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“The Perfect Portfolio”

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Abstract

Nowadays investors have a large number of choices of how they can invest their money. One of their biggest challenges is how to allocate their portfolio between equities, bonds and properties. It can only be shown afterwards, which was the best allocation. That is why it is so popular to look at historical mean-variance to predict the future. In this master thesis historical data is also used to see which patterns in the asset returns that repeat themselves. As an investor you can earn or save a lot of money by following some rules learned from historical data. Through multiple regressions these predictors are found and are called *views* in this master thesis. The historical mean-variance together with the views are inputs into the Black Litterman model which is used to optimise the asset portfolio.

In this work the diversity between different investments and how the returns have been during the last years is discussed. The investors' investment universe is important for the portfolio return. In this master thesis it is examined if the pretty low yielding Direct Property Investment can increase a portfolio return in comparison with a portfolio without this asset.

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1 Introduction

The portfolio optimisation has been well examined for a long time and several different proposals have been written by persons all over the world. The most common asset portfolio theories have been to examine portfolios consisting of stocks and bonds; among the stocks indirect property investments have played a small role too. Not many works have included Direct Property Investment as an asset. The few works that have included the asset have concluded that Direct Property Investment is under weighted as a portfolio asset in large and long term portfolios; according to Hoesli, Lekander and Witkiewicz (2004)

Why the asset has not been so theoretically examined as other assets is difficult to say but the lack of reliable historical data can be one reason. In this master thesis portfolios with and without the asset Direct Property Investment are examined. The asset is rather attractive since it has low correlation with other assets in the short term and a little too high risk adjusted return; because of the way the returns in the index is calculated the asset has a artificial low standard deviation that must be raised to get a proper index to work with. The disadvantage with the asset is that it has a high transaction cost and is considered as an active investment for the investor.

For me, properties are a very interesting asset, since they together form a city and the owner can rebuild and develop the building until it is not recognisable. The property asset is not as risky as stocks, but has a higher risk factor than bonds. Since the rent income from the tenants often is very reliable and predictable, it is common to have the properties debt financed. The risk with property asset increases with higher gearing and is one of the reasons Indirect Property Investments have higher risk.

In many pension funds and other long term investment portfolios today, property investments represent only a small part of the total portfolio according to Hoesli, Lekander and Witkiewicz (2004) and also Fisher (2002). Stocks and bonds distinguish a much larger part. Last years it has become more popular for pension funds etc., to directly or indirectly exposure their portfolios to real estate. This large inflow of capital among other things has driven up the prices on real estates and real estates shares, and thereby raising the total return for both assets, according to Figure 1. Since there is a lack of dependable historical data of Direct Property Investment Returns a new portfolio optimisation method is used called Black Litterman optimisation model. This model has a lower sensitivity to historical input data and the investor can use views to affect the outcome. The views in this master thesis are dynamic and come from multiple regressions of many variables from the last 60 months of historical data. The dynamic views help the investor to filter away noise in the historical data and optimise the return.

1.1 Background

All assets' returns shifts in cycles and some years all the macro factors are correct for one kind of asset and the return becomes high. In the same way it might sometimes be hard for the returns to reach the old peaks. This is the reason why one should not only look at the historical return, because that increases the risk to invest at the highest peak of return. Most theories today used for asset allocation only consider the historical return and risk and does not consider the expectations of the future return. In this work a quantitative model called the Black Litterman model is used; which combine the historical return and risk to the investors'

expected future return on the asset to give a better asset allocation, Litterman and Fischer (1992).

Among investors today the Markowitz model is rarely used for allocating portfolios. The Markowitz model lacks the opportunity for the investor to use his own expected return. The model can also return portfolios with extreme allocation positions. Another weakness with the model is that the better an asset has performed the last years; the more of it the Markowitz model suggest the investor should have in his portfolio. For example if the stock market has outperformed a long time, its historical mean return is on a high level as well, which increases the stock market allocation in the Markowitz model. But in reality an investor may be looking for another asset to invest in that has not performed that good the last years.

1.2 Purpose

The purpose of this thesis is to examine if Direct Property Investments should be an asset in optimal asset portfolio allocation. The portfolios are calculated by a portfolio optimisation; through maximising risk adjusted return. To calculate expected returns; multiple regressions are used to produce *views* as inputs in the Black Litterman model which combine the views with historical mean-variance. Through these calculations asset portfolios are made with dynamical variables.

1.3 Limitations

This master thesis is limited to the US-market and the assets; Direct Property Investments, stock traded property shares, US treasuries with maturity between 5-7 years and the S&P500 index. The available data is based upon the US-market, monthly data between 1983 and 2005, totally 264 data points.

1.4 Target Group

The target group should be larger institutions, companies and investors that have adequate resources to allocate between all the assets and also invest for long term. Especially the asset Direct Property Investment requires larger amount of invested capital since the unit value of a property is high.

2 Classes of assets

A portfolio can consist of many different assets. This work has limited the assets to direct property investments, equities, stock traded property shares and bonds. For a large investor who is investing world wide the investment universe is almost unlimited. I have scanned the market for data and limited the work to the US-market.

With indices of returns it is possible to compare different assets and the correlation coefficient between them. It is also possible to evaluate the return of the assets and standard deviation during different periods. With the indices it is feasible to gain knowledge to develop optimal investment portfolios. Indices can also be used to benchmark performance measurements and portfolio strategies.

2.1 *Direct Property investments*

Real estates are the most valuable asset in most countries. The value of the properties shows how the economy in the country changes. When an investor is buying a building this is known as a direct property investment. The building can be an industry, office or a residential building. Properties are heterogeneous and the variation in size, tenant, location, use, age and maintenance is big describes Hoesle and MacGregor (2000). Many people believe that one of the most important things to look at when buying a property is location. They think that that is all that matters because of the difficulty to move a building. The unit value of a property is large and makes it impossible to small investors to invest; it can even be hard for a large investor to get a well diversified portfolio. In this case equities are much easier to invest in, in view of the fact that only a small amount of money is needed.

Since the large unit value and the secure cash flow of a real estate investment, it is very common to borrow money in order to finance a large part of the investment. It is widely said that when buying a property; half of the time looking for the investment is spent on the setup for the financing, says Hoesle and MacGregor (2000), which is besides time costly also capital costly.

Direct property investments are active investments. Equities and bonds can be passive investments but direct property investments need management. The properties can be rebuilt, maintenance is always needed, tenants can move in and out, rent must be collected etc. This also opens up for big opportunities when rebuilding, the use of the property can be changed and maybe raise the rent and then also increase the value of the house. To get the same influence over a listed company a very large share of the voting rights is needed, according to Hoesle and MacGregor (2000). So the high transaction costs together with a low volatility on the market; compared to the stock market; and the longer transaction times makes the asset considered as a more long term investment compared to stocks or bonds.

2.1.1 *Direct Property investment indices*

To be able to use financial tools to create optimal asset portfolios with properties as an asset you need reliable indices of the return from the properties. Low transaction volumes, the lack of a central marketplace and the large diversity between different properties make the asset hard to value writes Hoesle and MacGregor (2000). The value of the property assets and

investment performance cannot be measured as easily as equity and bond assets. Hence, there is a lack of transaction based indices and instead appraisal based indices are used.

Appraisal based indices are the major source of empirical time-series data on commercial property returns. Appraisal based indices are constructed from a time-series valuation of several properties. Yearly or quarterly appraisers estimate the market value or the most likely transaction price of each property; consequently the indices have significant serial correlation in returns. It is well known that the indices are smoothed and have lower variance than transaction based indices according to Hoesle and MacGregor (2000). There are several reasons for this, among some are mentioned below:

1. It is harder for an appraiser to explain a larger than a small change in value. Appraisers often work for or are hired by investment managers that work for commission, based on how much the properties increase in value. This can make the properties overvalued when the prices are falling.
2. When valuing a property, the appraisers look at the latest estimated value of the property. They look at the value of the latest sold similar properties nearby, but also consider what have happened to the market since then. The uniqueness of every property makes it more difficult for the appraiser to trust the last transaction and they must therefore take several transactions into account. Another advantage of this is that it also gives a historical correlation. This make a certain lagging in the valuation and the whole index will be smoothed.

A common method of appraisal for a non-traded property is the use of comparable sales evidence says Brown and Matysiak (2000). Such an approach is backward-looking and may not reflect the current market information. Transaction based indices show greater volatility and less lagging than appraisal based according to Fisher, Miles and Webb (1999). To neutralise the serial correlation effects in appraisal based indices the Geltner filter; which is an inversed filter; is often used.

The US property market has one of the longest historical indices in the world. I have therefore chosen an appraisal based index from this market; the US property index goes back to 1984. The lack of historical return index from property investments is one reason why the investment is underweighted in many portfolios.

2.1.2 Forecasting property markets

Forecasting returns on the direct property market using econometric models started to be used in the mid 1980s in the US for a number of reasons according to Hoesle and MacGregor (2000). Some are mentioned below:

1. Major investors; using portfolio managing for their assets; needed property research to compare property assets to other assets. They also needed to find a link between the property market, the economy and the rest of the capital markets. When using cash flow analyses to calculate the value of the investment; the forecast of the rental growth is an essential requirement.
2. All the new results from research in economics and finance were used in the property market.

3. The use of computers enabled more econometric calculations.

Rent is very important when forecasting the property market and basic economic suggests that rent is determined by supply and demand. The difference in supply and demand can vary between countries, cities and even between different parts in one city. It can also be hard to forecast for a specific building since the building of a road can change the attractiveness of a building. Therefore the models become general and are better for large portfolios.

The demand for space is hard to measure, for example office space depends on the need of space for the employees and the profitability of the company. For example, in an economic boom when more people become employed the demand for office space increases. To attract people to work at the companies, the companies need to offer larger and better offices at good locations, which make the demand of office space and the rents to increase.

The supply increases as the space of office buildings increases; therefore the supply should be a function of new construction output. Building an office is a time demanding process and there is often a lag between when the demand for office space is at the peak and when the offices are built and ready to be let out. If the time is too long between these two parameters there is a risk that there is no demand for the new office spaces and the rents might then fall.

2.2 Indirect Property Investment

A more passive way of doing a property investment is to buy real estate shares on the stock market. There are many different real estate companies investing in various property types and using diverse degrees of leverage. The leveraged property company is a more risky investment compared to the un-leveraged direct property investment index.

2.2.1 Indirect Property Investment Index

An index covering the major real estates investment companies listed on the stock market.

2.2.2 Forecasting Indirect Property Investment Index

The index should be coupled with the underlying market, but the covariance matrix shows that it is more correlated to the stock market, see table 2. This may depend on when the stream of capital goes into the stock market it is spread widely over many different branches and when the capital flow goes out of the market all types of shares are sold. Short term it is more correlated to the stock market but in the long term it must be coupled to the underlying market, the direct property investments.

2.3 Equities

Equities (or stocks and shares) hold a higher risk than properties. The companies always have to pay rent before they pay dividends to their shareholders. If the company goes bankrupt all of the investment can be lost, but if an occupier goes bankrupt it is often possible to find another tenant for the property company. But over time equities have carried out the highest return among all different kinds of assets. The investment type is popular since it is easy to buy and sell. It is also possible to invest in many different branches without having to be

active in the companies. When owning a share you are not obliged to go and vote and no other work is expected of the share holder.

2.3.1 Equity index

Since the equities have such a high turn over, is traded at a central market place and are homogenous, it is easy to make indices of the return.

2.3.2 Forecasting stock market

There have been many people trying to forecast the stock market, with different methods. Selling when valuation is high has been a good rule. Also not owning equities during some months of the year has increased the historical return.

2.4 Bonds

A bond is a debt security, where the bond holders are contracted to receive interest (the coupon) and the principal at a later date. How late is determined by the maturity of the bond. A treasury bill has a fixed redemption price and has no coupons (zero coupon bonds), so the return on the Treasury bill is the difference between the market price when purchased and the redemption price according to Cuthbertson (1996). The size of outstanding bonds in the world in 2006 is estimated to be \$45 000 000 000 000 (\$45 trillion) as of 25 trillion on the US-market (Wikipedia, www.wikipedia.org, 2007-12-20).

Bonds and treasuries outstood by the government are often referred to as the risk free rate since the lack of credit risk. The markets are also well known because of the size and liquidity and are generally called the “bond market”. The bond markets do not often have a central market place like the stock markets, due to the large variety of different bonds and issuers.

2.4.1 Bond Index

The US-market had an average daily trading volume of nearly \$923 billion in the beginning of 2007 (Wikipedia, www.wikipedia.org, 2007-12-20). This large trading volume makes it very easy to construct a bond index.

2.5 Views to predict the future returns

To estimate the future returns for the assets that are studied in this master thesis; different historical indices of economic indicators can be used for example: office rents, office demand, office supply, inflation, GDP-growth, etc. The number of theoretical indicators could almost be infinite, but the marginal value growth for every new predictor sinks. Therefore a limited number of wide-ranging indicators are used to find views to predict the future returns. For example; since rents, demand for offices and supply for offices is a function of the companies profits these variables are replaced by S&P 12 months forward EPS. That is a measure of how the market think the S&P 500 companies' profits will change in the next 12 months.

By not using too many variables in the multiple regressions and instead being careful of which ones to use; the views become rather good.

3 Statistical modelling

Examining the historical data and modelling it to predict future returns can be done in many ways. In this work multiple regressions are used to find views to predict future returns. The Direct Property Investment data is only available in yearly returns but this master thesis demands monthly returns, therefore both a cubic spline interpolation and the Brownian Bridge is used to split the years into months. Geltner filtering is made to increase the standard deviation of the historical return from Direct Property Investment. Many of these models are described in Carter, Griffiths and Judge (2001).

3.1 Multiple regression

The purpose of regression is to learn more about the relationship between two or more predictor variables. The result of multiple regressions also describes how important every predictor variable is and how a regression equation can be formed. The equation makes it easy to construct a graph and predict a value. When more than one regression coefficients are used, it is called multiple regressions. The purpose of multiple regressions is to establish a quantitative relationship between a group of predictor variables and a response. This relationship is useful for:

1. Understanding which predictors has the greatest effect on the response.
2. Knowing the direction of the effect.
3. Using the model to predict future values.

