour and capital that should be assigned to extraction and how much of the low entropy that would enter production as an input.

The current model looks only at closed economies which gives it limited explanatory power in a global, interconnected economy. This is especially problematic in this model because there can be significant differences in returns to capital and labour depending on economic sector (which are not dependent on the productivity of capital or labour but of the relative scarcity of extraction or production in general). And if some countries specialise in either primary or secondary sectors (made possible by trade), which is the current norm, the differences in factor rents between sectors will help further explain differences in output levels between countries. This effect might be framed as an unequal exchange given a certain normative position, relaxation of the closed economy assumption is therefore not only warranted from a descriptive scientific perspective but might also suit a constructive ambition. Another related effect is the possible concentration of capital in countries which specialise in production if s_E is above the optimum (which would lead to even higher marginal returns to labour in those countries).

The final area that I want to recommend for future work is probably the least complex and difficult of the four. In most growth models, including this one, the economy is viewed as one aggregate process with one technological level, homogeneous inputs and outputs when in reality it is made up of innumerable processes, each one with it's own conditions. One step towards this more realistic view of the economy would be to rewrite all quantities in the model as sums of heterogeneous inputs, outputs and technologies. I do not believe that this would affect any of the main results of the model, but it might help illustrate how dependent our economy is on certain processes and certain inputs. It could also further shed light on the role of natural resources in our economy by providing examples and anecdotal proofs of the strengths of the assumptions imported from ecological economics, something that I believe is necessary to pave the ground for and inspire further studies that combine growth theory and ecological economics.

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