In a two-dimensional space multiple regressions can not be visualised, but are easily computed. If the data is not perfectly predictable, the difference between the predicted value and the true value is called the residual.

When deciding which variables the outcome depends on, one way is to plug-in as many relevant variables as possible and remove those that are not statistically significant.

If there is a linear correlation the equation to best estimate the values looks as:

$$y_i = \alpha + \beta x_i + \varepsilon_i \quad (1)$$

where α is the point where the line intersects the y-axis; it is also called the intercept, β is the regression coefficient, a constant deciding the slope of the line, and ε_i is the deviation (error) from the line:

$$\varepsilon_i = y_i - f(x_i) \quad (2)$$

To solve the equation The Method of Least-Square solves equation (1) by:

$$Q(\alpha, \beta) = \varepsilon_1^2 + \varepsilon_2^2 \dots + \varepsilon_i^2 = (y_1 - f(x_1))^2 + \dots + (y_i - f(x_i))^2 \quad (3)$$

where Q is the sum of the squared deviation. The parameters are chosen such that $Q(\alpha, \beta)$ is minimized. Therefore:

$$\frac{\partial Q}{\partial \alpha} = -2 \sum_{i=1}^n (y_i - \alpha - \beta(x_i - \bar{x})); \quad (4)$$

$$\frac{\partial Q}{\partial \beta} = -2 \sum_{i=1}^n (x_i - \bar{x})(y_i - \alpha - \beta(x_i - \bar{x})) \quad (5)$$

These equal to zero gives:

$$\alpha^* = \bar{y} \quad (6)$$

$$\beta^* = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (7)$$

The coefficient of determination can in words be expressed as:

$$\text{Total variation} = \text{Explained variation} + \text{Unexplained variation}$$

The coefficient of determination is the ratio:

$$R^2 = \frac{\sum (m_i^* - \bar{y})^2}{\sum (y_i - \bar{y})^2} \quad (8)$$

and is in words expressed as:

$$\text{Explained variation/Total variation}$$

This means the ratio of variation that is explained by the model. In this master thesis multiple regressions are used to predict the future returns from the assets which are combined with the historical returns in the Black Litterman model. But besides of that the Black Litterman model also needs the investor's certainness of the predicted returns; called the confidence level which the coefficient of determination is used as.

3.2 Cubic Spline Interpolation

To get monthly returns; on the Direct Property Investment index; instead of yearly returns, cubic spline interpolation is used. This function creates piecewise continuous curves passing through each of the yearly values; and on the curves the monthly returns are set.

For each interval between the yearly returns a separate cubic polynomial with separate coefficients is created:

$$S_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i \text{ for } x \in [x_i, x_{i+1}] \quad (9)$$

all pieces together form $S(x)$, the spline.

3.3 The Brownian Bridge

The Brownian Bridge is another way of getting monthly returns from the yearly returns. It is a continuous time stochastic process where the probability distribution is generated by a Wiener process. The Brownian Bridge can be required to satisfy conditions at both ends of the interval, which in this case is the yearly returns; in between the functions have the restrictions to have a normal distribution and an expected value of zero. The Brownian Bridge stretches between time $T1$ and $T2$ where the function is equal to a respective b . The Brownian Bridge $W(T)$ at time T between $T1$ and $T2$ is defined:

$$W(T) = W(T1) + \frac{T - T1}{T2 - T1} (W(T2) - W(T1)) \quad (10)$$

3.4 The Geltner Filter

To overcome the problem, that the direct property index is smoothed and thereby has too low volatility and too high serial correlation, a reverse filter (Geltner 1992) is used. The Geltner filter is an AR(1) filter used to de-smooth appraisal based total return property indices. After applying this filter to the data; the data should have more similar properties to the real asset in reality. The Geltner filter here is from this formula:

$$r_t^* = ar_t^U + (1 - a)r_{t-1}^* = ar_t^U + (1 - a)r_{t-1}^U + a(1 - a)^2 r_{t-2}^U + a(1 - a)^3 r_{t-3}^U + \dots \quad (11)$$

This is easy inverted into:

$$r_t^U = \frac{(r_t^* - (1 - a)r_{t-1}^*)}{a} \quad (12)$$

where r_t^U is the unobservable underlying property market return at time t ,

r_t^* is the appraisal based property total return, and a is the corrected factor and is determined by the formula:

$$a = \frac{\text{var}[r_t]}{(\sigma_e^2 + \text{var}[r_t])} \quad (13)$$

In this work different a :s are tested to see which one gives the best asset allocation. It is widely said that Direct Property Investments have a standard deviation between the standard deviation of stocks and bonds or has half of the stocks' standard deviation.

4 Theory of Portfolio optimisation

When constructing a portfolio the investor always has to choose between risk and return and the optimisation is to maximise the return without increasing the risk. The Markowitz model has a high sensitivity on inputs, which leads to large fluctuations between assets in the optimal portfolio. There can sometimes be some extreme portfolio weights while other assets have no positions at all. Some assets with smaller market cap can get unreasonably large portfolio positions. The Markowitz portfolio optimises the risk adjusted return, but it takes no care of how the investors predict the future. Therefore, this model is not very often used in reality by portfolios managers according to Litterman and Black (1999).

The goal to all portfolios is to get more return and less risk. To be able to achieve a correct portfolio optimisation it is very important to have the correct historical figures of returns, standard deviation and correlation; especially in the Markowitz optimisation where these are the only inputs that are used. The Markowitz portfolio optimisation has another weakness as well; if the investor has his own view of how the market will move (as he often has) he has no use of this view; when optimising his portfolio using the Markowitz optimisation.

That is why Goldman Sachs wanted to create a new model to use when optimising portfolios, and succeeded through the Black Litterman model. This new model does not only have the historical returns and risks as inputs, but also it has the market capitalisation weights of the assets which gives the equilibrium returns and the model can also have as input the investors view that depending on how certain he is; it affects the estimated return different much. This makes it less sensitive to the historical data and is a useful quantitative tool for investors.

There are some assumptions about the market used in the theories and this master thesis; they are:

1. There are no transaction costs in the buying and selling of capital assets.
2. There are no income or capital gains taxes.
3. There is always someone that wants to buy or sell all assets; so the supply and demand is always infinite and in equilibrium.
4. Short sales of assets are not allowed

4.1 *The Markowitz Model*

Harry Max Markowitz won 1990 the Nobel Prize in Economics for his establishing of today's modern portfolio theory. Markowitz thought the old theories of portfolio allocation lacked analyse of the impact of risk. The result of Markowitz' work is the Markowitz Efficient Frontier which is the set of all portfolios that will give the highest expected return for each given level of risk. It is from the Markowitz Efficient Frontier that the Capital Asset Pricing Model (CAPM) is developed. (Cuthbertsson (1996) and Wikipedia)

Markowitz theories are based on the mean-variance model; which is based on some assumptions taken from West (2004):

1. Expected return and risk are the basis for the investors' decisions. The expected return and risk are measured by the mean and variance of the returns on various assets.
2. All investors have the same time horizon.
3. Information is freely and simultaneously available to all investors which make them all in agreement as to the parameters necessary, the parameters value (means, variance and correlations of returns for all assets).
4. Financial assets are arbitrarily fungible.

The Markowitz' investor only needs the mean return, the standard deviation and the covariance of the assets. With this information the idea is to mix two or more assets to maximise the expected portfolio return for a given level of standard deviation; the risk; on the portfolio. The risk is calculated by the formula:

$$\sigma_p^2 = \sum w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \rho_{ij} \sigma_i \sigma_j \quad \text{alternative} \quad \sigma_p^2 = w^T \Sigma w \quad (14)$$

and the expected return is the sum of all the portfolio assets' weighted return:

$$E[R_p] = \sum_{i=1}^n w_i E[R_i] \quad \text{alternative} \quad E[R_p] = w^T \mu \quad (15)$$

So by diversifying the portfolio into different uncorrelated assets the return can increase without increasing the risk. Theoretically the risk of an equally weighted asset portfolio becomes zero when the number of assets goes toward infinity if the assets are uncorrelated with each other. But if the assets are correlated the risk goes toward the average covariance; which is a measure of the undiversifiable market risk. The Markowitz Efficiently Portfolio is found by optimising the risk adjusted return for all possible standard deviations:

$$\frac{E[R_p]}{\sigma_p} \quad (16)$$

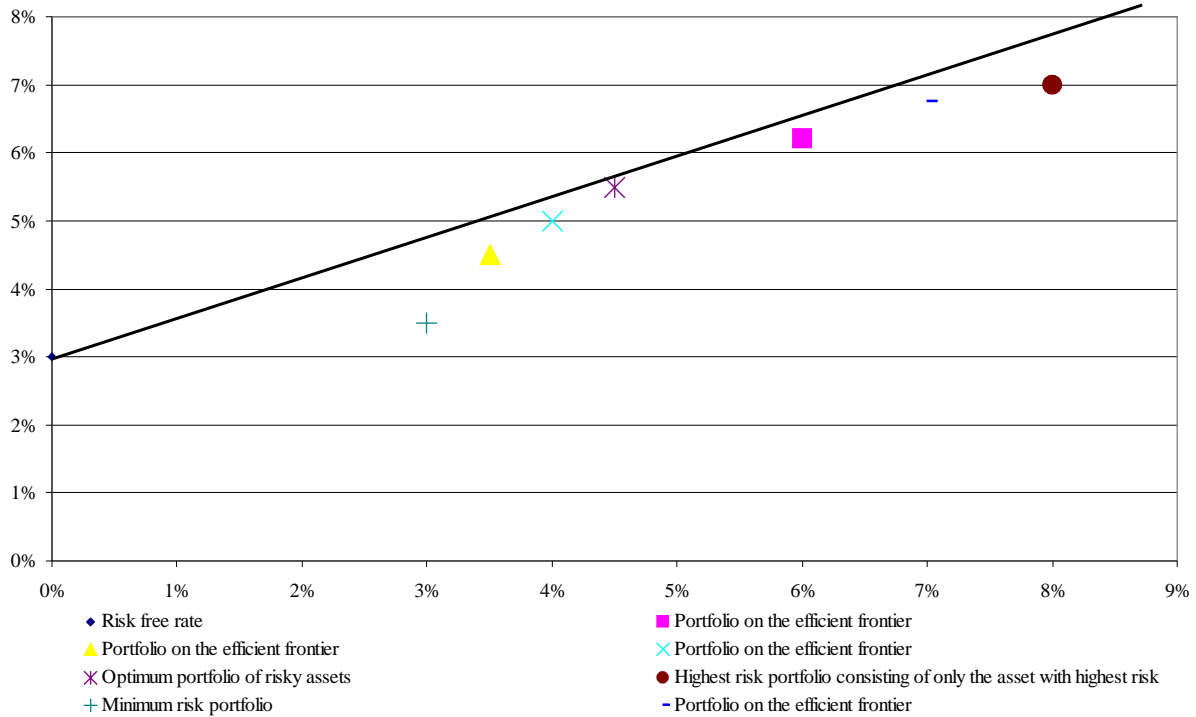


Figure 1. In the diagram examples of efficient portfolios are plotted with the return on the y-axis and the standard deviation on the x-axis.

In Figure 1, all the portfolios lie on the upper left side of the boundary of possible portfolios and are called the Markowitz Efficient Frontier. The frontier stretches between the minimum risk portfolio on the left, to the portfolio consisting of only the asset with the highest risk to the right. To evaluate an investment one can not only look at the return without looking at the risk taken to achieve that return. The Sharpe ratio is a measure of that; since it measures the return per unit of risk and is defined as:

$$sr_p = \frac{E[R_p] - r_f}{\sigma_p} \quad (17)$$

In the same way the Optimum Portfolio of Risky Assets (OPRA) is calculated. By drawing a straight line between the risk free asset and the OPRA one gets the Capital Market Line (CML). The CML represents how the investor at the most efficient way can invest depending on how much risk he is willing to take. The most careful investor buys only the risk free rate and his portfolio will only have the risk free rate as expected return. An investor that is more risk willing places himself more to the right on the CML, by mixing the OPRA with the risk free rate. A risk loving investor will lend to risk free rate and invest in the OPRA.

The serious lack in the model is that only historical returns are used as predictors. If the future won't be as the history the model delivers a poor result.

4.2 The Capital Asset Pricing Model

William Sharpe simplified Markowitz' portfolio model in 1964 to decrease the number of data requirements. The simplification of the model led to Sharpe receiving the Nobel Price in

Economics together with Harry Max Markowitz 1990. This model only demands the data of how the asset will move relative the market. The model is described with the formula:

$$R_i(t) = \alpha_i + \beta_i R(t) + e_i(t) \quad (18)$$

where

- $R_i(t)$ the return on asset i at time t
- α_i constant coefficient
- β_i sensitivity of the asset returns to the market returns
- $R(t)$ the return on the market, usually the arithmetic average of the historical returns on a market portfolio
- $e_i(t)$ a random variable, with the expected return zero and independent from $R(t)$

The value of the constants α_i and β_i can be found through regression analysis; but β_i can also be found through:

$$\beta_i = Cov(R_i, R_M) \quad (19)$$

The regression analysis also ensures that $e_i(t)$ will be uncorrelated to $R(t)$ and have a mean of zero and that $e_i(t)$ and $e_j(t)$ are independent. This gives the formula:

$$E[R_i(t)] = \alpha_i + \beta_i E[R(t)] \quad (20)$$

The formula implies that equities move only because of market moves, which is not the case in reality. That is why it is more common and generates better results when using a multifactor model. The variance for the single asset i is generated by:

$$\sigma_i^2 = \beta_i^2 \sigma^2(R) + \sigma^2(e_i) \quad (21)$$

For a portfolio with n number of assets; the expected return is calculated through:

$$E[R_p(t)] = \sum_{i=1}^n w_i \alpha_i + \sum_{i=1}^n \beta_i E[R(t)] \quad (22)$$

and the volatility of the portfolio P is given by:

$$\sigma^2(P) = \left(\sum_{i=1}^n w_i \beta_i \right)^2 \sigma^2(R) + \sum_{i=1}^n w_i^2 \sigma^2(e_i) = w' \Sigma w \quad (23)$$

where Σ is the $n \times n$ covariance matrix of the n assets. The optimum asset portfolio is calculated in the same way as Markowitz; where the risk adjusted return is maximised. In this master thesis the market is not defined; so instead of using the market return, the return from the other assets are used together with other variables to form a multifactor model to find the best way of estimating the expected return.

4.3 The Black Litterman model

The Black Litterman model was created by Fischer Black and Robert Litterman of Goldman Sachs & Company (1992). It is used to overcome the unintuitive, highly concentrated and input sensitivity portfolio, when using the Markowitz model. The Black Litterman model uses the market equilibrium portfolio as a basis and then lets the investor have his own view to tilt the portfolio towards certain assets. The more bullish and the more confident of a view the investor becomes; the more the weight of that asset will increase in the portfolio.

The market equilibrium occurs when prices have been adjusted to the expected returns of all assets (equilibrium returns); such that all investors hold the same belief and the demand for these assets will exactly equal the outstanding supply.

In this master thesis so few assets are used and the market capitalisation of these assets is so large that it will not be any problem even for the largest investors to shift their portfolios between the assets. Not all investors have the same investment universe or views and therefore their portfolios will not look the same.

The Market equilibrium portfolio means a portfolio with the assets weighted according to their market capitalisation value and all investors have the same expected returns which make them not wanting to trade assets. One of the weaknesses of the Markowitz model is that assets with a small market capitalisation can get an unrealistic large position in a portfolio and this is avoided by the market equilibrium. In this master thesis the Market portfolio is replaced with the historical mean-variance data as input.

The investor does not have to have a view; he can have a relative view (stocks will return 3% more than bonds) and he can have an absolute view (stocks will have a total return of 5%). Some assets can have a view and some can be without, it is all up to the investor. Depending on how sure the investor is on his view, the model will tilt the portfolio after his view. The investor's certainty of the view is known as the confidence level in the model. In this master thesis multiple regressions analyses have been used to estimate the expected returns; which are set to absolute views in the Black Litterman model. The absolute views of returns change the returns and weights for all the assets compared with the market portfolio. The same thing happens to the assets that are correlated with those assets with views.

The master Black-Litterman formula is:

$$E(R) = \left[(\tau \Sigma)^{-1} + P^T \Omega^{-1} P \right]^{-1} \cdot \left[(\tau \Sigma)^{-1} \Pi + P^T \Omega^{-1} Q \right] \quad (24)$$

where k is the number of views and n is the number of assets with $k \leq n$ and

$E(R)$	=nx1 new combined return vector, unknown
τ	=1x1 Scaling parameter that measures confidence in benchmark
Ω	=kxk diagonal covariance matrix of errors terms in expressed views representing the confidence in each view
P	=kxn matrix relating views to expected returns
Σ	=kxk covariance of estimated views
Π	=nx1 vector of implied equilibrium returns
Q	=kx1 vector of absolute / relative return expectations

Return estimates differing from the strategic equilibrium returns are an essential input to the Black Litterman estimation process and are called views. There can be absolute return expectations for individual assets and there can be relative expectations relating a number of assets or aggregates of assets. The formal constraint is that the number of views should be equal to or less than the number of assets. Every view has to be assigned a quantification of confidence and if there is no analysis available for some assets, those views can be restricted.

When there are no estimates the equation gives

$$P = 0 \Rightarrow E(R) = \Pi \quad (25)$$

which gives the Black Litterman model the same returns as the equilibrium returns. When there are no estimation errors the equation gives

$$\Omega^{-1} \rightarrow \infty \Rightarrow E(R) = (P^T \Omega P)^{-1} (P^T \Omega Q) \quad (26)$$

which gives Black Litterman-returns equal view returns. The relative view can be formulated as:

The stock market will outperform bonds by 5% with a confidence of 90%.

An absolute view can be formulated as:

The stock market will do better than stated by equilibrium return, 7% instead of 5%, with 80% confidence.

The equilibrium return vector is given by the CAPM-formula:

$$\Pi = \delta \sum w_{MCap} \quad (27)$$

where w_{MCap} is the asset's market capitalisation weight and δ is the risk aversion parameter representing the world average risk tolerance, calculated through:

$$\delta = \frac{r_M - r_f}{\sigma_M} \quad (28)$$

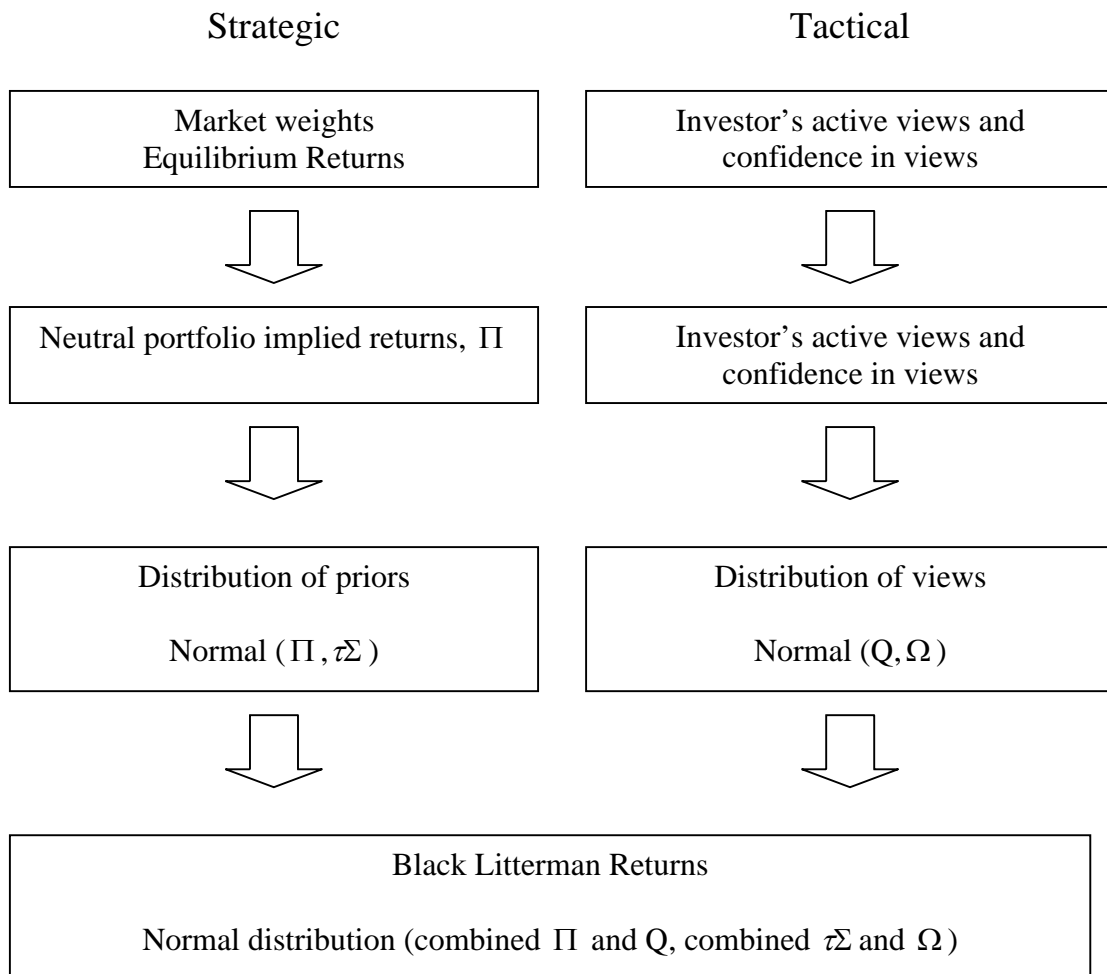
The equilibrium return vector makes all investors satisfied with their allocations and gives a neutral starting point where no one wants to reallocate.

But in this master thesis equilibrium returns are not used because of several reasons:

1. The combination of the four assets that are used in the master thesis; have no benchmark that can provide historical return or standard deviation
2. The equilibrium returns do not minimise the risk adjusted return; since this work is to find the most efficient portfolio it must minimise the risk adjusted return.
3. All the assets have so large capital market value that there is no problem for any large investor to swift his entire portfolio into one asset.

4. All investors have not the same expected return or information; which is needed for the equilibrium returns.
5. All investors do not have the same investment universe; some investors must be in certain assets, some can choose between much more assets; this makes the Implied Equilibrium Returns incorrect when using the market capitalisation weights. It is hard to say if the equilibrium portfolio weights should be equal to the portfolios owned by investors that this master thesis turns to or simply the market capitalisation of the assets.
6. The portfolio weights do not get extreme enough, which sometimes is needed for the highest risk adjusted return. Meanwhile, there are some constraints since it is impossible to go short in Direct Property Investments in reality, therefore no assets can be shortened. During some periods assets can be in a falling trend and then no investor should be investing in that asset.
7. In this work the investment universe consists of so few assets that large positions are needed sometimes to get the best risk adjusted return.
8. The mean-variance model is sensitive to data input but in this master thesis the calculations are on monthly basis and over several years time so the number of data points is large to spread the faults on.

Instead, the more traditional mean-variance model has been used to calculate the neutral weights in the portfolio. This gives the portfolios more extreme weights to some assets during some times, while it also maximises the risk adjusted return according to the historical data.



To combine the strategic return with the tactical return estimation; the Black-Litterman model uses the Bayesian probability theory. Thomas Bayes' theorem is applied in the probability theory and is used to determine the probability of a result given the probability of another result. The probability π for θ given X can be written as:

$$\pi(\theta|X) = \frac{\pi(\theta, X)}{\pi(X)} = \frac{\pi(X, \theta)\pi(\theta)}{\pi(X)} \quad (29)$$

and is called the Bayesian probability.

The Black Litterman model has been approved by the academic world, but is anyhow not used by investors. For an investor it is not always easy to define expected returns or relative expected returns, often they focus on small segments of their potential universe and there the investors who buy assets that they feel are undervalued or assets in positive momentum. Also investors do not balance expected returns against the contribution to portfolio risk, instead they think of weights in the portfolio; according to He Guangliang and Litterman Robert (1999). The Black Litterman solves one problem but creates another and both academics and investors understand this.

5 Presentation of data

The data comes from different sources; National Real Estate IF (NREIF), Merrill Lynch and Standard & Poors. The data is transformed from indices into monthly returns. The returns from Direct Property Investment were originally yearly returns, but through spline interpolation the returns have been changed into monthly returns. The returns from Direct Property Investment here are from appraisal based returns and are thereby smoothed and have too low volatility and too high serial correlation. To overcome this problem a Geltner filter is used. To determine what variance to apply, the first one used is the mean of the standard deviation between shares (S&P) and bonds (5-7 years maturity Treasury Bills); the other standard deviation that is tested is half of shares' standard deviation.

When data are all set; multiple regressions are made to predict future returns to use in the Black Litterman optimisation. When new returns are estimated, portfolios are calculated through maximising risk adjusted returns.

5.1 Presentation of data

The returns are from January 1984 to December 2005 in Figure 2, consists of 264 monthly returns.

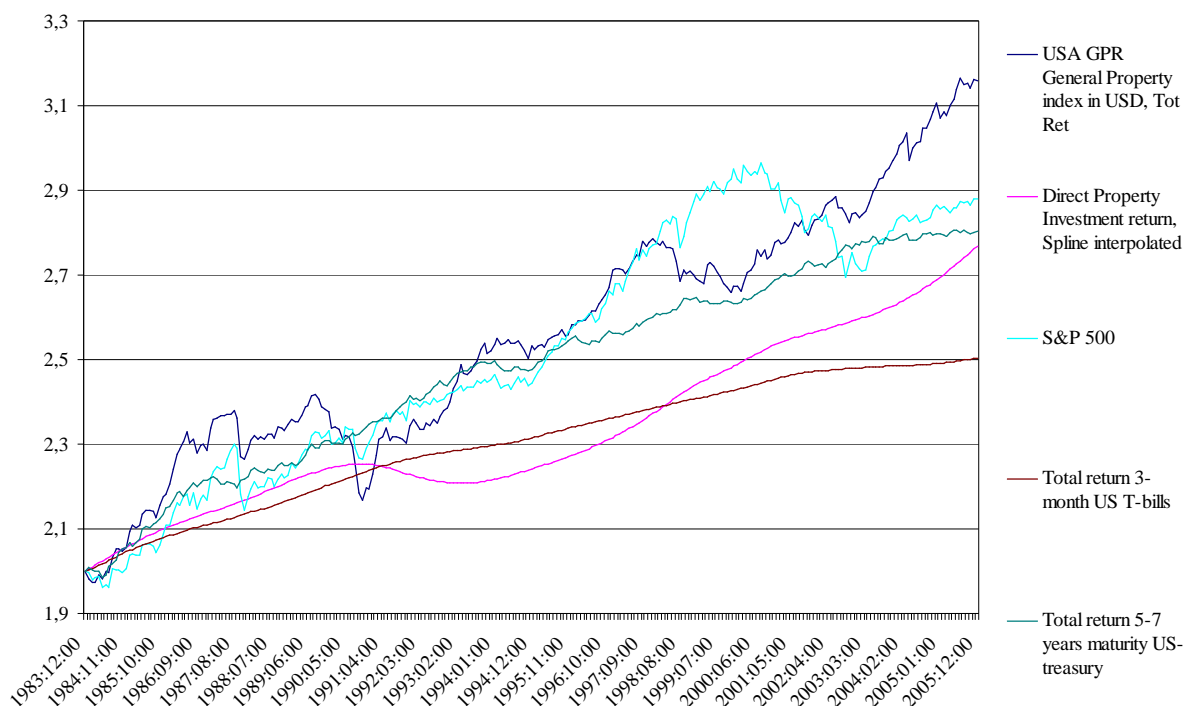


Figure 2. The figure shows a log arithmetic diagram of the total return indices from all the different assets used in the master thesis.

As the risk free rate in this Master Thesis the total return from 3-month T-bills is used. And as a bond asset and a more risky asset a benchmark from Merrill Lynch of total return from US-treasuries with a maturity between 5-7 years is used. The securities are swapped approximately every third month to fit the benchmark.

The Indirect Property Investment return comes from Global Property Research (GPR)-index. This index had in September 1998 based the return on 128 real estate securities with a market capitalisation of around \$118 billions, according to Hoesle and MacGregor (2000).

The Direct Property Investment returns come from National Council of Real Estate Investment Fiduciaries (NCREIF). This organisation was started in 1982 when it started to measure the yearly total appraisal based return from un-leveraged property investments.

	Standard deviation	Mean monthly Total Return
Total return 3-month US T-bills	0,00%	0,37%
Total return S&P 500	4,08%	0,78%
Total return 5-7 years maturity US-treasury	1,39%	0,63%
Total return USA GPR General Property index in USD	4,16%	1,00%
Tot ret Geltner mean of S&P and Bonds, Direct Prop Inve, BB	2,74%	0,69%

Table 1. The table shows the mean monthly total return and the mean standard deviation of all the assets used in this master thesis. After the Geltner filter the Direct Property Investment the asset has a standard deviation between stocks and bonds.

The stock market is represented by the S&P 500 index, introduced 1957; an index covering 500 large cap corporations and is the most notable index owned by Standard & Poor's. The companies included in the index are selected through a committee so the index represents different industries in the US-economy.

All the assets have varying properties of mean total return and standard deviation. Among the assets Indirect Property Investment has the highest return over time and risk free rate the lowest return; as it should be; which is shown in table 1. Before the Geltner filter; Direct Property Investment has a low standard deviation since it is an appraisal based index, shown in Table 2.

	USA GPR General Property index in USD, Tot Ret	Geltner filtered Direct Prop Inve Tot Ret, BB	S&P 500, Tot Ret	Total return 5-7 years maturity US-treasury
USA GPR General Property index in USD, Tot Ret	0,21%	-0,01%	0,21%	0,02%
Geltner filtered Direct Prop Inve Tot Ret, BB	-0,01%	0,05%	-0,01%	0,00%
S&P 500, Tot Ret	0,21%	-0,01%	0,27%	0,02%
Total return 5-7 years maturity US-treasury	0,02%	0,00%	0,02%	0,04%

Table 2. A co-variance matrix for the first 60 months of the analysed time is shown in this table. It is easy to see how low or sometimes negative correlation/covariance the Geltner filtered Direct Property Investment generated through the Brownian Bridge has to the other

assets. The table also shows the high variance Indirect Property Investments and S&P 500 have as well as these assets high correlation/covariance to each other.

5.1.1 Getting monthly returns through Spline Interpolation

All data shows monthly returns, except the Direct Investments in real estate. To un-smooth the yearly data; a cubic spline interpolation is used to receive monthly data. Matlab was used to get the data converted into monthly returns. As a result the variance on the monthly returns became artificially low. The diagram in Figure 3, shows the green line that is calculated between the old data points that is showed by a blue ring.

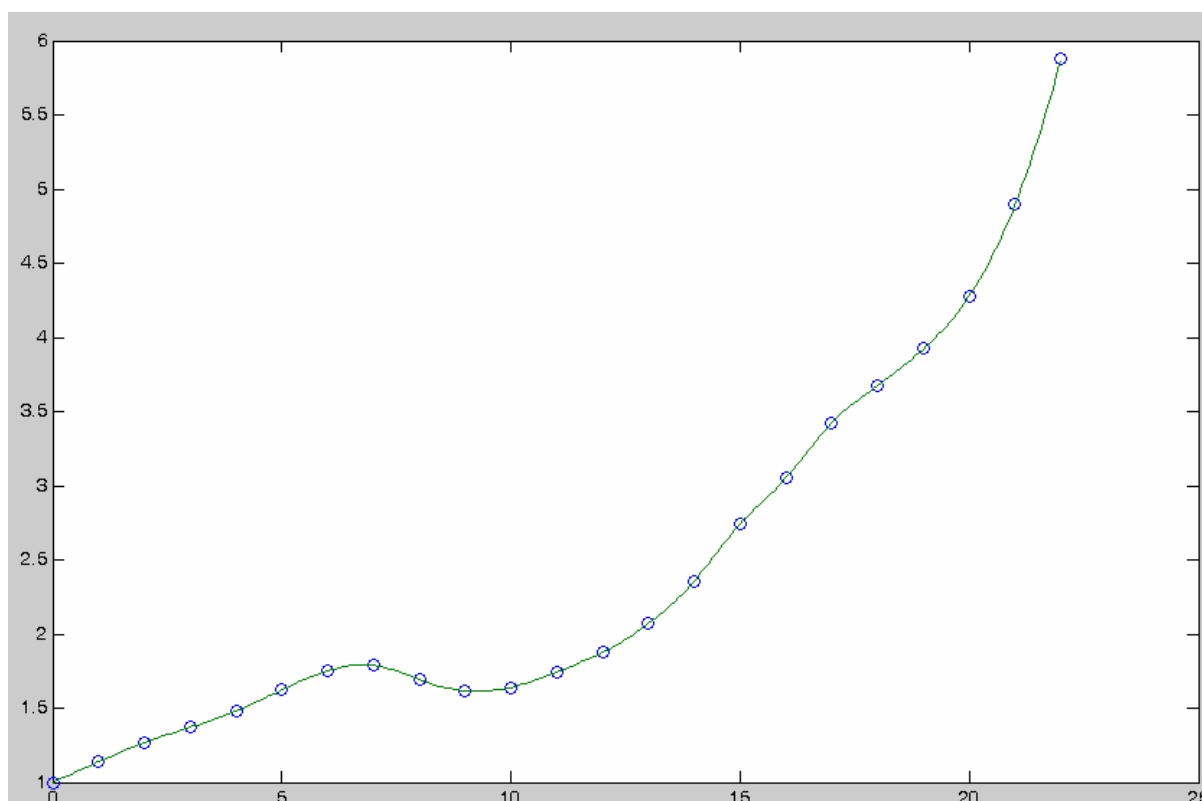


Figure 3. The figure shows the total return from Direct Property Investment Index on the y-axis and time on the x-axis. The blue rings are the yearly return index and the green line is the monthly returns smoothed by a cubic spline interpolation.

5.1.2 Increasing the risk on the Direct Property Investment Return

Since the index is appraisal based and after the spline interpolation the index gets an artificially low volatility. The asset is not this safe to invest into in reality as the data shows; therefore the risk must be increased. This is done by the Geltner filter; which gives the asset a risk more similar to the reality. Which standard deviation the asset should have is not easy to decide, one common appreciation is to use the mean between bonds and stocks according to Hoesli, Lekander and Witkiewicz (2004). This is the first value that is tested in this master thesis; which gave an a of $-0,0099$; the filter also lowered the mean monthly return. Another common approach is to assume that Direct Property Investments Return should have half of

the standard deviation of stocks' return, in this master thesis the S&P500 return (Geltner, 1993).

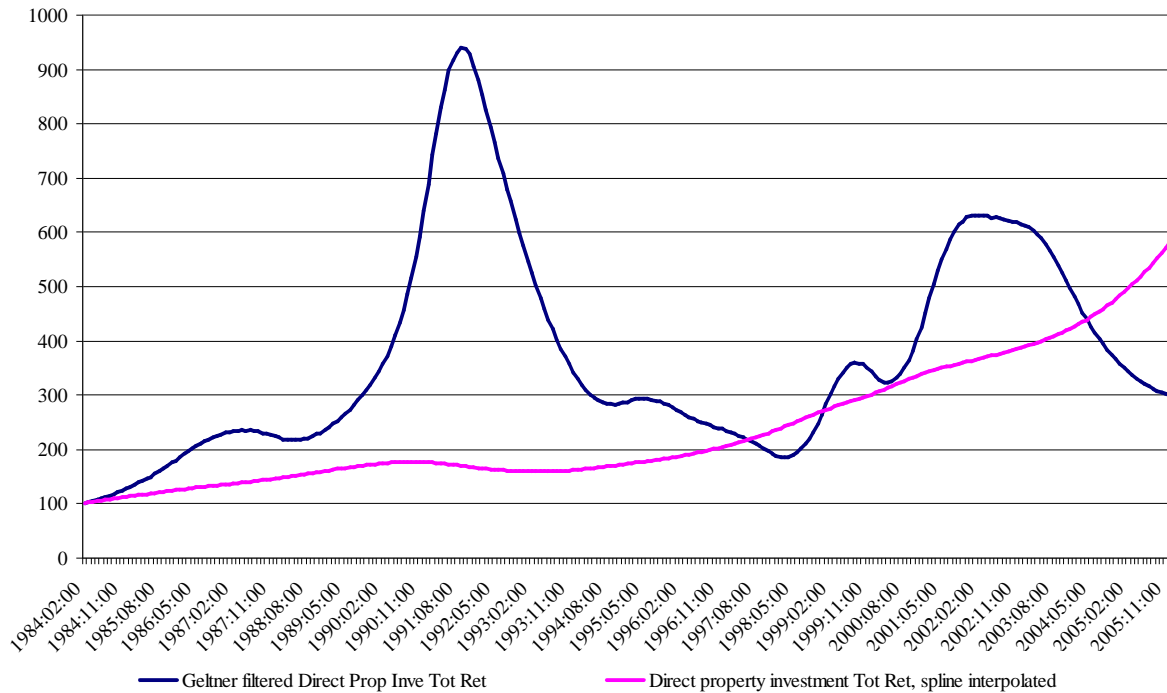


Figure 4. The pink line shows the return on the Direct Property Investment spline interpolated. The blue line is the same asset but with increased risk through the Geltner-filter; now the standard deviation is between S&P500 and Bonds; which gave an α of -0,0099. Between 1989 and 1992 there is an extreme total return; not like the reality at all.

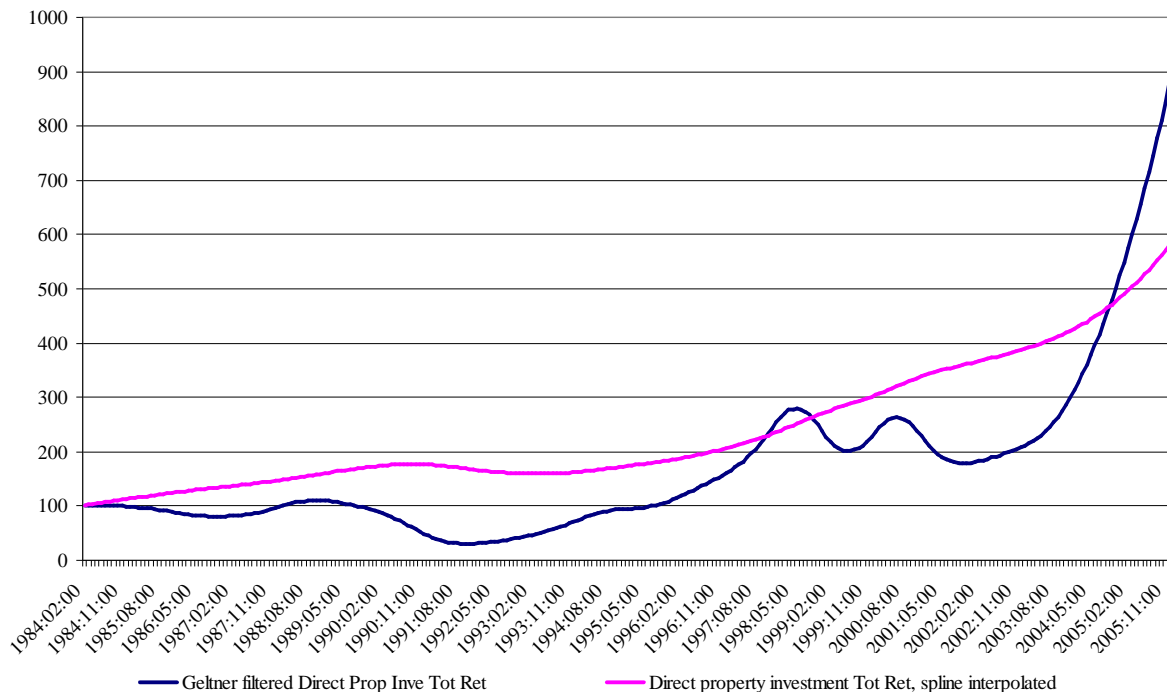


Figure 5. Here $a=0,010$ which also gives the same standard deviation as the one above but the total returns looks a bit different. The last two years of the index the total return gives a very high return.

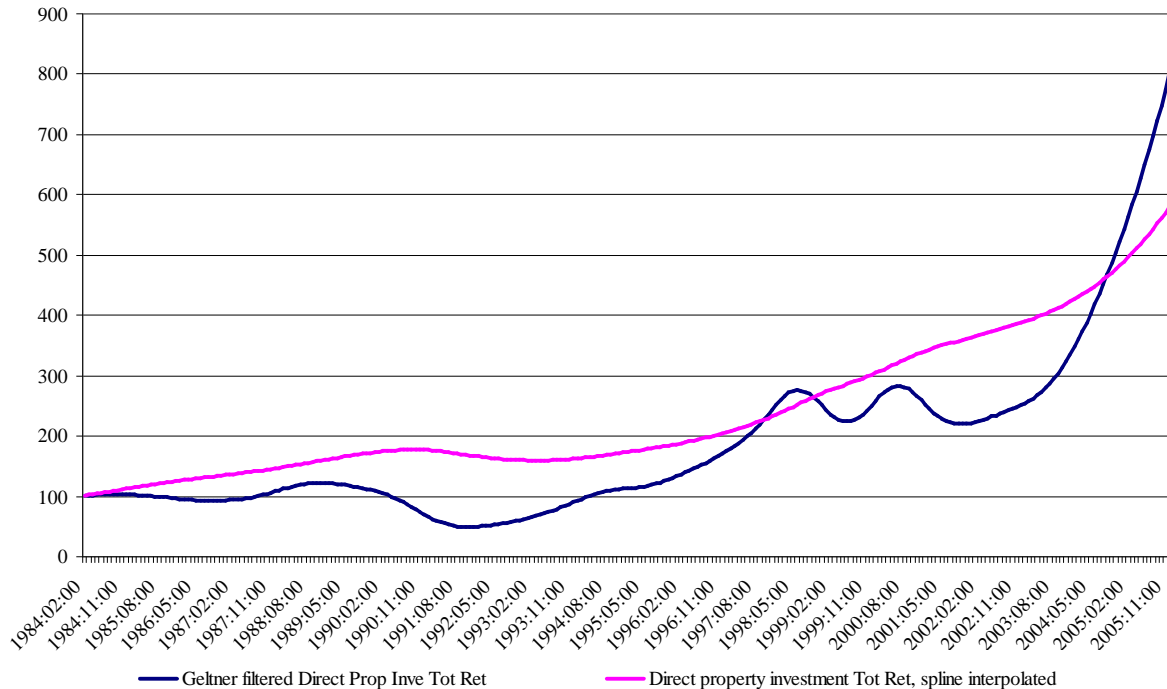


Figure 6. Here the Direct Property Investment Return has a standard deviation that is 50% of the S&P500's; which is lower than the mean between S&P500 and Bonds. The total return gets a bit lower than before, but the two last year's return is still extreme compared to the reality.

As shown in Figure 4, 5 and 6 the spline interpolation returned an index a bit too smooth and when raising the variance of the index through the Geltner filter it computed extreme total returns not similar to the reality at all. Therefore this would not give the correct properties to the asset direct property investment and the spline interpolation can not be used. Instead the Brownian Bridge is applied to receive monthly returns that are more similar to the reality.

5.1.3 Getting monthly returns through the Brownian Bridge

Here all the actual yearly total returns are used as restrictions and the monthly returns between the yearly returns are calculated through the Brownian Bridge function in Matlab, see Figure 7.

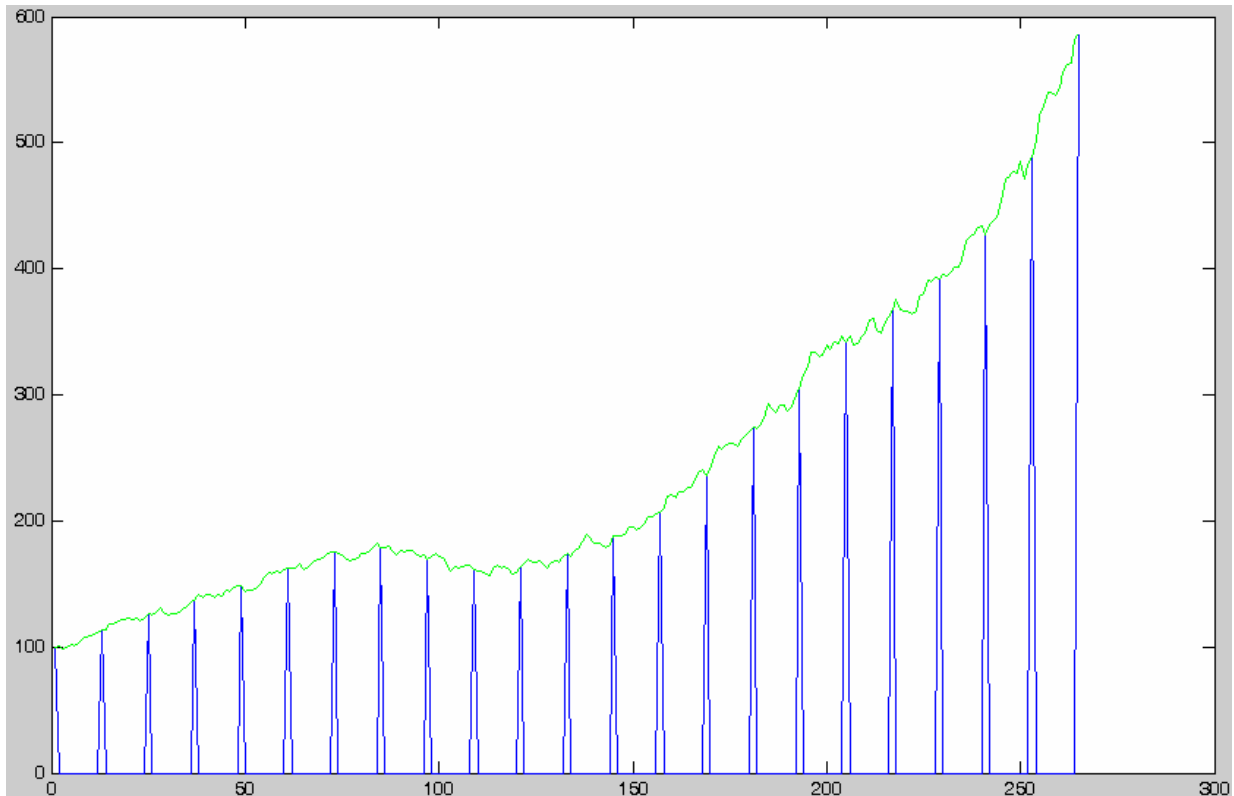


Figure 7 Plot from Matlab showing the Direct Property Return index through the blue staples. The green line is the Brown Bridge function that through a random way calculates the monthly returns between the yearly returns.

This does not make the index as smooth as when using the spline interpolation, see Figure 8. The total return for the whole period is equal to the real one from the appraisal based index.

5.1.4 Raising the risk on the Direct Property Investment Return

The Brownian Bridge had a higher standard deviation compared to the spline interpolation but still a bit too low and was therefore raised through the Geltner filter; first to the mean between S&P 500's and bond's standard deviation and thereafter to half of the S&P 500's standard deviation.



Figure 8 This diagram shows the index of monthly total returns of the Direct Property Investment asset. It is easy to see how smooth the spline interpolated index is compared to the Brownian Bridge function.

5.2 The Multiple Regressions

Predicting the future is never easy not even with multiple regressions. The multiple regressions' results are used as views in the Black Litterman optimisation and the coefficient of determination is used as a measure of the confidence for each view.

The results varied depending on which assets that were analysed. Assets that have high volatility are often harder to predict so it was not a surprise that the return from S&P 500 had the lowest coefficient of determination. The multiple regressions for every month were based only on the last 60 months to get dynamical variables.

5.2.1 Ideas for regression variables

The return from an investment shifts in cycles. Sometimes an investment has a high return while another has a low and then they might shift. The reason why the return is high for a specific investment is not always easy to understand and hard to predict. Looking in the mirror trying to explain the cycles is not straightforward. By looking at Figure 2 some conclusions are made about the correlations between the investments when they have high or low return. Below are some explanations of the correlations of when the return is high or low:

1. When equities have high valuations and thereby the P/E is on high levels; the return on property investments has been low. I think the explanation is that when the PE-ratio is on high levels there is an expected growth in the earnings of companies. Then, investors prefer to be in high growing sectors and not the property sector. For example, during 2000

the stock market had a very high valuation compared to earnings and investors preferred fast growing IT-companies instead of property companies. It is seldom a good timing buying stocks when the valuation is on a high level; either the stocks should go down or the profits up. Therefore S&P 12 months forward P/E is one variable in the regression.

2. When corporate earnings decline the returns from Direct Property Investments should also drop. I think the reason is that when the management of a company starts to see that its future earnings might decrease, they become more careful and do not expand or look for bigger offices. This makes the rents go flat or down and the vacancy rate might increase. Lower rents and higher vacancies lowers the return from real estates in two ways; the yield from the investment sinks and also the value of the property goes down. But since the return from stocks should go down even more, Direct Property Investment can attract some investors and capital flow to the asset can increase the return. The return from stocks should also decline with falling profits. So therefore S&P 12 months forward earnings is one variable in the regression.
3. Trend following, if the asset had a positive return the month before, it is more likely the return will be positive the month after. The up going cycles are often between 5-7 years and the corrections are often shorter. Because of that the return on the asset the month before works like one variable.
4. The flow of capital is important to follow for an investor. It is important since it determines a large part of the return. Investing before a large flow into an asset comes can pay off well. Thus all months have been taken as variables in the regression.
5. The total return on the risk free rate; 3 months US T-bills is very important for the return from other assets. Investors often use this as a base value for the valuation of other assets.

5.2.2 The Regression Results

The last 60 months returns were used in the multiple regressions and for each new month the input were moved, so the first 5 years of the historical data were only used as inputs for the first regressions. Inputs in the multiple regressions were:

1. The returns from the other assets in the portfolio the month before
2. The return from the same asset the month before
3. The return on 3 months Treasury Bill (risk free rate)
4. S&P 12 months forward P/E
5. S&P 12 months forward EPS
6. Which month in the year it was

In the multiple regressions the two least significant variables were removed for all the assets:

- For bonds they were the S&P 12 months forward EPS and the returns in August month.

- For S&P 500 they were the months May and June.
- For Indirect Property Investments they were the S&P 500 and the month July.
- For Direct Property Investments they were S&P 500 and Indirect Property Investments.

The high variance on the S&P 500 returns made it to the worst asset to predict and S&P 500 was also among the least significant variables in the regressions for the other assets. Indirect Property Investments was the asset with the highest mean on the coefficient of determination, almost 38%; Bonds had the second highest mean, 36%; Direct Property Investment 34% and S&P only 31%. When Direct Property was not used as a variable in the regression to predict the other assets' future return; the outcome of the mean of the coefficient of determination was a bit lower; Indirect Property Investment, 37%; Bonds, 33% and S&P 29%.

The coefficient of determination explains the ratio of variation that is explained by the model and was used in the Black-Litterman optimisation as confidence of the views. The results from all the variables were:

- 1 Direct Property Investment return had the highest negative correlation with S&P 12 months forward P/E; thereafter the total return from S&P. So in times when stocks have a high valuation it has not been a good time to investment in stocks nor directly into properties.
- 2 Bonds had surprisingly such a low significant correlation with S&P 12 months forward earnings that it was not used as a variable in the multiple regressions. Direct Property Investment returns had a negative correlation with S&P 12 months forward EPS. This might depend on when company earnings decline Direct Property Investments are considered a safe investment and have a low correlation to company earnings. The stock market also had a negative correlation to the S&P 12 months forward EPS but is not statistically significant.
- 3 All assets except stocks have a positive correlation with the return the month before. Stocks may be too volatile and have too low statistical significance to apply this relationship.
- 4 This method really works in the multiple regressions. It is easy to see that investors should not own equities during July and August, these months have historically a negative mean return; this time during the year many investors are on a holiday and only a few people working decides the prices on the stocks. This time of year investors also starts to worry about the corporate profits in the third quarter. Owning stocks November to April is good for the return on the portfolio. The property equities have bad performance in September. The multiple regressions also says that one should not own bonds during January to February, maybe investors move capital from bonds to stocks during this time and the capital flow explains the difference in returns. This relationship is also seen in the negative correlation between returns from S&P and bonds.

- 5 Risk free rate has a positive correlation with all the assets except the Indirect Property Investment return. Why is hard to explain, maybe short-term investors see the connection between listed property companies' higher funding rates and higher costs; which lowers the return.

The multiple regressions' results were used in the Black-Litterman optimisation as views. As a market view for one month; the last 60 months historical returns were used. The 60 months average returns were moved one step for every new month. Since no parameters are constant over time they were renewed to be as new as possible.

6 The portfolio Optimisation

The general way to optimise an asset portfolio is to use the historical return and standard deviation to maximise the risk adjusted return. That is made in this Master Thesis as well; but that portfolio is compared to a portfolio that combined expected returns with the historical returns. The outcome is two different portfolios with different asset allocations, returns and risks. First the portfolios generated by the Markowitz model are analysed and thereafter the portfolios generated by the Black-Litterman portfolio. For the two models different standard deviations on the Direct Property Investment is tested and also a portfolio without Direct Property Investment.

6.1 The Markowitz Model

Here are the means from the last 60 months of returns, co variances and volatility used to generate the optimal portfolio, shown in Figure 9, 10 and 11. The use of Geltner filter and 60 months' returns shortens the number of data/time to valuate; the outcome goes down from 264 to 200 data points/months.

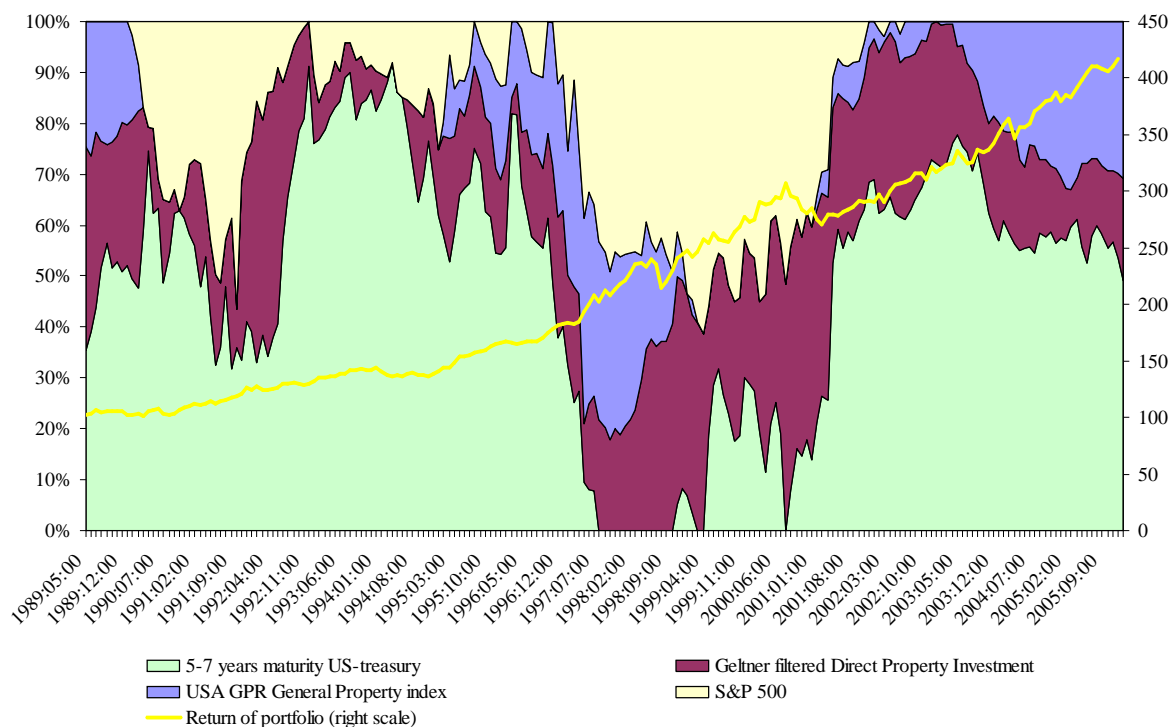


Figure 9. The diagram shows the asset allocation of a portfolio generated by the Markowitz model where the asset Direct Property Investment is generated by the Brownian Bridge and thereafter Geltner-filtered to reach a standard deviation between S&P 500 and bonds. The asset allocation varies over time, but the 5-7 years maturity US-treasury is the largest portfolio asset. During this time the portfolio generates a return about four times the money. The largest drop in value is between August 2000 and March 2001 the portfolio lost 12% in value, at the same time the S&P 500 lost twice as much.

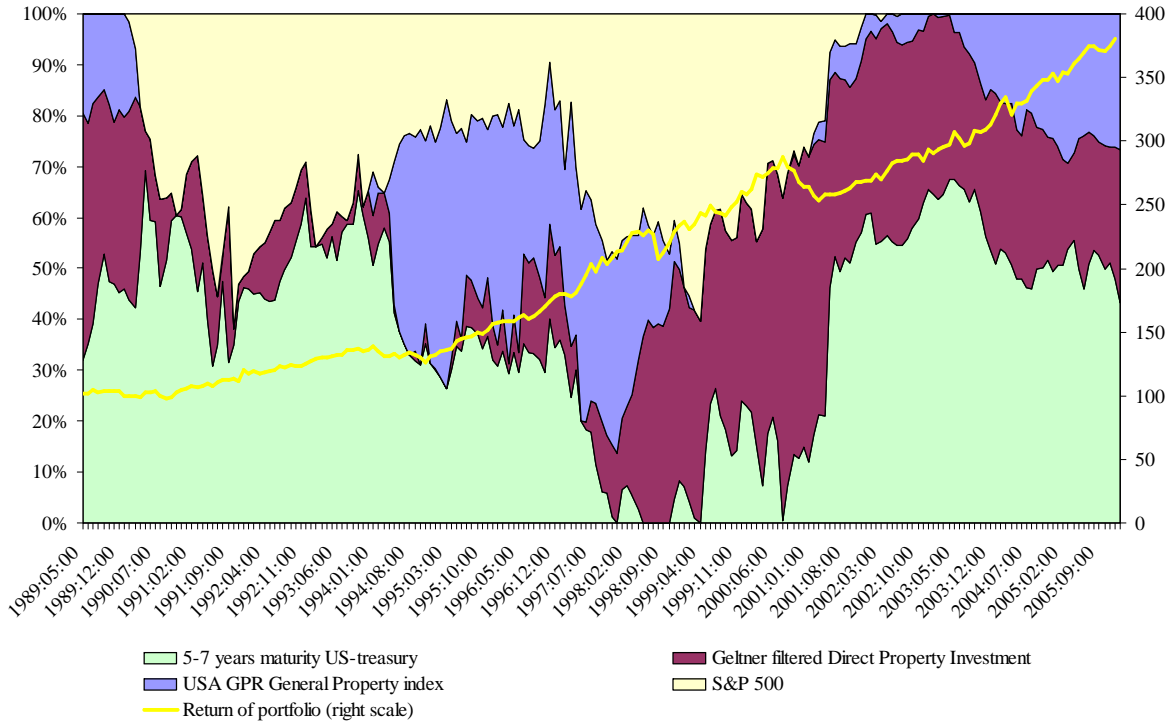


Figure 10. This diagram shows the same generated Markowitz portfolio as above but the asset Direct Property Investment has a lower standard deviation; half of the S&P 500's.

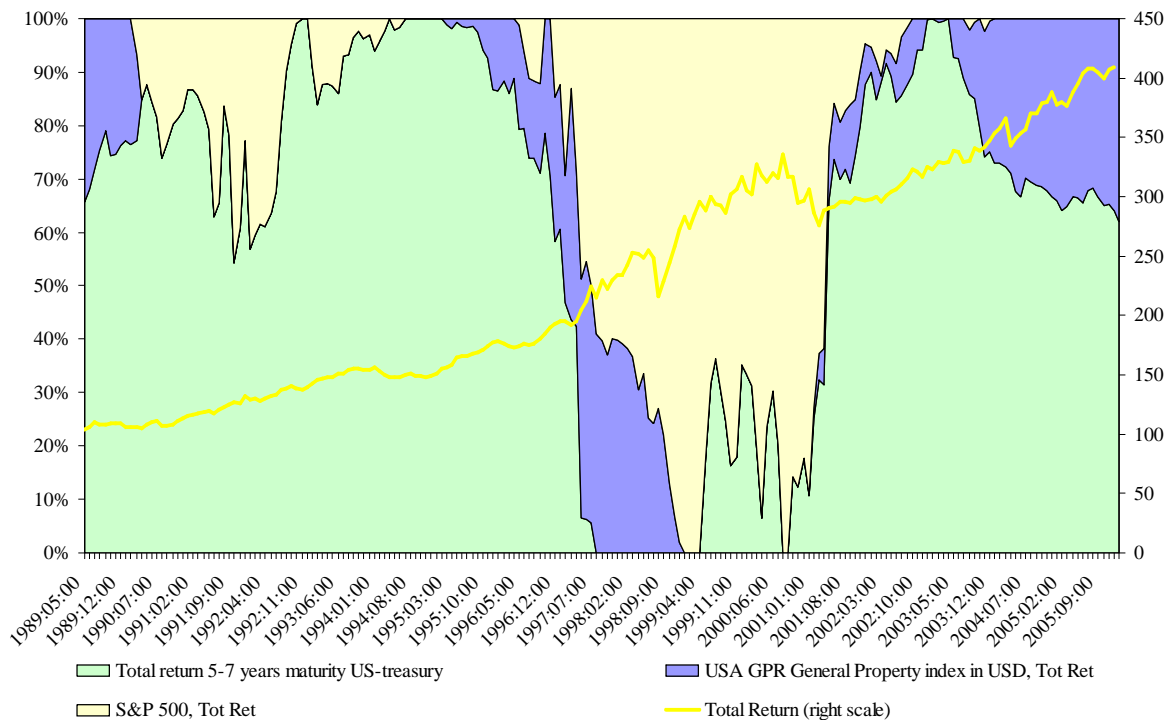


Figure 11. The diagram shows the asset allocation and total return for a Markowitz portfolio without the asset Direct Property Investment. The worst performance has the portfolio between August 2000 and March 2001; the portfolio drops 18% because of a 24% drop in the S&P500 during the same time. S&P continue to fall and not until September 2003 the index

reaches the bottom; but the portfolio reallocates before that so the drop in value limits. The S&P dropped 47% from the top to the bottom.

6.2 Black Litterman Optimisation

Here the multiple regressions result comes in use as the views and are combined with the historical mean returns to generate the optimal portfolio, shown in Figure 12, 13 and 14.

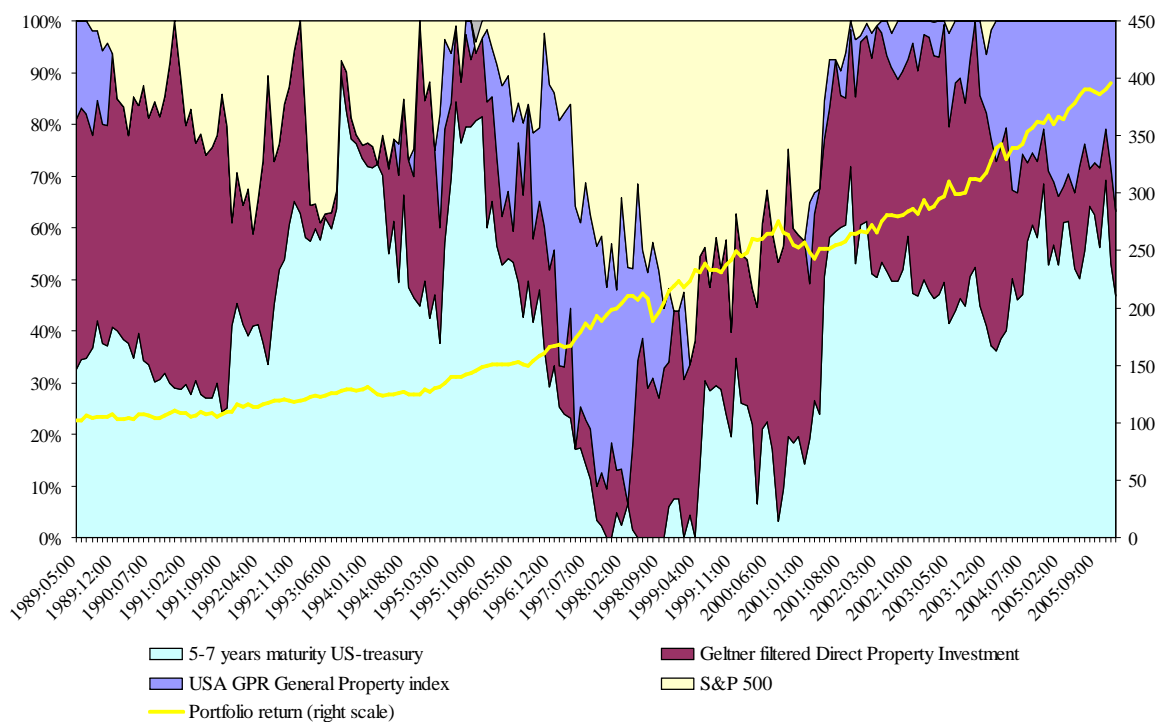


Figure 12. The portfolio from the Black Litterman optimisation shows a more varying asset allocation. The Direct Property Investment has a Geltner-Filter generated standard deviation equal to the mean between the S&P 500 and bonds. Between August 2000 and March 2001 the portfolio had its largest and longest drop in value. At the worst; the portfolio had lost 12% in value.

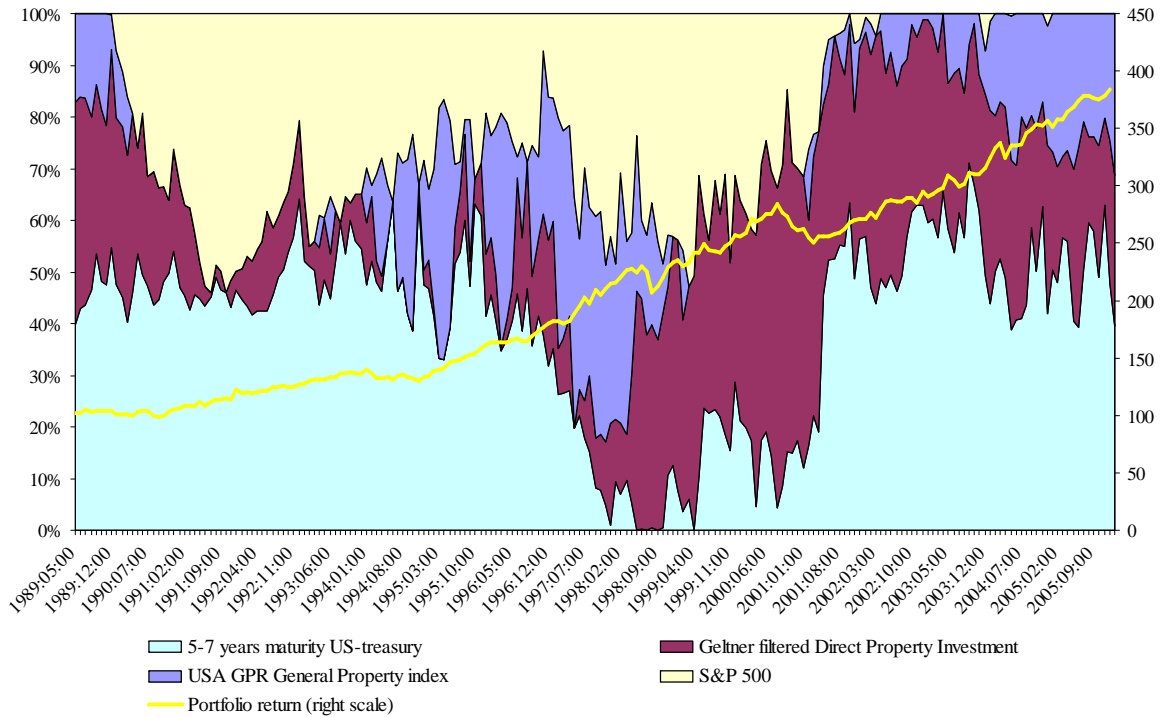


Figure 13. When using a lower standard deviation on the Direct Property Investment -half of the S&P 500's; the outcome gets a bit different. The total return does not get as high as before.

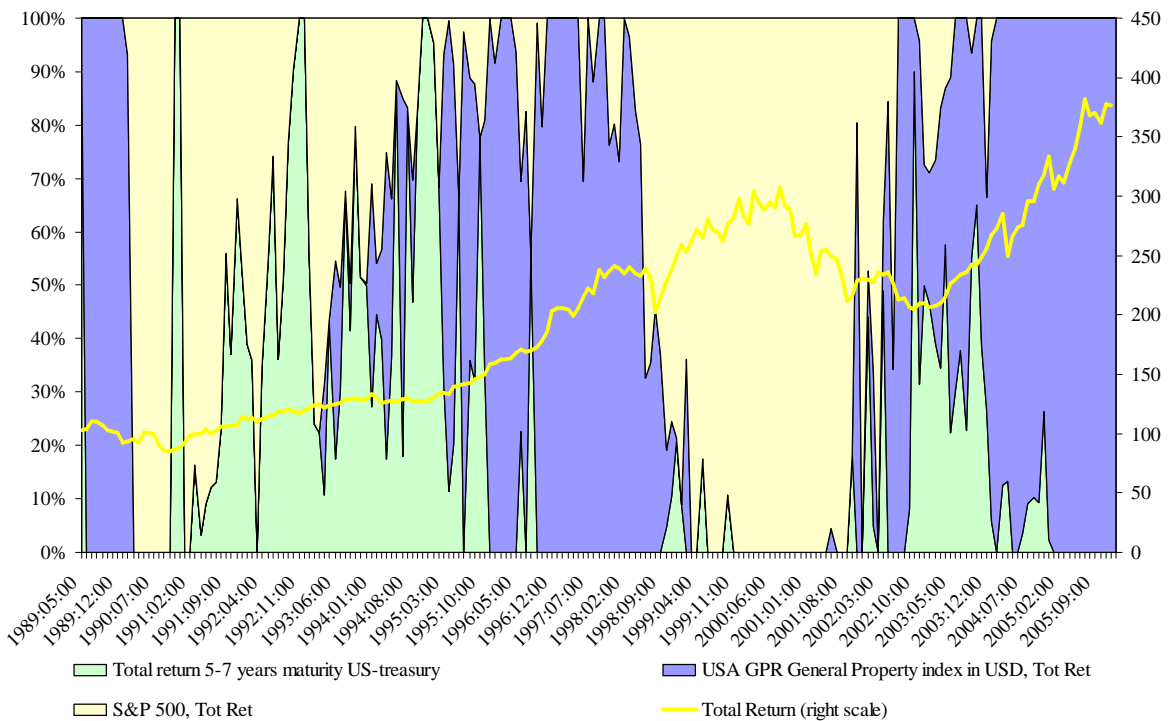


Figure 14. Not having Direct Property Investment as an asset in the portfolio makes the asset allocation varies a lot; often to extreme weights. The volatility of the total return increases substantially and the return decreases. Between August 1989 and October 1990 the portfolio

lost 23% of its value and between August 2000 and January 2003 it lost 33% in value. So the portfolio has two big drops during the analysed time and a very varying asset allocation. If there was a transaction cost involved it would have been very costly.

6.3 Analysis

The low correlation between Direct and Indirect property and also S&P investments shows that the financial market is a poor indicator for the underlying market. It also indicates that property stocks are not a substitute for direct property investments in a portfolio since listed Property stocks have much higher standard deviations and are much more correlated to the stock market.

When using a Brownian Bridge function the coefficient of determination calculated by the multiple regressions was lower for the Direct Property Investment; compared to when using the spline interpolation. The spline interpolation smoothed out the curve and by looking at the return the month before the multiple regressions could reach a higher confidence level when calculating the next month's return. When using the lower value of the coefficient of determination as the confidence level of the views in the Black Litterman model these views do not get much of influence on the expected returns. The model does not work as good as when the model have higher confidence levels.

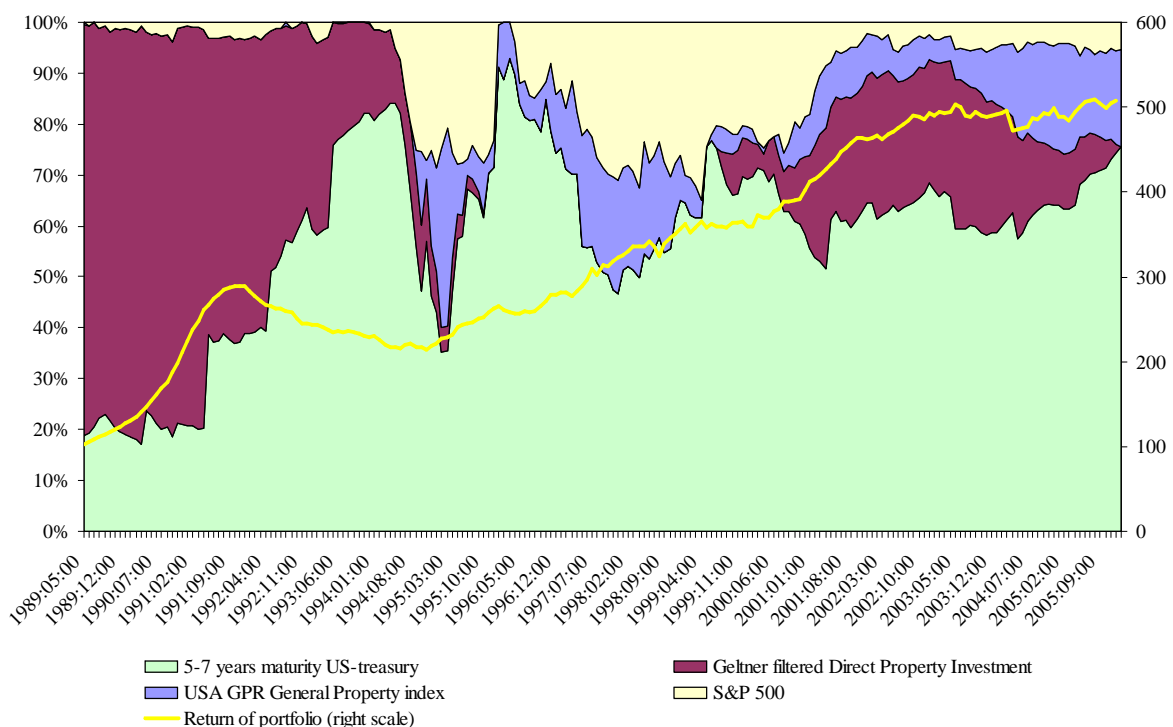


Figure 15. Here is the Markowitz portfolio with the Direct Property Investment Return generated by the spline interpolation as an asset. After this asset was spline interpolated it was filtered in a Geltner filter to increase the standard deviation to the mean of S&P 500 and bonds. This made the total return index look totally different from the actual total return from the asset giving the whole portfolio a too high total return.

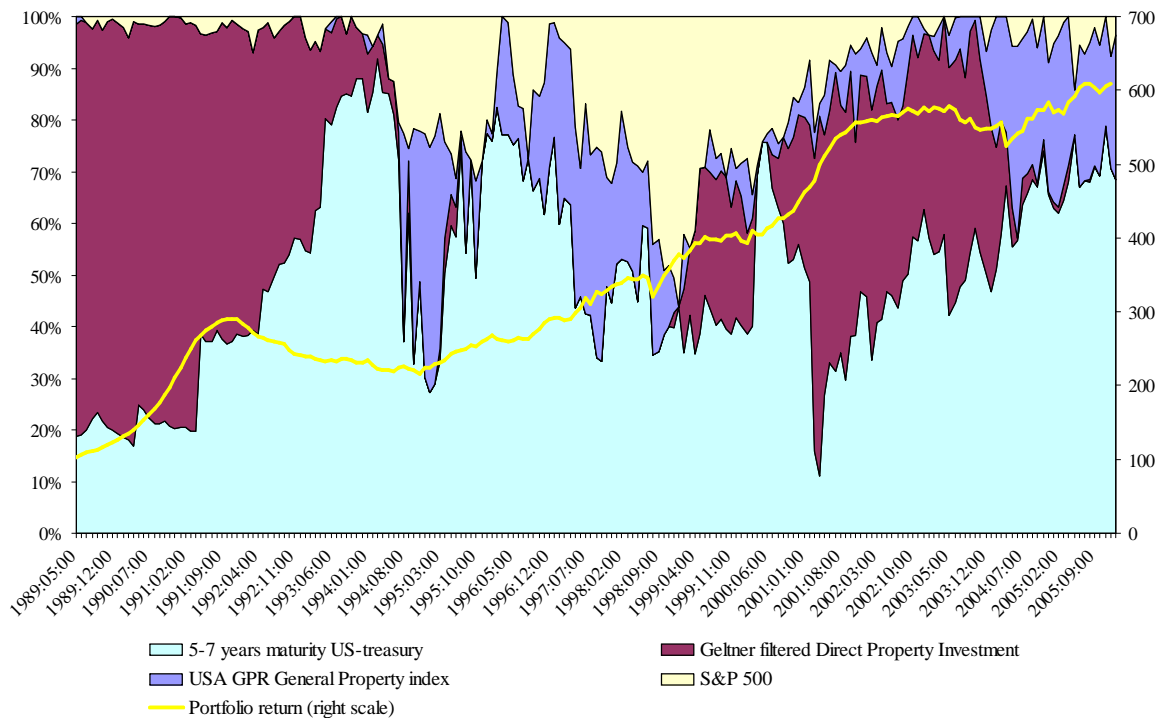


Figure 16. Here is the Black Litterman portfolio with the asset returns of the Direct Property Investment spline interpolated. The multiple regressions gave a very high value of the coefficient of correlation; a mean of almost 93%. This helped the Black Litterman model to give a much better expected returns compared to the Markowitz model and the total return of the portfolio was over 500 percentages; compared to the Markowitz portfolio with a return of just over 400 percentages.

The two diagrams in Figure 15 and Figure 16 show how much better the Black Litterman model works when it has strong views of confidence. The spline interpolated Direct Property Investment has an artificial high serial correlation which gives a high coefficient of correlation when analysing it with multiple regressions. When instead the Brownian Bridge is used to receive the monthly returns on the Direct Property Investment the serial correlation for the index falls and the result from the multiple regressions analysis gets much worse with a mean of the coefficient of correlation of only 34% which is about the same for all the other assets in the portfolio. This makes the Black Litterman model to work much worse when there is no strong view. The result between the Markowitz portfolio and the Black Litterman portfolio; when using the Brownian Bridge generated returns of the Direct Property Investment; had no big difference.

6.4 Out of sample valuation

The outcome varied a lot between the portfolios, shown in Figure 17. The portfolios generated by the Markowitz model show the highest returns. The Black Litterman generated portfolio without Direct Property Investment in the investment universe has the worst total return, much depending on the big 33% drop that starts in August 2000.

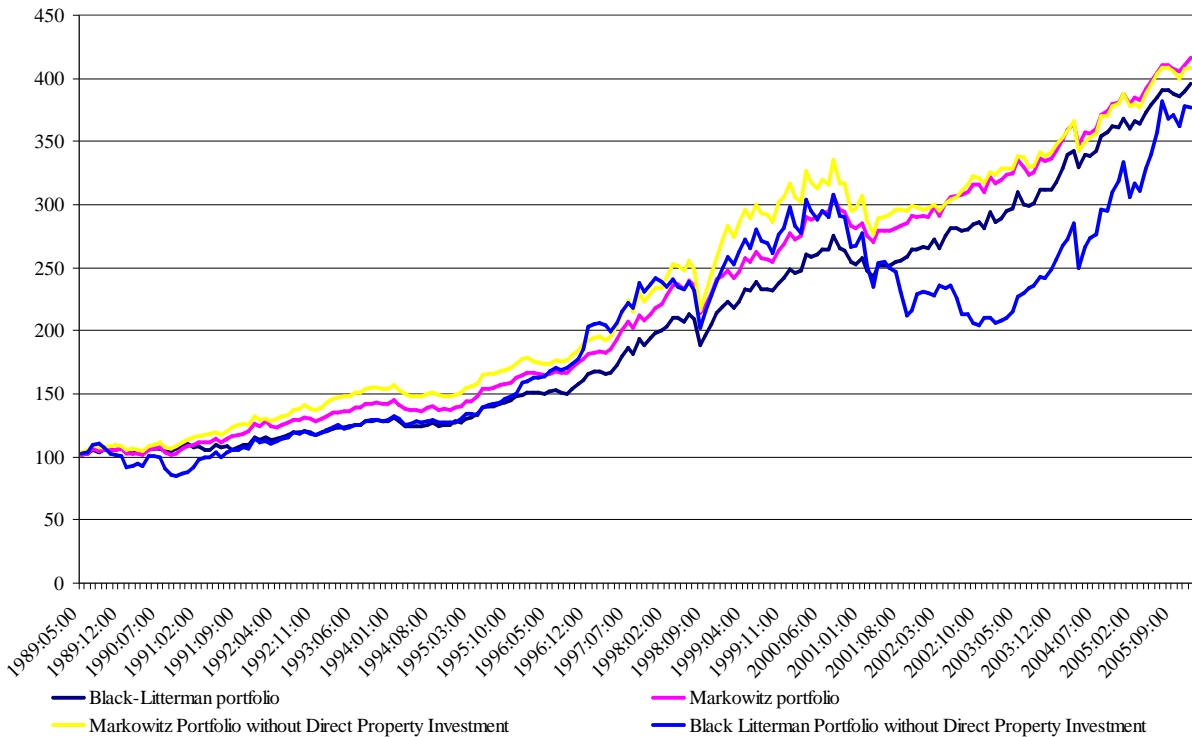


Figure 17. The figure shows the indices of the total return of the portfolios with and without Direct Property Investment as an asset.

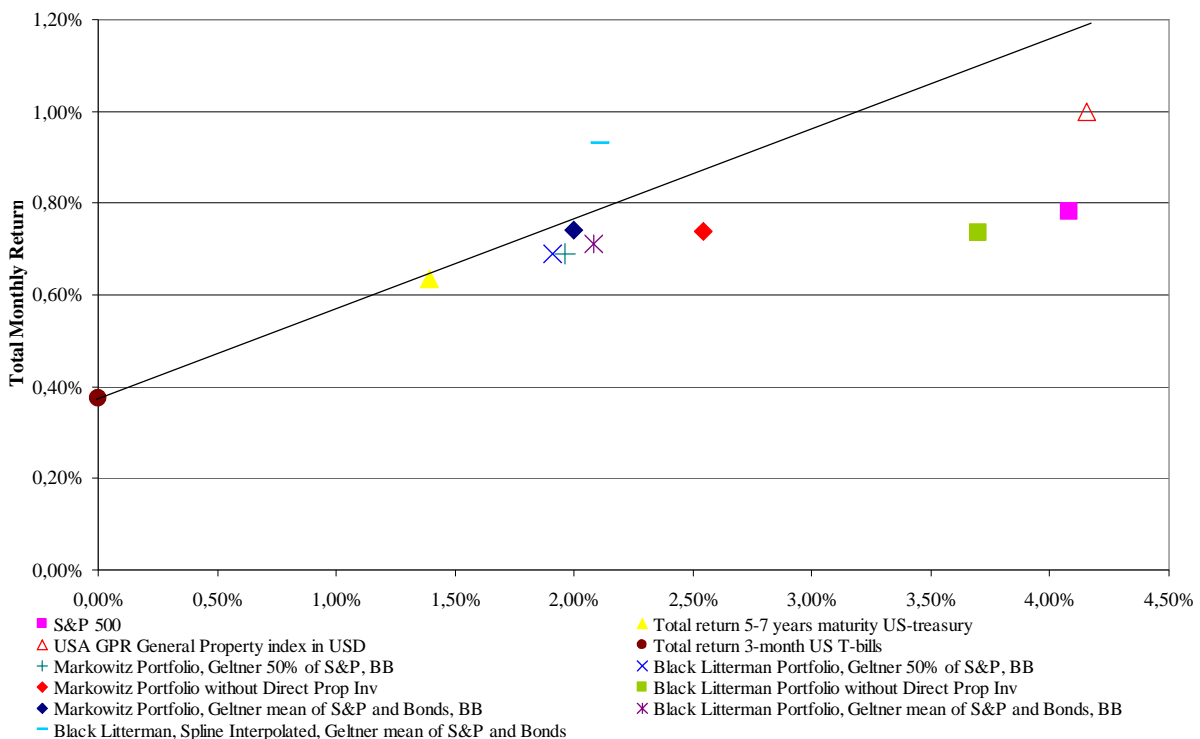


Figure 18. The diagram shows the mean monthly return on the y-axis and the standard deviation on the x-axis. The risk free rate is to the left in the diagram and from that point the Capital Market Line stretches. Several portfolios lies close together so it is no big difference

between them, but it is easy to see that portfolios with Direct Property Investments as an asset has much lesser standard deviation but still the same return compared to the portfolios without that asset.

	Sharpe-ratio	Significance
Black Litterman, Spline Inter, Geltner mean of S&P and Bonds	26,35%	100,00%
Markowitz Portfolio, Geltner mean of S&P and Bonds, BB	18,30%	99,47%
Black Litterman Portfolio, Geltner 50% of S&P, BB	16,54%	99,98%
Black Litterman Portfolio, Geltner mean of S&P and Bonds, BB	16,15%	98,89%
Markowitz Portfolio, Geltner 50% of S&P, BB	16,12%	99,96%
Markowitz Portfolio without Direct Prop Inv	14,37%	97,89%
Black Litterman Portfolio without Direct Prop Inv	9,74%	91,49%

Table 3. The table shows the portfolios with the highest Sharpe ratio and the probability that the portfolio return larger than risk free rate is larger than zero, assuming the returns are normal distributed. The significance test shows that all portfolios except the Black Litterman Portfolio without Direct Property Investment have a very high significance.

Both Figure 18 and table 3 show that the Black Litterman portfolio with the spline interpolated Direct Property Investment is the optimal portfolio of risky assets but is disqualified because of the unrealistic returns generated by the Geltner-filter. But it is interesting to see that despite the Geltner-filter generated such a low mean monthly return (only 0,12%) for the spline interpolated Direct Property Investment; the portfolio with this asset in its investment universe; has the highest mean monthly return, shown in Figure 19.

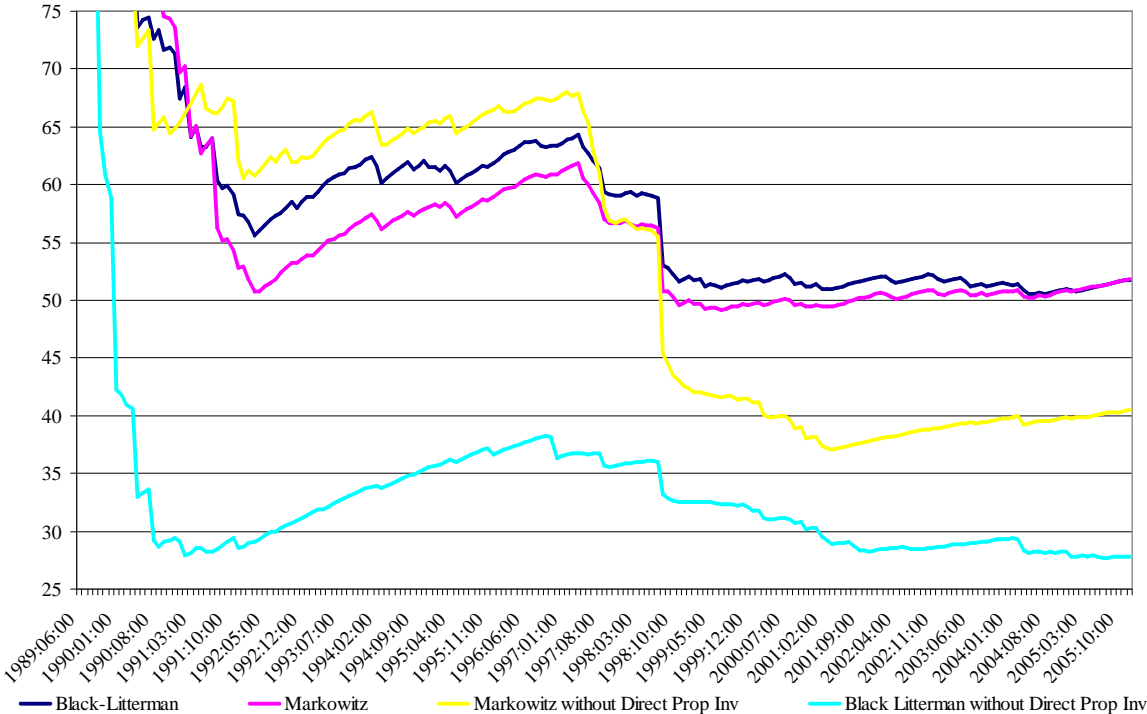


Figure 19. This diagram shows the risk adjusted return's change over time for the Black Litterman portfolio and the Markowitz portfolio with and without Direct Property Investment. The Black Litterman portfolio with Direct Property Investment as an asset has the highest

average of risk adjusted return. The Black Litterman portfolio without Direct Property Investment as an asset has the lowest average of risk adjusted return; probably because of the lower confidence level of the investor's expected returns.

6.5 Sources of faults

Transaction costs are not included. For a share or a bond transaction the cost is low but for a direct property investment it is high. The time to shift assets depends on the assets, shares and stocks are easy to trade but direct investments in properties can though take more time.

Tax rates are not included. Some transactions can imply tax costs and maybe not be profitable.

It is important to note what time an index starts and stops; since they all goes in cycles maybe one more period of high returns will be included. For example property shares has had big drops in value since the index used in this master thesis was updated.

The property return consists of both the actual income return and the capital return. The actual income return is not time-correlated and should not be de-smoothed. But the difficulty of getting the data with both these data series made it impossible.

The number of monthly returns is important, a larger number results in a more significant test result.

When calculating the Brownian Bridge and the spline interpolation, yearly returns are used to get monthly returns. This leads to the returns for the whole year being used to calculate each month's return. So the return for February is based on a return that in reality is not known at that time. The Brownian Bridge is reconstructed when knowing a certain stochastic process in two points but not between them.

With an actual monthly transaction based return index instead of the yearly appraisal based return index on the Direct Property Investment one would not have had to estimate the standard deviation on the index and the data to work with would be more reliable.

7 Conclusions and future research

In this master thesis the thought was to find a portfolio that was better than the one generated by the traditional Markowitz model where bonds and stocks are the whole investment universe. Therefore Direct Property Investment and listed property shares are included as optional assets and are examined in different portfolios generated by the Black Litterman portfolio model. In the thesis the Black Litterman model combines views; of the future returns of all the assets; to the mean historical returns and generates new expected returns. These new combined returns are used when optimising the risk-adjusted return to create the optimal portfolio.

It is always good to have as many uncorrelated assets as possible in a portfolio to lower the risk and increase the return. Direct Property Investments is an asset like that and this master thesis has shown that. It is not the same thing at all to invest indirectly into property assets through property shares on the stock market; since they are much more correlated to the stock market and have a significantly higher volatility.

When including the investor's views it is important that he is confident enough on the outcome and not only guesses; because then the investments can be totally wrong and instead lowering the return.

7.1 Conclusions

It is not always best to just use the historical returns to get the optimal portfolio. By using the investors own views the return can become higher. If an investor could know which asset would have the highest return the coming month and allocate his whole portfolio into that asset; the investor would have had an astonishing return on 235 458% during the analysed time. This of course no one can achieve but it is an enormous difference and can be worth trying to reach.

The cubic spline interpolation gave a too smooth index; when creating monthly returns from the yearly returns on the Direct Property Investment; so when using the Geltner-filter it did not work. The Geltner-filtering returned an index too dissimilar to the actual returns. Instead the Brownian Bridge gave a much better result.

By using the historical return vector instead of the implied equilibrium return vector the Black Litterman model did not give that well-balanced portfolio as it should have done. The historical return vector created an optimum risk adjusted portfolio with sometimes large asset positions. The multiple regressions have given expected returns to use as views in the Black Litterman model; though with a rather low coefficient of correlation. This gave an intuitive portfolio with an asset allocation varying a lot with sometimes rather extreme positions. Because of the low confidence level in the views, the portfolio did not become very different from the portfolio generated by the Markowitz model.

The portfolio allocation did not change much when the standard deviation for the Direct Property Investment was altered between a standard deviation equal to the mean of S&P 500's and bond's and one equal to half of the S&P 500's standard deviation. So the conclusion was that no matter what the exact standard deviation for the asset Direct Property Investment is set to; the asset is a good asset allocation in the portfolio since it has such a low correlation to the other assets. The Direct Property Investment asset also gave a more

stabilised and less varying portfolio allocation compared to the portfolios without the asset in their investment universe.

The best portfolio is attained when the views are more confident and the Black Litterman model creates returns that generate more high yielding portfolios. This was seen in the difference of return between the Markowitz and the Black Litterman portfolio having the spline interpolated Direct Property Investment as an asset. The good results in the multiple regressions generated high confidence level in the Black Litterman model which generated a portfolio with the highest return and Sharpe ratio.

7.2 Future Research

The more I wrote on this master thesis the more I wanted to examine and include into the work. But the time has its limits and therefore all ideas were not examined and here are some of them:

It would be nice to find economic variables that increase the coefficient of correlation in the multiple variables; this would make the Black Litterman model work even better and perhaps beat the returns generated by the Markowitz model. There are almost infinitely many variables that could be tested; besides new variables they could also be tested if they really should be dynamical. Is 60 months the optimum time to calculate with?

To create better portfolios maybe a larger number of assets should be included in the investment universe. The S&P 500 could be divided into more asset classes and maybe gold, oil and other raw materials should be included? In the stock market there are also opportunities to hedge the portfolios to secure it against falling share prices which can raise (or decrease) the return on the portfolio.

It is also interesting to experiment on how the results would have been if equilibrium returns had been used. Through some smart assumptions the equilibrium allocation weights could generate the equilibrium returns used instead of the historical mean returns in the Black Litterman model.

The estimations on the standard deviation of the Direct Property Investments returns could be more examined to find where the asset's real standard deviation is.

References

- Black Fischer and Litterman Robert, 1992. *Global Portfolio Optimization*, Financial Analysts Journal, Sep/Oct 1992; 48, 5; ABI/Inform Global pg. 28.
- Brown, G. R. and G. A. Matysiak, 2000. *Real Estate Investment: A Capital Market Approach*, Harlow: Financial Times Prentice Hall.
- Cuthbertson Keith, 1996. *Quantitative Financial Economics, Stocks, Bonds and Foreign Exchange*, John Wiley & Sons Ltd, West Sussex, 2002.
- Fisher Jeffrey D., 2002. *Real Time Valuation*, Journal of Property Investment & Finance, Vol 20, No 3.
- Geltner David, 1993. *Estimating Market Values from Appraised Values without Assuming an Efficient Market*, The Journal of Real Estate Research, volume 8, number 3, May 1993.
- Graeme Dr. West, 2004. *An introduction to Modern Portfolio Theory: Markowitz, CAP-M, APT and Black Litterman*, CAM Dept, University of the Witwatersrand, November 28, 2004.
- He Guangliang and Litterman Robert, 1999. Investment Management Research, Goldman Sachs Quantitative Resources Group.
- Hill Carter R., Griffiths William E. and Judge George G., 2001. *Undergraduate Econometrics*, 2:nd edition, John Wiley & Sons.
- Hoesli Martin, Lekander Jon and Witkiewicz Witold, 2004. *International Evidence on Real Estate as a Portfolio Diversifier*, The Journal of Real Estate Research; Apr-Jun 2004; 26, 2; ABI/INFORM Global, pg. 161.
- Hoesle Martin and MacGregor Bryan D., 2000. *Property Investment Principles and practice of portfolio management*, Pearson Education.
- Idzorek Thomas, 2002. *A step-by-step guide to the Black-Litterman model*, February 10, 2002.
- Koch Dr. Werner, 2003. *Consistent asset return estimates via Black-Litterman – Theory and Application*, CEFA Cominvest, PM Balanced, Asset Allocation, June 2003.
- Madsen Henrik, Nygaard Nielsen Jan, Lindström Erik, Baadsgaard Mikkel and Holst Jan, *Statistics in Finance*, KF Sigma, Lund 2004.
- Miles Fisher, J., M. and Webb R., 1999. *How Reliable are Commercial Property Appraisals? Another Look*, Real Estate Finance, 1999, 16:3, 9-15